Semi–Classical Approach To Nuclear Collisions:

(or, *Melting a* Color Glass Condensate *in Heavy Ion Collisions)*

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Outstanding Phenomenological Questions

- *Is high energy density Quark–Gluon matter formed at RHIC?*

- *Does this matter thermalize to form a QGP? What can we learn about the properties of the QGP?*

- *Can we learn about universal properties of hadronic wavefunctions at high energies?* (Color Glass Condensate)

- Very likely Partonic Matter at High Energy Densities produced at RHIC

- The CGC describes the earliest (most easily calculable) stage of the collision:

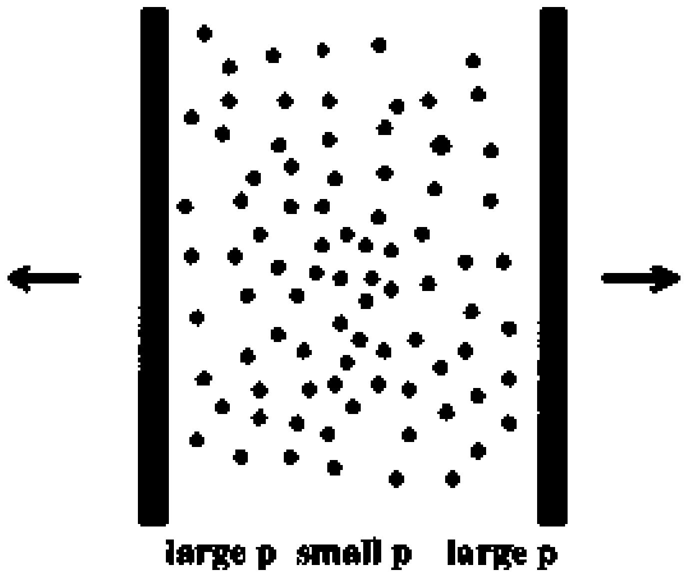
  Re–scattering (energy loss) is essential–most sensitive to CGC rather than QGP!

- No conclusive evidence for thermalized QGP yet.

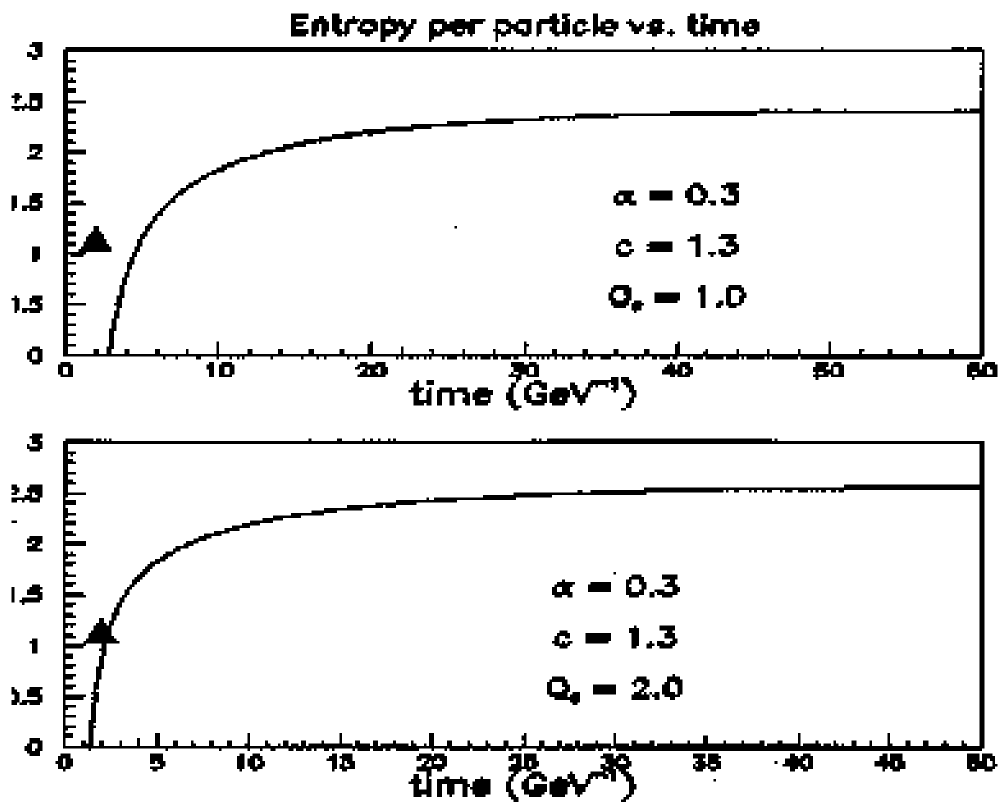
  Both KLM version of CGC & ideal hydro + "jet quenching" are problematic

Heavy–Ion Collisions–violent dynamical system...

Is Thermalization achieved ?

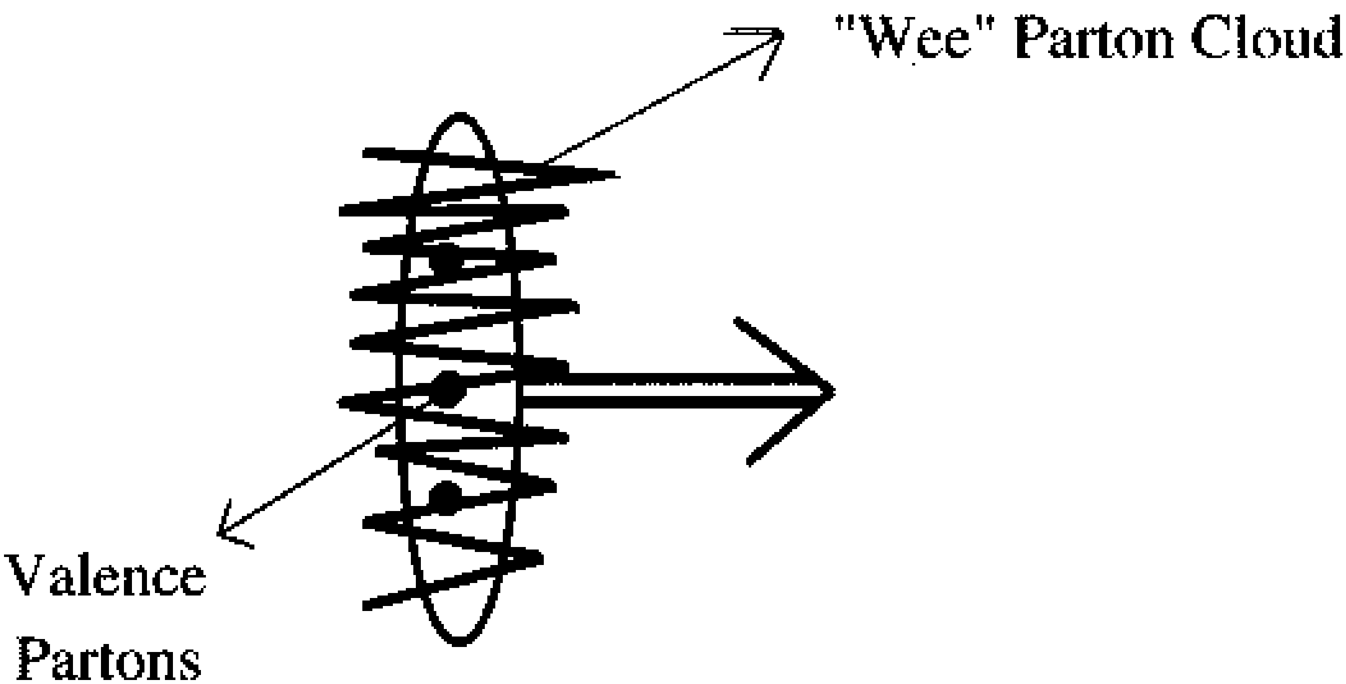


- Require ratio of rates: $\frac{\Gamma_{exp.}}{\Gamma_{coll.}} < 1$



- Thermalization very sensitive to initial conditions!

A Hadron at High Energies

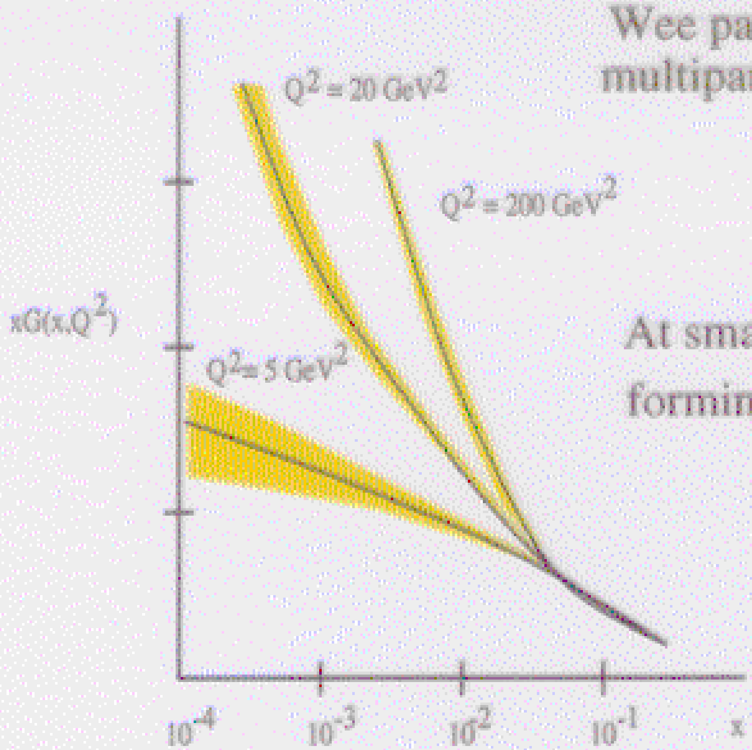


$$|h> = |qqq> + |qqqg> + \cdots + |qqq \cdots gggq\bar{q}g>$$

Each Wee Parton carries only a small fraction $x = k^+ / P^+ << 1$ *of momentum* P^+ *of the hadron/nucleus*

- What is the behavior of Wee Partons in a High Energy Hadron ?

Parton Distributions at small x:



Wee partons (x <<1) responsible for multiparticle–production at high energies

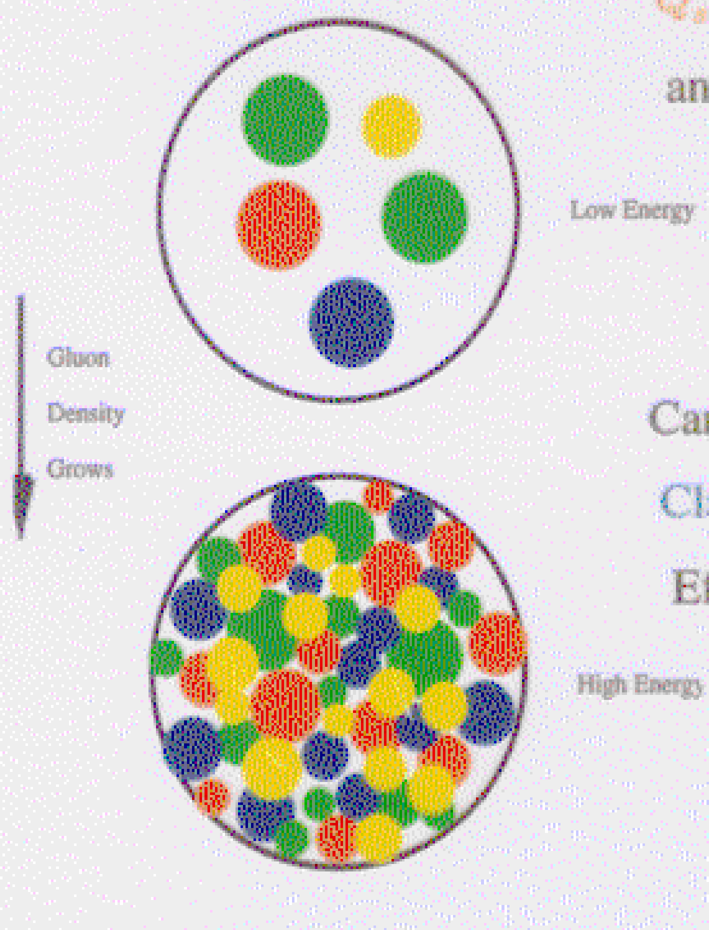
At small x, the gluon distribution saturates forming a Color Glass Condensate

Gluon density per unit area

$$Q_s^2 = \alpha_s N_c \frac{1}{\pi R^2} \frac{dN}{dy}$$

$Q_s^2 >> \Lambda_{QCD}^2$ for small x

and large nuclei $Q_s^2 \propto A^{1/3}$



$$\alpha_s(Q_s^2) << 1$$

Can compute initial conditions in

Classical $\left(f \sim \frac{1}{\alpha_S} > 1 \right)$

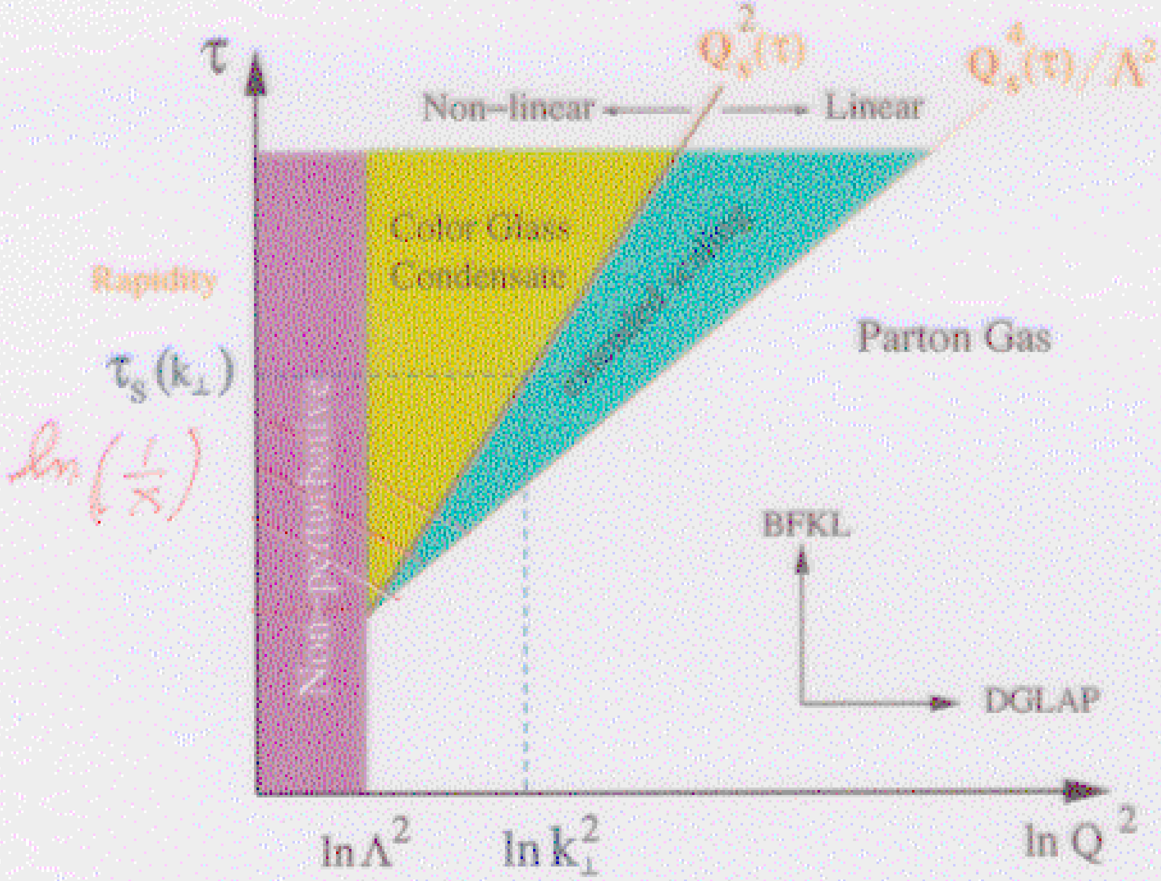
Effective Theory

Gribov, Levin, Ryskin
Mueller, Qiu
McLerran, Venugopalan

Jalilian–Marian, Kovner, Leonidov, Weigert ; Kovchegov

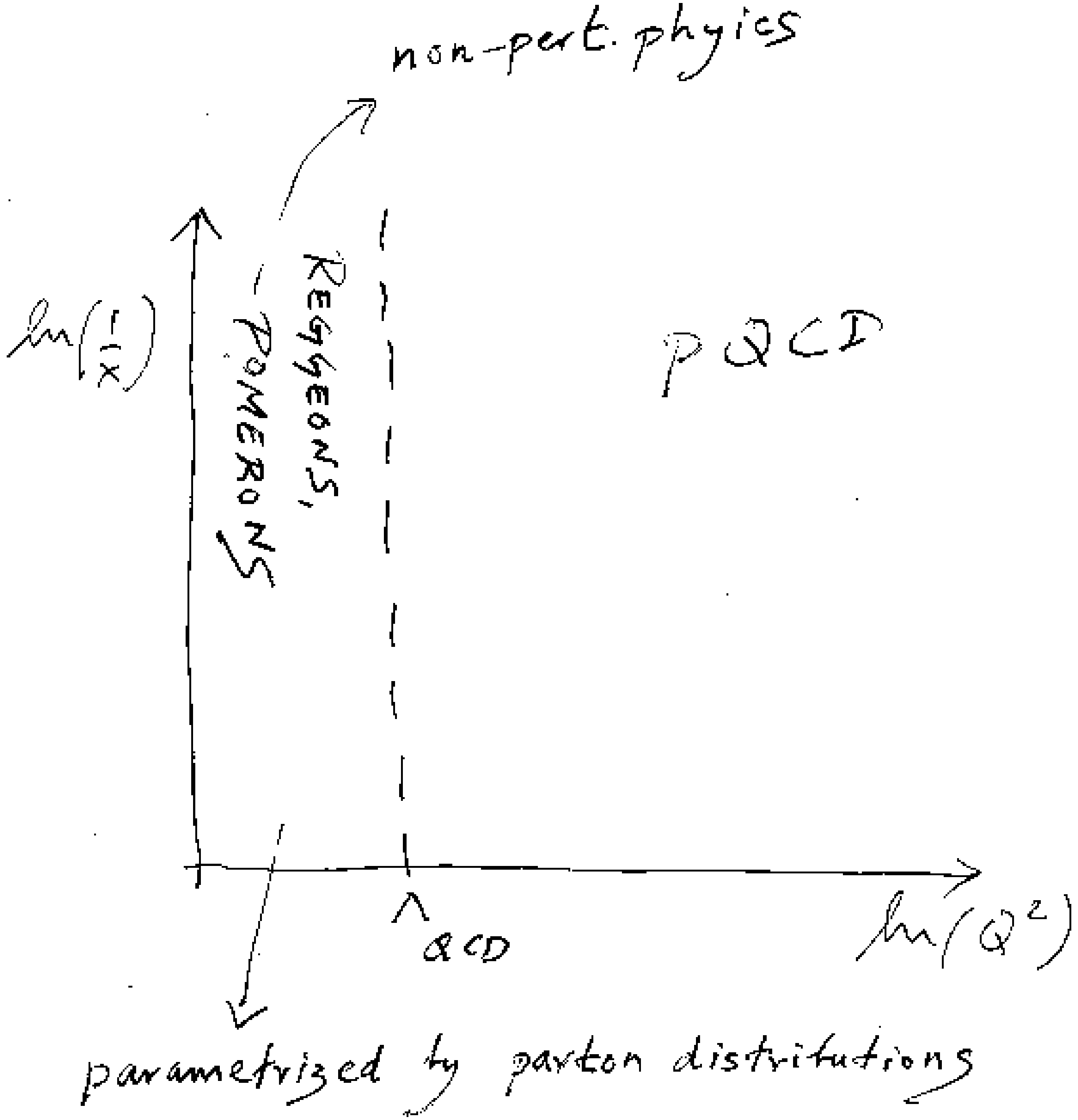
Iancu, Itakura, McLerran

Phase Diagram of Hadron Wavefunction



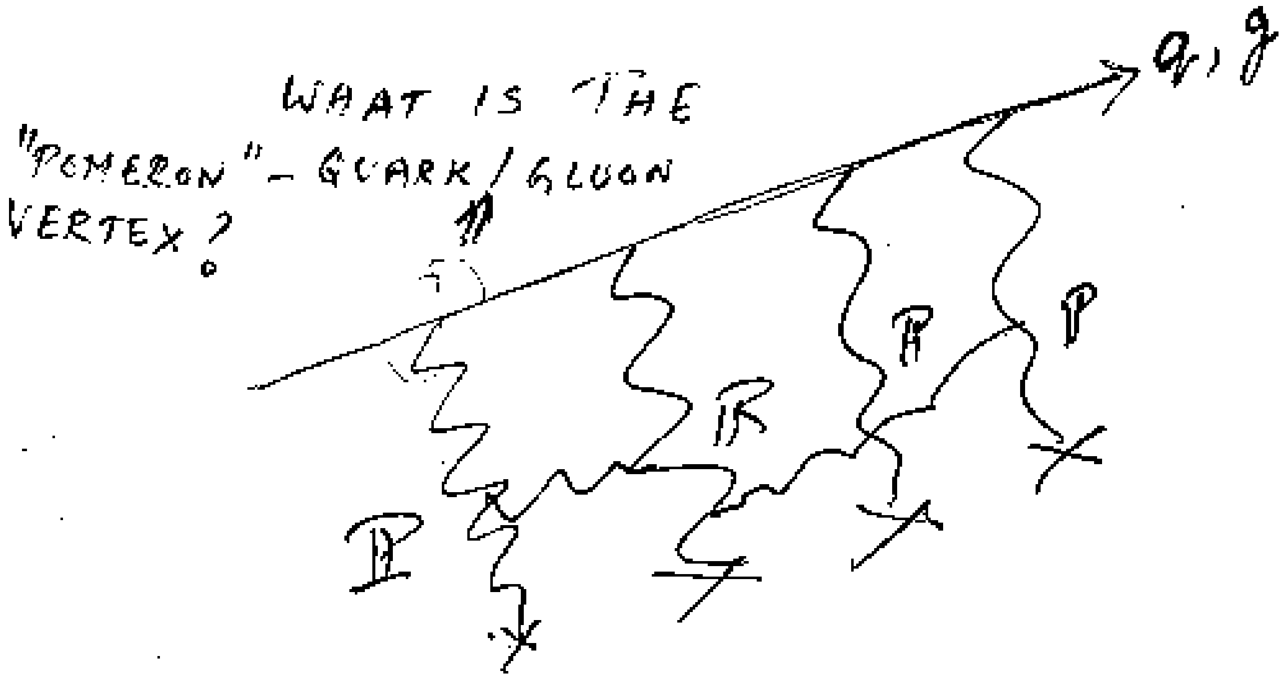
- "Color Glass Condensate" => Low p_t physics at RHIC
- KLM (Kharzeev-Levin-McLerran)
  "Extended Scaling" => Moderate p_t : $Q_s^2 << p_T^2 << \frac{Q_s^4}{\Lambda_{QCD}^2}$
  (analogous to "leading twist" shadowing but with different anomalous dimensions)
- " Parton Gas" => Usual pert. QCD physics

ALTERNATIVE "PHASE DIAGRAM" OF WAVE FN



- NO RELIABLE THEORY OF POMERONS / REGGEONS AT LOW OR HIGH ENERGIES.

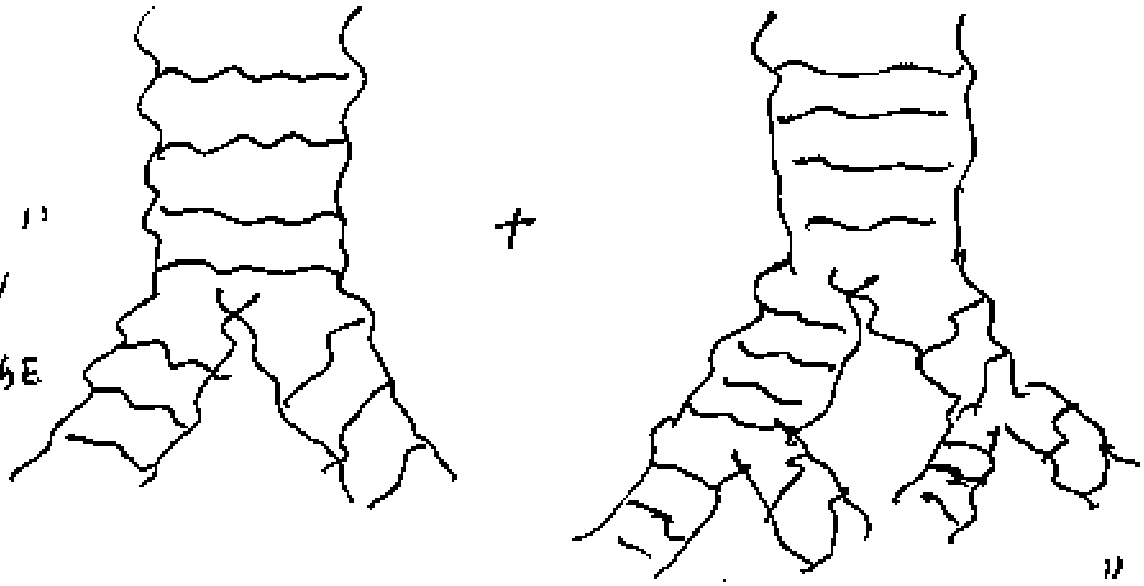
⇒ CANNOT RELIABLY COMPUTE PROPERTIES OF MEDIUM <u>OR</u> PROBES OF MEDIUM.



- ALL JET QUENCHING MODELS IMPLICITLY ASSUME WEAK COUPLING WITH MEDIUM (BDMPS, GLVW, ...)

- CGC PROVIDES A <u>PERTURBATIVE</u> "HIGHER TWIST" MECHANISM TO UNDERSTAND SHADOWING IN QCD

"TRIPLE POMERON" IN OLD LANGUAGE



"FAN DIAGRAMS"

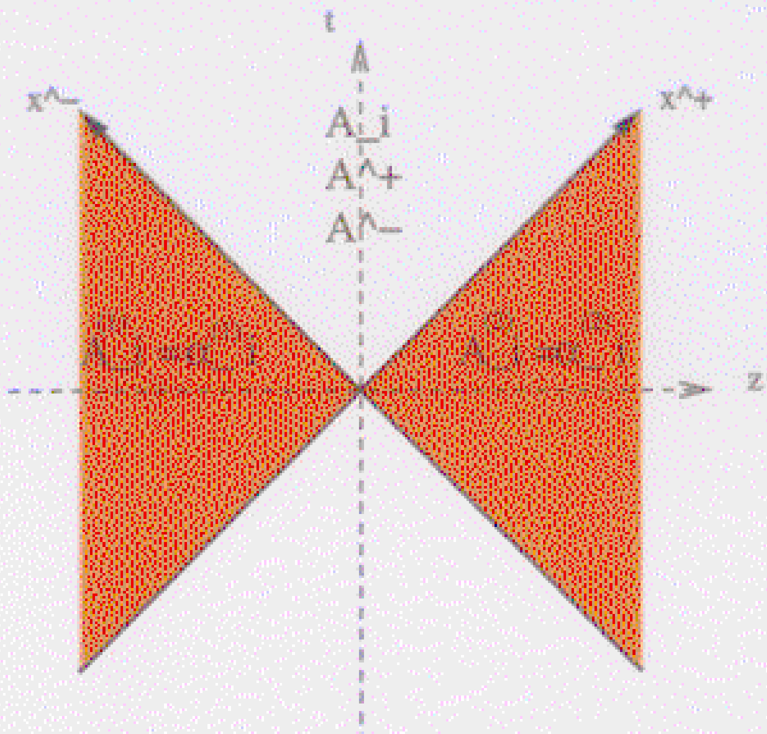
- SYSTEMATIC APPROACH TO GO FROM LOW PARTON DENSITIES TO HIGH PARTON DENSITIES.

NOT JUST APPLICABLE IN THE "SATURATION" LIMIT!

→ REPRODUCES DGLAP + BFKL AT LOW PARTON DENSITIES.

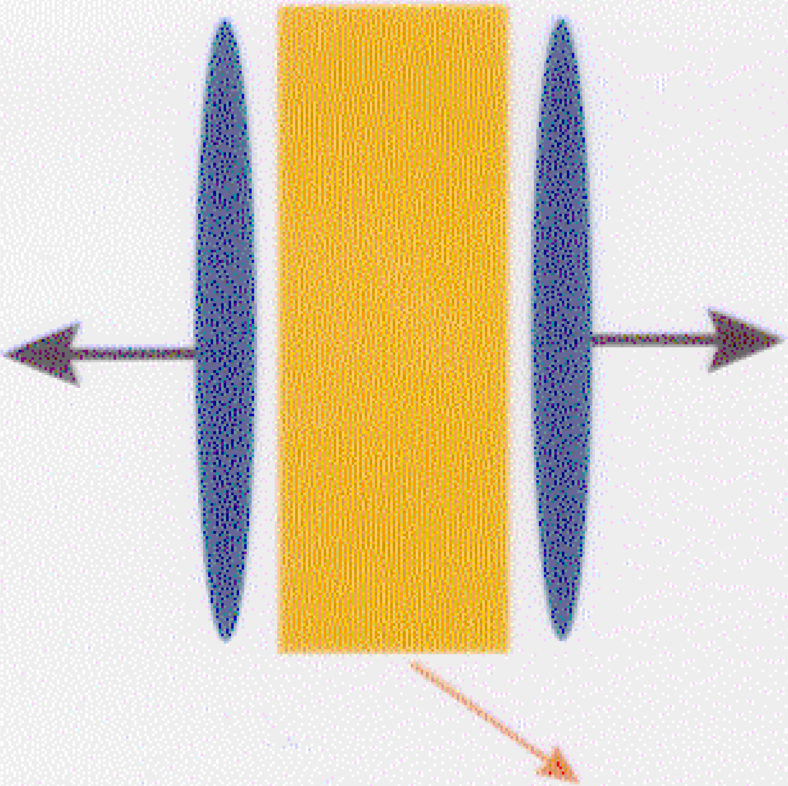
Melting the Color Glass Condensate

- Classical equations of motion
- Two sources of color charge ρ_1, ρ_2
- Can be numerically solved on a lattice
- Can be analytically solved in the weak field limit



- Initial energy density, number density, etc.

Real Time Gluodynamics of Nuclear Collisions



Kovner,McLerran,Weigert
Krasnitz,Nara,Venugopalan
Lappi

Classical Fields with occupation # $f = \frac{1}{\alpha_s}$

- Non–perturbative formulae for initial glue distributions

$$\frac{1}{\pi R^2}\frac{dE_T^{glue}}{d\eta} = \frac{0.25}{g^2}Q_s^3$$

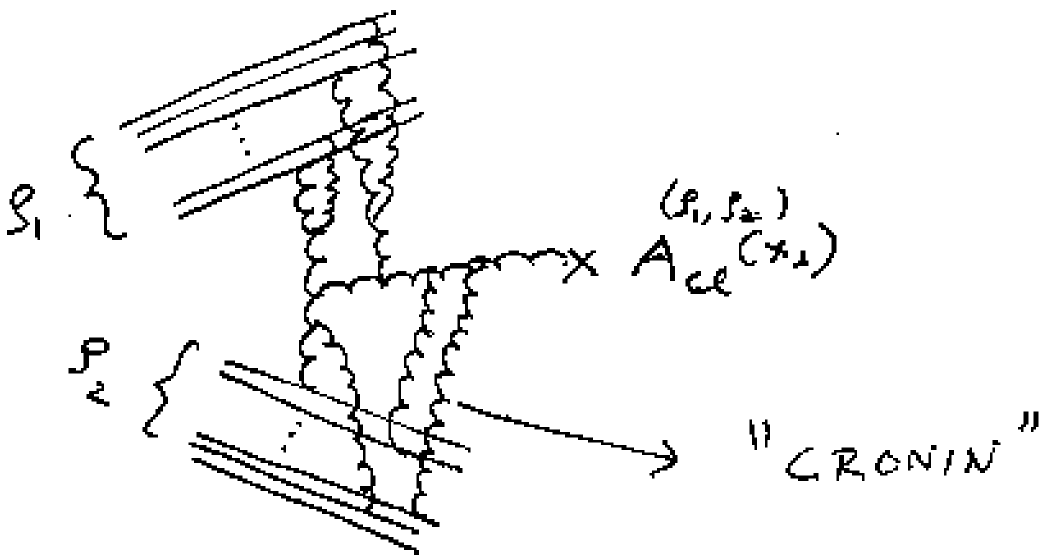
$$\frac{1}{\pi R^2}\frac{dN^{glue}}{d\eta} = \frac{0.3}{g^2}Q_s^2$$

- Classical approach breaks down at late time when f << 1...

$\tau >> \frac{1}{Q_s}$ but $\tau << R$

- SOLVE $D_\mu F^{\mu\nu,a} = \rho_1^a(x_\perp)\delta^{\nu+}\delta(x^-) + \rho_2^a(x_\perp)\delta^{\nu-}\delta(x^+)$

— COMPUTE ALL TREE LEVEL GRAPHS.

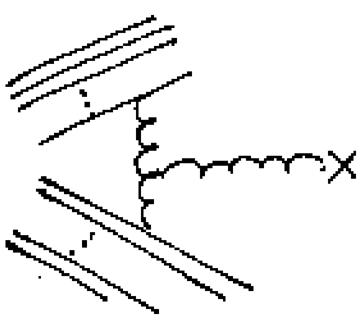


- AVERAGE OVER COLOR CHARGE DISTRIBUTIONS IN BOTH NUCLEI

$$\int [\mathcal{D}\rho_1][\mathcal{D}\rho_2] \exp\left[-\int d^2x_\perp \left(\frac{\rho_1^a\rho_1^a + \rho_2^a\rho_2^a}{\Lambda_s^2}\right)\right]$$

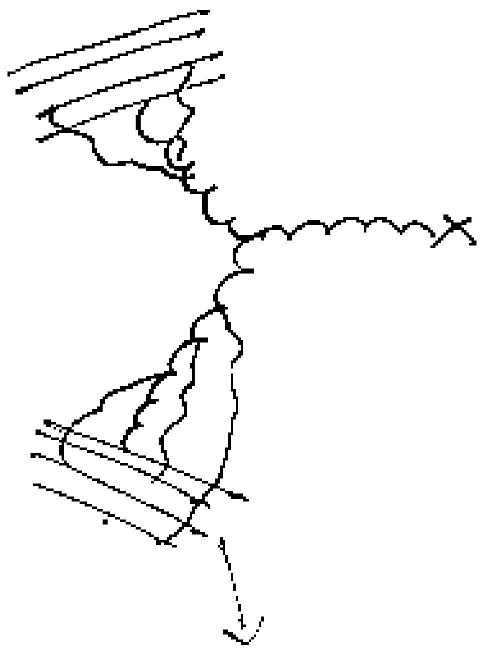
— RESULTS TO ALL ORDERS IN $\Lambda_s/k_\perp$

- TO LOWEST ORDER...



$$\propto \frac{\Lambda_s^2}{k_\perp^2}\frac{\Lambda_s^2}{k_\perp^2}$$

KLM SATURATION ("$k_\perp$ FACTORIZATION")



INCLUDES EVOLUTION ("SHADOWING")

– BUT NOT CRONIN

OR ENERGY LOSS.

BOTH SHOULD BE THERE EVEN IN CGC

FRAMEWORK.

$\tau = \sqrt{2x^+x^-}$

$\eta = \frac{1}{2}\ln\left(\frac{x^+}{x^-}\right)$

Lattice formulation

$A^\tau = 0$

The Hamiltonian formalism is better suited for numerical work. In the continuum

$$H = \frac{\tau}{2}\int \mathrm{d}\eta\mathrm{d}^2r_t \left[p^\eta p^\eta + \frac{1}{\tau^2}p^r p^r + \frac{1}{\tau^2}F_{\eta r}F_{\eta r} + F_{xy}F_{xy}\right].$$

For "perfect pancake" nuclei we only consider boost-invariant configurations. Hence

$$A_r(\tau,\eta,\vec{r}_t) = A_r(\tau,\vec{r}_t), \quad A_\eta(\tau,\eta,\vec{r}_t) = \Phi(\tau,\vec{r}_t)$$

(this resembles a finite-T dimensional reduction: an adjoint scalar emerges).

Per unit rapidity

$$H = \frac{\tau}{2}\int \mathrm{d}^2r_t \left[p^\eta p^\eta + \frac{1}{\tau^2}E_r E_r + \frac{1}{\tau^2}(D_r\Phi)(D_r\Phi) - F_{xy}F_{xy}\right]$$

Discretize on a 2d lattice

$$H_L = \frac{1}{2\tau}\sum_l E_l E_l + \tau \sum_{\mathrm{pl}} \left(1 - \frac{1}{N_c}\Re \mathrm{Tr} U_{\mathrm{pl}}\right) + \frac{\tau}{2}\sum_j p_j p_j + \frac{1}{4\tau}\sum_{j,n} \mathrm{Tr}\left(\Phi_j - U_{j,n}\Phi_{j+n}U^\dagger_{j,n}\right)^2$$

and solve (numerically) the resulting equations of motion for $x_\pm > 0$.

Interested in soft modes $\rightarrow$ use classical approximation.

Just as in the continuum

- Average over the static color charge
- Determine initial conditions by matching

Relation to continuum physics

Dimensional quantities in the classical lattice theory:

- Λ_s
- R, the nuclear radius
- l, the color neutrality scale (a recent development!)
- a, the lattice cutoff

Hierarchy of scales (ideal): $1/a \gg \Lambda_s \gg 1/l \gg 1/R$

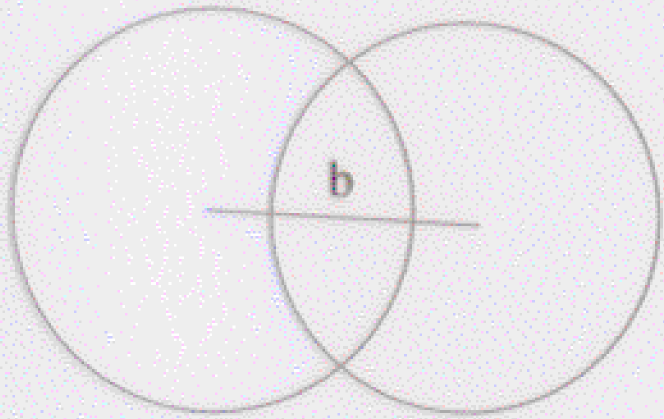
In the units of a, in the continuum limit $\Lambda_s \to 0$, $R \to \infty$, but $\Lambda_s R$ is constant.

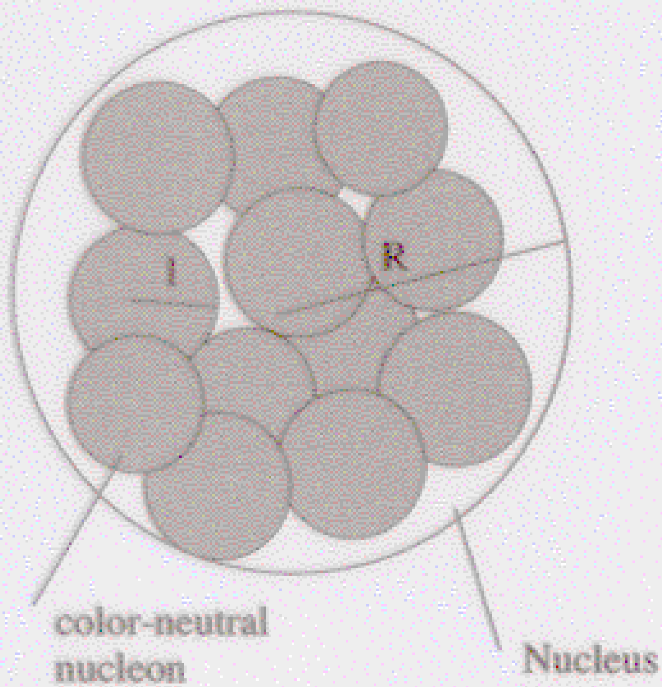
For any well-defined P of dimension d

$$P = (\Lambda_s)^d f_P(\Lambda_s R).$$

where $f_P(\Lambda_s R)$ contains all the non-trivial physical information.

- **RHIC** – $\Lambda_s \approx 1.4$ GeV
- **LHC** – $\Lambda_s \approx 2.2$ GeV





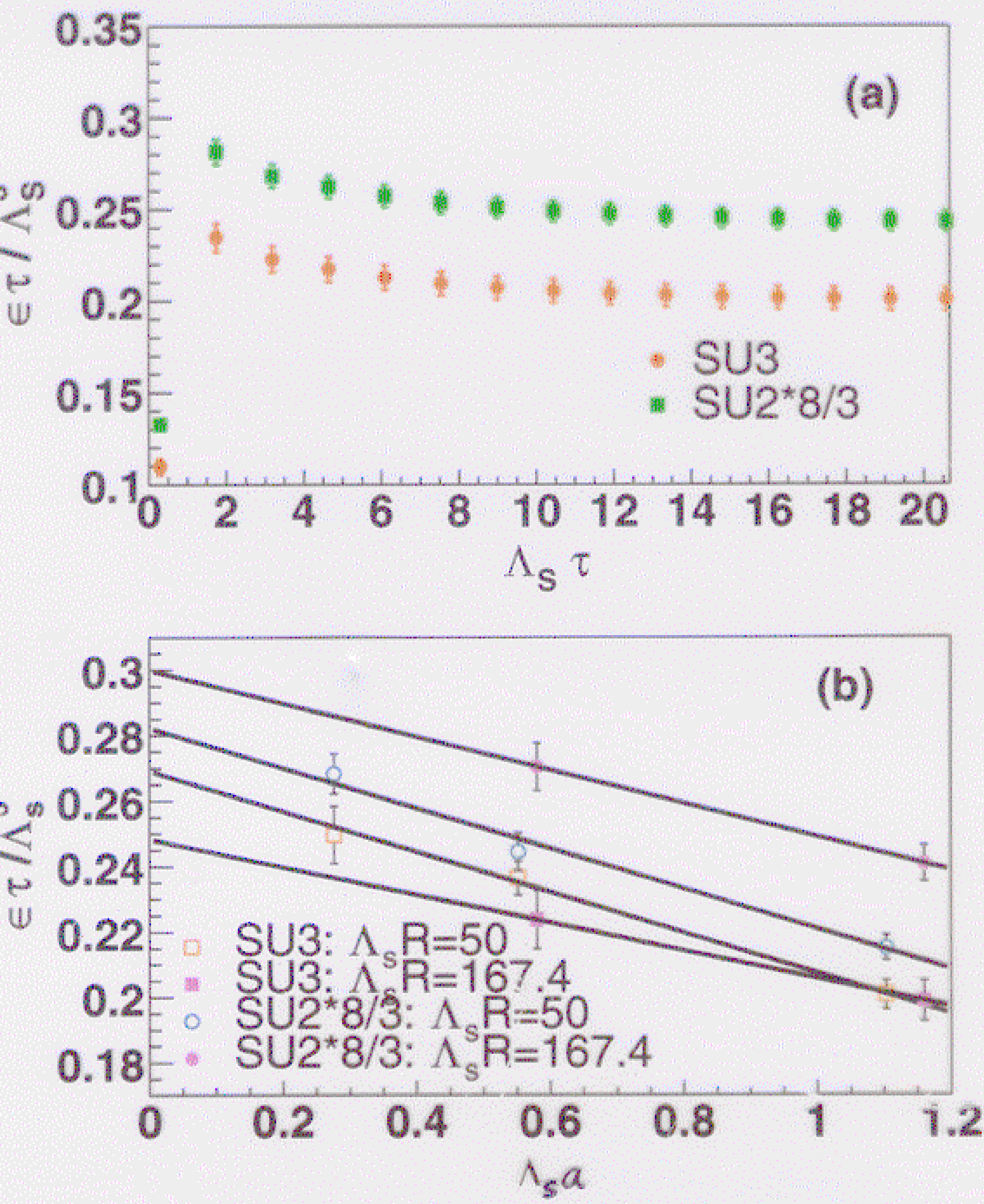
Refining the initial conditions

Impose neutrality w.r.t. color charge and color dipole moment of each nucleon.
In a nucleon, begin with

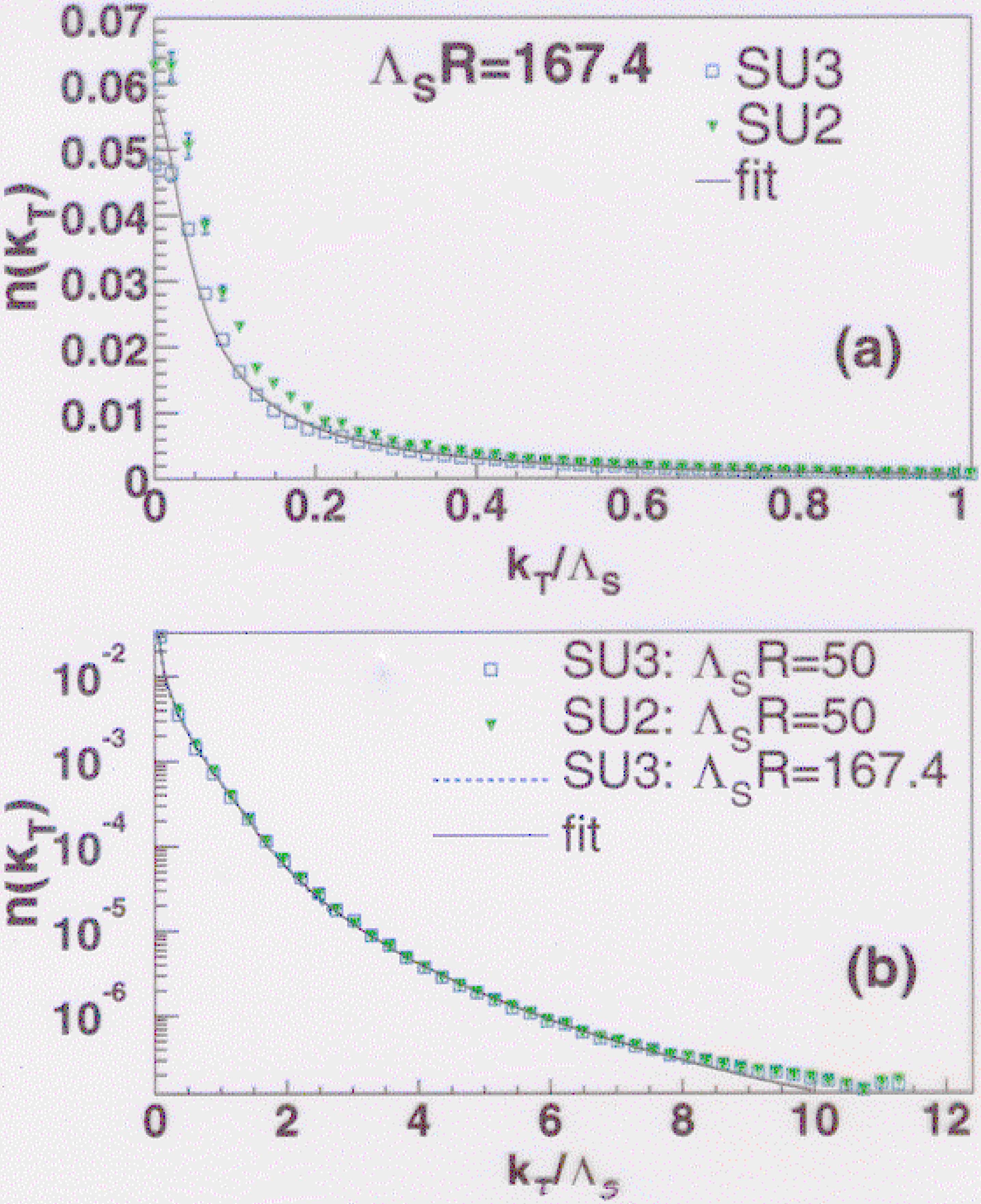
$$\langle \rho^a(\vec{r})\rho^b(\vec{r}')\rangle = \Lambda_n^2 \delta^{ab}\delta(\vec{r}-\vec{r}')$$

and remove the total color carge and dipole mc by subtracting uniform distributions.
Nucleons uniformly distributed within a spherica nucleus:

$$\Lambda_s^2(r) = \frac{2}{l}\Lambda_n^2\sqrt{R^2-r^2}$$



(a)
0.35
0.3
0.25
0.2
0.15
0.1
0
2
4
6
8
10
12
14
16
18
20
$\epsilon\tau/\Lambda_s^3$
$\Lambda_s\tau$
SU3
SU2*8/3
(b)
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0
0.2
0.4
0.6
0.8
1
1.2
$\epsilon\tau/\Lambda_s^3$
$\Lambda_s a$
SU3: $\Lambda_s R$=50
SU3: $\Lambda_s R$=167.4
SU2*8/3: $\Lambda_s R$=50
SU2*8/3: $\Lambda_s R$=167.4



$\Lambda_s R=167.4$
SU3
SU2
fit
(a)
$n(k_T)$
k_T/Λ_s
SU3: $\Lambda_S R=50$
SU2: $\Lambda_S R=50$
SU3: $\Lambda_S R=167.4$
fit
(b)
$n(k_T)$
k_T/Λ_s

- Transverse Energy

$$\left.\frac{1}{\pi R^2}\frac{dE_T}{d\eta}\right|_{\eta=0} = \frac{1}{g^2} f_E(\Lambda_s R)\,\Lambda_s^3$$

$f_E \simeq 0.25$ (previously $\simeq 0.5$)

Lappi, hep-ph/0303076.

- $$\varepsilon\tau = \alpha + \beta \exp(-\gamma\tau)$$

"formation time" $\tau_D = 1/\gamma / \Lambda_s$

$\gamma \simeq 0.3$

- Energy Density

$$\varepsilon = \frac{0.08}{g^2}\Lambda_s^4$$

- **Transverse Energy Per Gluon**

$$\frac{\left.\frac{1}{\pi R^2}\frac{dE_\perp}{d\eta}\right|_{\eta=0}}{\left.\frac{1}{\pi R^2}\frac{dN}{d\eta}\right|_{\eta=0}} = \frac{f_E(\Lambda_s R)}{f_N(\Lambda_s R)}\,\Lambda_s \simeq \boxed{0.88\,\Lambda_s}$$

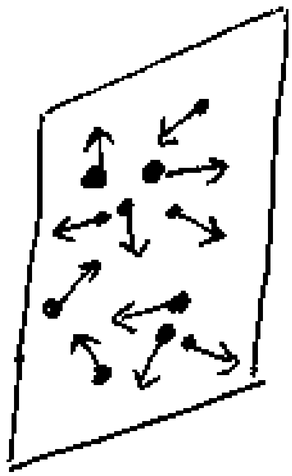
- **Hadron Multiplicity** At $\eta \simeq 0$

  For $\sqrt{s} \simeq 130$ GeV $\sim$ 1000

  Hadron Transverse Energy

  For $\sqrt{s} \simeq 130$ GeV $\sim$ 500 GeV

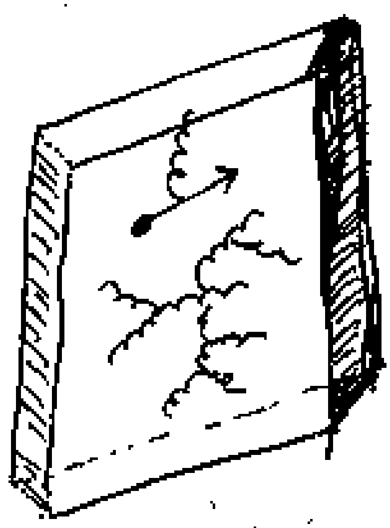
- THE CGC DESCRIBES ONLY THE INITIAL STATE – PRODUCED GLUONS MAY THERMALIZE



$\tau = \# / \Lambda_s$

$P_\perp \sim \Lambda_s$

$P_z \sim 0$

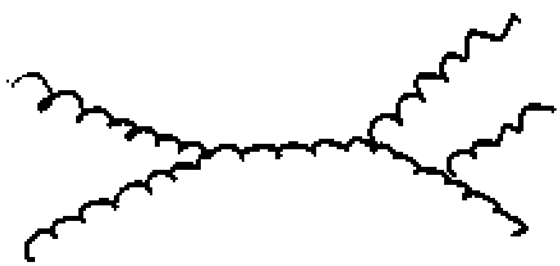


$\frac{\#}{\Lambda_s} < \tau < R$

$P_\perp \sim P_z \sim T$

* THERMALIZATION?

SMALL ANGLE 2→2 DRIVES THE SYSTEM SLOWLY TOWARDS EQUILIBRIUM



2→3 PROCESSES MAY BE MORE EFFICIENT

T_i^{QGP}, $t_{equil} \propto \Lambda_s$

* MANY OPEN QUESTIONS...

Tight Constraints on Final State Models from Classical Field results and RHIC data

$$E_T^{\text{glue}} > E_T^{\text{hadrons}}$$

$$N^{\text{glue}} \leq N^{\text{hadrons}}$$

==> 1.3 GeV < Q_s < 2 GeV at RHIC energies

$$1.14\ \text{GeV} < \frac{E_T{}^{\text{glue}}}{N} < 1.76\ \text{GeV}$$

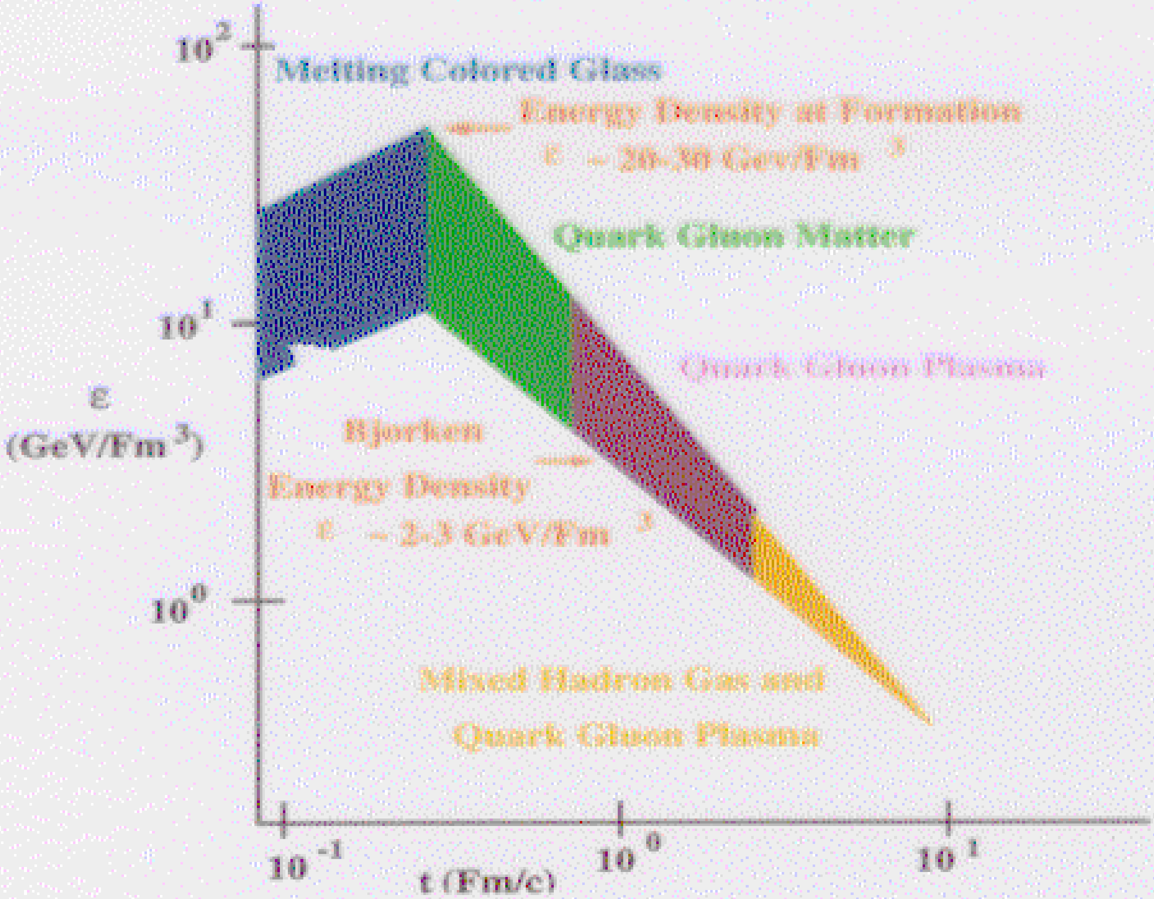
$$7.1\ \frac{\text{GeV}}{\text{fm}^3} < \epsilon^{\text{glue}} < 40\ \frac{\text{GeV}}{\text{fm}^3}$$

$$0.3\ \text{fm} < \tau_{\text{form}} < 0.45\ \text{fm}.$$

- **E_t ~ 500 GeV ; N ~ 1000 at central rapidities in Au–Au at RHIC**

- **Golec–Biernat–Wusthoff Parametrization of HERA data extrapolated to RHIC gives Q_s ~ 1.4 GeV**

From Classical fields towards Thermalized QGP...



- Monte–Carlo simulations problematic due to Quantum Mechanical Coherence...
- ''Bottom–Up'' Thermalization–can follow evolution from classical stage up to thermalization–requires $\alpha_s << 1$

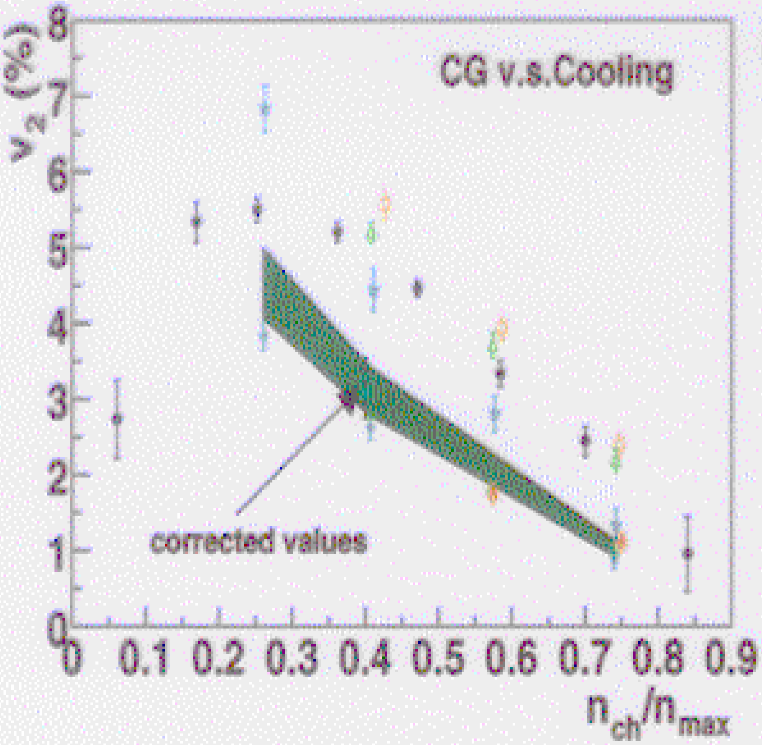
  Baier,Mueller,Schiff,Son

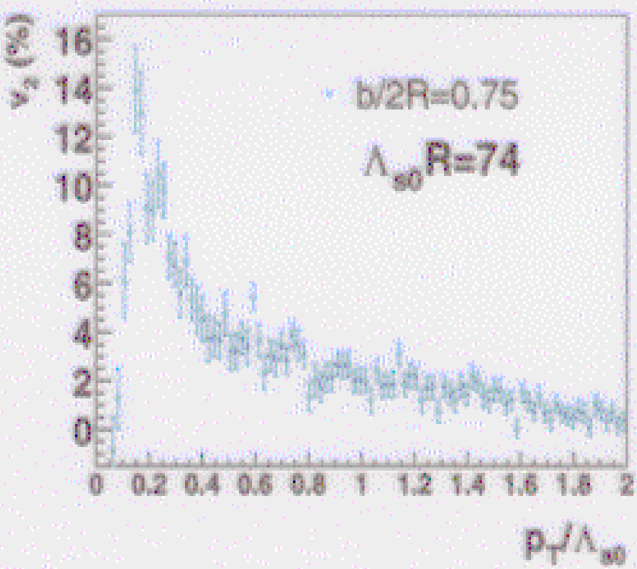
- Two Strategies for Phenomenology:

  a) CGC + Hadronization –''ignore'' final state interactions

  b) Ideal Hydro+ Mini–Jets–''ignore'' initial state interactions

The azimuthally anisotropic flow of Colored Glass

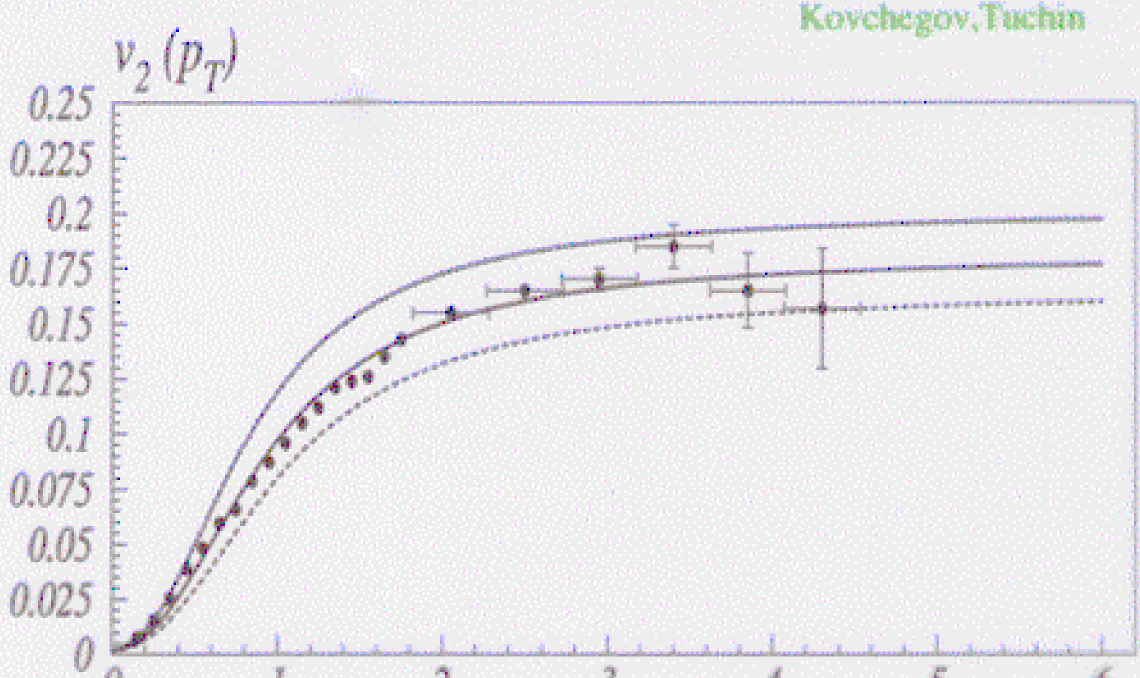




Krasnitz,Nara,Venugopalan

Non–flow Correlations?

Kovchegov,Tuchin



RHIC Phenomenology: Current Status

- The CGC Scenario:

[illegible]

A) CGC + Parton–Hadron Duality :

- Explains Global Features–Energy, Rapidity, Centrality Dependence

  (also, see BMSS)

- Right p_T dependence at moderate p_T (~2–9 GeV)

  (via Geometrical Scaling)

  [illegible]

- Problems: v_2 !

  [illegible]

  — Possible way out–"Non–Flow Correlations"

  [illegible]

B) CGC + Hydro:

[illegible]

—Combines nice features of both approaches–phenomenology needs further study

The Ideal Hydro + Energy Loss Scenario

(A QGP Scenario...)

Braun-Munzinger, Stachel, Redlich, Xu

- **Does very well with low p_T spectra /particle ratios**

[illegible]

- **v_2, for charged hadrons, flavor**

  (requires very early thermalization times of 0.6 fm...)

- **HB–T poses problems–opaque, short–lived source?**

Gyulassy, Levai, Vitev, Wang

- **Energy Loss explains suppression qualitatively–recent quantitative results**

  need to be better understood.

- **v_2 at high p_T is problematic**

- **Independent Fragmentation fails for Baryons (see Kretzer talk tomorrow)**

  –recent work on "recombination" models.

- Open Conceptual and Phenomenological issues in all approaches

- Implement $Q_s \equiv Q_s(x, r_\perp, b)$

with determined from HERA data and nuclear

geometry a la Kowalski–Teaney

–no free parameters

Hirano,Krasnitz,Nara,R.V.

- Extend formalism to 3+1—D: essential for

–transport studies of thermalization

–implementation of Wilsonian RG.

Jeon/ Krasnitz,Nara, R.V