Baryon and Anti-Baryon Production at RHIC

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We use a form of the fluctuation-dissipation theorem to derive formulas giving the rate of production of the spin-1/2 baryon octet in terms of the fluctuations of vector and axial-vector currents at finite temperature. An estimate is made of the corresponding rates for producing the spin-3/2 baryon decuplet. Rate equations are solved for hydrodynamically expanding matter to obtain the final observed baryon ratios.

Nucleons and SU(2)

We write the coupling of nucleons to vector V_μ^a and axial-vector A_μ^a currents or fields as

$$\mathcal{L} = -\overline{\psi}\gamma^\mu \frac{\tau^a}{2}\psi V_\mu^a + g_A \overline{\psi}\gamma^\mu\gamma^5 \frac{\tau^a}{2}\psi A_\mu^a$$

where $g_A \approx 1.26$ is the axial coupling constant relative to the vector.

Using vector meson dominance, the Goldberger-Treiman relation, the KSFR relation, and perturbative QCD for the spectral densities, we find that the differential rate is quite remarkable in that it is inversely proportional to the fourth power of f_π and does not depend on any other hadronic parameters except the nucleon mass.

$$E_1 E_2 \frac{dR}{d^3p_1 d^3p_2} = \frac{3}{8(2\pi)^7} \frac{1}{e^{\beta(E_1+E_2)} - 1} \frac{s(s-m_N^2)}{f_\pi^4} \left(1 + \frac{\alpha_s(s)}{\pi} + \cdots\right)$$

Integration gives the total rate.

$$\begin{aligned} R(\bar{N}N) &= \frac{9}{(2\pi)^5} \left(1 + \frac{\alpha_s(4m_N^2)}{\pi} + \cdots\right) \frac{m_N^4 T^4}{f_\pi^4} \left[\frac{m_N^2}{T^2} K_1^2\left(\frac{m_N}{T}\right)\right. \\ &+ \left. 4\frac{m_N}{T} K_1\left(\frac{m_N}{T}\right) K_2\left(\frac{m_N}{T}\right) + \left(8 + \frac{m_N^2}{T^2}\right) K_2^2\left(\frac{m_N}{T}\right)\right] \end{aligned}$$

The nonrelativistic limit is

$$R(\bar{N}N) = \frac{9}{2(2\pi)^4} \left(1 + \frac{\alpha_s(4m_N^2)}{\pi} + \cdots\right) \frac{m_N^5 T^3}{f_\pi^4} \exp\left(-2m_N/T\right)$$

This is the total rate for the production of $\bar{p}p$, $\bar{p}n$, $\bar{n}p$ and $\bar{n}n$. The individual rates are related as: $R(\bar{p}n) = R(\bar{n}p) = 2R(\bar{p}p) = 2R(\bar{n}n)$. According to the latest analysis $\alpha_s(m_\tau^2) = 0.35 \pm 0.03$.

Baryon Octet and SU(3) Flavor

Consider the baryon octet $\mathcal{B}$, vector meson nonet $\mathcal{V}$, and axial- vector nonet $\mathcal{A}$ represented by 3×3 matrices. Assume ideal mixing between the ω and ϕ. There are three types of SU(3) invariant couplings: the F and D types plus a singlet coupling.

$$\begin{aligned}\mathcal{L}_{\rm int} &= \frac{g_{\rho NN}}{\sqrt{2}}\Big[(1-\alpha_V)\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\left[\mathcal{V}_\mu,\mathcal{B}\right]\right)+\beta_V\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\mathcal{B}\right)\mathrm{Tr}\left(\mathcal{V}_\mu\right)\\ &+ \alpha_V\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\left\{\mathcal{V}_\mu,\mathcal{B}\right\}\right)+g_A\alpha_A\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\gamma^5\left\{\mathcal{A}_\mu,\mathcal{B}\right\}\right)\\ &+ g_A(1-\alpha_A)\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\gamma^5\left[\mathcal{A}_\mu,\mathcal{B}\right]\right)+g_A\beta_A\mathrm{Tr}\left(\bar{\mathcal{B}}\gamma^\mu\gamma^5\mathcal{B}\right)\mathrm{Tr}\left(\mathcal{A}_\mu\right)\Big]\end{aligned}$$

Close and Roberts; Klingl, Kaiser and Weise: $\alpha_A = 2/3$

$g_{\omega NN} = 3g_{\rho NN}$ (empirical, quark model): $\beta_V = 1$

Nucleons do not couple to ϕ (OZI rule): $\alpha_V = (1-\beta_V)/2 = 0$

Nucleons do not couple to $f_1(1420)$ (generalized OZI rule): $\beta_A = (1-2\alpha_A) = -1/3$

Quark currents and spectral densities

$$\begin{aligned} j^\mu_{\rho^0} &= \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d) \;\to\; \rho_{\rho^0}(s) = \frac{s}{8\pi^2}\left(1+\frac{\alpha_s(s)}{\pi}\right)\\ j^\mu_{\omega} &= \frac{1}{6}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d) \;\to\; \rho_{\omega}(s) = \frac{s}{72\pi^2}\left(1+\frac{\alpha_s(s)}{\pi}\right)\\ j^\mu_{\phi} &= -\frac{1}{3}\bar{s}\gamma^\mu s \;\to\; \rho_{\phi}(s) = \frac{s}{36\pi^2}\left(1+\frac{\alpha_s(s)}{\pi}\right)\end{aligned}$$

Define r as shorthand notation for $E_1 E_2 dR/d^3p_1 d^3p_2$ and

$$r_\pm(m_1, m_2) = \frac{2}{(4\pi)^7} \frac{F^2_{\rm ANN}(s)}{e^{\beta(E_1+E_2)} - 1} \left(1 + \frac{\alpha_s}{\pi}\right) \times \frac{2s^2 - (m_1^2 + m_2^2)s - (m_1^2 - m_2^2)^2 \pm 6m_1 m_2 s}{f_\pi^4}$$

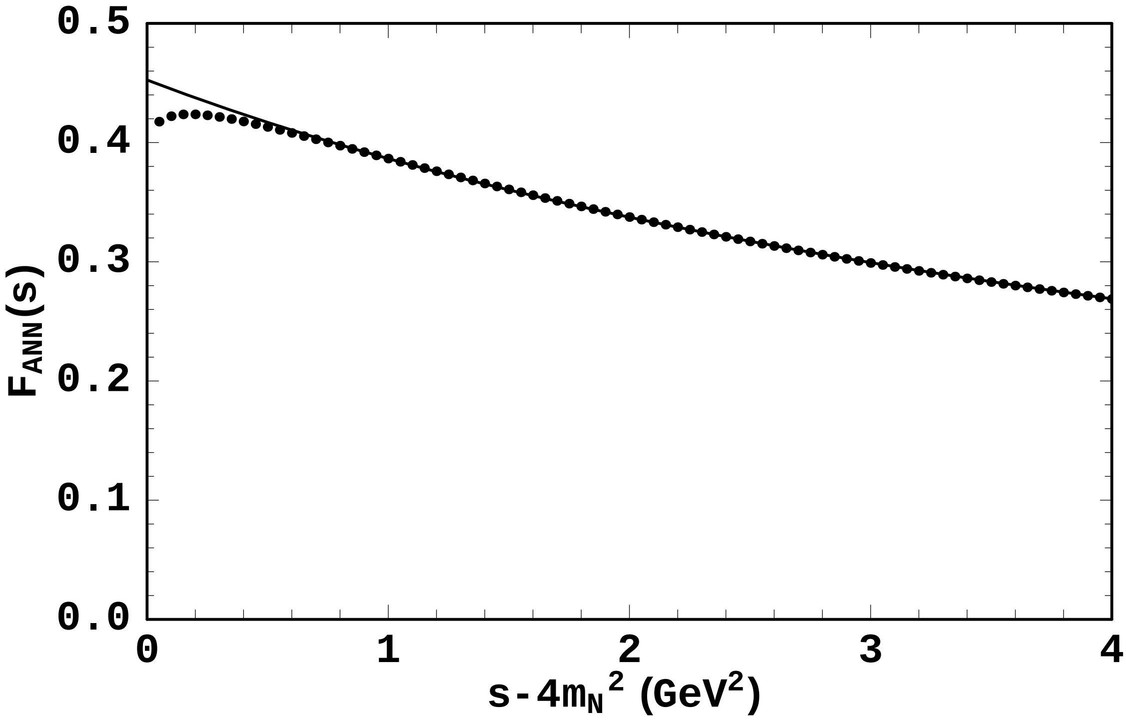
where the $\pm$ corresponds to vector/axial-vector contributions. The function $F_{\rm ANN}(s)$ is a form factor. Some examples are

$$\begin{aligned} r(n\overline{p}) &= 2r_+(m_N, m_N) + 2r_-(m_N, m_N) \\ r(p\overline{p}) &= 2r_+(m_N, m_N) + \frac{82}{81} r_-(m_N, m_N) \\ r(\Lambda\overline{p}) &= 3r_+(m_\Lambda, m_N) + \frac{25}{27} r_-(m_\Lambda, m_N) \\ r(\Xi^-\overline{\Lambda}) &= 3r_+(m_\Lambda, m_\Xi) + \frac{1}{27} r_-(m_\Lambda, m_\Xi) \end{aligned}$$

Altogether there are 46 nonvanishing combinations of baryon/anti-baryon pairs. If $F^2_{\rm ANN}(s)$ is evaluated at the average value $\bar{s} = (m_1 + m_2)^2 + 3(m_1 + m_2)T$ then the integral can be done in closed form.

$$\begin{aligned} R_\pm = {} & \frac{9(1 + \alpha_s/\pi)T^8}{4(2\pi)^5 f_\pi^4} z_1^2 z_2^2 \left\{4z_1 K_1(z_1) K_2(z_2) + 4z_2 K_1(z_2) K_2(z_1) \right. \\ & \left. \pm (z_1 \pm z_2)^2 K_1(z_1) K_1(z_2) + [16 + (z_1 \pm z_2)^2] K_2(z_1) K_2(z_2) \right\} F^2_{\rm ANN}(\bar{s}) \end{aligned}$$

where $z_i = m_i/T$.



The monopole fit (solid curve) to the annihilation form factor as extracted from experimental data (dotted curve.)

Spin-3/2 Decuplet

We estimate the octet-decuplet and decuplet-decuplet production rates using the same functions $r_\pm$ as before with the appropriate masses. SU(3) flavor symmetry is used, but this is not sufficiently stringent to uniquely determine the coefficients. The order of magnitude is based on the octet-octet coefficients, and the relative coupling strengths between different isospin combinations is adjusted to yield equal (or near equal) summed rates when all baryon masses are degenerate within the multiplet.

Table III: Relative strengths of coefficients in the expressions for the rates.

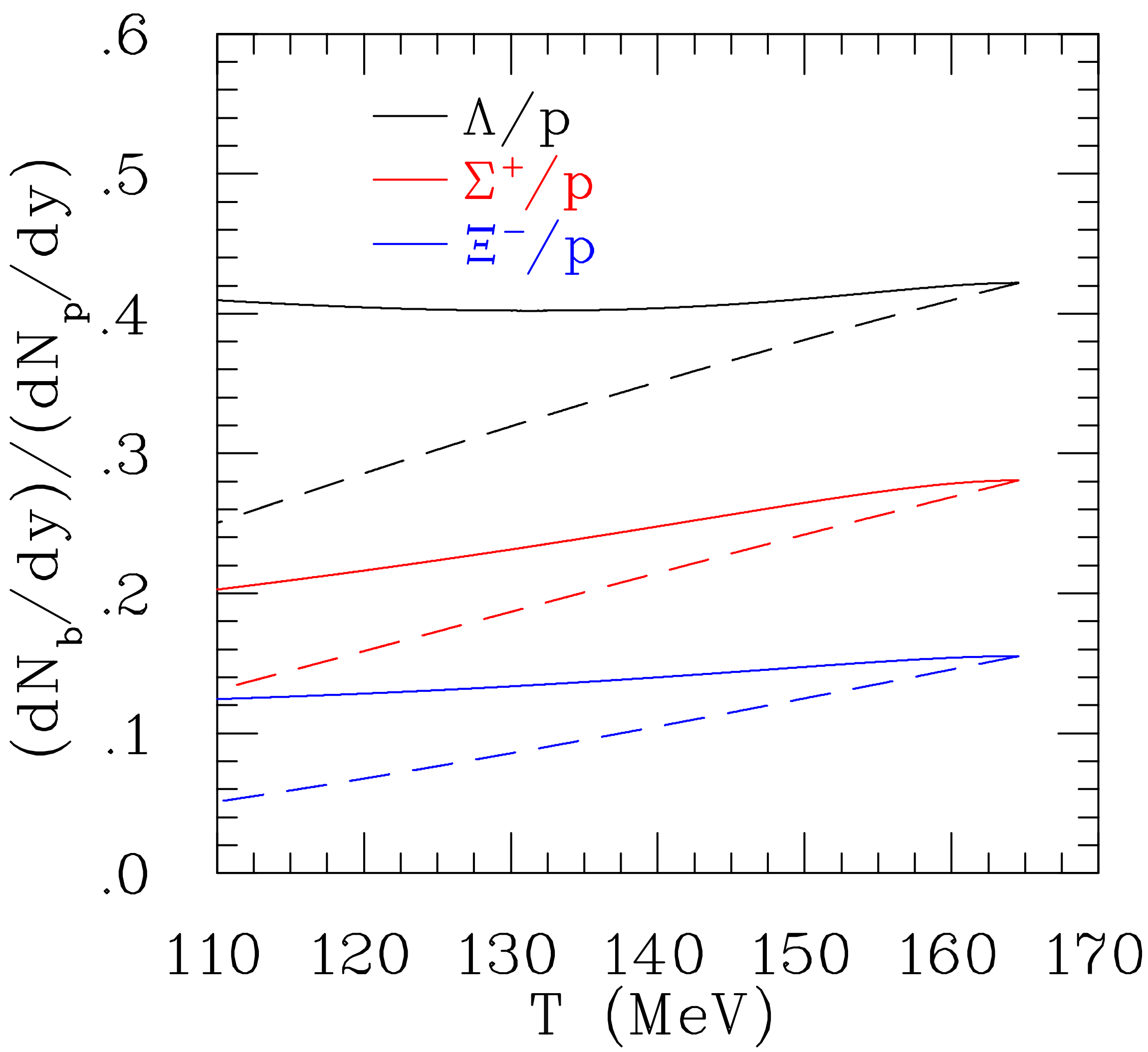
	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω
$\overline{p}$	2	4/3	2/3	0	1	1/2	0	0	0	0
$\overline{n}$	0	2/3	4/3	2	0	1/2	1	0	0	0
$\overline{\Lambda}$	0	0	0	0	1	1	1	1	1	0
$\overline{\Sigma}^{+}$	2	2/3	0	0	1/2	1/2	0	1	0	0
$\overline{\Sigma}^{0}$	0	4/3	4/3	0	1/2	0	1/2	1/2	1/2	0
$\overline{\Sigma}^{-}$	0	0	2/3	2	0	1/2	1/2	0	1	0
$\overline{\Xi}^{0}$	0	0	0	0	1	1/2	0	1	1/2	1
$\overline{\Xi}^{-}$	0	0	0	0	0	1/2	1	1/2	1	1

Rate Equations

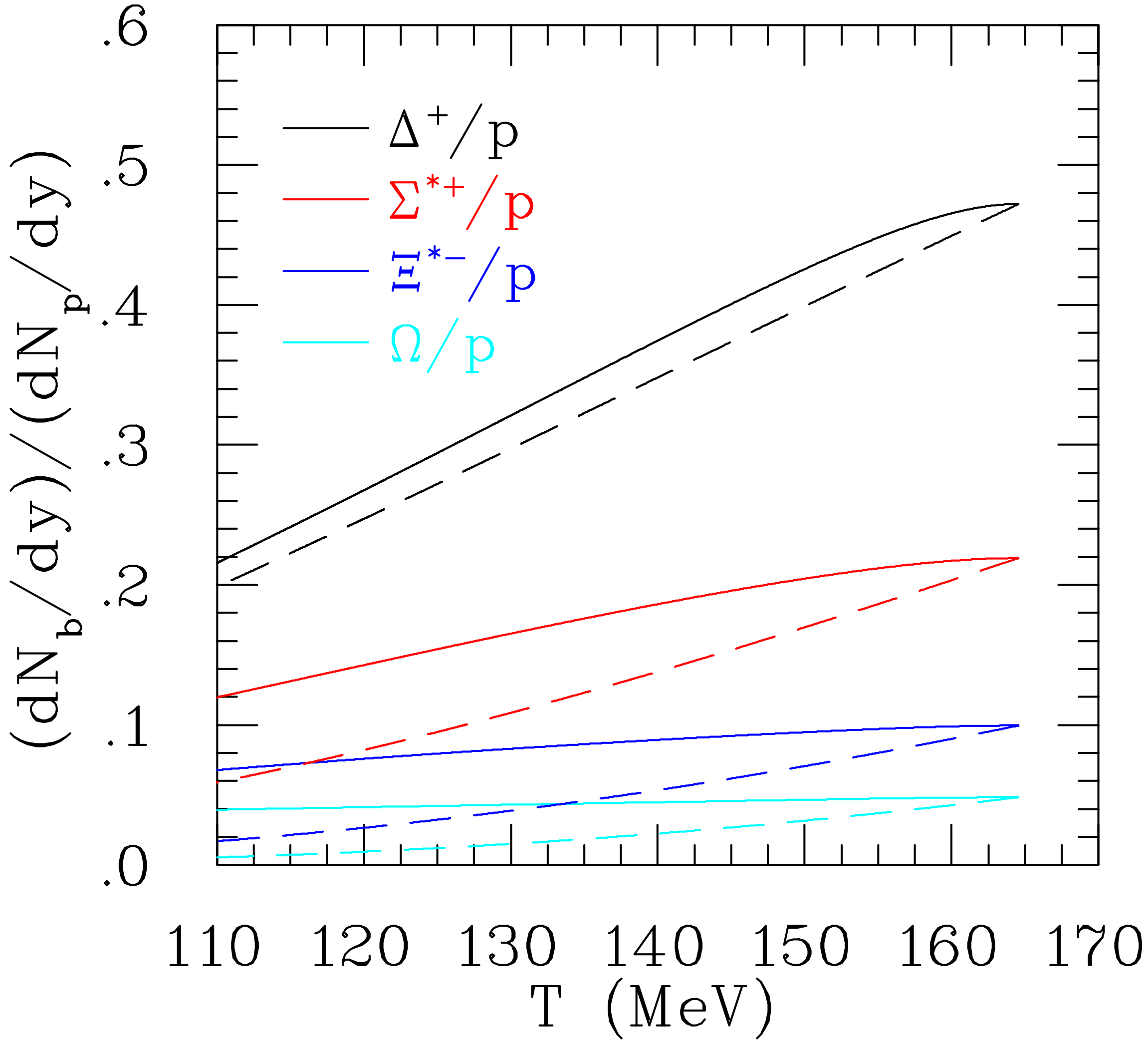
The rate equation for the density of some baryon, the anti-proton to be specific, is

$$
\begin{aligned}
\frac{dn_{\bar{p}}}{dt} &= \sum_b R(b\bar{p}) \left[1 - \frac{n_{\bar{p}} n_b}{n_{\bar{p}}^{\text{equil}} n_b^{\text{equil}}} \right] \\
&\quad + \sum_{\bar{b}} \Gamma(\bar{b} \to \bar{p} + X) \left(n_{\bar{b}} - \frac{n_b^{\text{equil}} n_{\bar{p}}}{n_{\bar{p}}^{\text{equil}}} \right) - \frac{n_{\bar{p}}}{V} \frac{dV}{dt}
\end{aligned}
$$

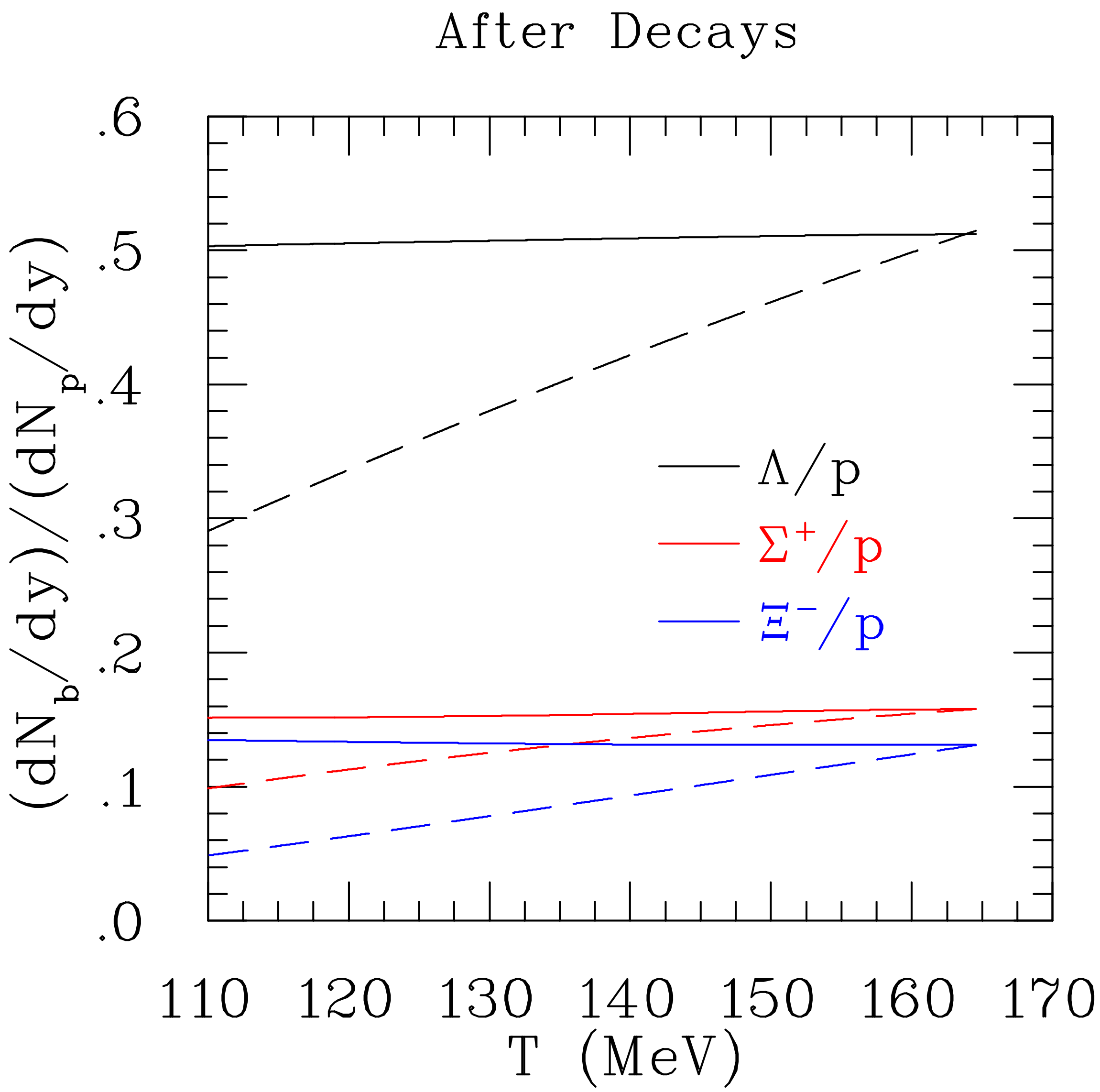
This set of equations is solved in a simple hydrodynamical model where the volume V is proportional to time and the temperature T is determined by entropy conservation together with an equation of state.



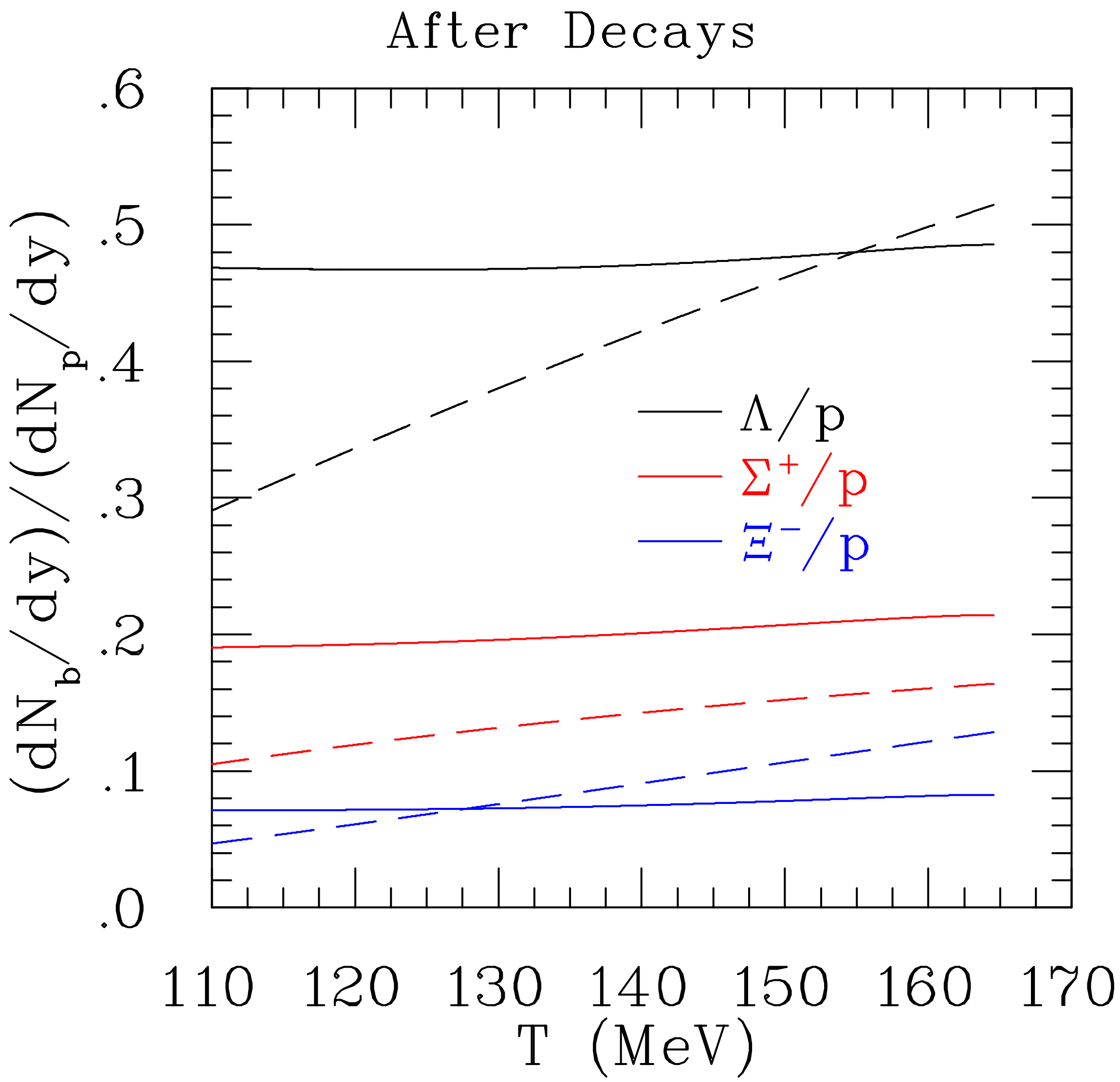
Solution to the rate equations (solid curve) assuming full initial chemical equilibrium compared to instantaneous local chemical equilibrium (dotted curve.)



Solution to the rate equations (solid curve) assuming full initial chemical equilibrium compared to instantaneous local chemical equilibrium (dotted curve.)



Solution to the rate equations (solid curve) assuming full initial chemical equilibrium compared to instantaneous local chemical equilibrium (dotted curve.)



Solution to the rate equations (solid curve) assuming zero initial abundances compared to instantaneous local chemical equilibrium (dotted curve.)

Conclusions and Tasks

- We have calculated the production of spin-1/2 octet and spin-3/2 decuplet baryon/anti-baryon pairs through fluctuations in the strong interaction currents. The formulation used a version of the fluctuation-dissipation theorem that does not rely on the system being in thermal equilibrium. If one has a model for these fluctuations those formulas may be used directly. We evaluated them in thermal equilibrium and made quantitative predictions for the rates.

- Rate equations were solved in a simple model of hydrodynamic expansion with initial conditions chosen to be either full chemical equilibrium or with no baryons at all.

- Do the analogous calculations for a realistic 3d hydrodynamic expansion.

- Compare to experimental data.