Quenched large $p_\perp$ π^0 spectra

and the transport coefficient $\hat{q}$

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mainly based on more recent work together with **Dominique Schiff**

"Deciphering the properties of the medium produced in heavy ion collisions at RHIC by a pQCD analysis of quenched large $p_\perp$ π^0 spectra"

[JHEP 0609 (2006) 059 (arXiv: hep-ph/0605183)]

and with **Al H. Mueller and Dominique Schiff**

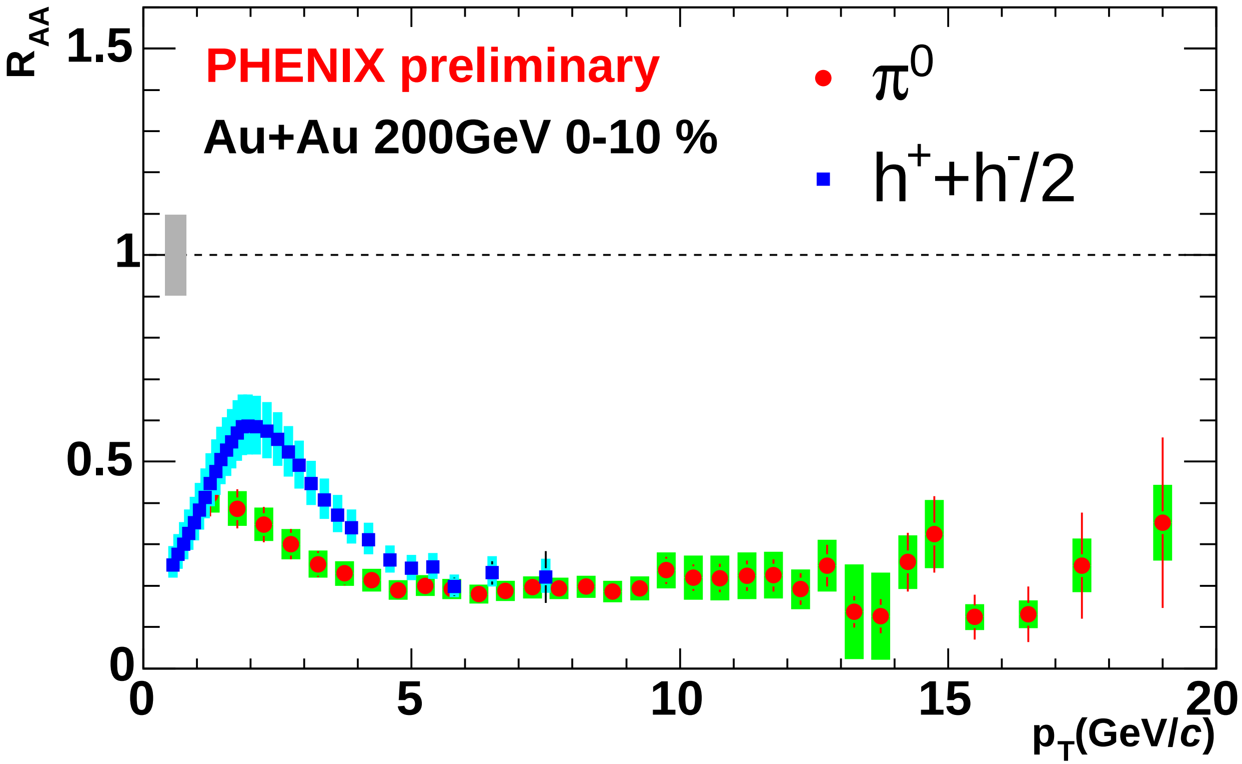
"How does transverse (hydrodynamic) flow affect jet-broadening and jet-quenching ?"

[Phys. Lett. B 649 (2007) 147 (arXiv: nucl-th/0612068)]

NOT A REVIEW !

RHIC discovery: suppression of large $p_\perp$ hadrons
in high energy $Au - Au$ collsions

FINAL STATE EFFECT



[from M. Tannenbaum - review (2007)]

Is it true that

"Theory ties strings round jet suppression ?"

[CERN Courier, May 2007]

"Jet quenching is one of the most dramatic pieces for the *strong-coupling* nature of the quark-gluon matter produced at RHIC"

[see: H. Liu, K. Rajagopal and U. A. Wiedemann (2006 - 2007)] my answer is: **NO !**

at least in the pQCD framework of

medium-induced gluon radiation

CONTENT

- reminder:

explanation in perturbative leading order QCD framework: medium-induced radiative energy loss by gluon radiation

(following BDMPS, Zakharov, Wiedemann, Salgado, .. . approach)

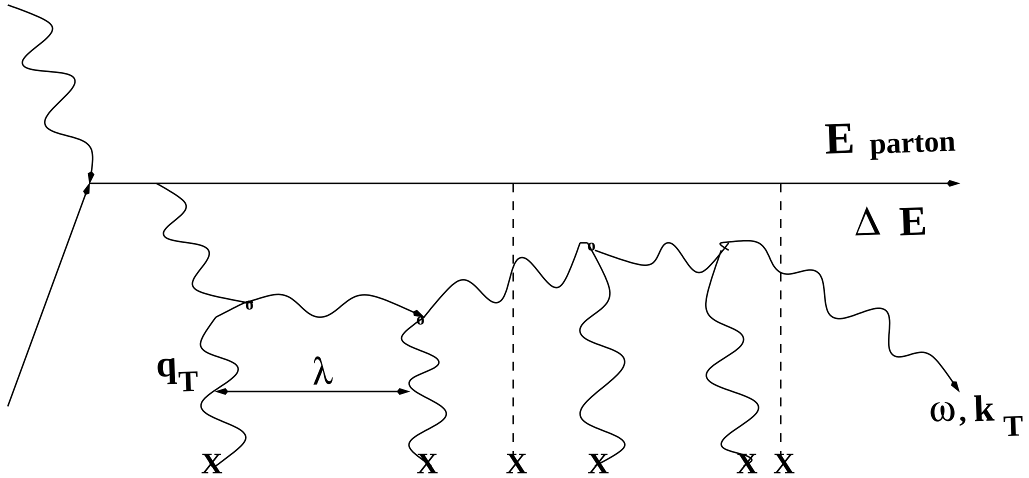
- basic quantity: transport coefficient $\hat{q}$
- discussion of hard scale necessary to

resolve the deconfined medium

- Bethe-Heitler versus LPM radiation:
  central role of cut-off ω_{BH}
- quenching factor and (Poissonian) energy distribution of primary gluons
- nuclear geometry and parton (transverse) path length L
- results including radial flow for central collisions
- summary/conclusion

ZIG-ZAG gluon in finite size L medium

$E_{\rm parton} \to \infty$, loss ΔE



typical dominant gluon radiation diagram

(non-abelian gluon properties at LO pQCD)

requirements: mean free path $\lambda_g = \lambda > \frac{1}{\mu}$ range of screened gluon interaction, μ.. screening mass

and $L >> \lambda$, i.e. many scatterings

medium-induced GLUON RADIATION

- N_{coh}: number of scattering centers that fall inside the formation length t_{coh} of the emitted gluon and which act coherently as a single scatterer, with $1 << N_{coh} \simeq t_{coh}/\lambda < L/\lambda$
- coherence/formation time: $t_{coh} \simeq \omega/k_\perp^2 \simeq N_{coh}\,\lambda$
- random walk due to multiple scatterings: accumulated $k_\perp^2 \simeq N_{coh}\mu^2 >> \mu^2$
- transport coefficient:

$$\hat{q} \simeq \mu^2/\lambda \simeq \rho \int d^2q_\perp q_\perp^2 d\sigma/d^2q_\perp$$

ρ ... density of medium, σ ... gluon-medium (nucleus, partons) interaction

COMBINING:

$$t_{coh} \simeq \sqrt{\frac{\omega}{\hat{q}}}\,,\;\; k_\perp^2 \simeq \sqrt{\omega\hat{q}}\,,\;\; N_{coh} \simeq \sqrt{\frac{\omega}{\mu^2\lambda}} \to \omega >> \omega_{BH} = \mu^2\lambda$$

$$characteristic\ energy:\ t_{coh} \simeq L \;\to\; \omega_c \simeq \hat{q}L^2$$

characteristic MOMENTUM SCALE

$$\hat{q}L \simeq \sqrt{\omega_c \hat{q}} > k_\perp^2 \simeq N_{coh}\mu^2 >> \mu^2$$

compare with *hard/saturation scale:*

$$\hat{q}L \simeq Q_s^2 \simeq \hat{q}A^{1/3}$$

multiple scattering environment in nucleus:
small distance physics
(large scale $k_\perp \simeq 1/x_\perp$.. small size of the system)
$\rightarrow$ deep inelastic/hard process $\rightarrow$ pQCD description by
only one large scale in $\alpha_s(k_\perp)$!

therefore $\hat{q}$ is calculated in pQCD framework

temperature T is NOT the characteristic scale !

(soft) BDMPS medium-induced GLUON SPECTRUM

radiation spectrum per unit path length

characteristic behaviour:

- totally incoherent Bethe-Heitler regime:
  $\omega \leq \omega_{BH} = \lambda \mu^2$

$$\frac{\omega dI}{d\omega dz} \propto \frac{\alpha_s}{\pi} \frac{1}{\lambda} \quad \rightarrow \quad \frac{\omega dI}{d\omega} \propto \frac{\alpha_s}{\pi} \frac{L}{\lambda}$$

- coherent LPM regime:
  $\lambda < t_{coh} < L, \;\; N_{coh} >> 1, \;\; \omega > \omega_{BH}$

$$\frac{\omega dI}{d\omega dz} \propto \frac{\alpha_s}{\pi} \frac{1}{t_{coh}} \quad \rightarrow \quad \frac{\omega dI}{d\omega} \propto \frac{\alpha_s}{\pi} \sqrt{\frac{\omega_c}{\omega}}$$

CUT-OFFS

kinematic cut: $k_\perp \leq \omega$ and $k_\perp^2 \simeq \sqrt{2\hat{q}\omega} \;\rightarrow$

[C. A. Salgado and U. A. Wiedemann]

$$effective\ IR\ cut-off\ :\quad \omega \geq \hat{\omega} \simeq (2\hat{q})^{\frac{1}{3}} \simeq \omega_c(2/R)^{\frac{2}{3}}$$

additional parameter: $R = \omega_c L$

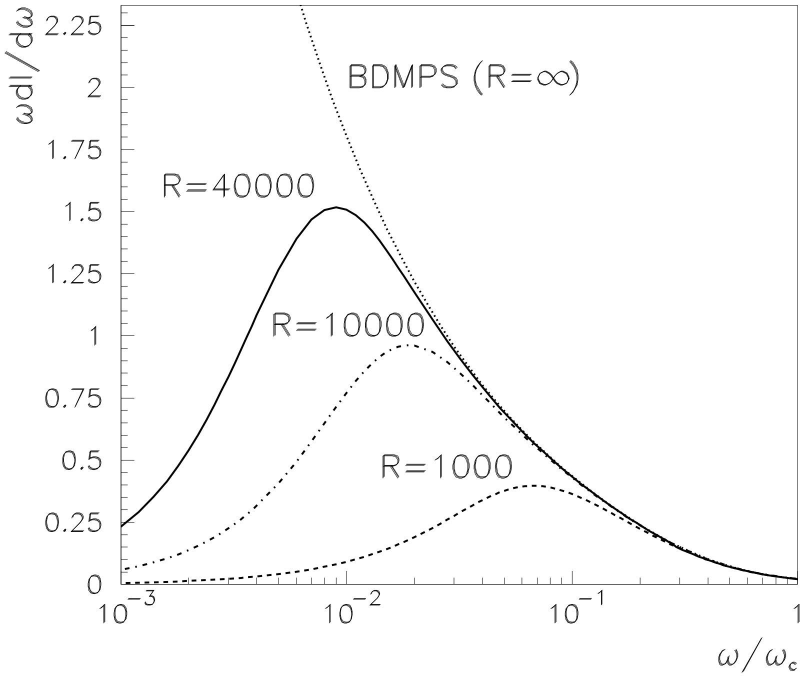
IMPORTANT:

$$\hat{\omega}/\omega_{BH} \simeq \frac{2^{1/3}}{(\lambda\mu)^{4/3}} \;<<\; 1$$

ω_{BH} energy is the proper IR limit of the medium-induced LPM gluon emission spectrum

SPECTRUM [C. A. Salgado and U. A. Wiedemann]

The medium-induced gluon energy distribution $\omega \frac{dI}{d\omega}$ in the multiple soft scattering approximation for different values of the kinematic constraint $R = \omega_c\, L$



typical values for RHIC large $p_\perp$ pions:

$$R \simeq 1000,\ \hat{\omega}/\omega_c \simeq 1.5\ 10^{-2},\ \omega_{BH}/\omega_c \geq 3\ 10^{-2}$$

QUENCHING EFFECT

$$R_{AA} = Q(p_\perp) \; = \; \int d\epsilon \, D(\epsilon) \; \left(\frac{d\sigma^{\mathrm{vacuum}}(p_\perp + \epsilon)/dp_\perp^2}{d\sigma^{\mathrm{vacuum}}(p_\perp)/dp_\perp^2} \right)$$

approximation: power behaved vacuum spectrum

$$Q(p_\perp) \simeq \int_0^\infty d\epsilon \, D(\epsilon) \, \exp\left\{ -\frac{n\epsilon}{p_\perp} \right\}, \; n \simeq 12$$

crucial assumption ("trigger bias"): probability $D(\epsilon)$ for emitting the energy ϵ into the medium by a Poissonian energy distribution by *primary gluons* in terms of the inclusive medium induced spectrum

$$D(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \, \frac{dI(\omega_i)}{d\omega} \right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i \right) \; \exp\left[- \int d\omega \frac{dI}{d\omega} \right]$$

RESULT

suppression is dominated by the NO-emission probability (i.e. by virtual contribution)

$$p_0(p_\perp) = Q_{min}(p_\perp) \; = \; \exp\left[-N(\omega_{BH})\right]$$

with number of gluons $N(\omega) \equiv \int_\omega^\infty d\omega' \frac{dI(\omega')}{d\omega'}$

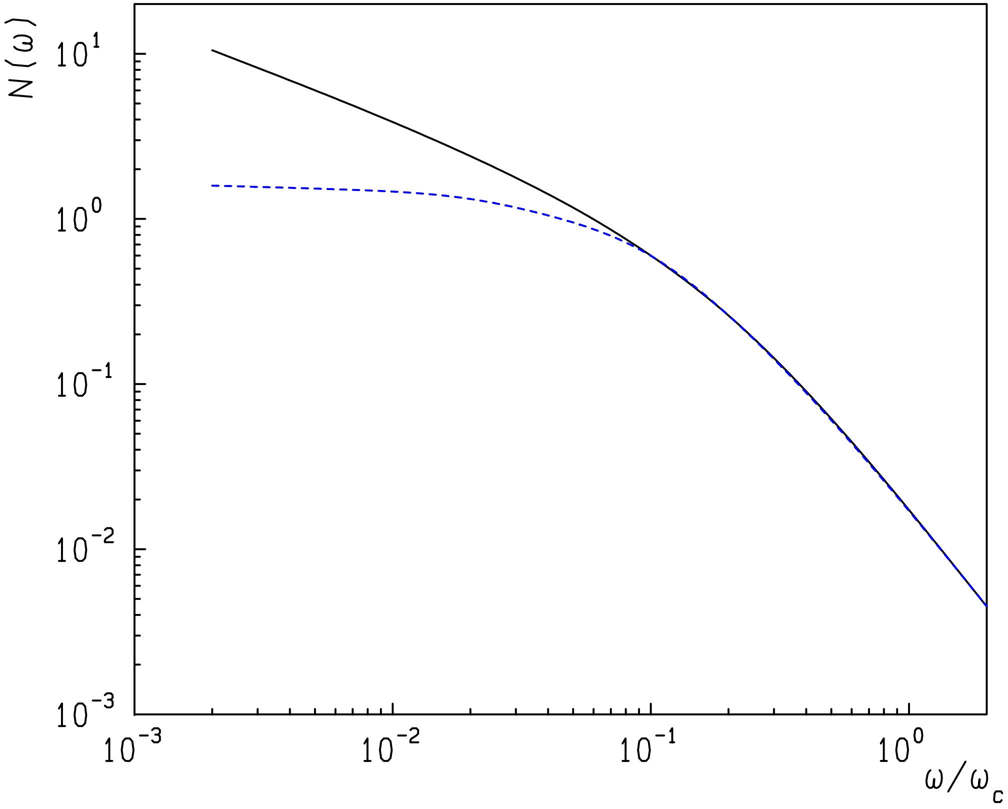
- p_0 increases when replacing $\hat{\omega}$ by ω_{BH}
- ω_{BH} reduces amount of real emission due to hardness of gluons $\omega \geq \omega_{BH}$:

$$Q(p_\perp) \leq Q_{max}(p_\perp) \; = \; \exp\left\{-N(\omega_{BH})\left[1 - \exp\left(-\frac{n\omega_{BH}}{p_\perp}\right)\right]\right\}$$

e.g. $Q_{real}/Q(p_\perp = 15\ GeV) < 20\ \%$

- $\rightarrow$ properties of the trigger jet ($\Delta\phi$ distribution)

GLUON MULTIPLICITY



$N(\omega)$ for $R = \infty$ (solid curve) and $R = 1000$ (dashed curve)

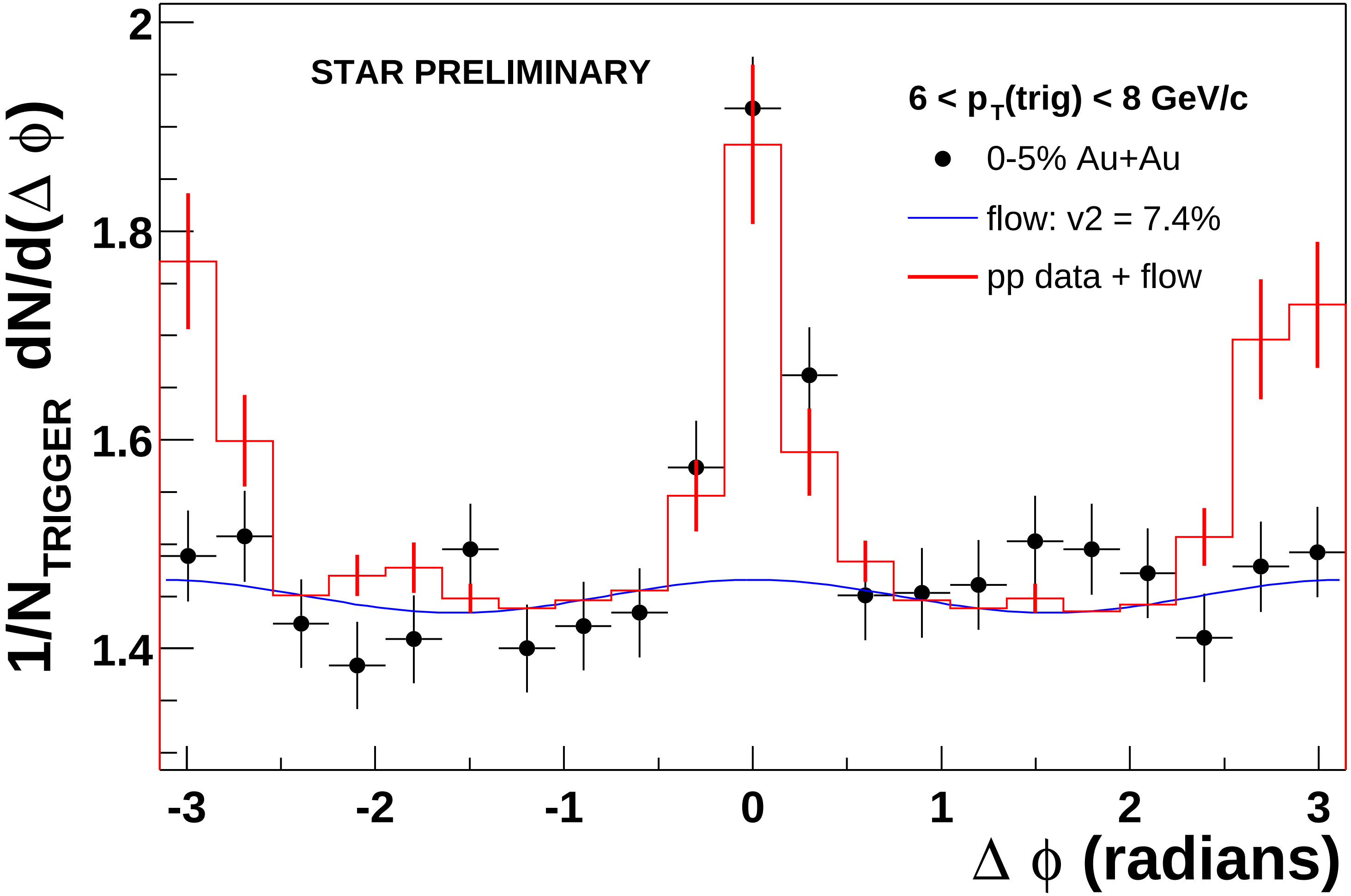
[C. A. Salgado and U. A.Wiedemann] for $\alpha_s = 1/2$.

NOTE: for $\omega_{BH}/\omega_c \simeq (3-4)\ 10^{-2}$

the $R-$dependence is not significant - e.g. important for $L-$ distribution

$\Delta\phi$ DISTRIBUTION

Note: same-side jet is not modified in $Au - Au$ vs. $p - p$ collisions



[from M. Tannenbaum - review (2007)]

COMPARISON

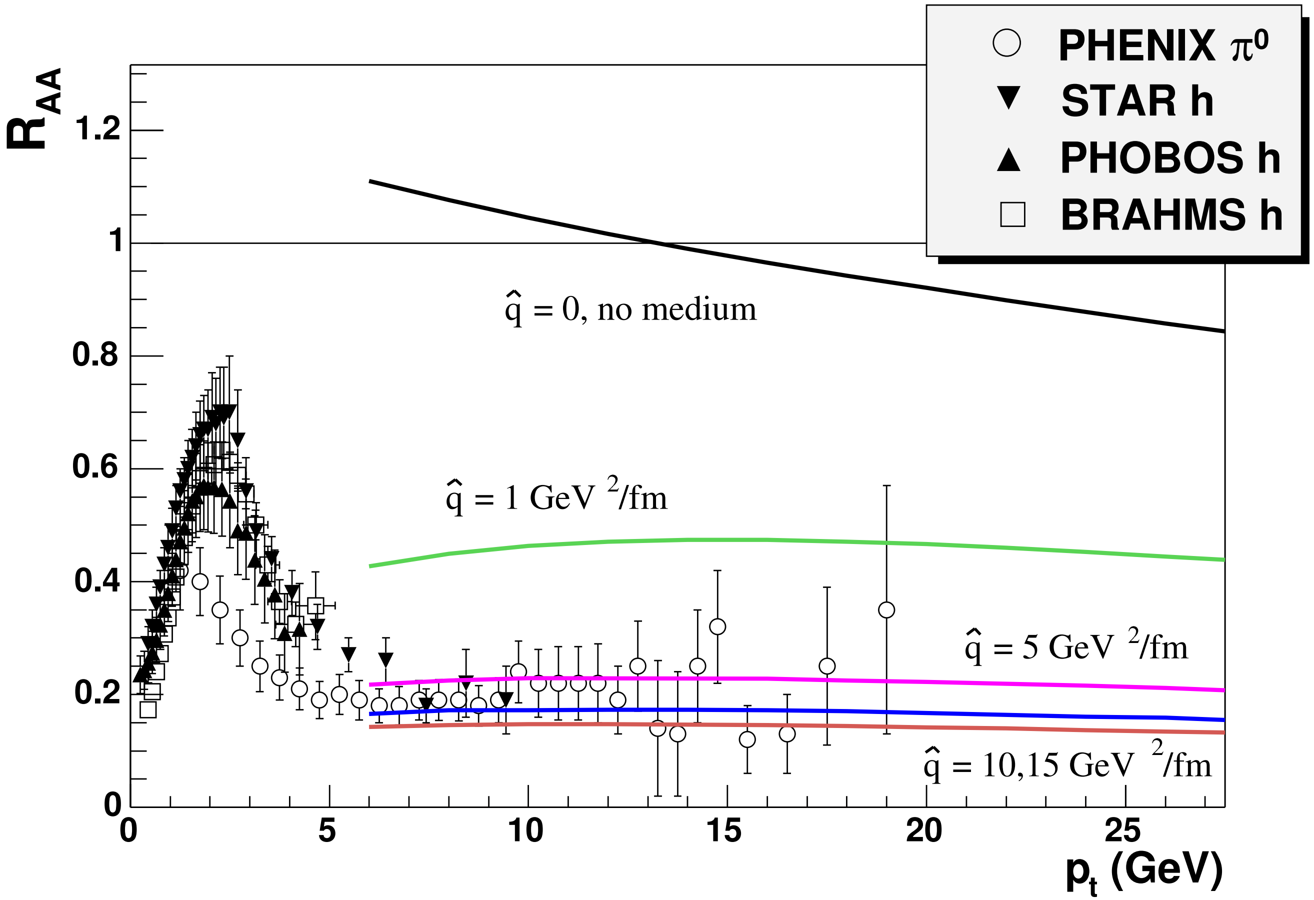
Suppression for $R = 1000$ (dominating quark jet)

$\hat{q} = 2\,GeV^2/fm,\;\; L = 3.5\,fm,$ $\hat{\omega} = 0.9\,GeV,\;\; \omega_{BH} = 1.6\,GeV$			$\hat{q} = 10\,GeV^2/fm,\;\; L = 2\,fm,$ $\hat{\omega} = 1.6\,GeV,\;\; \omega_{BH} = 3.5\,GeV$	
	$Q_{\rm min} = p_0$	$Q(p_\perp)$	$Q_{\rm min} = p_0$	$Q(p_\perp)$
$p_\perp = 10.4\,GeV$	0.291	0.321	0.322	0.329
$p_\perp = 20.4\,GeV$	0.291	0.359	0.322	0.345

[private communication by C. A. Salgado and N. Armesto]

NOTE: almost the same suppression - still compatible with the data, but *uncomfortably* **large** $\omega_{BH} \simeq 4\,GeV$ for jets of $O(20\,GeV)$, when $\hat{q}$ is **large** !

THEREFORE PREFER: $\hat{q} < 3\,GeV^2/fm,\;\; L > 3\,fm$



analysis by K. J. Eskola et al. (2005) - without ω_{BH} cut-off !

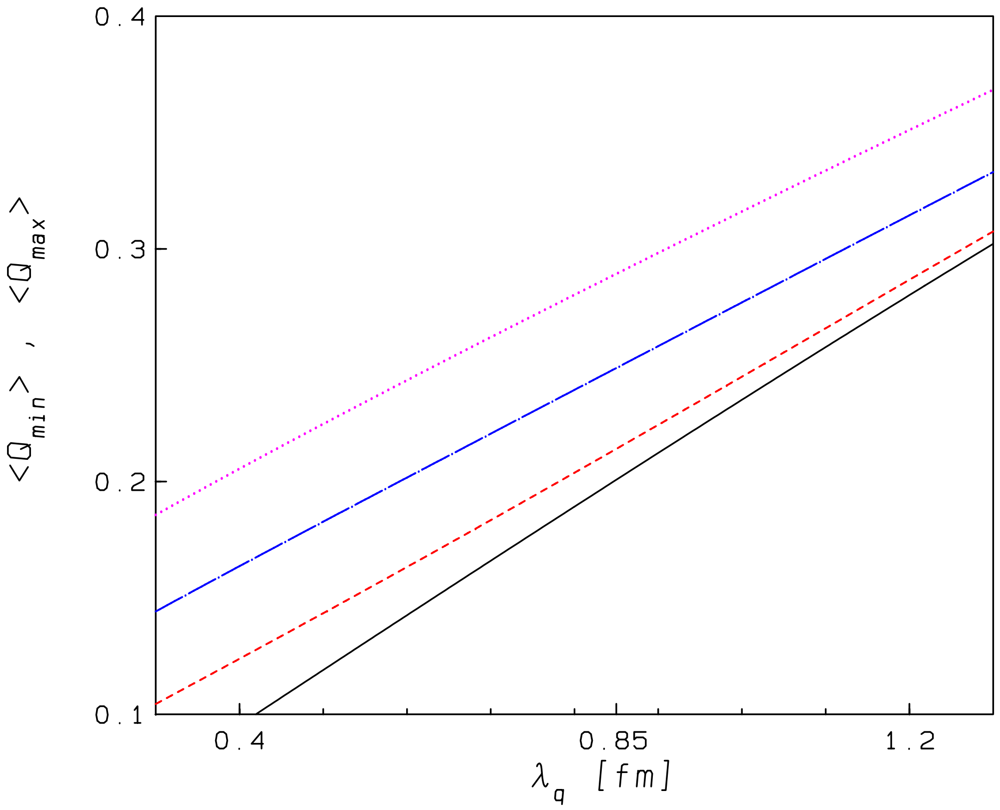
[taken from C. A. Salgado (2007)]

MAIN RESULT: average $\hat{q} = \; 5 ... 15 \;\; GeV^2/fm$

PATH LENGTH L

$$< Q >= Q_{min}|_{<geometry>} \quad : \; Q_{min}(\omega_{BH}/\omega_c) = Q_{min}(L/\lambda_q)$$

geometry : $L = L_{geom}(\vec{s})$, $\vec{s}$... position of jet production in transverse plane



$$< L > \simeq 3.5 \; \lambda_q \qquad \lambda_q \simeq 1 \; fm$$

$\langle Q_{min} \rangle$ (solid curve) and $\langle Q_{max} \rangle$ [screening mass μ: 0.65 (dotted), 0.8 (dashed-dotted) and $1.1 \; GeV$ (dashed curve)]

THERMAL GLUONIC MEDIUM at $T = 400\ MeV$

$$\hat{q} \simeq \frac{8\zeta(3)}{\pi}\alpha_s^2 N_c^2\, T^3 \simeq 2.2\ GeV^2/fm$$

($\alpha_s = \frac{1}{2}, N_c = 3$)

[compare: $\hat{q}|_{SYM} = 26.69\sqrt{\alpha_{SYM} N_c}\, T^3 \simeq 9.5\ GeV^2/fm$ (Hong Liu et al. (2006))]

screening mass $\mu \simeq 1\ GeV$, gluon mean free path $\lambda_g \simeq 0.45\ fm$, energy density $\epsilon \simeq 17\ GeV/fm^3$

leading to

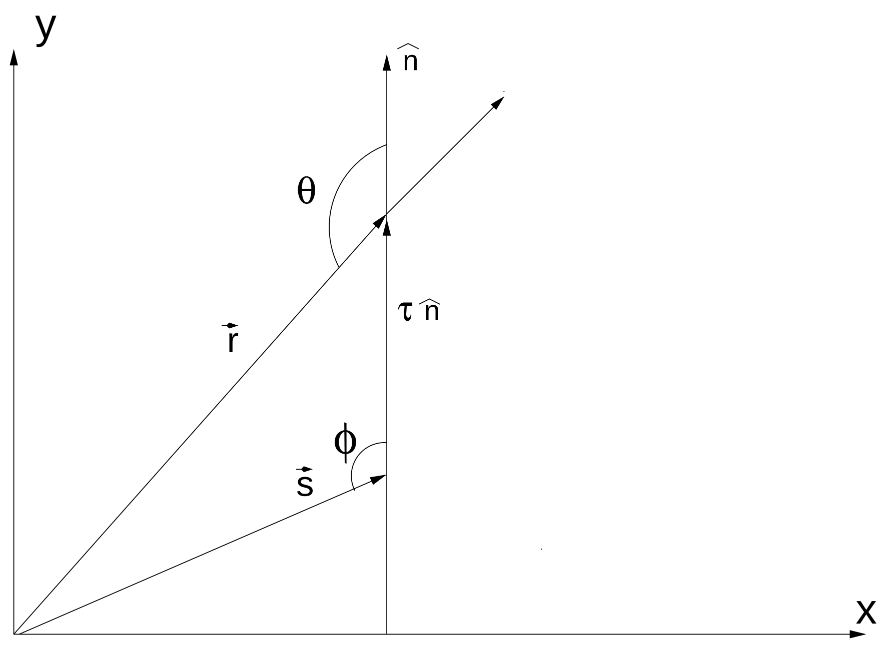
$\omega_{BH} \simeq 1.6\ GeV$, average path length < L >$\simeq 6\ \lambda_g \simeq 3\ fm$ and suppression $Q(p_\perp \simeq 10\ GeV) \simeq 0.32$

remark: $Q_{absorption}\ Q_{BH} \geq 0.7$ at $p_\perp = 10\ GeV$, i.e. neglect altogether contribution of the Bethe-Heitler and the absorption process [S. Turbide et al. (2005)]

FLOW: KINEMATICS in the TRANSVERSE PLANE

jet direction $\hat{n}$

transverse flow velocity $\vec{v} \parallel \vec{r}$

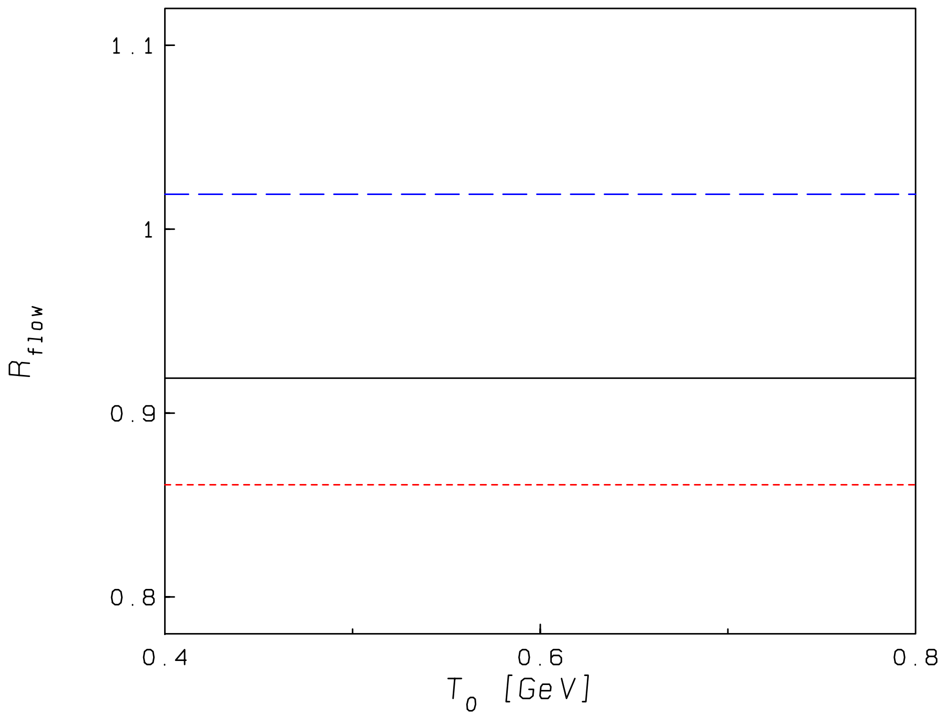


moving medium (by Lorentz boost): $\vec{v}$ from (ideal/viscous) hydro

$$\hat{q}|_{flow} = \hat{q}_0\ \gamma(v)\ (1 + v\cos\theta), \quad \gamma(v) = 1/\sqrt{1 - v^2}$$

[also H. Liu, K. Rajagopal and U. A. Wiedemann (2007)]

JET-BROADENING



small effects: $R_{flow} = (\Delta p_\perp^2)_{Bj+flow}/(\Delta p_\perp^2)_{Bj} \simeq 0.9$ (initial temperature T_0)

$$(\Delta p_\perp^2)_{Bj+flow} = \frac{1}{\pi R_A^2} \int d^2 s \int d\tau\ \hat{q}|_{flow}[T(\vec{s},\ \tau),\ \vec{v}(\vec{s},\tau)]$$

estimate effect of transverse flow by assuming scaling law:

$$\omega \frac{dI}{d\omega} \;=\; \tilde{I}(\omega/\omega_c)$$

with

$$R_{\omega_c} = (\omega_c)_{Bj+flow}/(\omega_c)_{Bj} \simeq 0.85$$ .. small effect

$$(\omega_c)_{Bj+flow} = \frac{1}{\pi R_A^2} \int d^2s \int d\tau\, \tau\, \hat{q}|_{flow}[T(\vec{s},\, \tau),\, \vec{v}(\vec{s},\tau)]$$

WARNING: NO SCALING in GENERAL

- $\hat{q}$ appears at different times [BDMS (1998)] !

$$\omega \frac{dI}{d\omega} \sim \int dt_1 \int dt_2\; \hat{q}(t_1)\; \hat{q}(t_2) \ldots\ldots$$

has to be evaluated in case of **large** effects due to flow

SUMMARY/CONCLUSIONS

- energy loss by gluon radiation in dense environment is a hard process, and therefore described in pQCD [so far only in LO]
- by one reasonably large scale: a perturbatively interacting system (QGP ?) is resolved - you see what you resolve
- in contrast to measurements of e.g. v_2 at low $p_\perp$: resolving sQGP ?
- no real need for extensive numerical work
- but carefull analysis of IR cut-offs: validity of LPM versus Bethe-Heitler spectrum (ω_{BH})
- affects the path length L distribution
- enhances dominance of NO gluon emission probability (same side jet ?)
- transport coefficient is determined by pQCD - at RHIC $\hat{q} \leq 3\ GeV^2/fm$, together with typical average path length of $L \geq 3\ fm$ - guarantees many scatterings: $L >> \lambda_g$
- large value of $\hat{q}$ is not compatible with pQCD in a hot medium, and NOT with a reasonably "soft" cut-off ω_{BH}

SUMMARY/CONCLUSIONS, cont.

- probability distribution of primary emitted gluons:
  Poissonian distribution ?
  what about two-gluon medium-induced correlations ?
- small influence of radial flow on jet-broadening and jet-quenching
  - beyond longitudinal Bjorken expansion
- equilibration time versus (average) path length:
  fast thermalization or perturbative scenario
  e.g. “bottom-up” time-scale $\tau_{eq} \simeq \ < L > \ \simeq 3\, fm$
  $\longrightarrow$ which dense medium is actually probed by quenching
  when saturated/CGC $\longrightarrow$ thermalized medium ?

EXTRAS

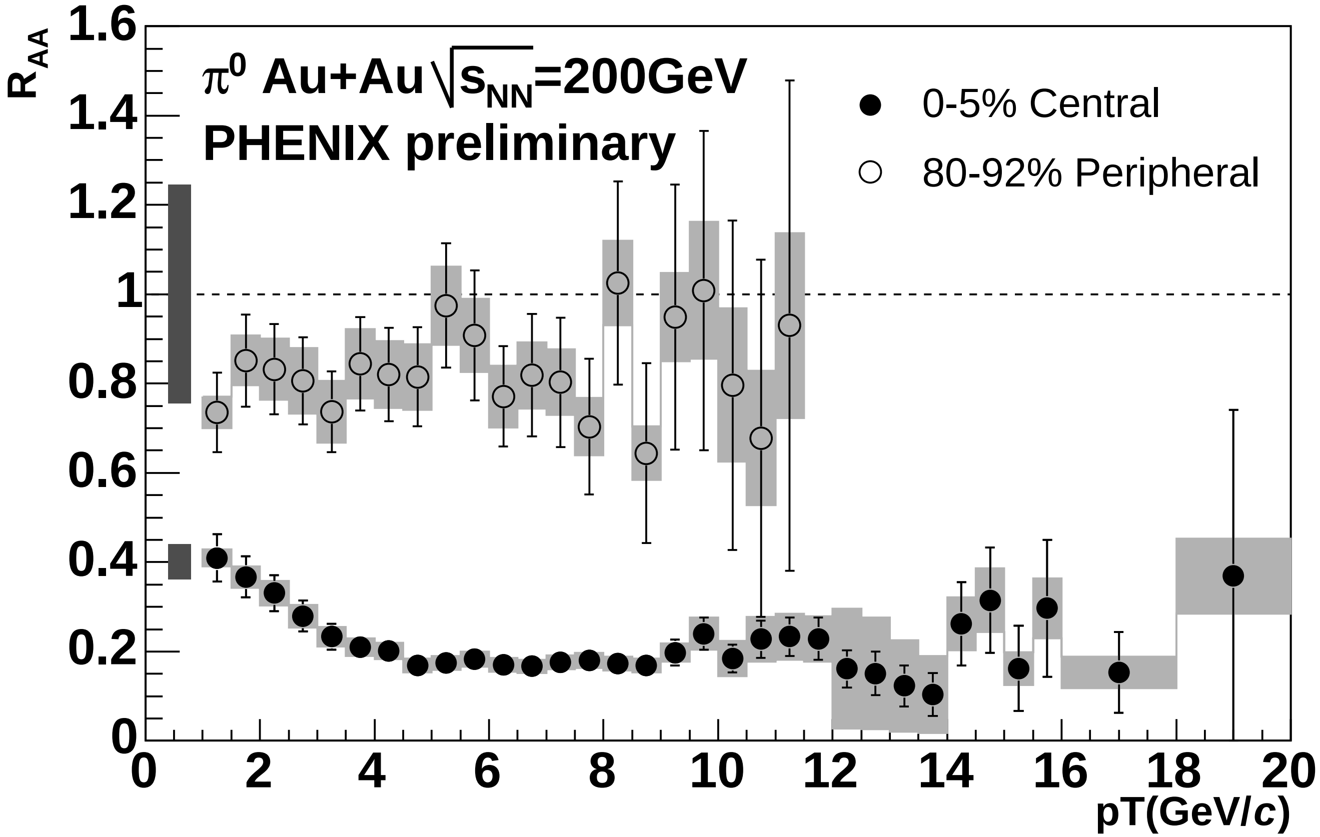
ABSTRACT

The suppression of large $p_\perp$ hadron spectra observed in $Au - Au$ collisions at $\sqrt{s} = 200\ GeV$ at RHIC is dominantly attributed to medium-induced gluon radiation. Information on the nature of the medium is extracted from data: a widely spread suggestion is that it is a sQGP. We question this statement in the context of quenching, and discuss a few points:

- the legitimate assumption of a hard scale for the coupling, allowing the leading order pQCD treatment,
- the multiple scattering BDMPS framework, including coherent LPM emission for gluons above a given energy threshold and the extraction of the transport coefficient $\hat{q}$ characterizing the medium,
- the parton path length L in the medium,
- effects due to longitudinal expansion and transverse flow of the hot medium.

The conclusion is that the resulting (average) $\hat{q}$ should not exceed $3\ GeV^2/fm$,

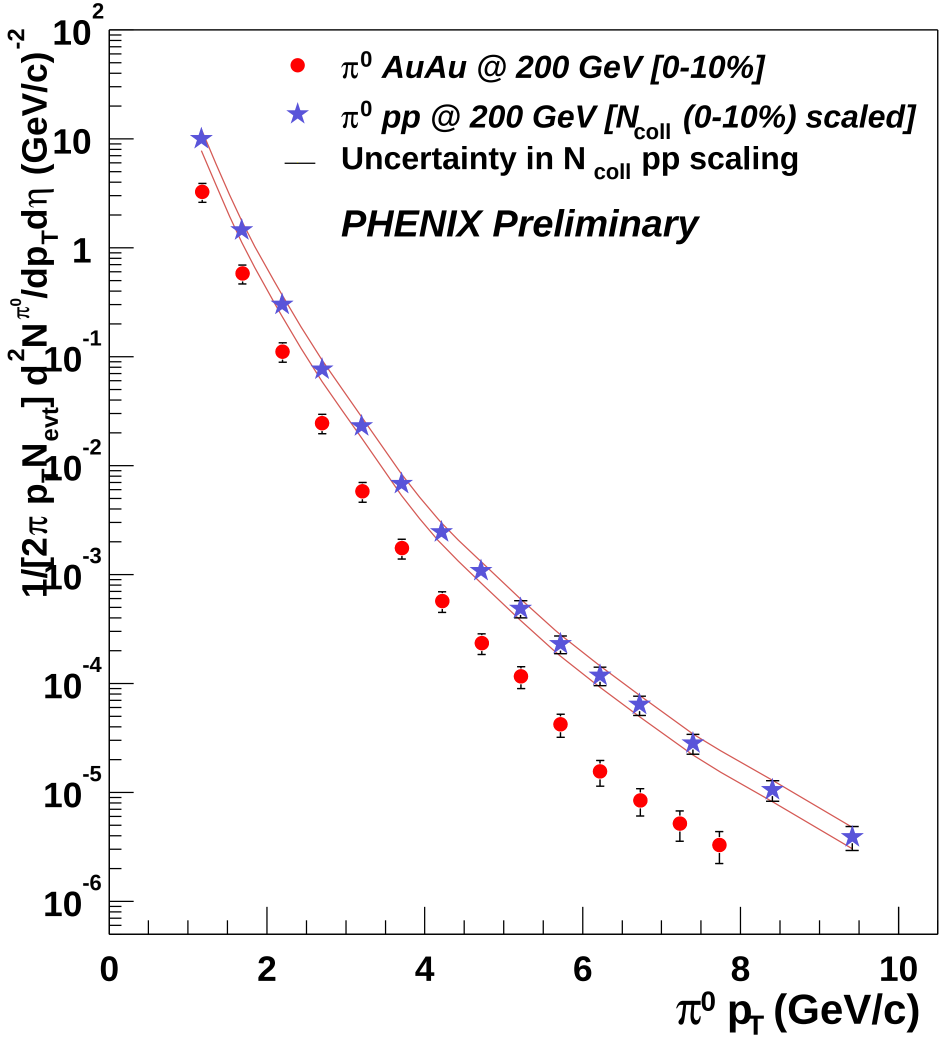
with $L \geq 3\ fm$!



π^0 R_{AA} as a function of $p_\perp$

[from T. Isobe, nucl-ex/0510085]

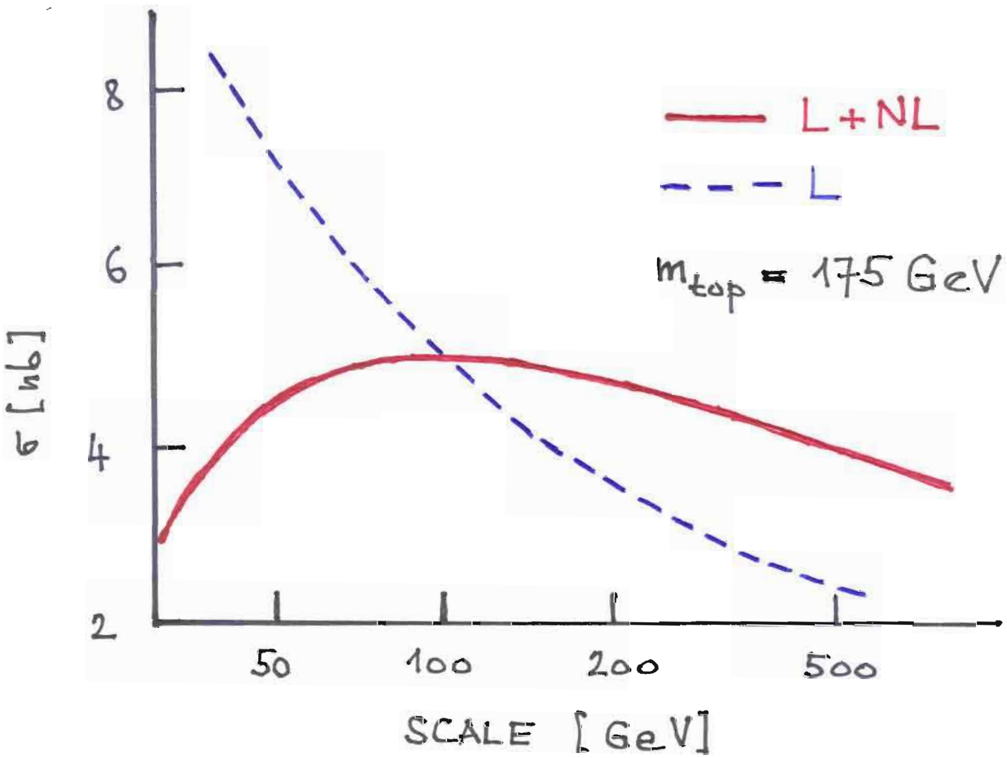
RHIC data



significant jet quenching observed at RHIC energies

top quark production

pQCD example:



scale dependence of the top quark cross section in LO and NLO pQCD:

$LO + NLO \simeq LO$: optimal scale $\mu \simeq m_{top} \simeq 1$/size of the system

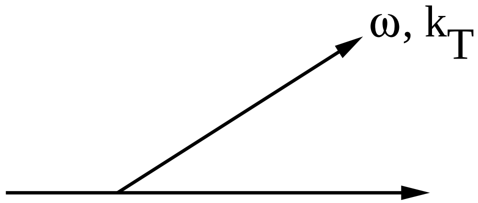
[from R. K. Ellis et al., “QCD and Collider Physics”]

time scales

formation time and coherence length

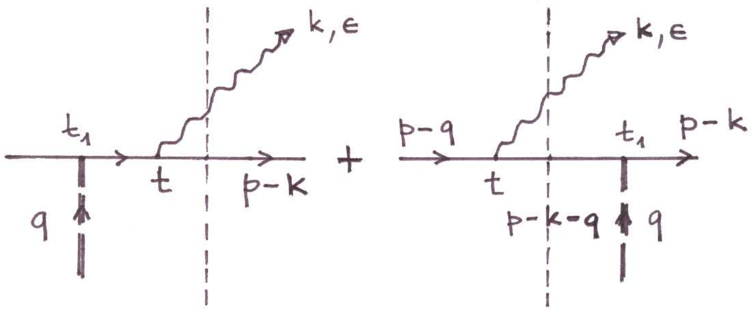
t_{form} : on-shell quark and gluon well separated

$E >> \omega >> k_\perp,\ E \to \infty$



$$t_{form} \sim \frac{E}{\sqrt{p \cdot k}} \frac{1}{\sqrt{p \cdot k}} \sim \frac{2\omega}{k_\perp^2}$$

phase:



$$\exp it[\omega + |\vec{p} - \vec{k}| - |\vec{p}|] = \exp\,[it/t_{form}]$$

$(|\vec{p} - \vec{k}| \simeq E - \omega + k_\perp^2/2\omega)$

multiple interactions

- group of scattering centers act as ONE source of radiation

- defines t_{coh}

$$t_{form} \equiv \underline{t_{coh}} \simeq \frac{\omega}{\langle k_\perp^2 \rangle|_{t_{coh}}} \simeq \frac{\omega}{\mu^2 t_{coh}/\lambda}$$

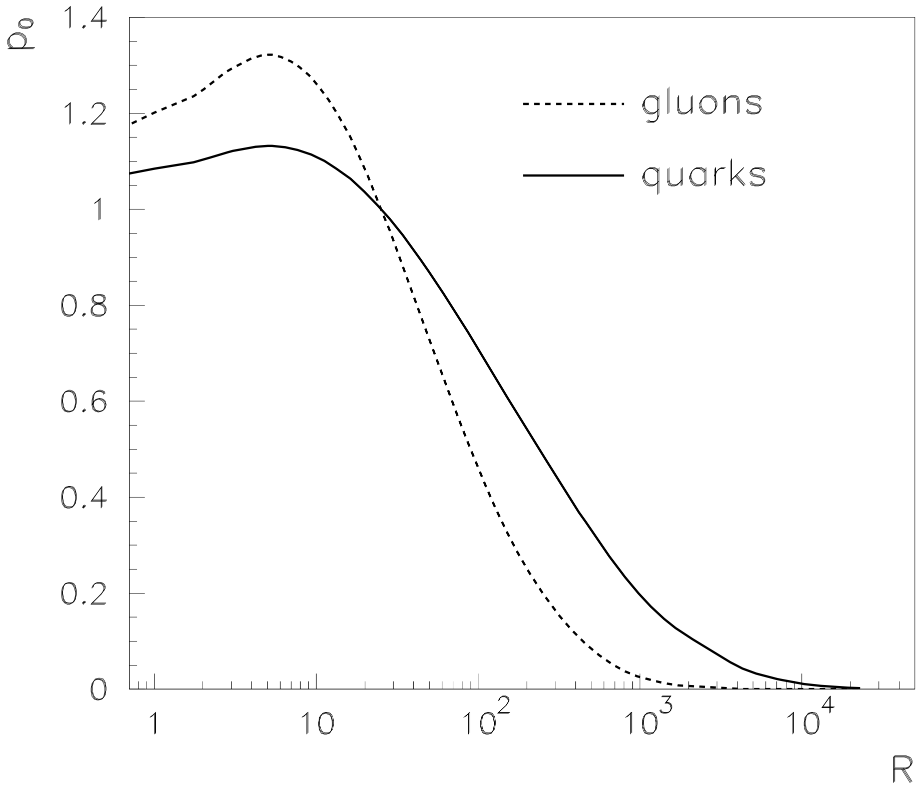
random walk:

$$\langle k_\perp^2 \rangle|_{t_{coh}} \simeq N_{coh}\mu^2 \simeq \frac{t_{coh}}{\lambda}\,\mu^2$$

$\Longrightarrow$

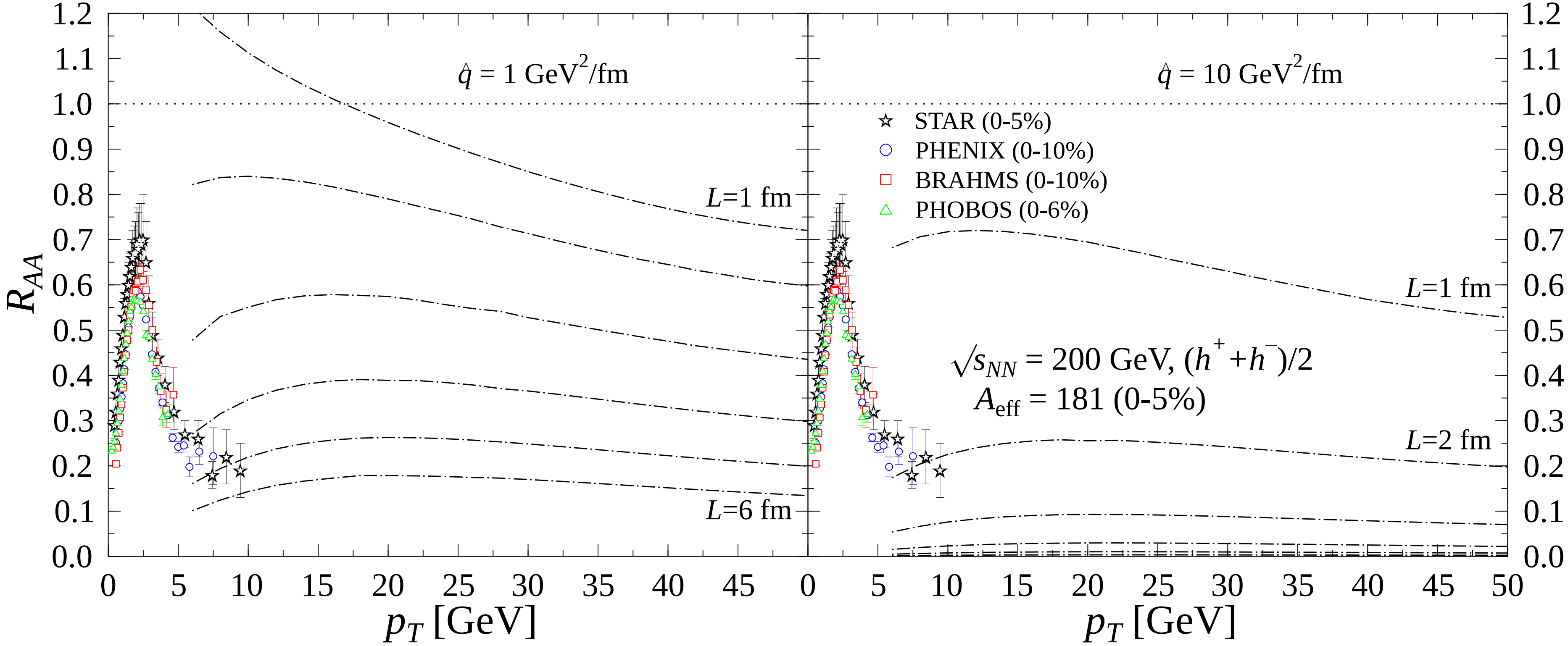
$$t_{coh} \simeq \sqrt{\frac{\lambda\omega}{\mu^2}}, \quad N_{coh} \simeq \sqrt{\frac{\omega}{\lambda\mu^2}}$$

$N_{coh} =$ number of coherent scatterings
$\hat{=}$ scattering centers which participate coherently in the gluon emission with energy ω



Quenching factor $p_0 = Q_{min} = \exp[-N(\omega)]$ for massless quarks and gluons

[C. A. Salgado and U. A.Wiedemann (2003)]

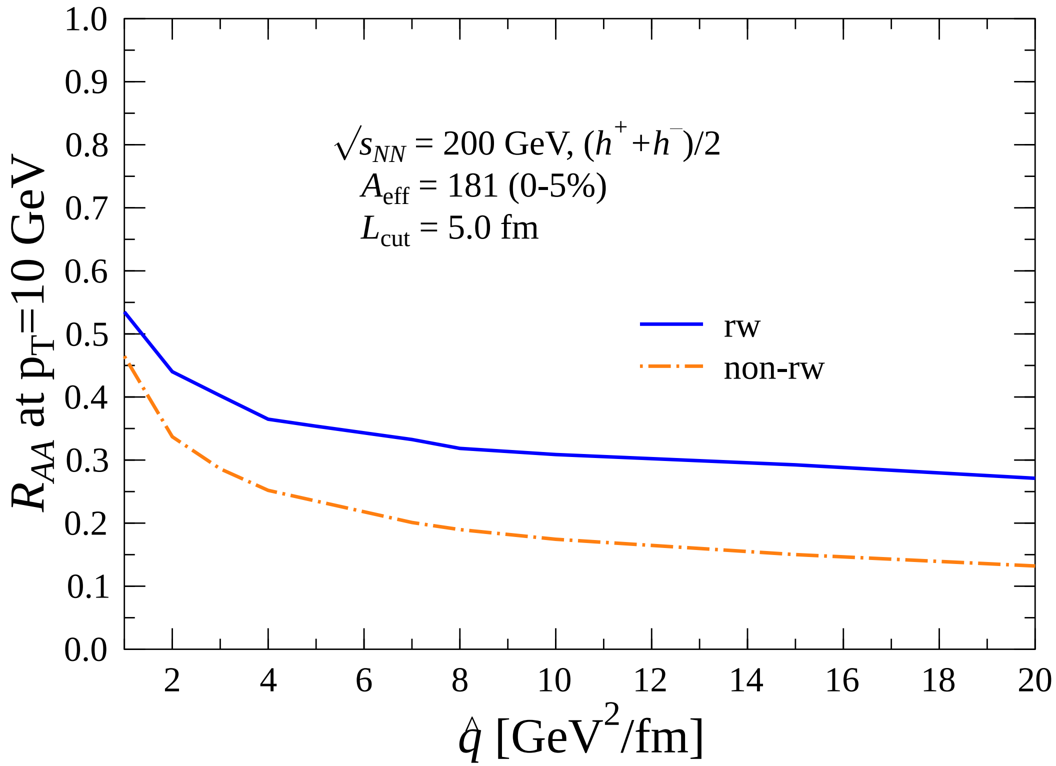


R_{AA} for different in-medium path length L

comment: $< L_{geom} >= 5.2\ fm$ for central for $Au - Au$ collisions

[K. J. Eskola et al. , Nucl. Phys. A747 (2005) 511]

Sensitivity on choice of gluon emission probability:

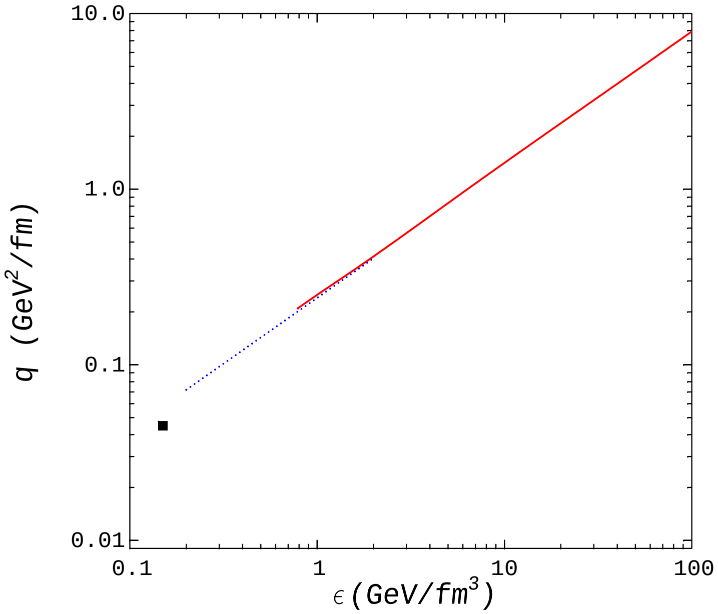


R_{AA} as a function of $\hat{Q}$ for $p_\perp = 10\ GeV$

[K. J. Eskola et al. , Nucl. Phys. A747 (2005) 511] equilibrated media:

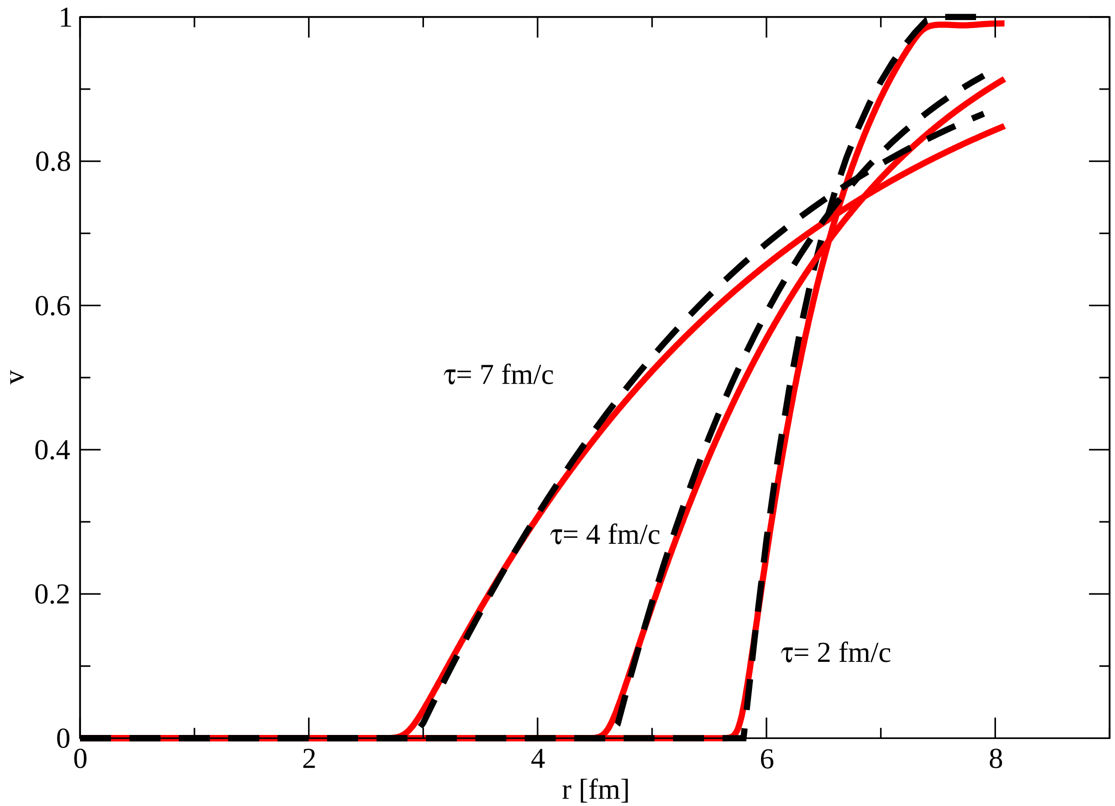
nuclear matter - (massless) pion gas - (ideal) QGP

pQGP : density $\rho(T) \sim T^3 \sim$ energy density $\epsilon^{\frac{3}{4}}$



"smooth" increase of $\hat{q}$ with increasing energy density of the medium, and

$$\hat{q}|_{\text{hot}} \simeq 2\,\epsilon^{\frac{3}{4}} >> \hat{q}|_{\text{nuclear matter}}$$

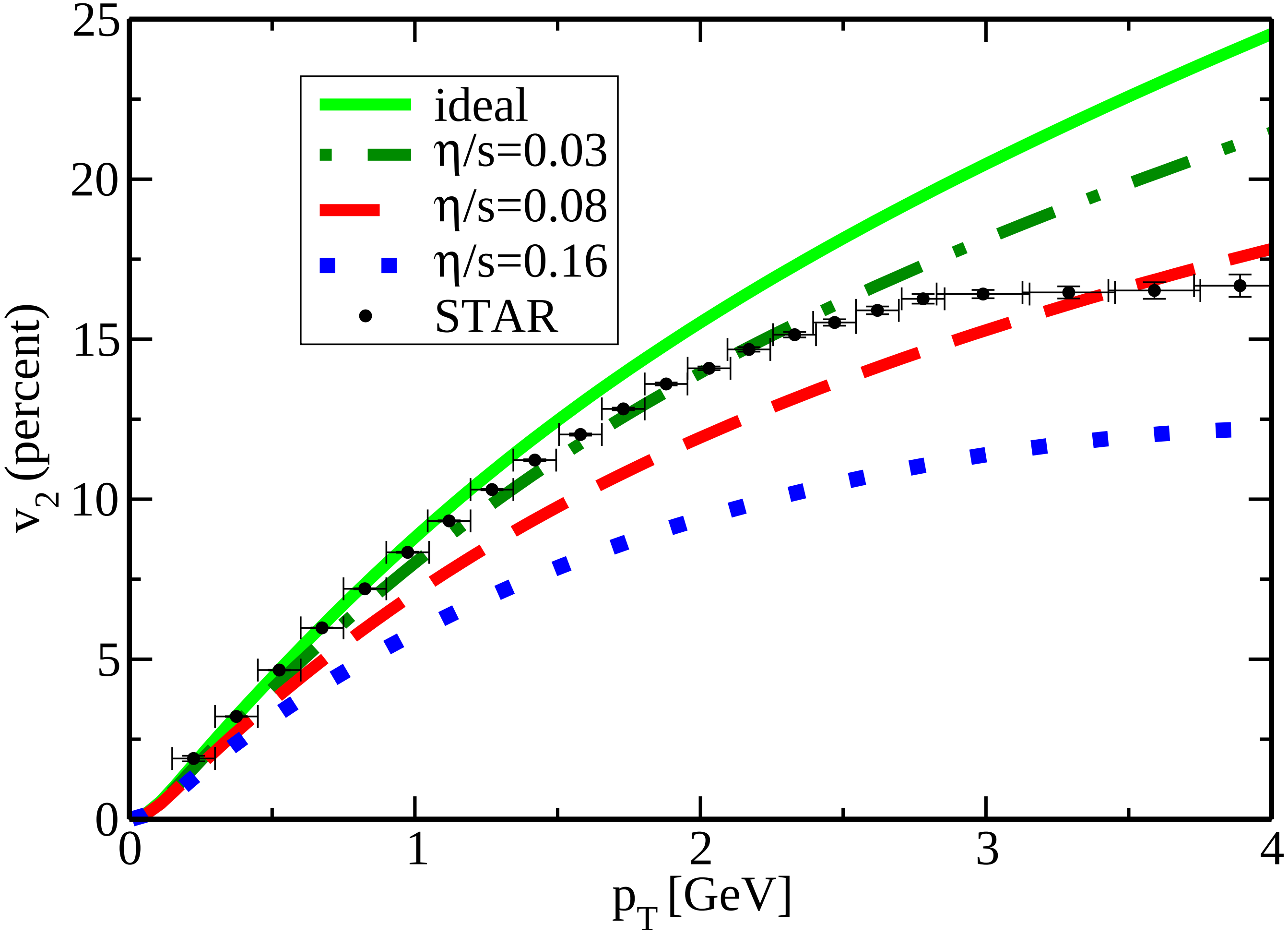


ideal hydro: transverse flow velocity as a function of r and Bjorken's τ

flow has a non-negligible effect only for large enough values of r, where v differs significantly from 0: realized when jet is moving with the flow

[G. Baym et al. (1983); R. Baier and P. Romatschke (2006)]

v_2 DISTRIBUTION



[P. Romatschke and U. Romatschke (2007)]