# Event-by-event fluctuations in high-energy heavy-ion collisions

- Why?
- How?
- Results
- Problems
- Perspectives

Fluctuations are sensitive to dynamics!

# Equilibrium fluctuations

- Energy fluctuations *)

$$\langle (U - \langle U \rangle)^2 \rangle = \langle T \rangle^2 C_V$$

$C_V \equiv \left(\frac{\partial U}{\partial T}\right)_{V,N}$ - heat capacity

- Multiplicity fluctuations **)

$$\langle (N - \langle N \rangle)^2 \rangle = \frac{T \langle N \rangle^2}{V^2 \chi_T}$$

$\chi_T \equiv -\left(\frac{\partial p}{\partial V}\right)_{N,T}$ - compressibility

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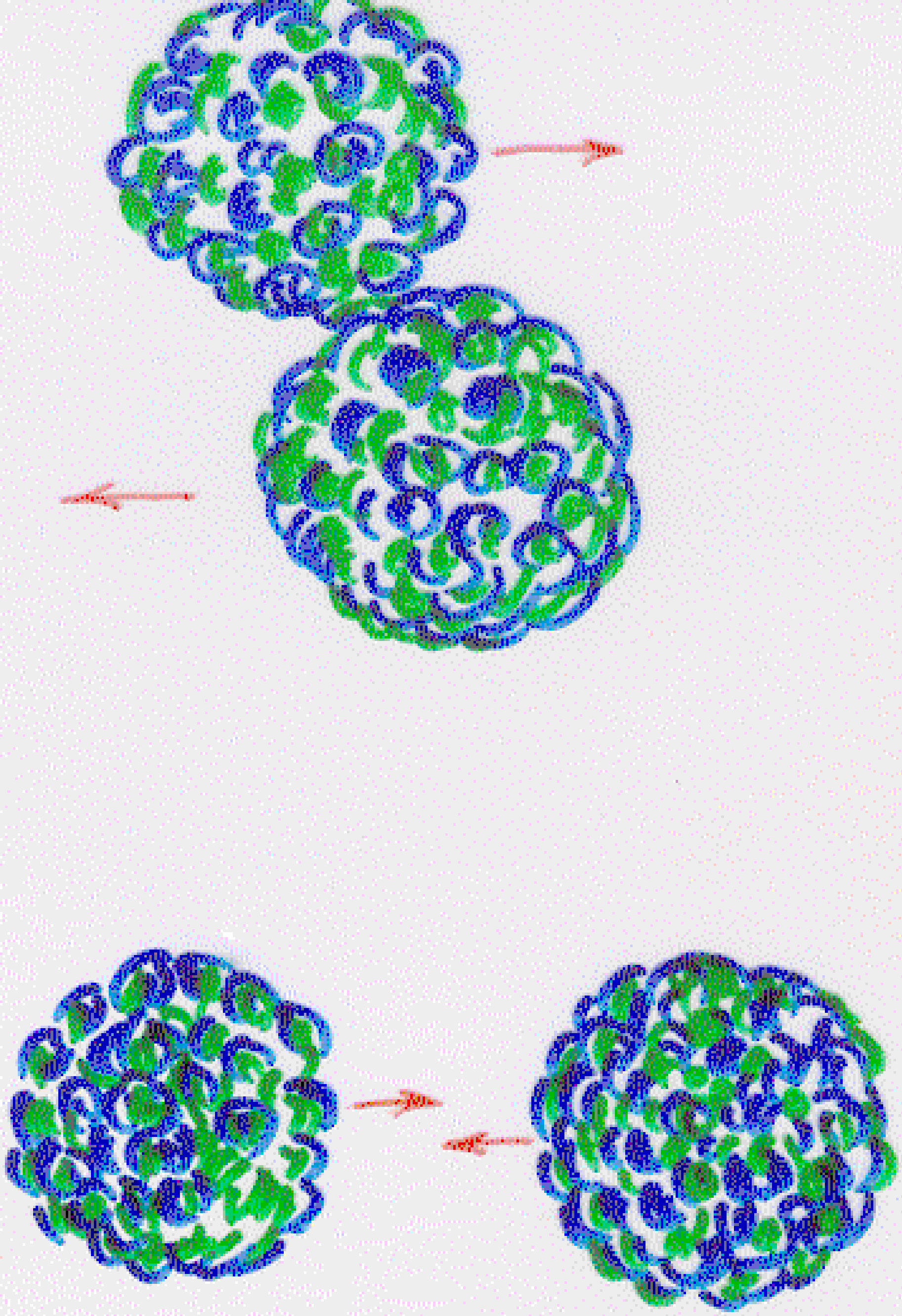
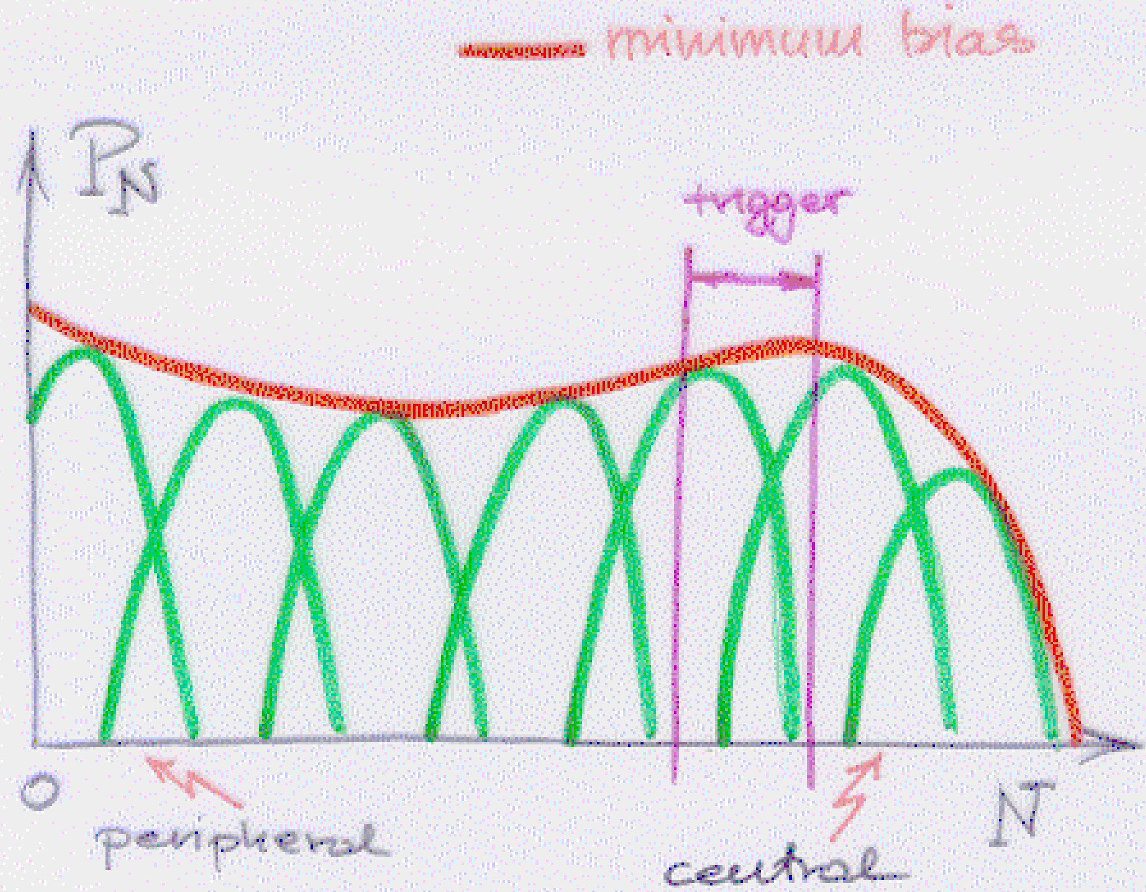
*) L. Stodolsky, Phys. Rev. Lett. 75 ('95) 1044;
E. Shuryak, Phys. Lett. B423 ('98) 9.

**) St. Mrów..., Phys. Lett. B430 ('98) 9.

# Impact parameter variation



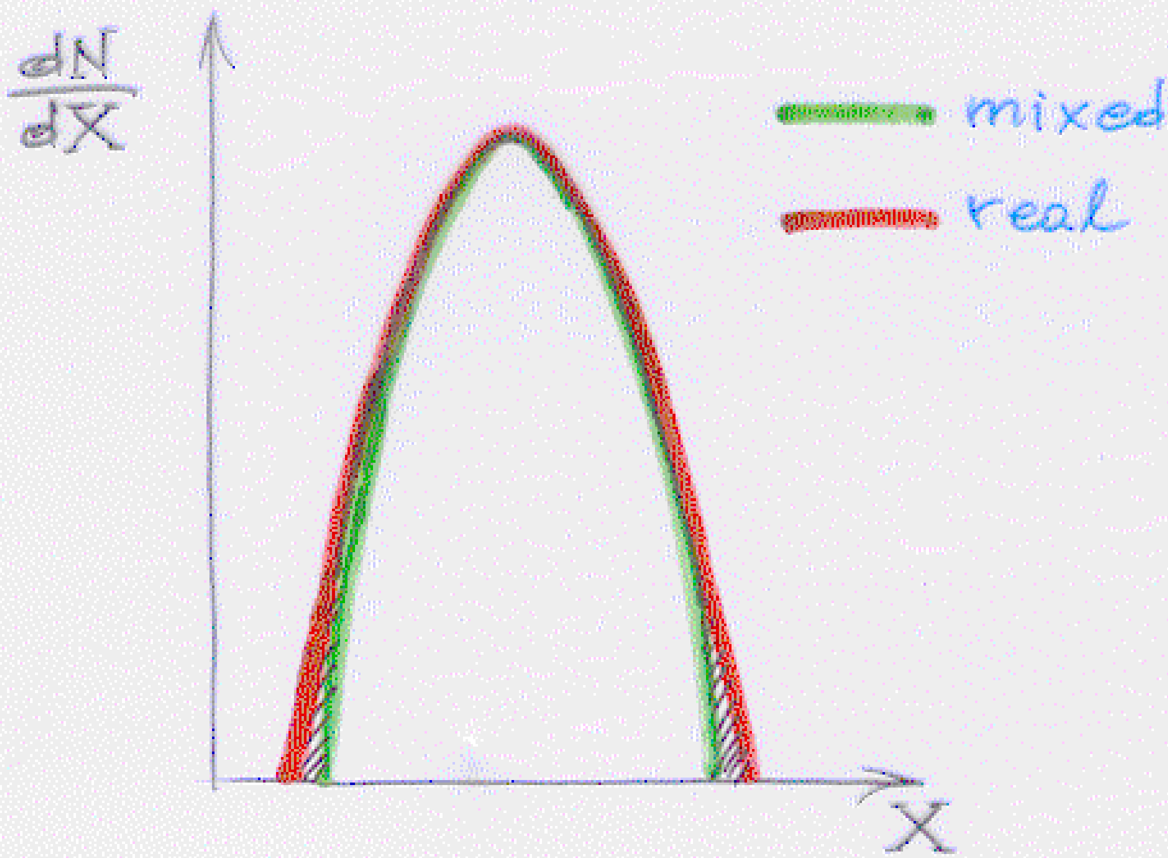
# Multiplicity distribution




$$\frac{\langle N \rangle_{central}}{\langle N \rangle_{peripheral}} \sim 100$$

# Statistical Noise

## Mixed vs. Real Events




X – event average of $p_\perp$, $\frac{K}{\pi}$, Q ....

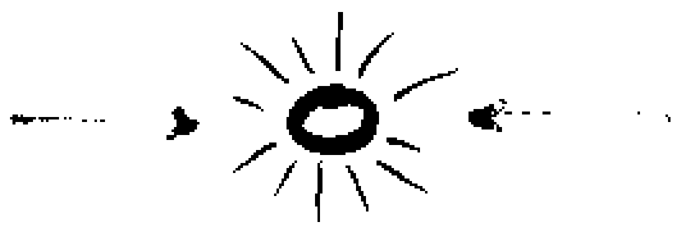
## Fluctuation measure should be:

- 'blind' to centrality,
- 'deaf' to statistical noise.

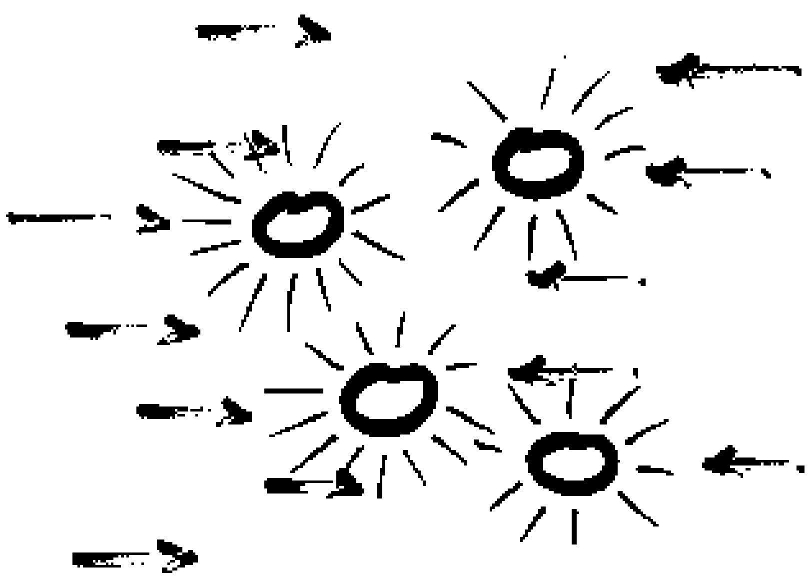
# "Background" model of A-A collision

- superposition of N-N interactions

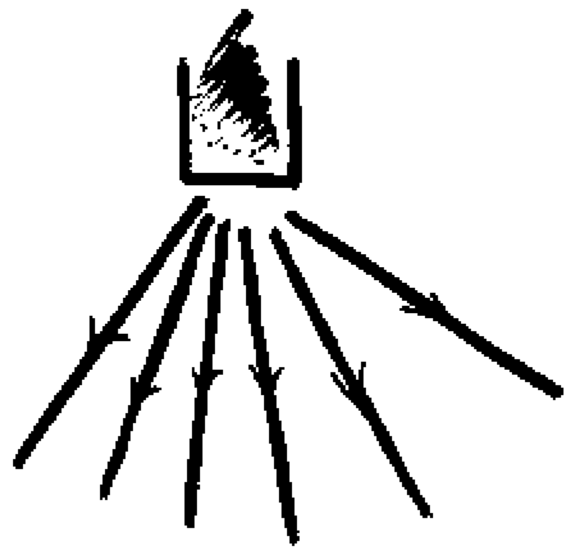
## N-N



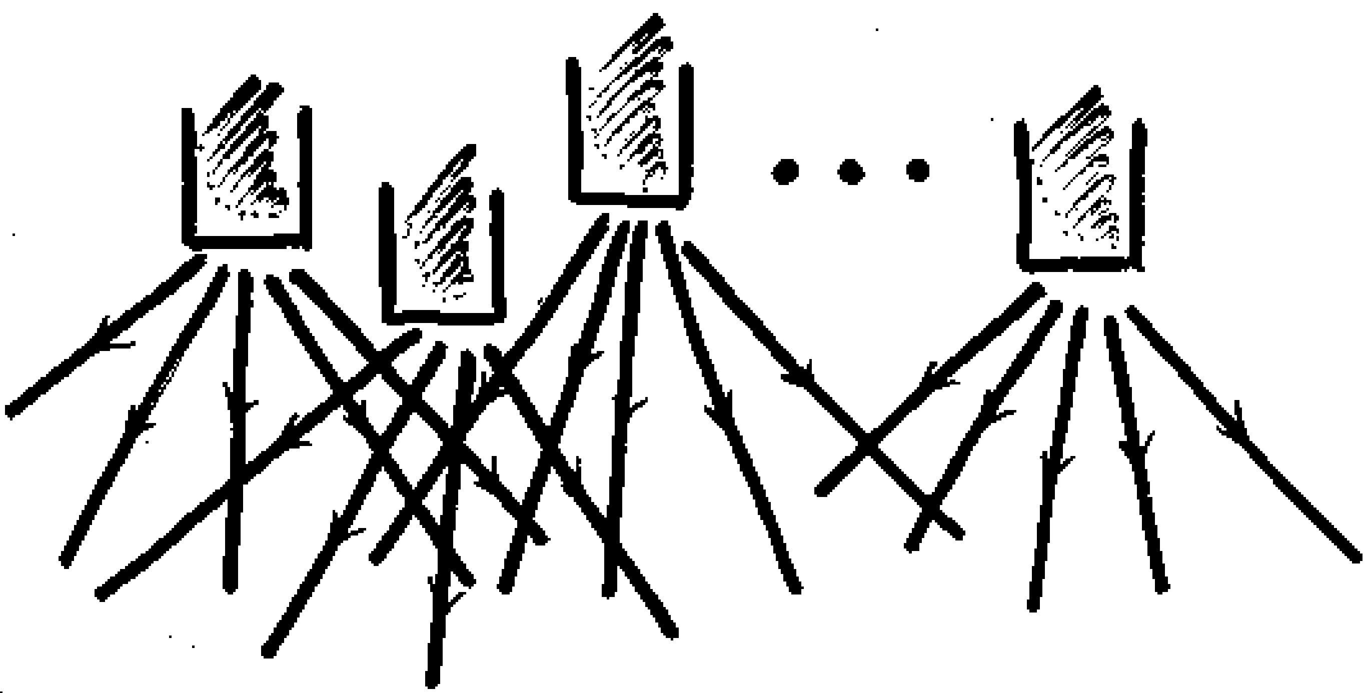
## A-A



1 - source



k - sources



k is random variable!

$E$ – energy from a single source

$U$ – energy from $k$ sources

$$\boxed{\begin{aligned} \langle (U-\langle U\rangle)^2\rangle &= \langle k\rangle \langle (E-\langle E\rangle)^2\rangle \\ &\quad + \langle E\rangle^2 \langle (k-\langle k\rangle)^2\rangle \end{aligned}}$$

$$\frac{\langle (U-\langle U\rangle)^2\rangle}{\langle N\rangle} = \frac{\langle (E-\langle E\rangle)^2\rangle}{\langle M\rangle} + \underbrace{\frac{\langle E\rangle^2}{\langle M\rangle}\,\frac{\langle (k-\langle k\rangle)^2\rangle}{\langle k\rangle}}_{?}$$

$M$ – multiplicity from a single source

$N$ – multiplicity from $k$ sources

$$U - \langle U \rangle \rightarrow U - \frac{\langle U \rangle}{\langle N \rangle} N$$

$$\boxed{\frac{\left\langle \left( U - \frac{\langle U \rangle}{\langle N \rangle} N \right)^2 \right\rangle}{\langle N \rangle} = \frac{\left\langle \left( E - \frac{\langle E \rangle}{\langle M \rangle} M \right)^2 \right\rangle}{\langle M \rangle}}$$

# Eliminating Statistical Noise

Recepie:

Compute $\frac{1}{\langle N\rangle}\left\langle\left(U-\frac{\langle U\rangle}{\langle N\rangle}N\right)^2\right\rangle$ for independent particles and the result subtract from $\frac{1}{\langle N\rangle}\left\langle\left(U-\frac{\langle U\rangle}{\langle N\rangle}N\right)^2\right\rangle$.

independent particles

$$\boxed{\frac{1}{\langle N\rangle}\left\langle\left(U-\frac{\langle U\rangle}{\langle N\rangle}N\right)^2\right\rangle = \overline{\varepsilon^2} - \overline{\varepsilon}^2}$$

$\varepsilon$ - single particle energy

$$\overline{\varepsilon^n} \equiv \int d\varepsilon\, \varepsilon^n P_{ind}(\varepsilon)$$

Statistical noise

# The correlation measure *)

- $z_x \stackrel{df}{=} x - \bar{x}$

$x$ - single particle variable, $p_\perp$, $E$, .....

$\bar{x}$ - inclusive average, $\bar{z}_x = 0$

- $Z_x \stackrel{df}{=} \sum_{i=1}^{N} z_x^i = \sum_{i=1}^{N} (x_i - \bar{x})$

event variable

$N$ - number of particles in a given event

- $$\boxed{\Phi_x \stackrel{df}{=} \sqrt{\frac{\langle Z_x^2 \rangle}{\langle N \rangle}} - \sqrt{\bar{z_x^2}}}$$

$\langle \ldots \rangle$ - average over events

*) M. Gaździcki, St.M., Z. f. Phys. 54 ('92) 127

# Properties of $\Phi_x$

- **No correlations**

$$\frac{\langle Z_x^2 \rangle}{\langle N \rangle} = \overline{z_x^2}$$

$$\boxed{\Phi_x = 0}$$

- **"N-N limit"**

1) $\langle Z_x^2 \rangle_{AA} = \langle k \rangle \langle Z_x^2 \rangle_{NN}$

$k$ - number of N-N sources

$\langle Z_x \rangle = 0$

2) $\langle k \rangle = \frac{\langle N \rangle_{AA}}{\langle N \rangle_{NN}}$

3) $\frac{\langle Z_x^2 \rangle_{AA}}{\langle N \rangle_{AA}} = \frac{\langle Z_x^2 \rangle_{NN}}{\langle N \rangle_{N}}$

4) $\overline{z_x^2}\Big|_{NN} = \overline{z_x^2}\Big|_{AA}$

$$\boxed{\Phi_x^{AA} = \Phi_x^{NN}}$$

# Other measures

- $$\Phi = \sqrt{\frac{\langle (X - N\bar{x})^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{(x-\bar{x})^2}}$$

- $$\Delta\sigma = \sqrt{\langle N \rangle \left\langle \left( \frac{X}{N} - \left\langle \frac{X}{N} \right\rangle \right)^2 \right\rangle} - \sqrt{\overline{(x-\bar{x})^2}}$$

$$\left\langle \frac{1}{N} \right\rangle = \frac{1}{\langle N \rangle} \left( 1 + \frac{\langle (N - \langle N \rangle)^2 \rangle}{\langle N \rangle^2} + \dots \right)$$

- $$\sigma^2_{DYN} = \left\langle \left( \frac{X_a}{N_a} - \left\langle \frac{X_a}{N_a} \right\rangle \right) \left( \frac{X_b}{N_b} - \left\langle \frac{X_b}{N_b} \right\rangle \right) \right\rangle$$

$a, b$ - subevents

$$\Delta\sigma_{noise} = \Big( \underbrace{\sqrt{\langle N \rangle \left\langle \frac{1}{N} \right\rangle} - 1}_{\cong \frac{1}{2\langle N \rangle}} \Big) \sqrt{\overline{(x-\bar{x})^2}}$$

Poisson →

# $\Phi(p_\perp)$ - experimental results

Colliding systems:

pp, CC, ... Pb-Pb central

Collision energy:

SPS, RHIC

Experiments:

NA22, NA49, CERES, PHENIX, STAR

$$\boxed{\Phi(p_\perp) \lesssim 10\ \mathrm{MeV}}$$

$$\Phi(p_\perp) = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}$$

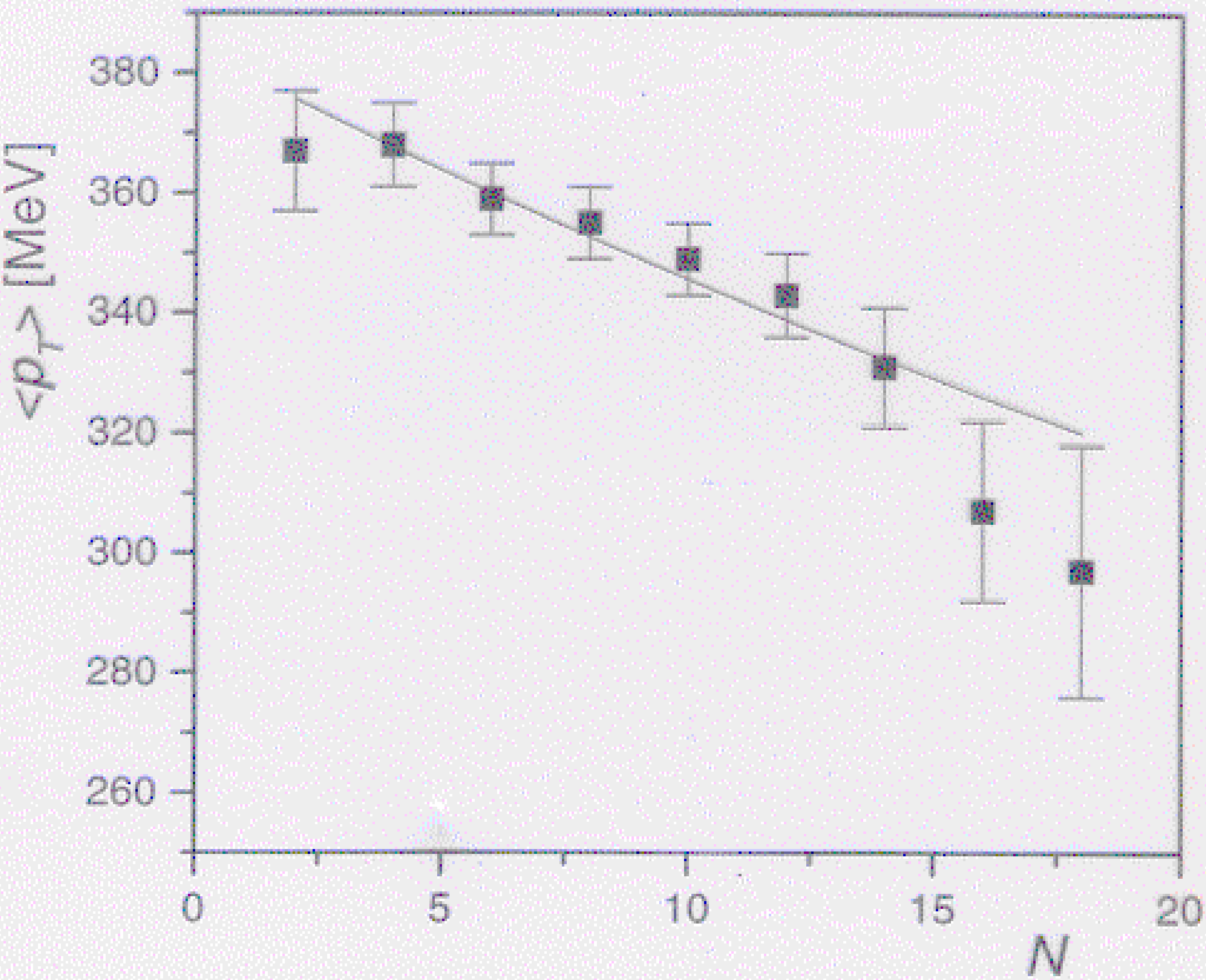
$\sigma(p_\perp) \approx 200\ \mathrm{MeV}$

- Less than 5% effect
- Small acceptance (1-20%) measurements

# Fluctuations in P.P.

# $\langle p_\perp \rangle$ vs. N. correlation

p-p data @ 205 GeV *)




$T = 167 \pm 1.5$ MeV

$\Delta T = 1.25 \pm 0.25$ MeV

*) T. Kafka et al., Phys. Rev. D16 (1977) 1261

## The model

$$P_{(N)}(p_\perp) \sim p_\perp e^{-\frac{\sqrt{m^2+p_\perp^2}}{T_N}}$$

$$T_N = T + \Delta T(\langle N\rangle - N)$$

$$T_{\langle N\rangle} = T$$

$P_N$ - multiplicity distribution

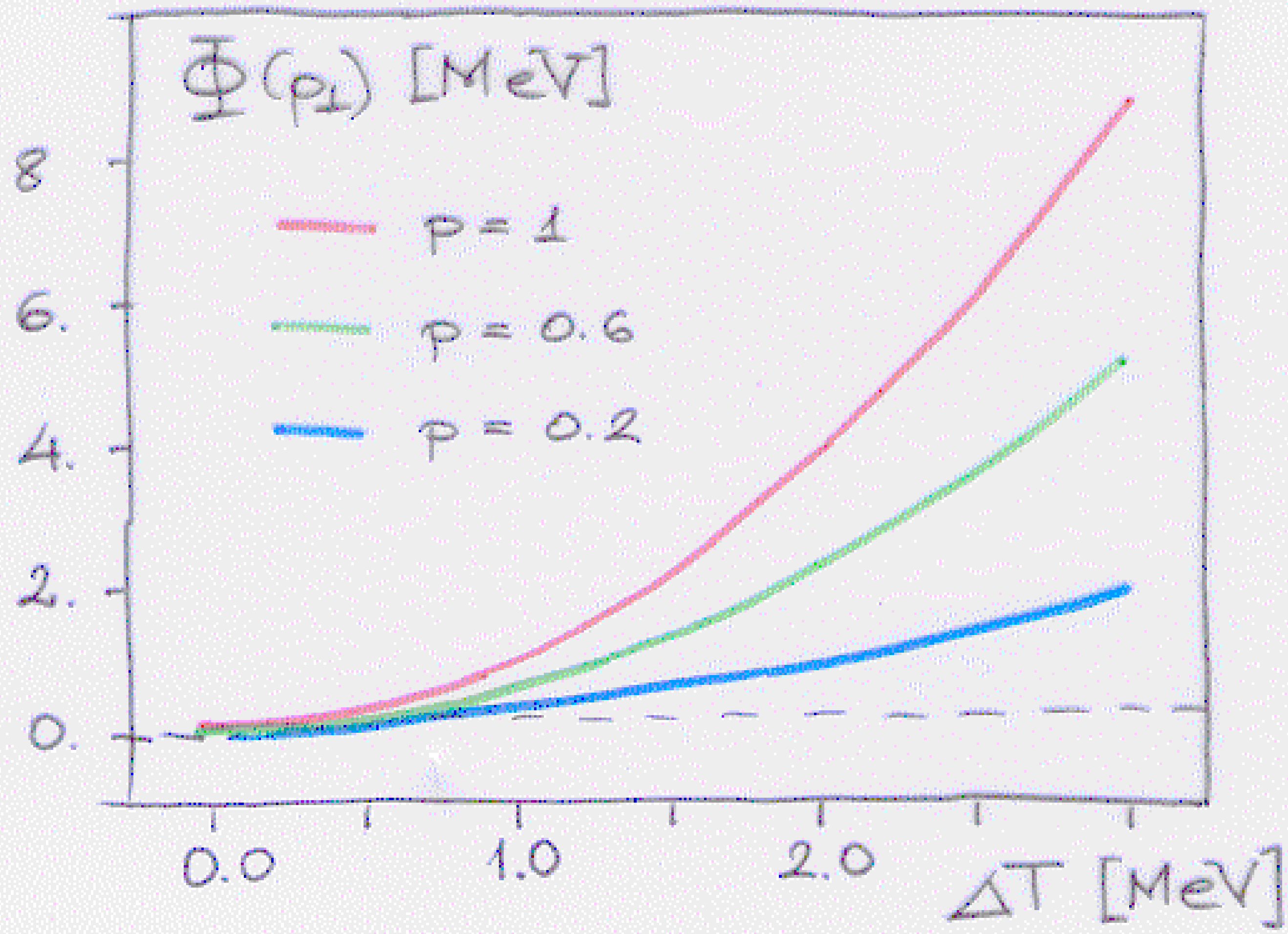
## Analytical result

$m=0$, $P_N \sim$ Poisson

$$\boxed{\Phi(p_\perp) \approx \sqrt{2}\,\frac{\Delta T}{T}^2 (\langle N\rangle^2 + \langle N\rangle)}$$

$$\Phi \sim \langle N\rangle^2$$

# $p_\perp$ vs. $N$




p - fraction of registered particles

What is the source of $P_\perp$-fluctuations in central Pb-Pb collisions?

Bose-Einstein correlations?

# $\Phi$ correlation in equilibrium*)

Ideal quantum gas in equilibrium

$x = E$ – particle energy

$$\boxed{\overline{Z_E^2} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} \frac{(E-\overline{E})^2}{\lambda^{-1} e^{\beta E} \pm 1}}$$

$\lambda = e^{\beta\mu}$ – fugacity

$$\rho = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\lambda^{-1} e^{\beta E} \pm 1}$$

$$\overline{E} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3} \frac{E}{\lambda^{-1} e^{\beta E} \pm 1}$$ – average single particle energy

$$Z_E = U - N\overline{E}$$

$U$ – total system energy
$N$ – particle number

$$\langle Z_E \rangle = \left[-\frac{\partial}{\partial\beta} - \overline{E}\lambda\frac{\partial}{\partial\lambda}\right] \ln \Xi(V,T,\lambda)$$

$\Xi(V,T,\lambda)$ – grand canonical partition function

*) St.M., Phys. Lett. B439 ('98) 6.

$$\langle Z_E^2 \rangle = \frac{1}{\Xi}\left[\frac{\partial^2}{\partial\beta^2} + 2\bar{E}\lambda\frac{\partial^2}{\partial\lambda\,\partial\beta} + \bar{E}^2\left(\lambda\frac{\partial}{\partial\lambda}\right)^2\right]\Xi(V,T,\lambda)$$

$$\ln \Xi(V,T,\lambda) = \pm V \int \frac{d^3p}{(2\pi)^3} \ln\left[1 \pm \lambda e^{-\beta E}\right]$$

$$\boxed{\frac{\langle Z_E^2 \rangle}{\langle N \rangle} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\,(E-\bar{E})^2\,\frac{\lambda^{-1}e^{\beta E}}{\left(\lambda^{-1}e^{\beta E} \pm 1\right)^2}}$$

Fermions

$$\frac{\langle Z_E^2 \rangle}{\langle N \rangle} < \overline{Z_E^2} \quad \Rightarrow \quad \Phi_E < 0$$

Bosons

$$\frac{\langle Z_E^2 \rangle}{\langle N \rangle} > \overline{Z_E^2} \quad \Rightarrow \quad \Phi_E > 0$$

Classical limit $(\lambda^{-1} \gg 1)$

$$\frac{\langle Z_E^2 \rangle}{\langle N \rangle} = \overline{Z_E^2} \quad \Rightarrow \quad \Phi_E = 0$$

$\underline{\Phi_{p_\perp}}$ $\quad x = p_\perp$

$p_\perp = p \sin \Theta$

- $$\overline{z^2_{p_\perp}} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{(p_\perp - \overline{p}_\perp)^2}{\lambda^{-1} e^{\beta E} \pm 1}$$

- $$\frac{\langle Z^2_{p_\perp} \rangle}{\langle N \rangle} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{(p_\perp - p_\perp)^2 \lambda^{-1} e^{\beta E}}{(\lambda^{-1} e^{\beta E} \pm 1)^2}$$

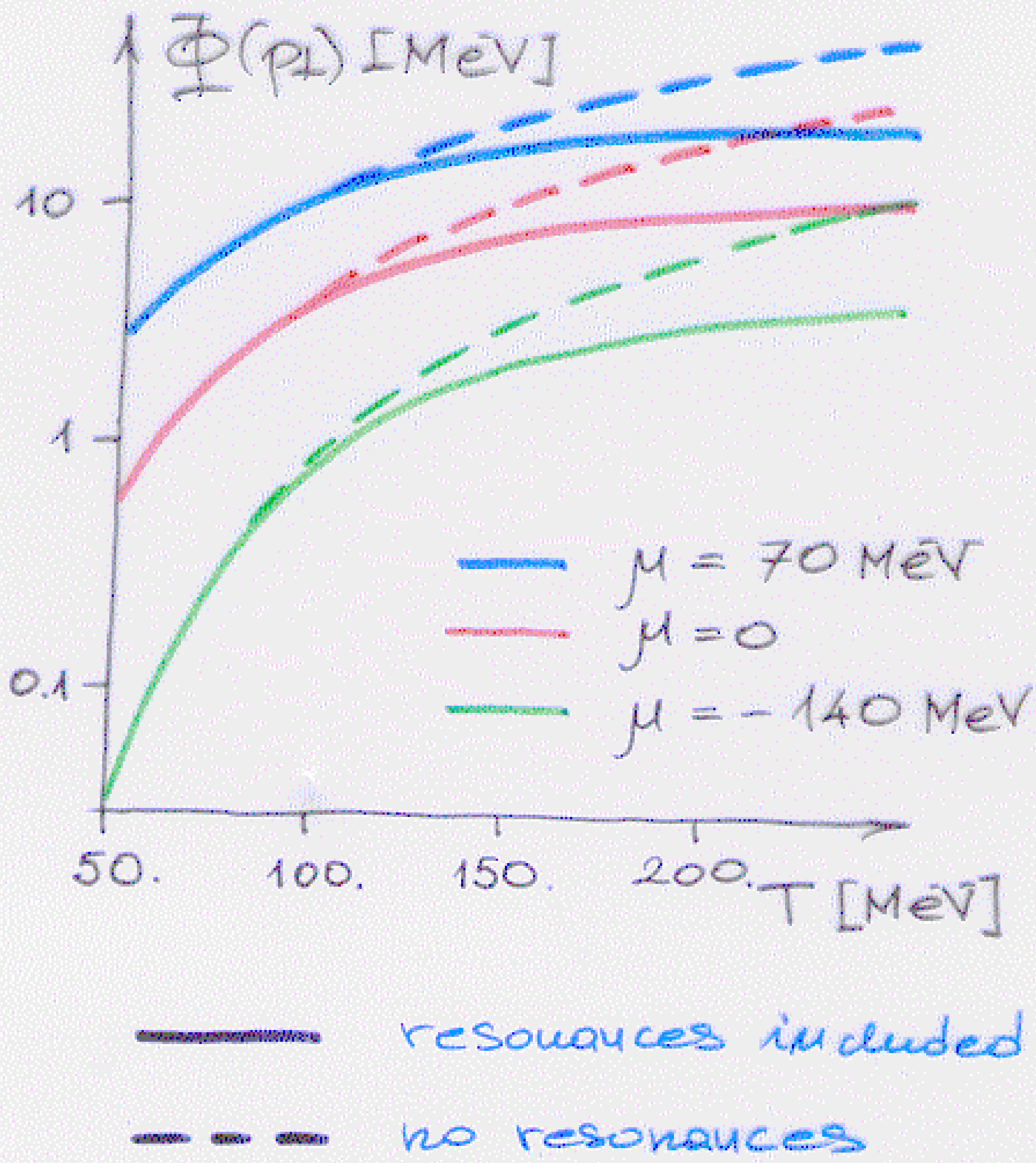
$$\Phi_{p_\perp} = \sqrt{\frac{\langle Z^2_{,p_\perp} \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2_{p_\perp}}}$$

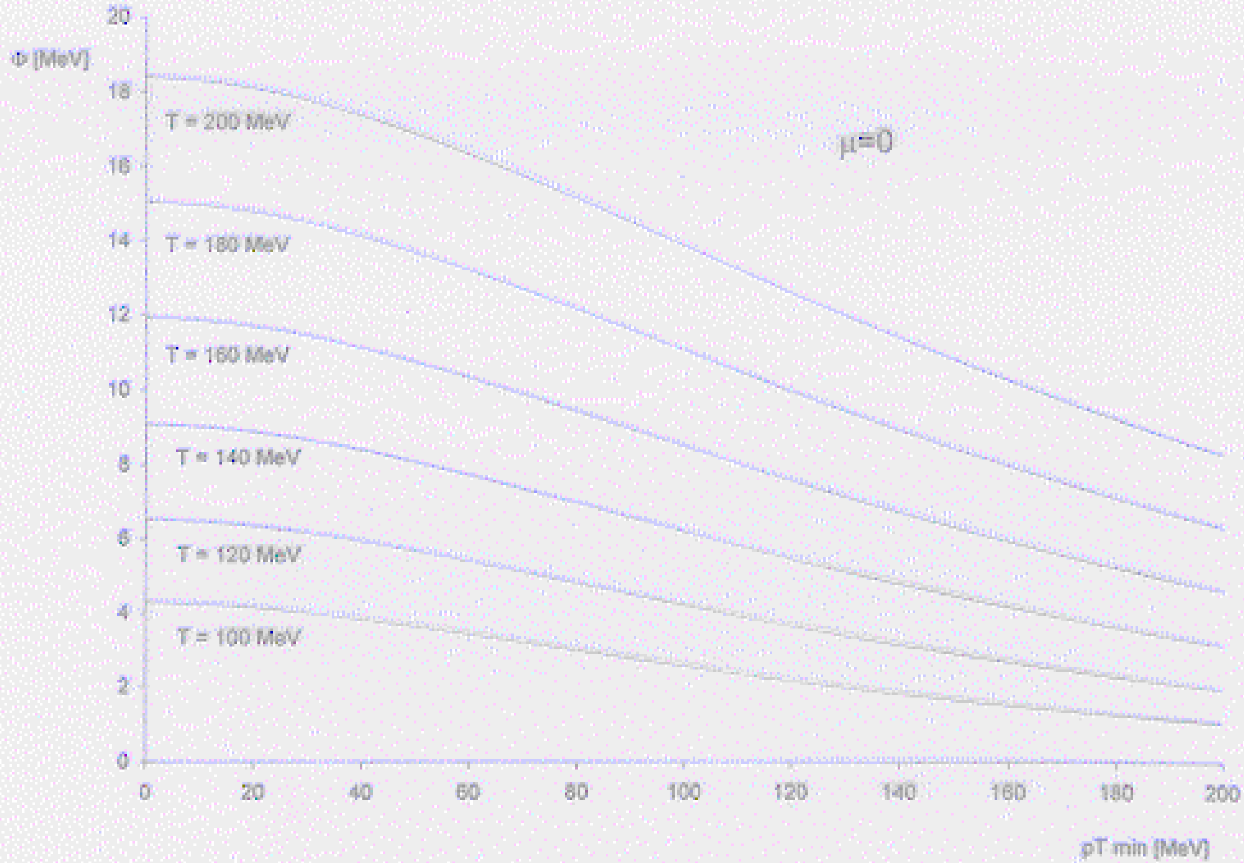
$m = 0 \;,\; \mu = 0$

$$\Phi_{p_\perp} \cong \begin{pmatrix} -0.05 \\ 0.29 \end{pmatrix} T$$

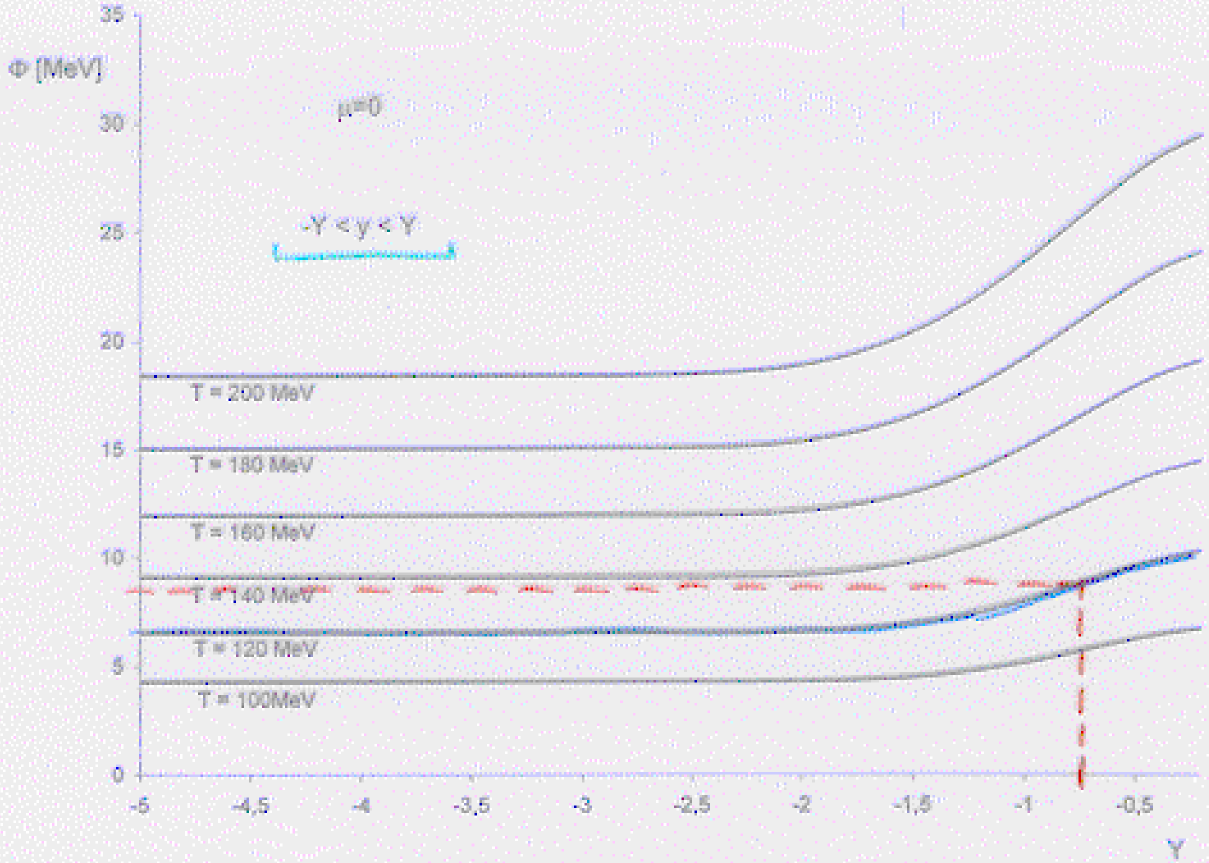
$\Phi$ - independent of $\langle N \rangle$

# Bose-Einstein statistics






$p_\perp \in [p_\perp^{min}, \infty]$




fireball CM frame

# T-fluctuations

$$P_{(T)}(p_\perp) \sim p_\perp e^{-\frac{m_\perp}{T}}$$

$P(T)$ - temperature distribution

$P_N$ - multiplicity distribution

## Analytical result

$m = 0$ , $P_N$ - Poisson

$$\boxed{\Phi(p_\perp) \approx \sqrt{2}\, \langle N \rangle \frac{\langle T^2 \rangle - \langle T \rangle^2}{\langle T \rangle}}$$

$$\Phi \sim \langle N \rangle$$

# $\Phi$ - measure of chemical fluctuations *)

Two component system

$N_a$ – number of "a" particles
$N_b$ – number of "b" particles

- $z \stackrel{df}{=} x - \bar{x}$ – single particle variable

$$x_i = \begin{cases} 1 & \text{i-th particle is of "a" type} \\ 0 & \text{i-th particle is not of "a" type} \end{cases}$$

- $Z \stackrel{df}{=} \sum_{i=1}^{N} z_i$ – event variable

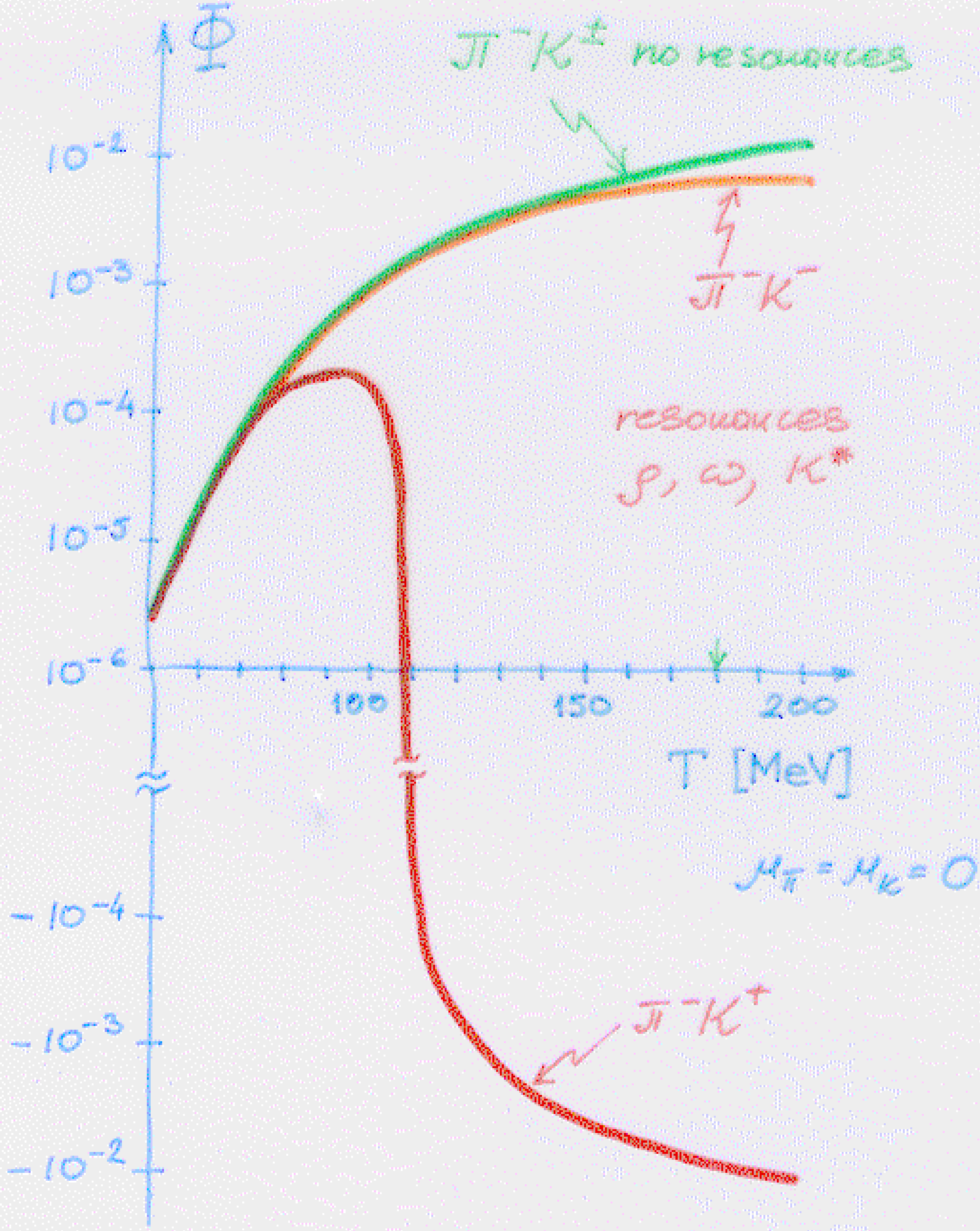
$N = N_a + N_b$

- $\Phi \stackrel{df}{=} \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}$

$\overline{\ldots}$ – inclusive average

$\langle \ldots \rangle$ – average over events

*) M. Gaździcki, Euro. Phys. J. C8 ('99) 131;
St. Mrów..., Phys. Lett. B459 ('99) 13.




St. M. Phys. Lett. B459 ('99) 13

# Azimuthal fluctuations

Are directed & elliptic flows the only (main) sources of azimuthal fluctuations ?

$$z = \varphi - \bar{\varphi}$$

$$Z = \sum_i (\varphi_i - \bar{\varphi})$$

$$\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}$$

$\bar{\varphi} = \pi$

$\overline{\varphi^2} = \frac{4}{3}\pi^2$

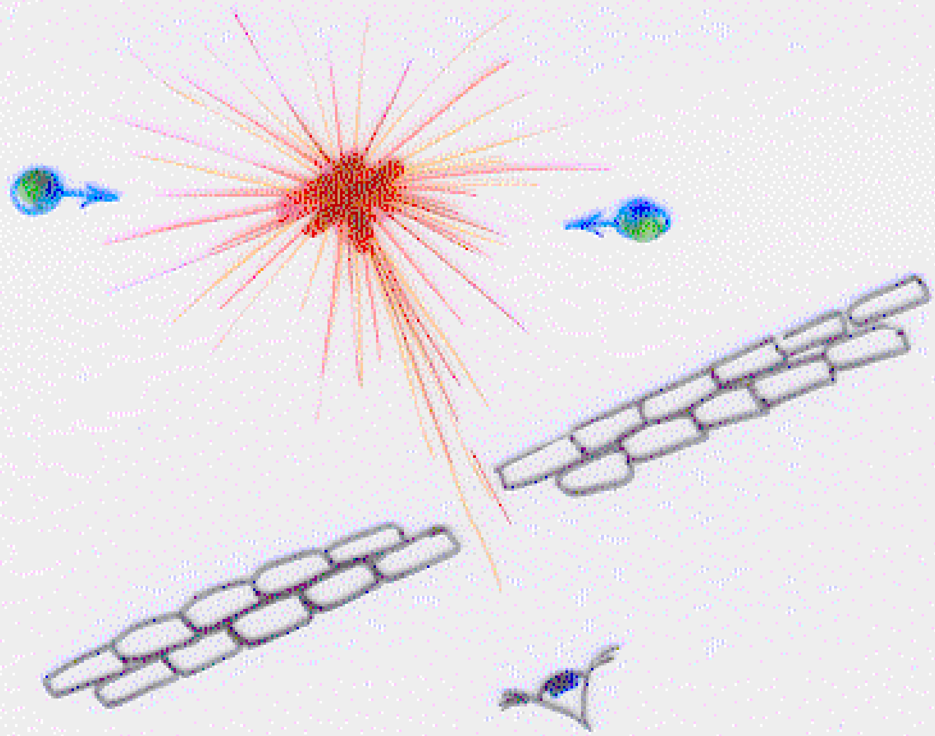
$\overline{z^2} = \frac{1}{3}\pi^2$

$\Phi$ due to flow:

$$\Phi = \frac{3}{\pi^2} \langle N \rangle \sum_n \frac{v_n^2}{n^2}$$

$v_n$ - amplitude of n-th harmonics of azimuthal distribution

# Effect of small acceptance



$$\mathcal{P}_N = \sum_M P_M \binom{M}{N} p^N (1-p)^{M-N}$$

observed multiplicity distribution

real multiplicity distribution

p – detection probability

$$\langle N^2\rangle - \langle N\rangle^2 = p\langle M\rangle + p^2\left(\langle M(M-1)\rangle - \langle M\rangle^2\right)$$

$p \to 0$

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} \approx 1$$

Poisson !

# Conclusions

- Dynamical fluctuations are small.

- Large acceptance measurements are needed.