Elliptic flow fluctuations
in 200 GeV Au+Au collisions

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for the* PHOBOS *collaboration

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Early Time Dynamics

in Heavy Ion Collisions

Montreal, Canada

PHOBOS collaboration

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46 scientists, 8 institutions, 9 PhD students

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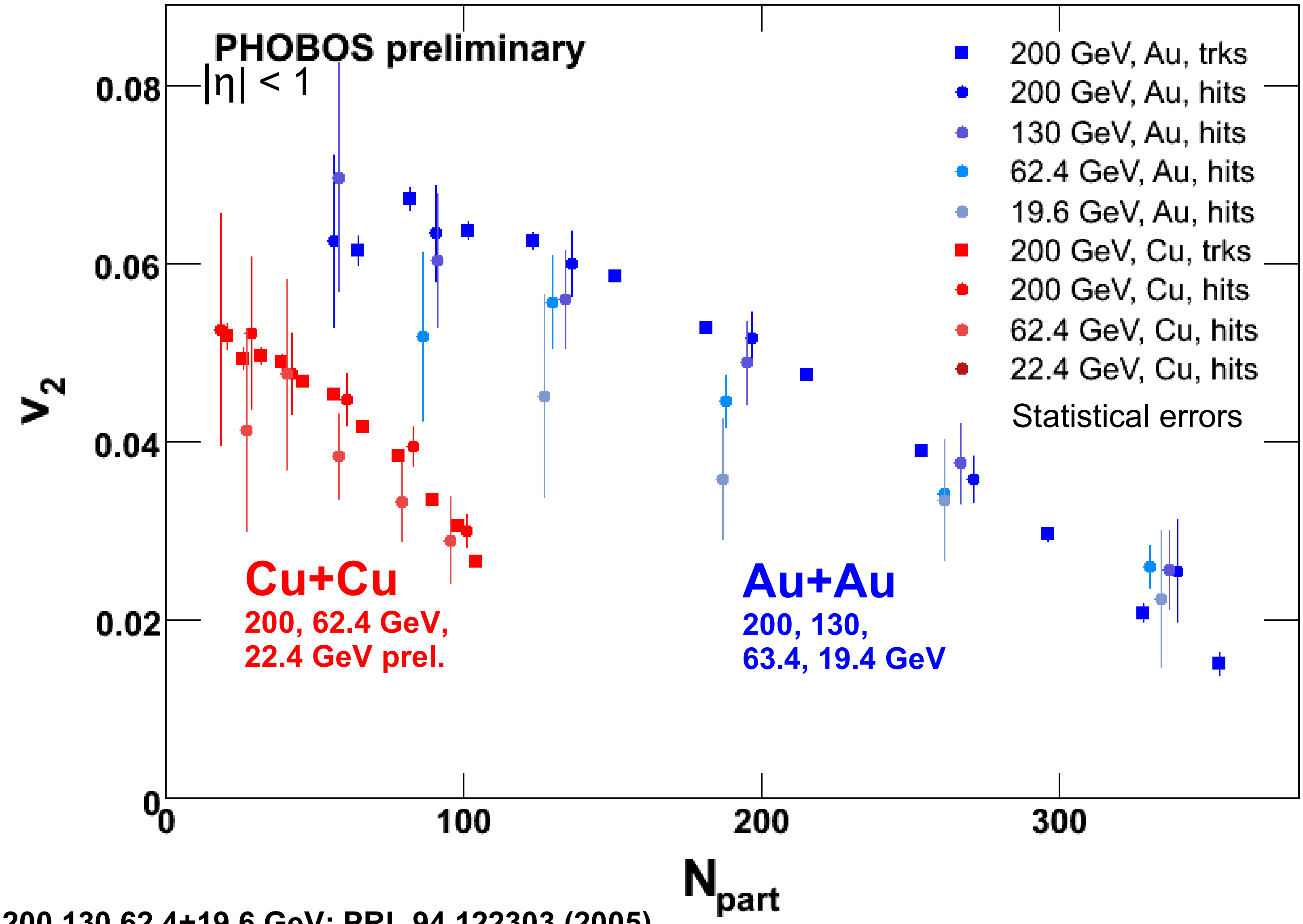
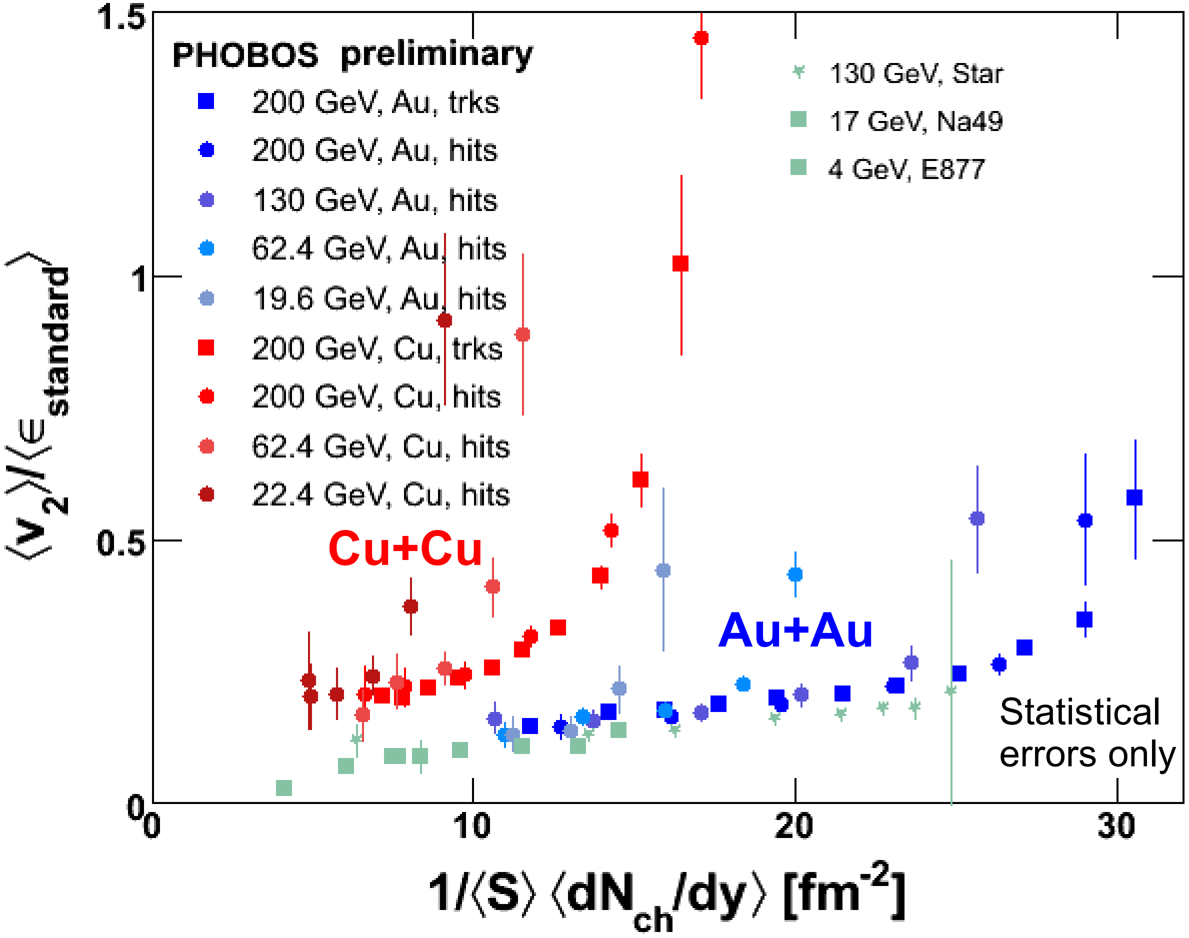
INSTITUTE OF NUCLEAR PHYSICS PAN, KRAKOW
NATIONAL CENTRAL UNIVERSITY, TAIWAN
UNIVERSITY OF MARYLAND
BROOKHAVEN NATIONAL LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
UNIVERSITY OF ILLINOIS AT CHICAGO
UNIVERSITY OF ROCHESTER

Elliptic flow for different species



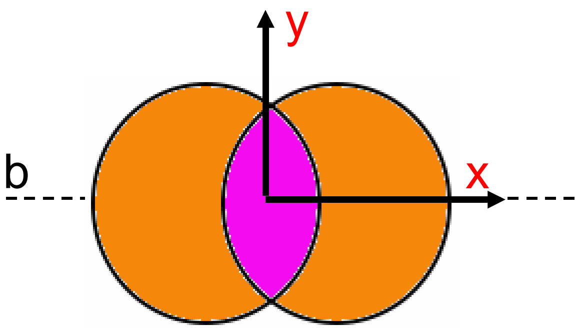
Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (acc. to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

Elliptic flow and Standard Eccentricity



No scaling between Cu+Cu and Au+Au

Standard Eccentricity



$$\epsilon_{standard} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

Fine print:

- ➢ Scale $v_2(\eta)$ to $v_2(y)$ (10% lower)
- ➢ Scale $dN/d\eta$ to dN/dy (15% higher)
- ➢ S is overlap area (MC Glauber)

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
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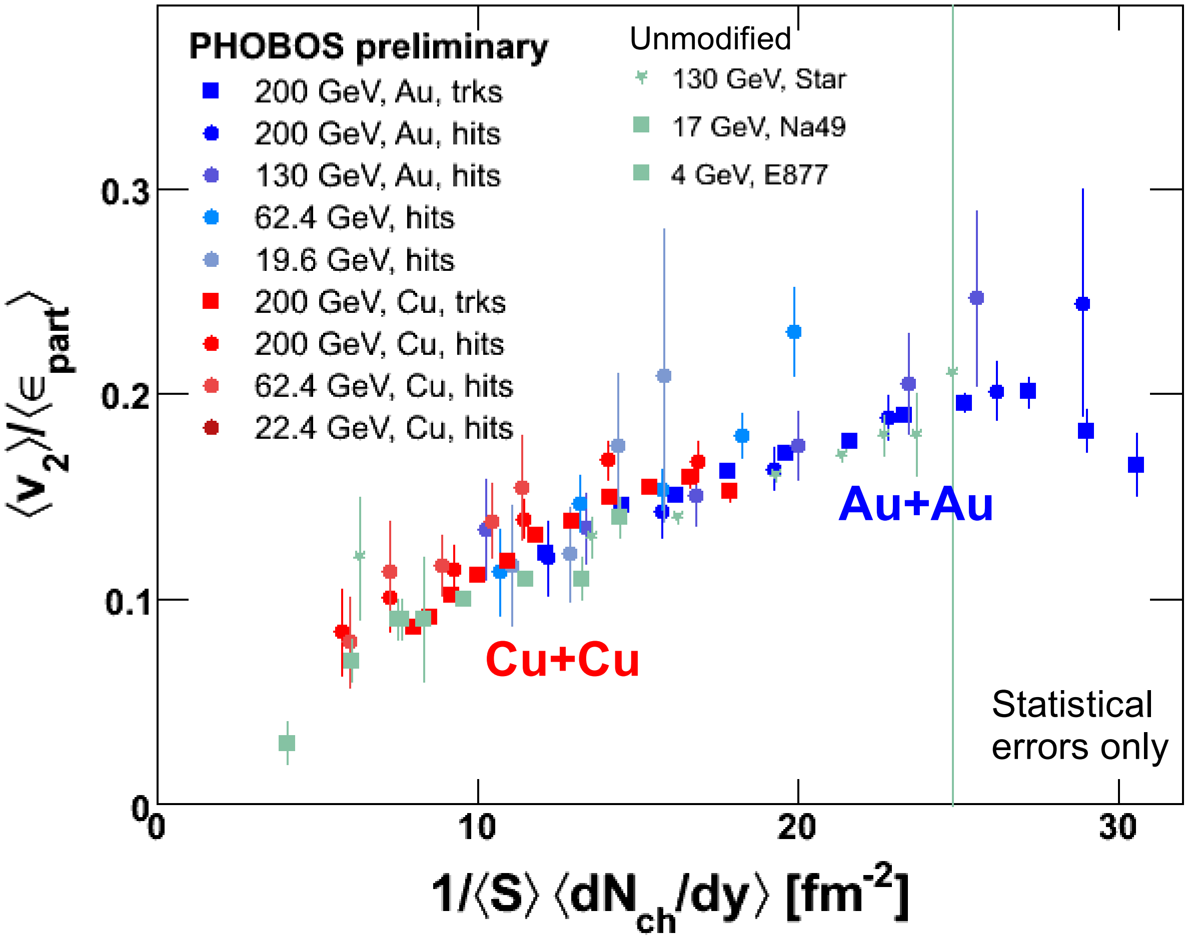
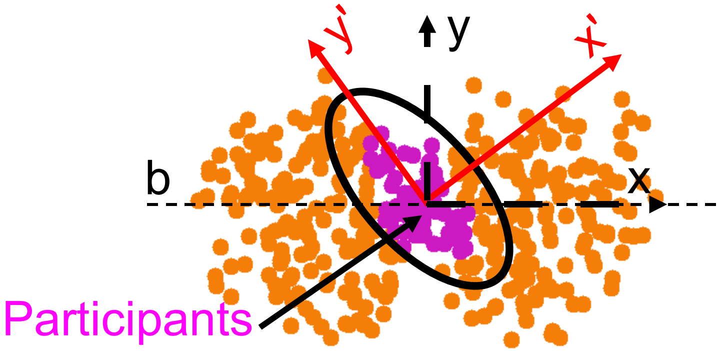
STAR, PRC 66 034904 (2002)
Voloshin, Poskanzer, PLB 474 27 (2000)
Heiselberg, Levy, PRC 59 2716, (1999)

Elliptic flow and participant eccentricity



Scaling between Cu+Cu and Au+Au

Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\boxed{\sigma_{xy}^2}}}{\sigma_y^2 + \sigma_x^2}$$

Fine print:

- Scale $v_2(\eta)$ to $v_2(y)$ (10% lower)
- Scale $dN/d\eta$ to dN/dy (15% higher)
- S is overlap area (MC Glauber)

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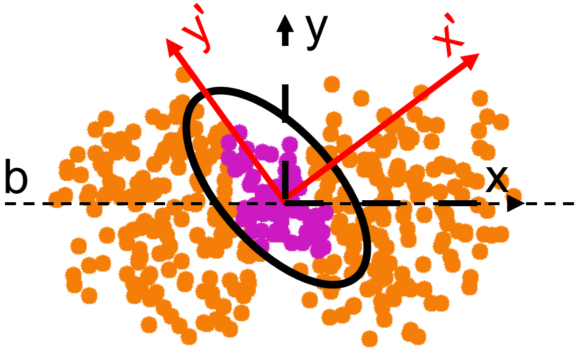
Expected elliptic flow fluctuations



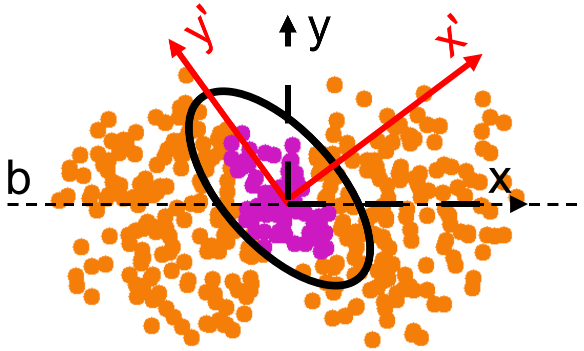
- Participant eccentricity model makes a prediction:
  - Assuming $v_2 \propto \epsilon_{part}$

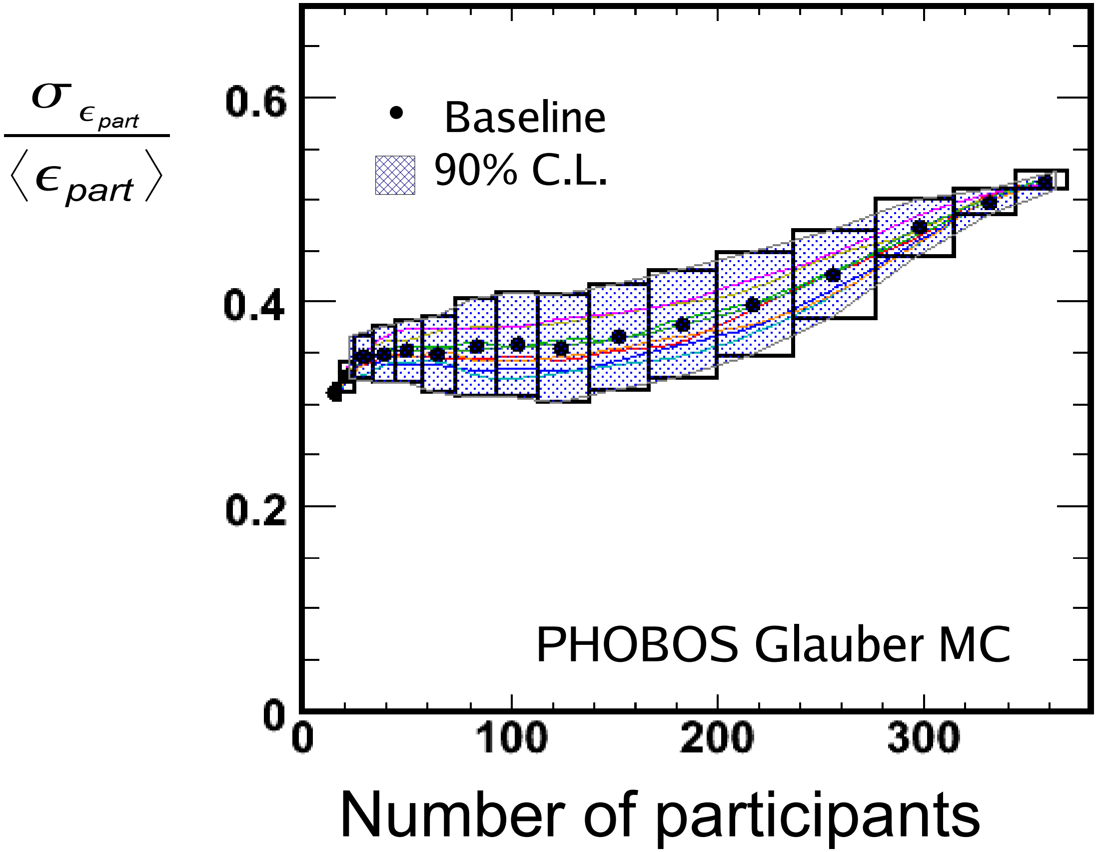
- For fixed impact parameter, if eccentricity fluctuates event-by-event, so should v_2.

$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \propto \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$

Expected elliptic flow fluctuations

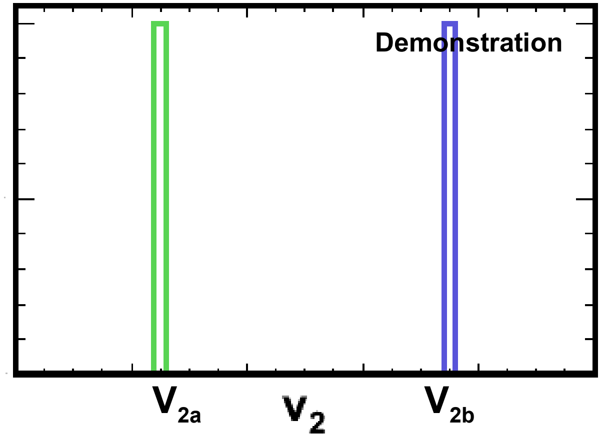
- Participant eccentricity model makes a quantitative prediction:



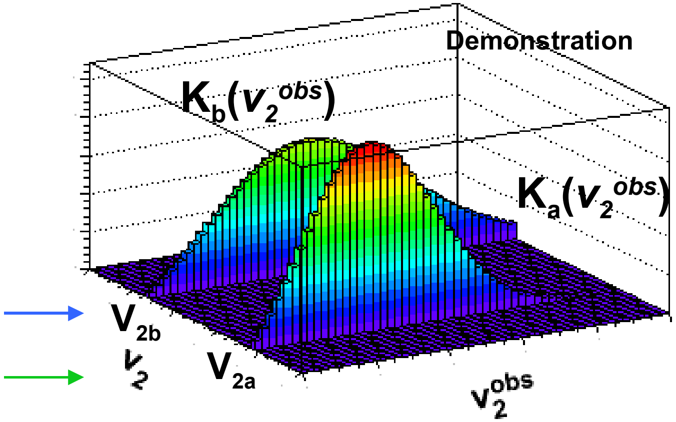


Method overview

2 possible v_2 values

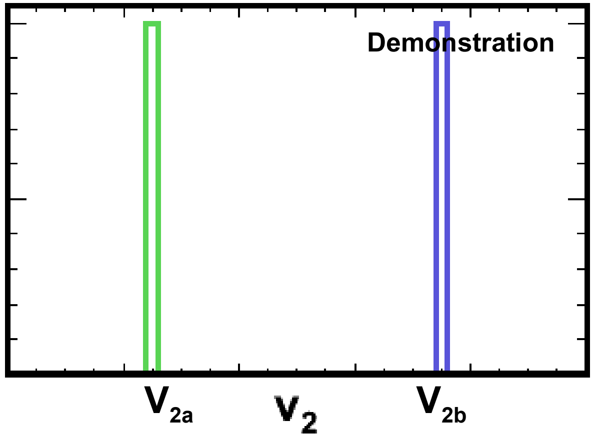


Event by Event measurement



Method overview

2 possible v_2 values



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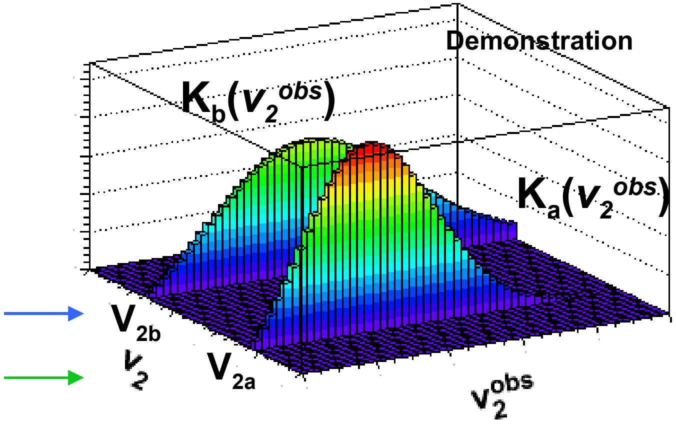
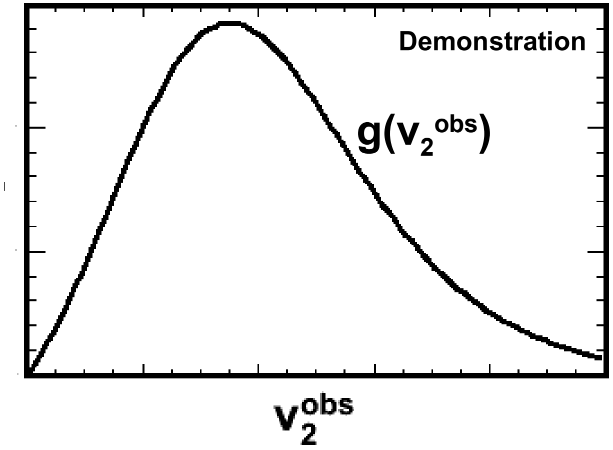


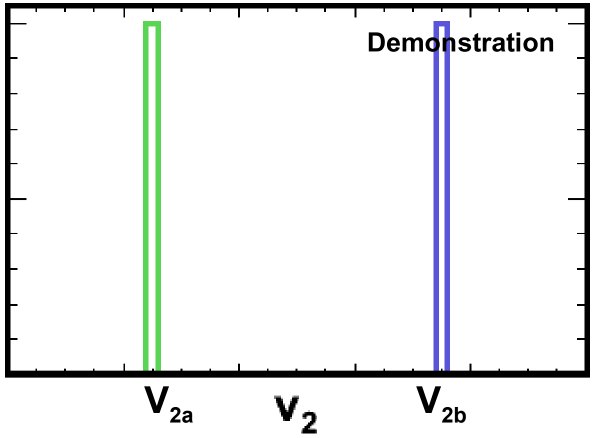
v_2^{obs} distribution in "data"



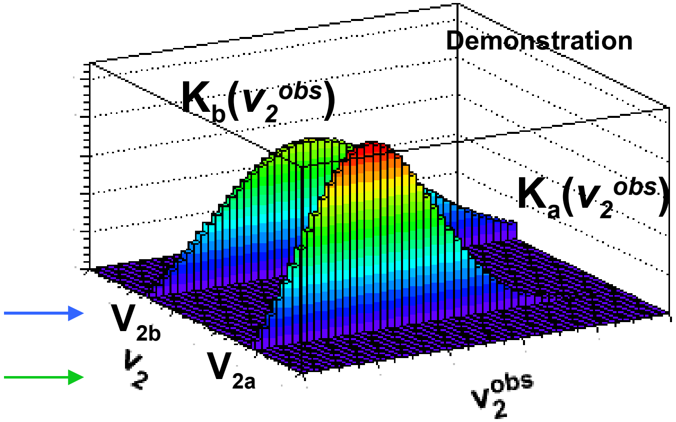
Question: What is the relative abundance of 2 v_2's in "data"?

Method overview

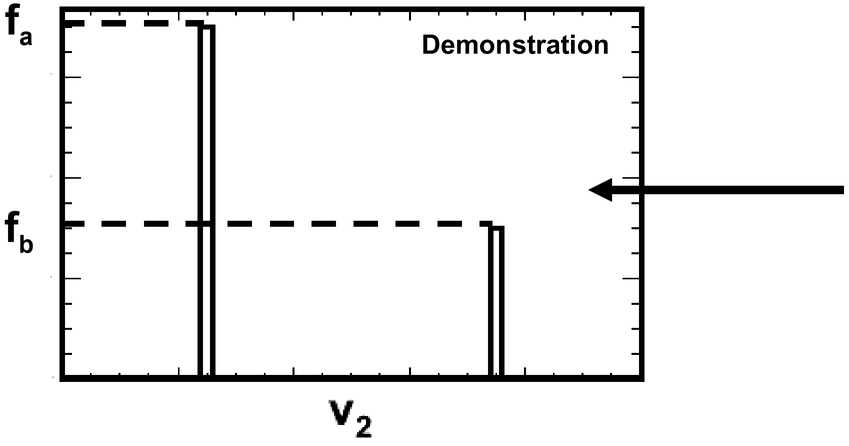
2 possible v_2 values



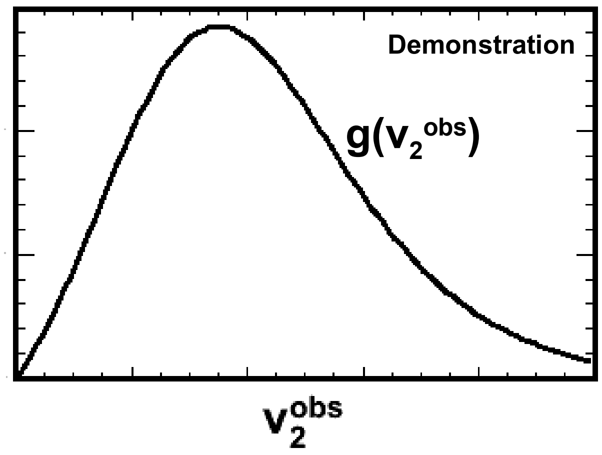
Event by Event measurement



Relative abundance in "data"



v_2^{obs} distribution in "data"



Question: What is the relative abundance of 2 v_2's in "data"?

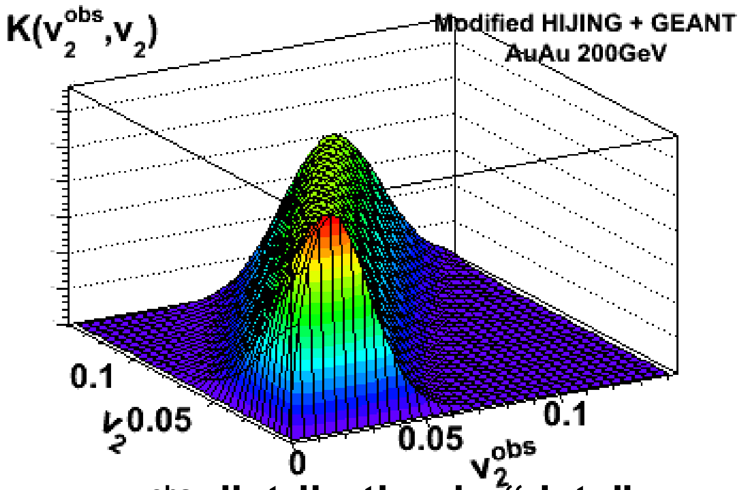
$$g(v_2^{obs}) = f_a K_a(v_2^{obs}) + f_b K_b(v_2^{obs})$$

Method overview

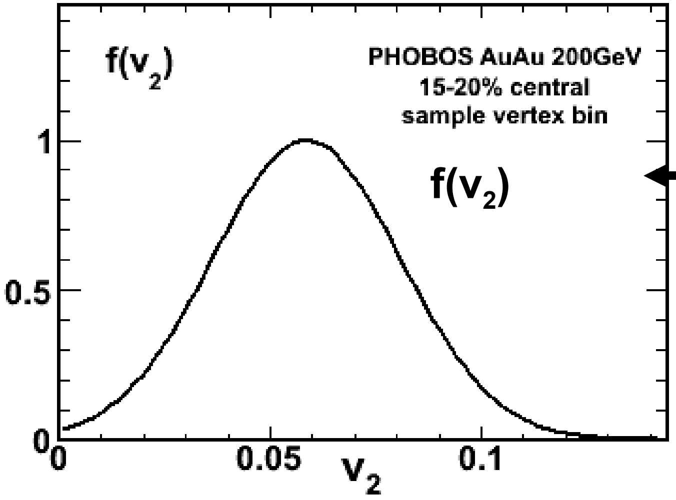
In real life v_2 can take a continuum of values

$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

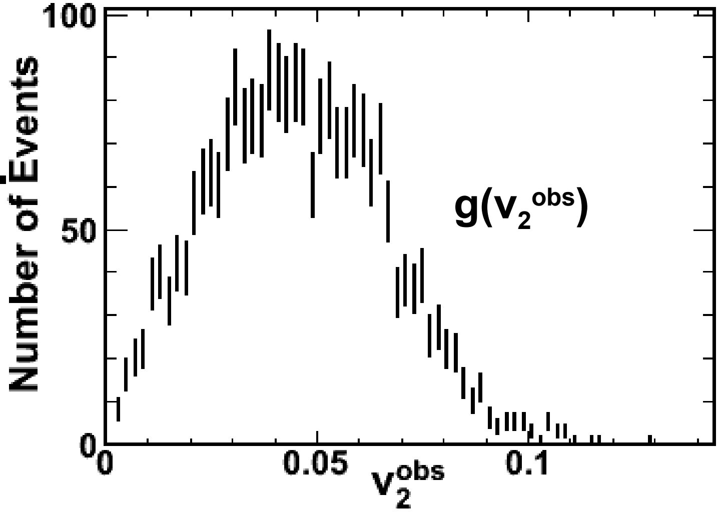
Kernel – Response Function



Extracted true v_2 distribution



v_2^{obs} distribution in “data”



Method Overview

If $K(v_2^{obs}, v_2) = \exp(\frac{-(v_2^{obs} - v_2)^2}{2\sigma_{stat}^2})$ **Then** $\sigma^2 = \sigma_{dyn}^2 + \sigma_{stat}^2$

Method Overview

~~**If** $K(v_2^{obs}, v_2) = \exp(\frac{-(v_2^{obs} - v_2)^2}{2\sigma_{stat}^2})$ **Then** $\sigma^2 = \sigma_{dyn}^2 + \sigma_{stat}^2$~~

(J.-Y.Ollitrault, PRD (1992) 46, 226)

However $K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma_n^2} e^{-\left(\frac{v_2^{obs} + v_2^2}{2\sigma_n^2}\right)} I_0(\frac{-v_2^{obs} v_2}{\sigma_n^2})$

Method Overview

(J.-Y.Ollitrault, PRD (1992) 46, 226)

However $$K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma_n^2} e^{-\left(\frac{v_2^{obs} + v_2^2}{2\sigma_n^2}\right)} I_0\left(\frac{-v_2^{obs} v_2}{\sigma_n^2}\right)$$

The analysis has 3 main steps:

Measuring v_2^{obs} event-by-event in data: $g(v_2^{obs})$

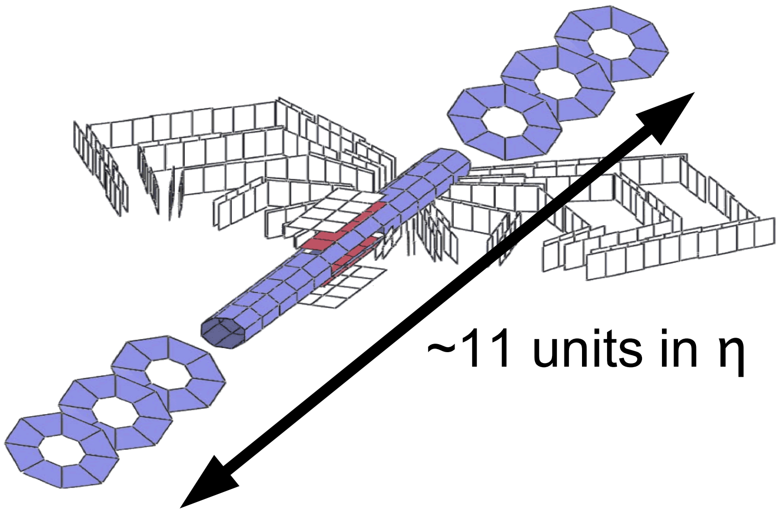
Calculating the Kernel: $K(v_2^{obs}, v_2)$

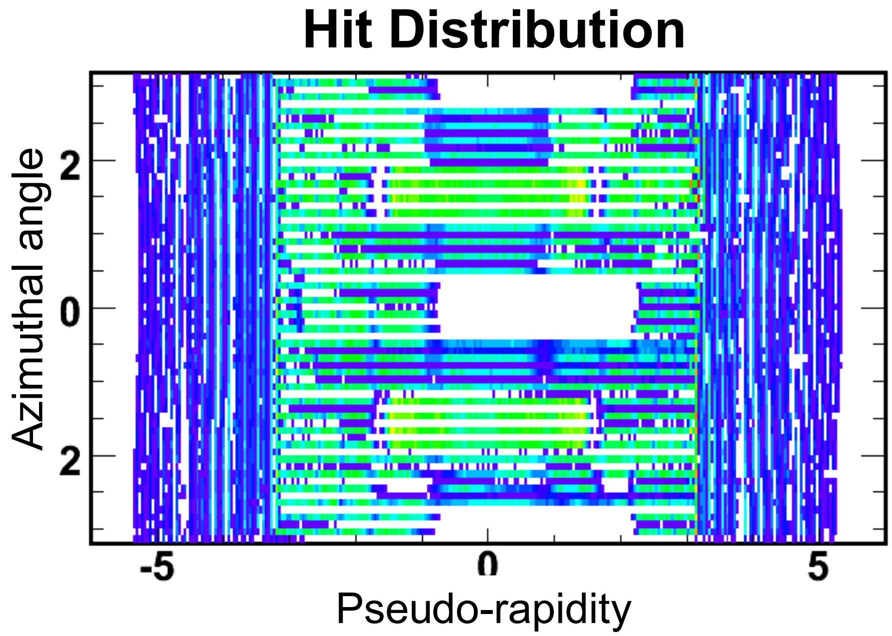
Extracting dynamical fluctuations: $f(v_2)$

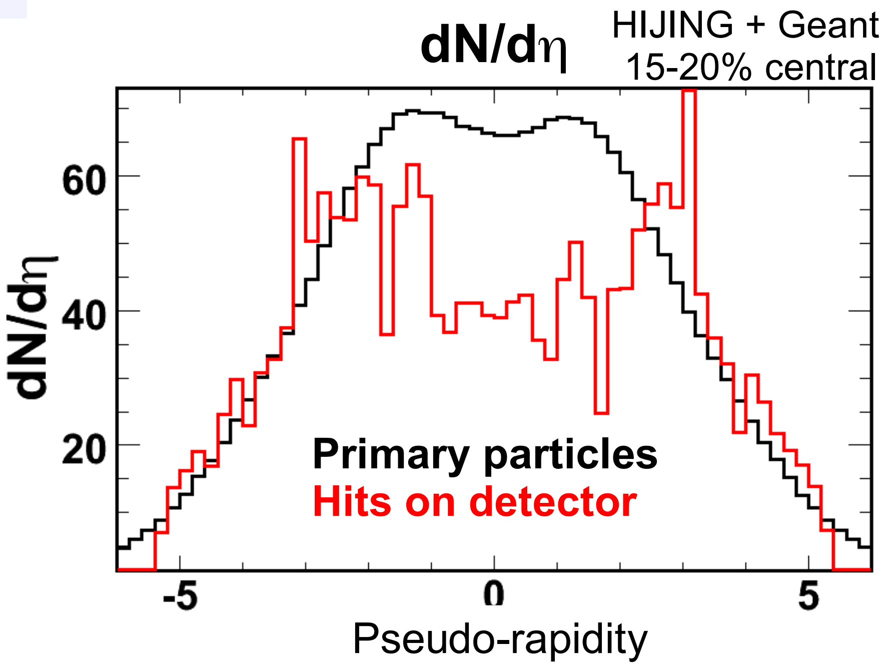
$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

Event-by-event measurement of v_2^{obs}

- PHOBOS Multiplicity Array
  - $-5.4<\eta<5.4$ coverage
  - Holes and granularity differences
- Usage of all available information
in event to determine event-by-event a single value for v_2^{obs}







Event-by-event measurement of v_2^{obs}

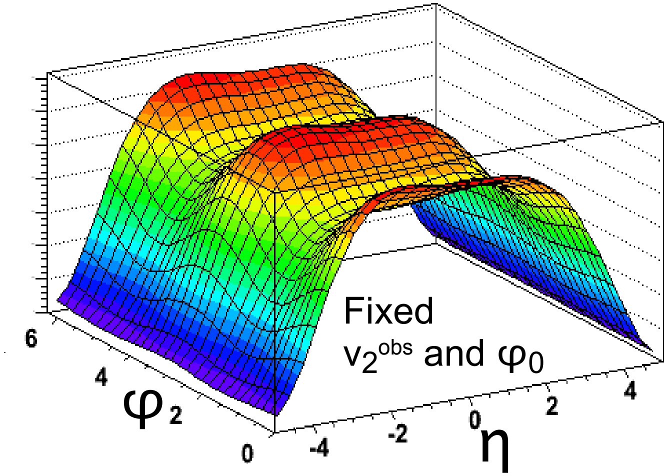
Define probability distribution function (PDF)for hit positions:

$$P(\eta,\phi;\, v_2^{obs},\phi_0)=\frac{1}{s(v_2^{obs},\phi_{0,}\eta)}[1+2v_2(\eta)\cos(2\phi-2\phi_0)]$$

Normalization

Normalization assures integral of PDF folded with the acceptance is the same for different values of v_2^{obs} and ϕ_0.

Probability distribution function



$$s(v_2^{obs},\phi_0;\eta)=\int A(\eta,\phi)[1+2v_2(\eta)\cos(2\phi-2\phi_0)]\,d\phi$$

Acceptance

Event-by-event measurement of v_2^{obs}

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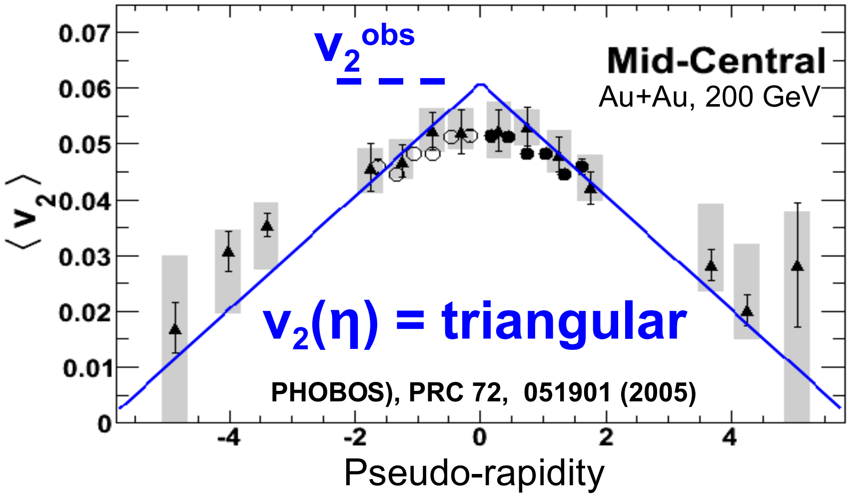
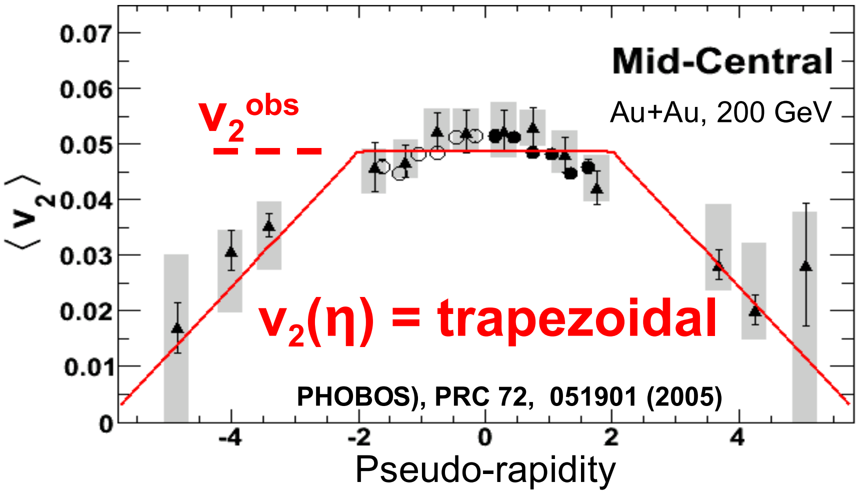
We parameterize $v_2(\eta)$ using known shape from previous measurements:



PHOBOS), PRC 72, 051901 (2005)



PHOBOS), PRC 72, 051901 (2005)

Event-by-event measurement of v_2^{obs}

Define probability distribution function (PDF) for hit positions:

$$P(\eta,\phi;\ v_2^{obs},\phi_0)=\frac{1}{s(v_2^{obs},\phi_{0,}\eta)}[1+2v_2(\eta)\cos(2\phi-2\phi_0)]$$

For a given event with n hits, the likelihood of v_2^{obs} and ϕ_0:

$$L(v_2^{obs},\ \phi_0)=\prod_{i=1}^{n}P(\eta_i,\phi_i;\ v_2^{obs},\phi_0)$$

Maximizing L allows a measurement of v_2^{obs} and ϕ_0 event-by-event.

Determining the kernel

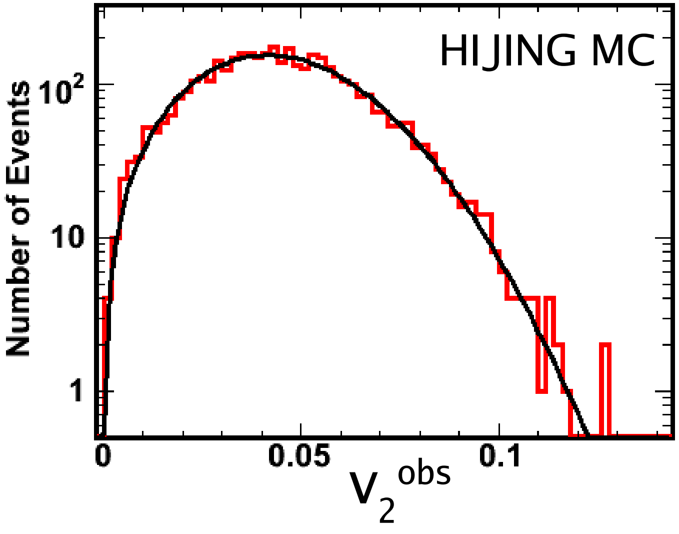
$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

Reminder: Kernel is the response of the measurement to input value of v_2.

Response also depends on the observed multiplicity n.

Determining the kernel = “measuring” v_2^{obs} distributions in MC in bins of v_2 and n.

v_2^{obs} distribution for fixed v_2 and n



Determining the kernel

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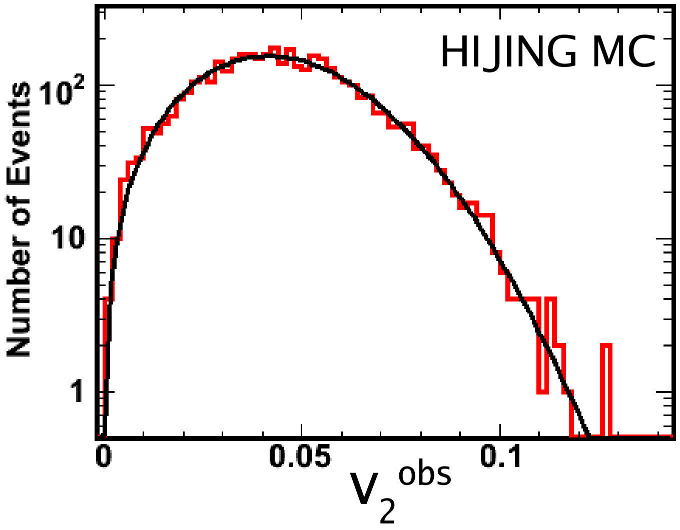
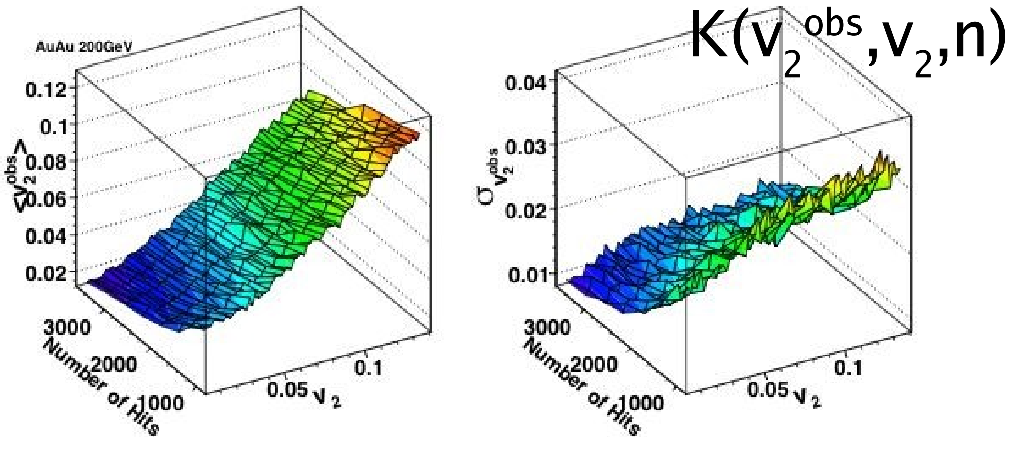
v_2^{obs} distribution for fixed v_2 and n



$\langle v_2^{obs} \rangle$ and $\sigma(v_2^{obs})$



$K(v_2^{obs}, v_2, n)$

$1.5 \cdot 10^6$ HIJING events
Modified φ to include
triangular or trapezoidal flow

Determining the kernel

Fitting $K(v_2^{obs},v_2,n)$ with smooth functions reduces bin-to-bin fluctuations.

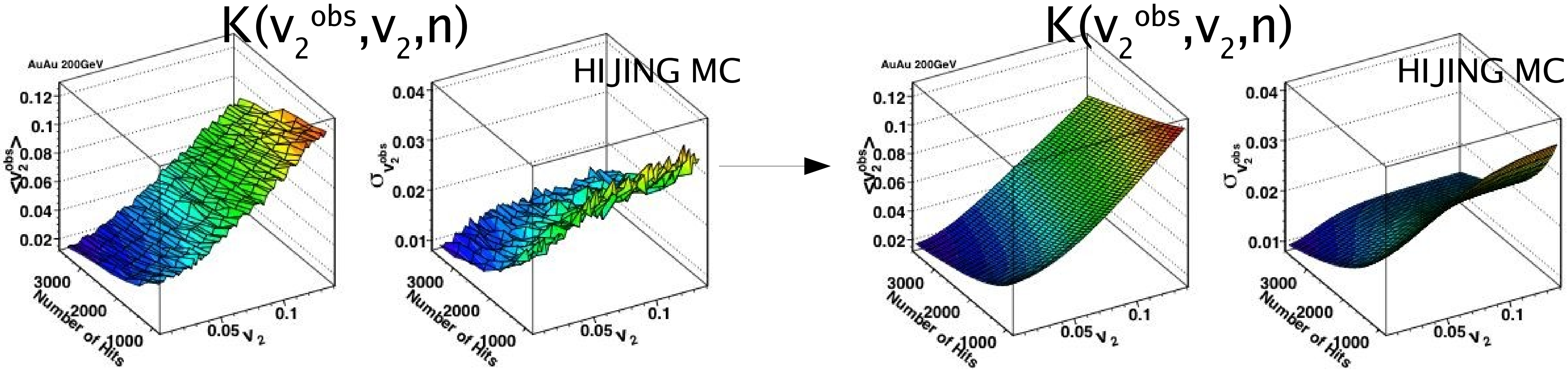
Theoretical distribution of $K(v_2^{obs},v_2,n)$ modified for experimental effects is used as fit function:

$$K(v_2^{obs},v_2,n)=\frac{v_2^{obs}}{\sigma^2}e^{-\left(\frac{v_2^{obs}+v_2^2}{2\sigma^2}\right)}I_0\left(\frac{-v_2^{obs}v_2}{\sigma^2}\right)$$

(J.-Y.Ollitrault, PRD (1992) 46, 226)

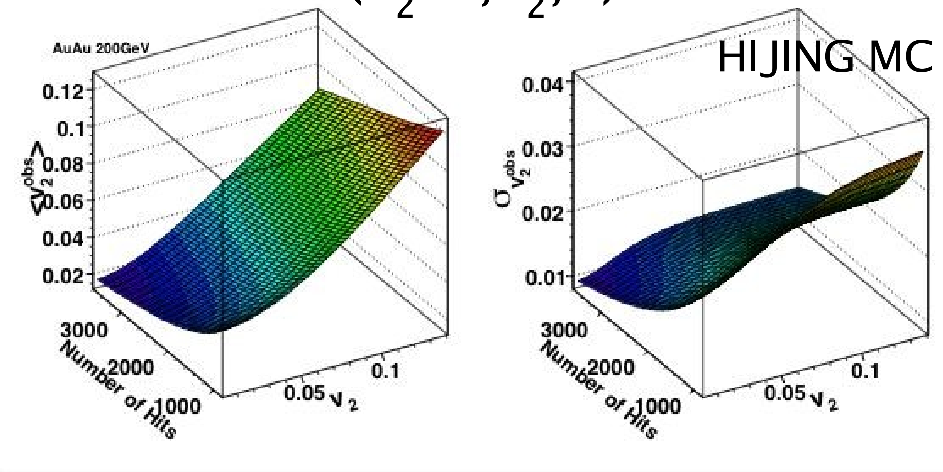
$$v_2\rightarrow(An+B)v_2$$ (suppression)

$$\sigma=\frac{C}{\sqrt{n}}+D$$ (finite resolution)



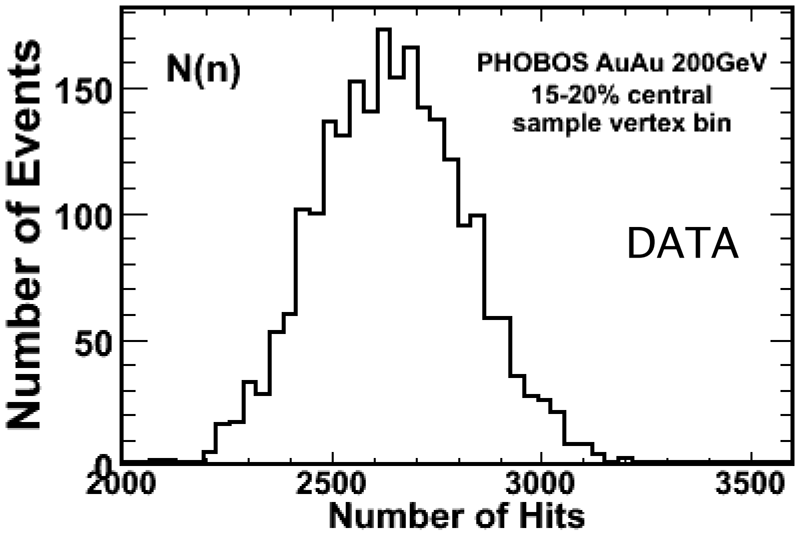
Determining the kernel

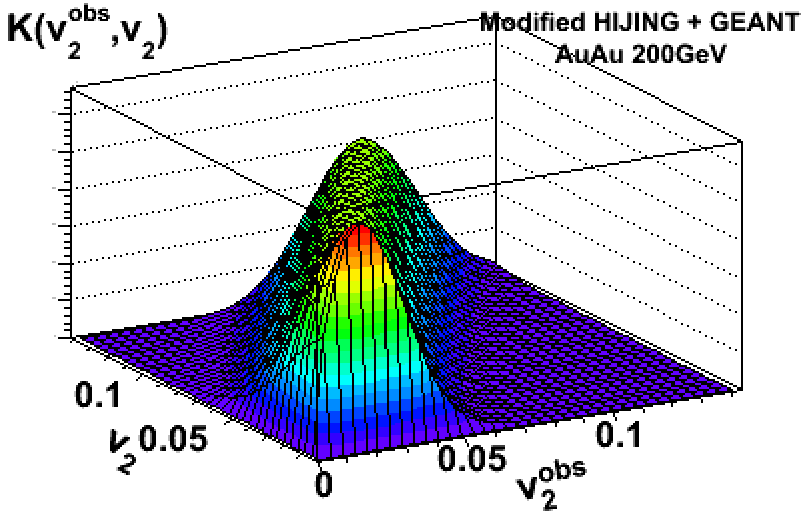
$K(v_2^{obs},v_2,n)$



Assuming that the true v_2 distribution for a set of events in a given centrality class is independent of n, it is possible to integrate out the multiplicity dependence:

$$K(v_2^{obs},v_2)=\int K(v_2^{obs},v_{2,}n)N(n)dn$$





Extracting dynamical fluctuations

$$\underbrace{g(v_2^{obs})}_{\text{Measured}} = \int_0^1 \underbrace{K(v_2^{obs},}_{\text{Constructed from MC}} v_2) f(v_2) dv_2$$

Extracting dynamical fluctuations

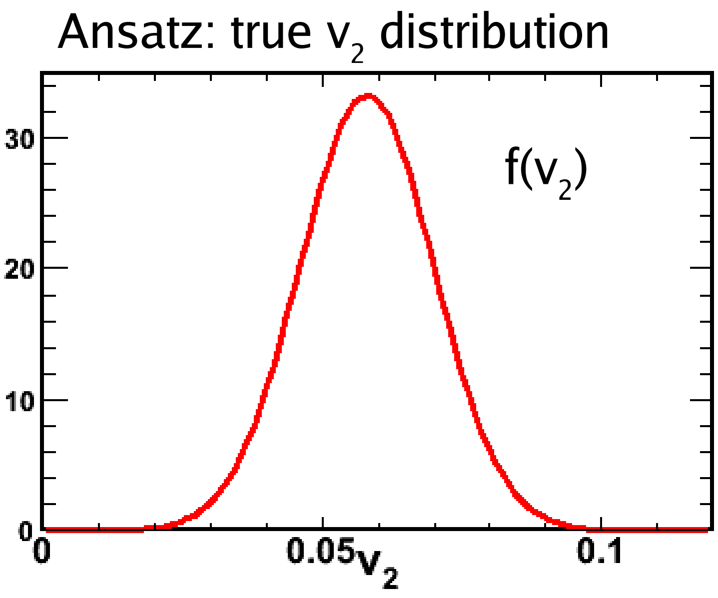
$$\underline{g\left(v_2^{obs}\right)} = \int_0^1 \underline{K\left(v_2^{obs}, v_2\right)}\,\underline{f\left(v_2\right)}\,dv_2$$

Measured

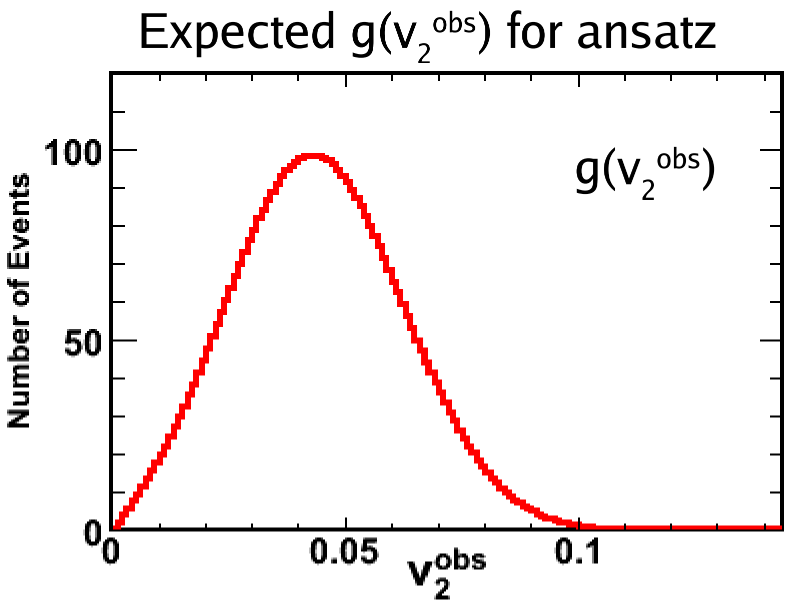
Constructed from MC

Gaussian Ansatz:

$$f\left(v_2\right) = \exp\left[\frac{-\left(v_2 - \langle v_2 \rangle\right)^2}{2\sigma_{v_2}^2}\right]$$



Ansatz: true v_2 distribution



Expected $g(v_2^{obs})$ for ansatz

Extracting dynamical fluctuations

$$\underline{g(v_2^{obs})} = \int_0^1 \underline{K(v_2^{obs}, v_2)}\,\underline{f(v_2)}\,dv_2$$

Measured

Constructed from MC

Gaussian Ansatz:

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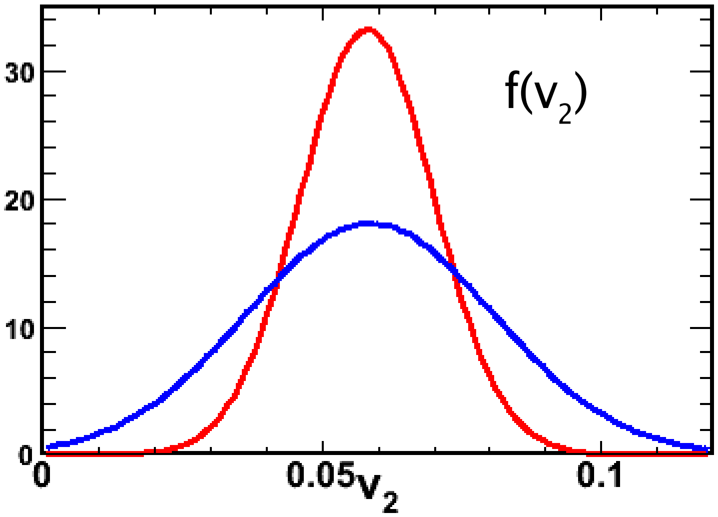
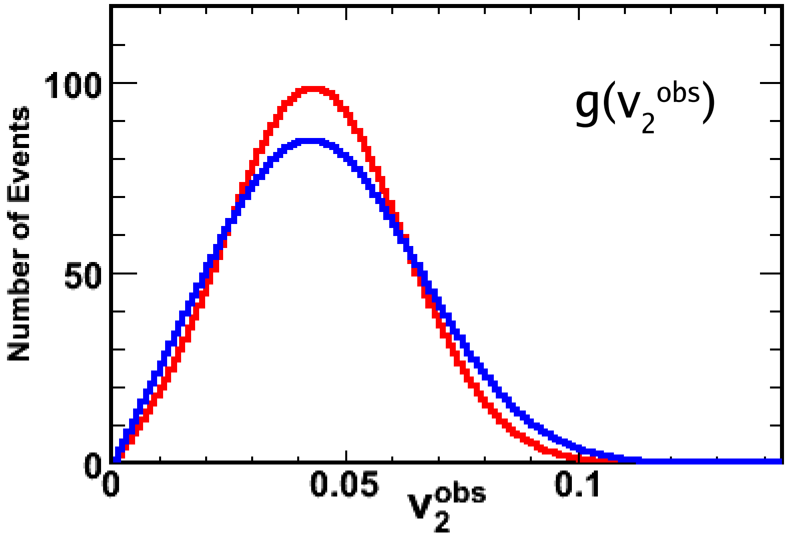


Expected g(v_2^{obs}) for ansatz



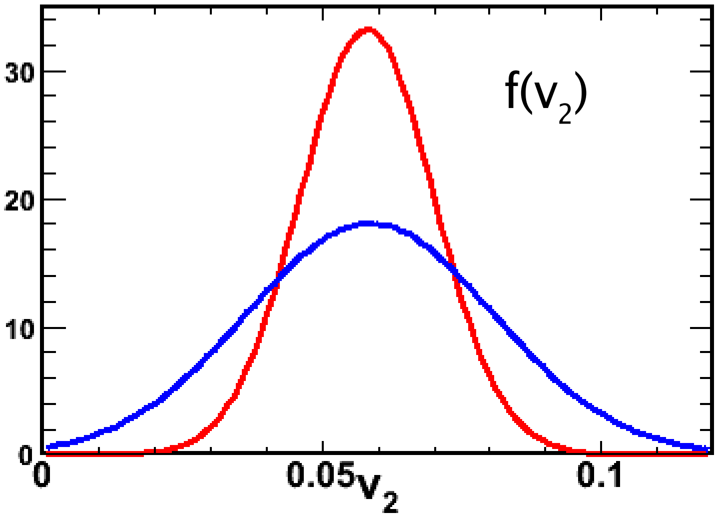
Extracting dynamical fluctuations

$$\underbrace{g\left(v_2^{obs}\right)}_{\textbf{Measured}}=\int_0^1 \underbrace{K\left(v_2^{obs},v_2\right)}_{\textbf{Constructed from MC}} \underbrace{f\left(v_2\right)}_{\textbf{Gaussian Ansatz:}} dv_2$$

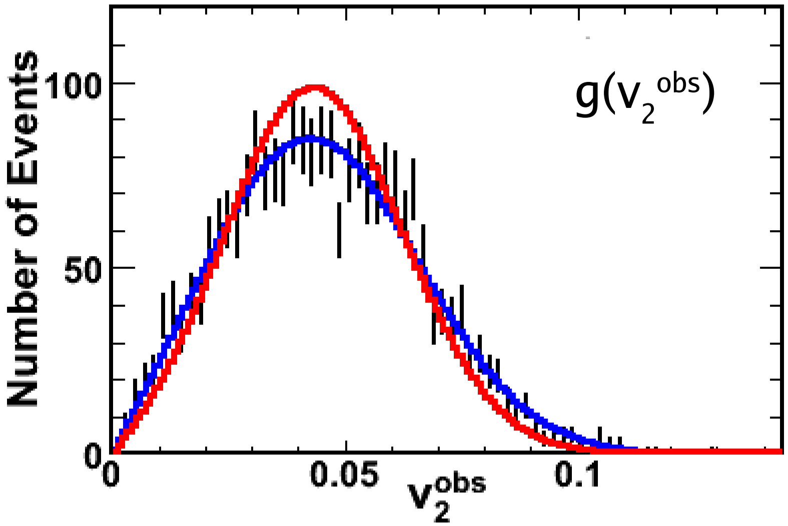
Gaussian Ansatz:

$$f\left(v_2\right)=\exp\left[\frac{-\left(v_2-\langle v_2\rangle\right)^2}{2\sigma_{v_2}^2}\right]$$

Ansatz: true v_2 distribution



Comparison with data

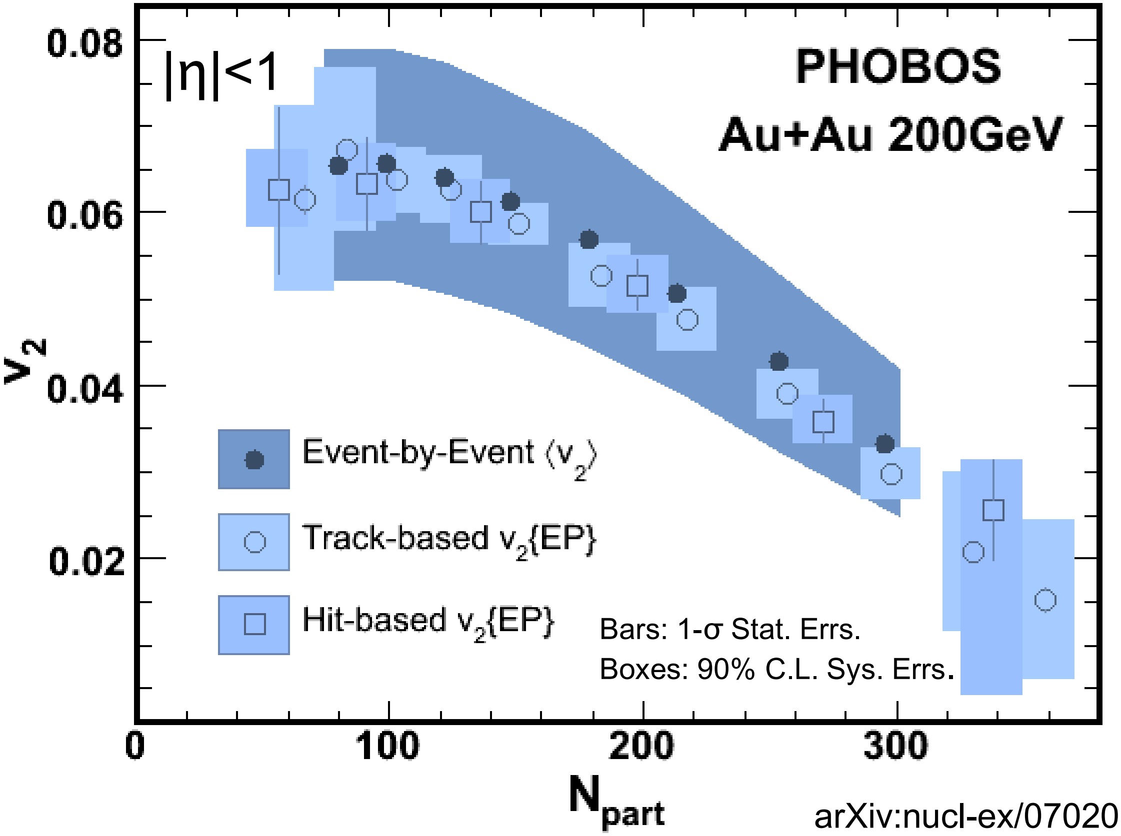


Compare expected g(v_2^{obs}) for trials with data:
Maximum-Likelihood fit → <v_2> and σ_{v_2}

Event-by-event mean v_2 vs published results

- Standard methods
  - Hit- and track-based
  - Use reaction plane sub-even technique
- Event-by-event:
  - PR04 Au+Au data
    - No magnetic field
    - 500.000 events
    - 10 vertex bins ($-10\text{cm}<z_{\text{vertex}}<10\text{cm}$)
  - Relate v_2^{obs} to $<v_2>$:

$$<v_2>(|\eta|<1) = 0.5 \times (11/12 <v_2^{\text{triangular}}> + <v_2^{\text{trapezodial}}>)$$



arXiv:nucl-ex/0702036
Submitted to PRL

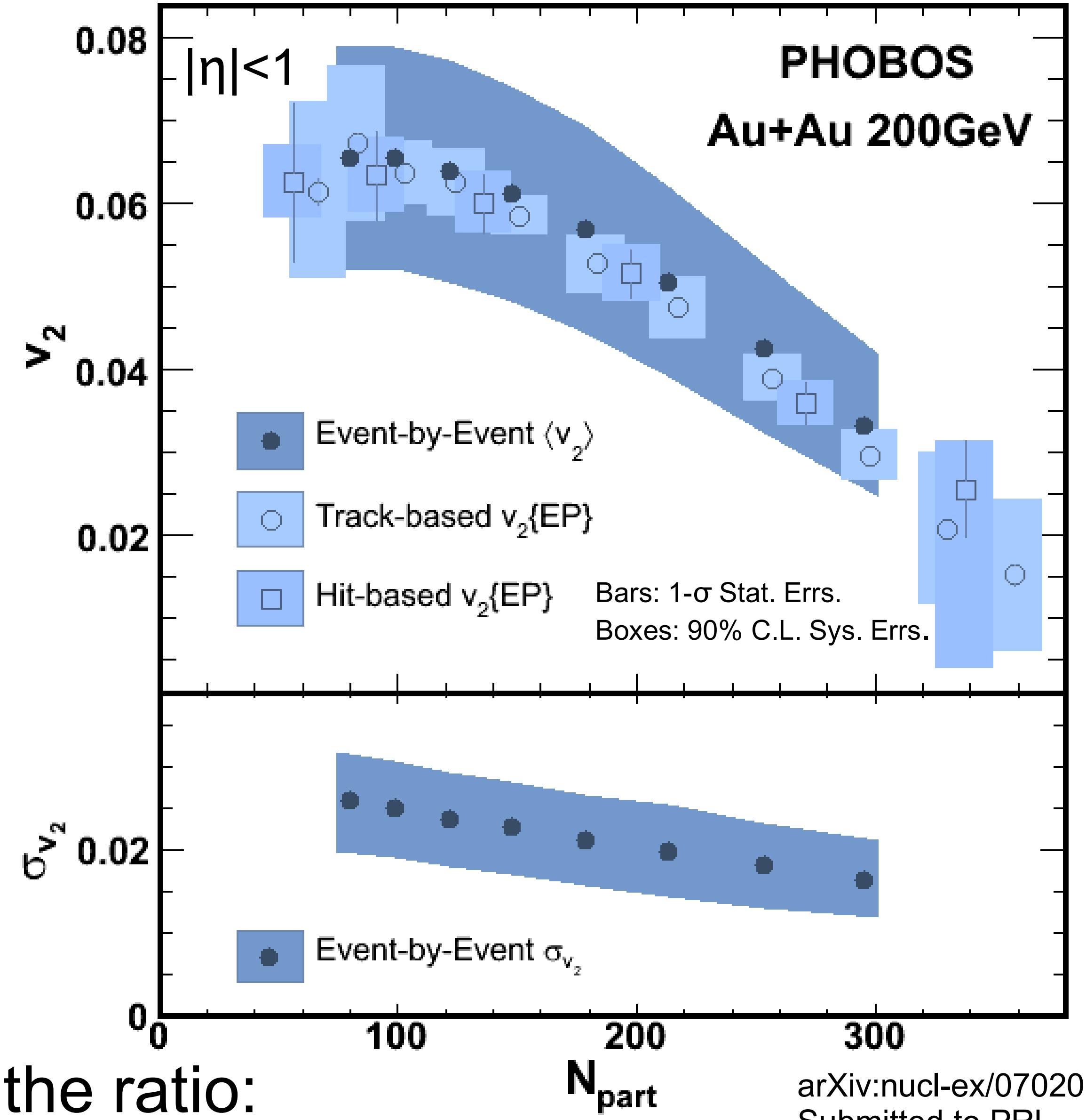
Very good agreement of the event-by-event measured v_2 with the hit- and tracked-based published results

Elliptic flow fluctuations: $\langle v_2 \rangle$ and σ_{v_2}

– Two parameter fit to event-by-event results: $\langle v_2 \rangle$ and σ_{v_2}

Systematic errors:

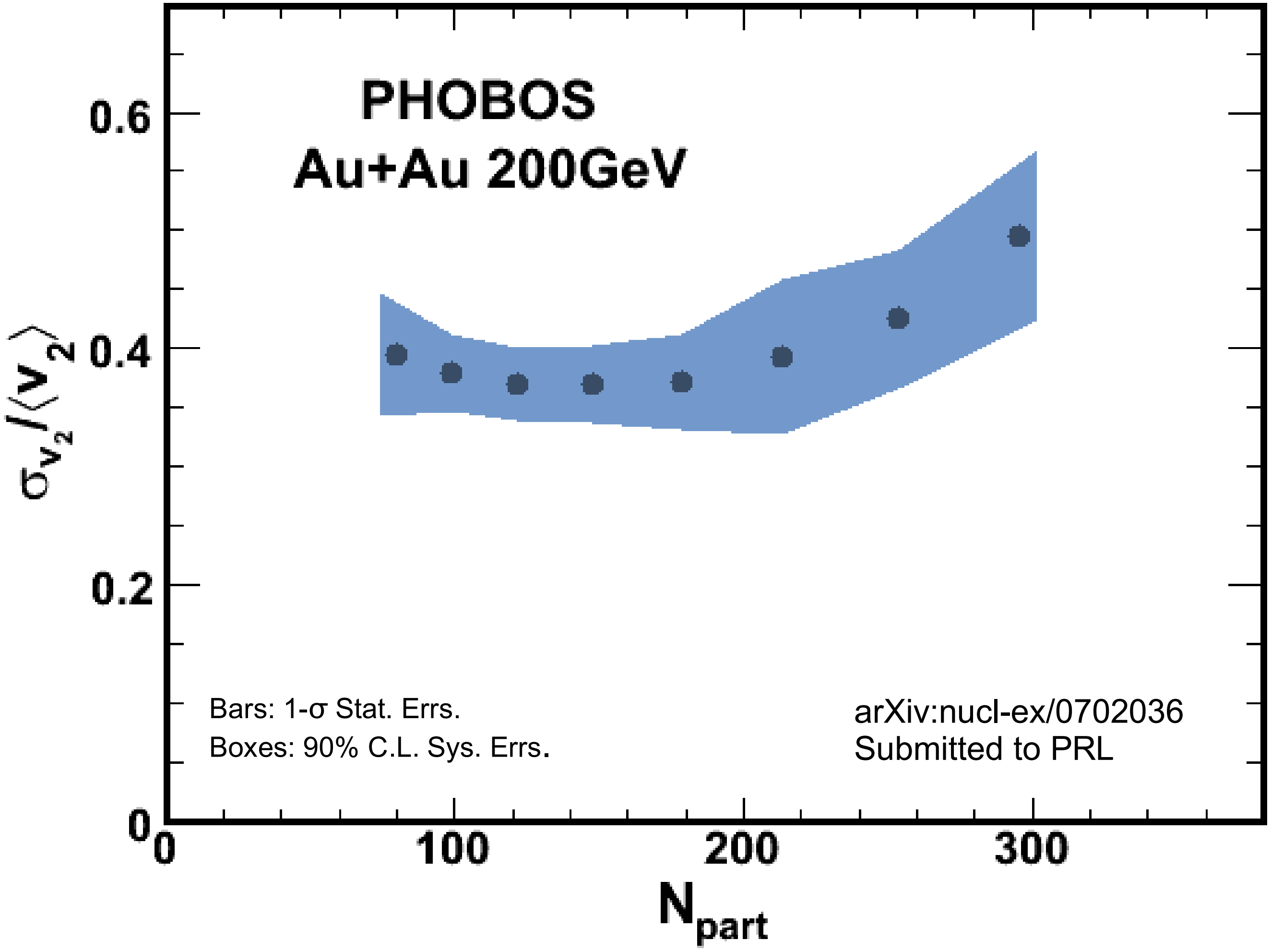
- Variation in $v_2(\eta)$
- Variation in dN/dη
- Variation of $f(v_2)$
- MC response
- Vertex binning
- Φ_0 binning
- Non-flow correlations



arXiv:nucl-ex/0702036
Submitted to PRL

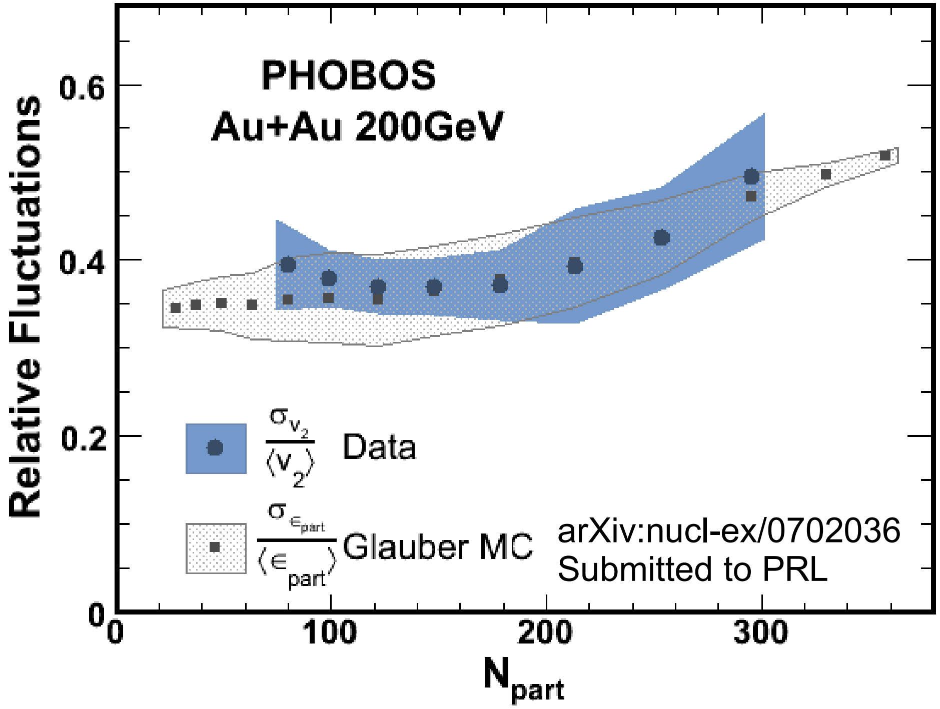
“Scaling” errors cancel in the ratio: relative fluctuations, $\sigma_{v_2}/\langle v_2 \rangle$

Elliptic Flow Fluctuations



Relative Fluctuations ($\sigma_{v2}/<v_2>$) are about 40%

Comparison to Participant Eccentricity Prediction



arXiv:nucl-ex/0702036
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- Relative fluctuations ($\sigma_{v2}/<v_2>$) are about 40%
- Striking agreement with predictions from participant eccentricity.

- Modeling of interaction points with MC Glauber interpreted event-by-event, the participant eccentricity model, appears to be able to explain:
  - The magnitude of the mean elliptic flow in Cu+Cu wrt Au+Au
  - The magnitude of the elliptic flow fluctuations in Au+Au

Non-flow correlations

- Non-flow : all particle correlations other than flow
- Particle correlations may appear as flow fluctuations
  - Rephrase: the “statistical” fluctuations may be underestimated in the kernel
- We have undertaken two studies to understand non-flow effects
  - A kernel with non-flow correlations
    - included in systematic errors
  - Comprehensive study of nonflow effect as a function of correlation strength

Correlation Function

- To study effect of non-flow correlations on flow fluctuation measurement
  - Define correlation strength:

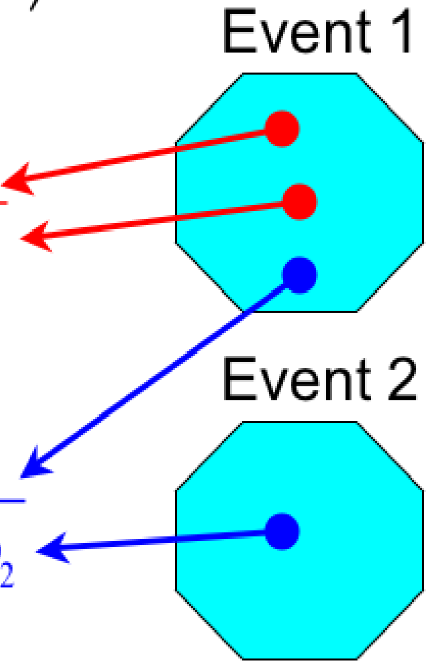
$$R(\Delta\eta,\Delta\phi) = <(n-1)\left(\frac{F_n(\Delta\eta,\Delta\phi)}{B_n(\Delta\eta,\Delta\phi)}-1\right)>$$

Foreground:

$$F_n(\Delta\eta,\Delta\phi) \sim \rho_n^{II}(\eta_1,\eta_2,\phi_1,\phi_2) = \frac{1}{n(n-1)\sigma_n}\frac{d^4\sigma_n}{d\eta_1 d\eta_2 d\phi_1 d\phi_2}$$

Background:

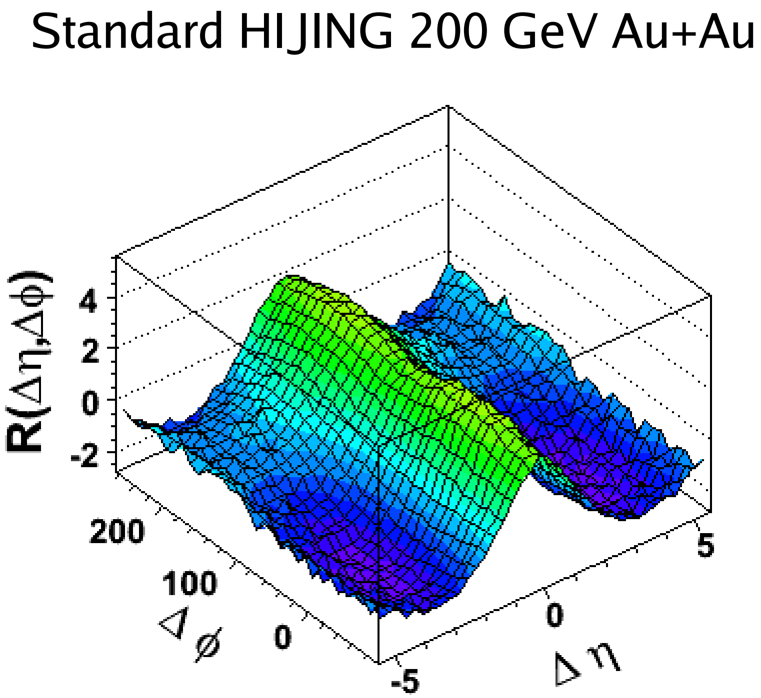
$$B_n(\Delta\eta,\Delta\phi) \sim \rho_n^{I}(\eta_1,\phi_1)\rho_n^{I}(\eta_2,\phi_2) = \frac{1}{n\sigma_n}\frac{d^2\sigma_n}{d\eta_1 d\phi_1}\bullet\frac{1}{n\sigma_n}\frac{d^2\sigma_n}{d\eta_2 d\phi_2}$$

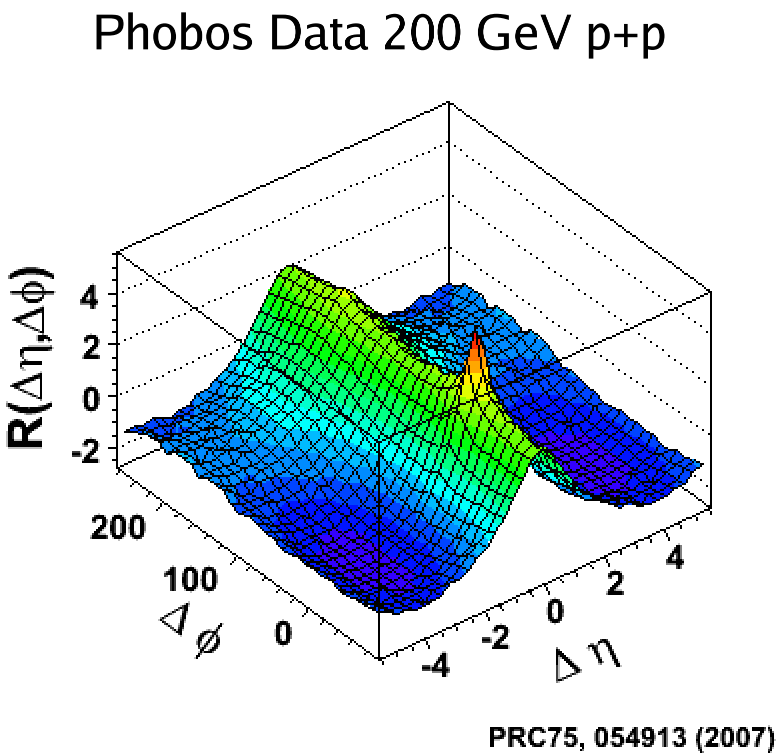


arXiv:nucl-ex/0704.0966
Accepted for PRC

Correlation Function in HIJING

- Hijing has some of the correlation structure features in data
  - Compare with p+p collisions


Standard HIJING 200 GeV Au+Au


Phobos Data 200 GeV p+p

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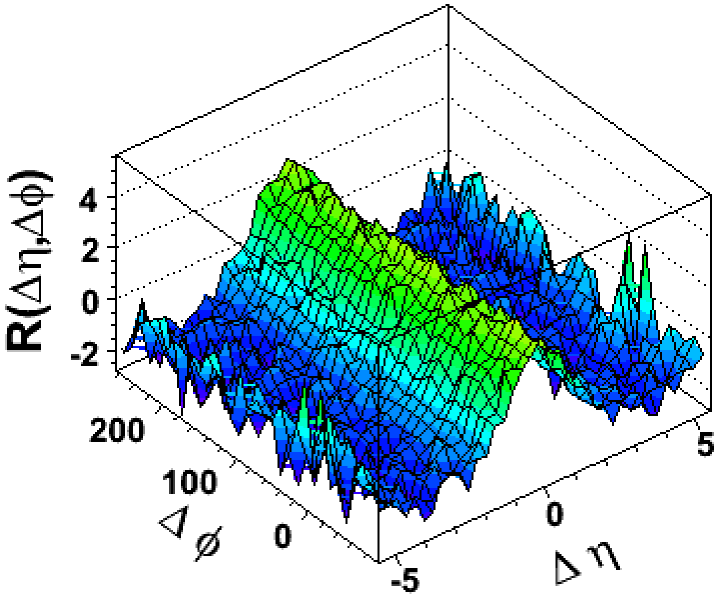
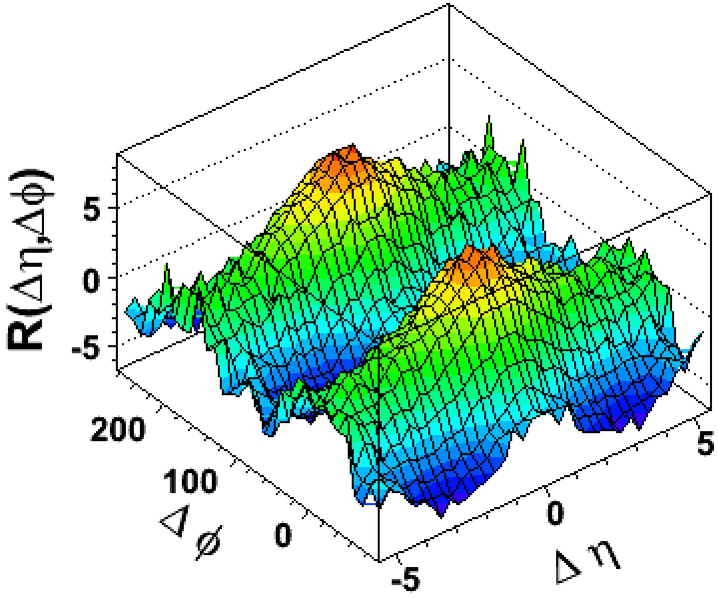
Correlation Function in HIJING

- For baseline measurement, we use modified HIJING
  - No correlations in φ
  - Particles randomly given φ values from a PDF with flow

Modified HIJING
No φ correlations
No Flow



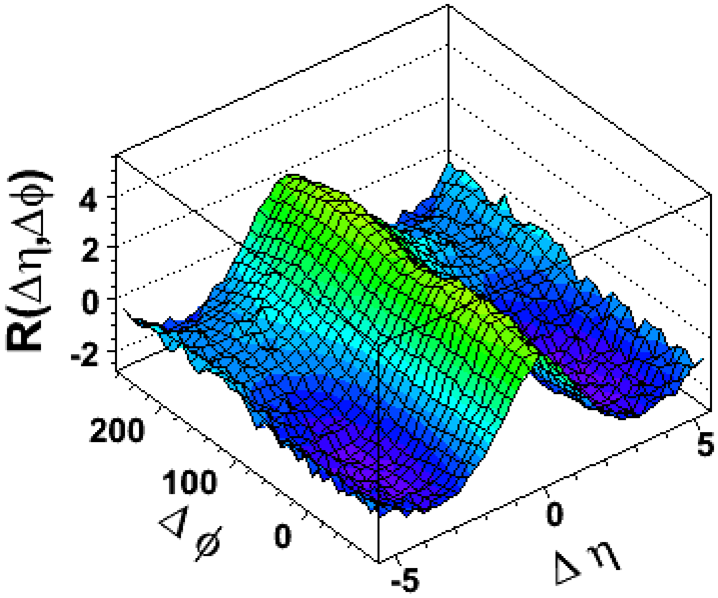
Modified HIJING
No φ correlations
With Flow



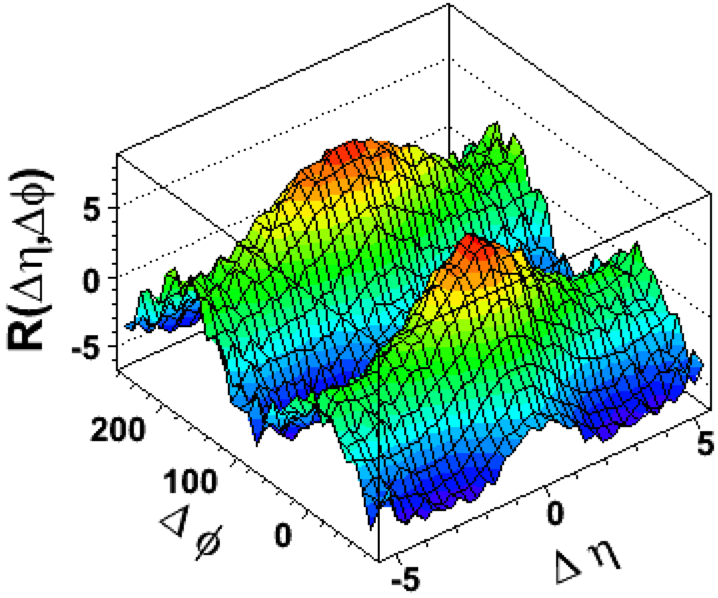
Correlation Function in HIJING

- For systematic studies, we preserve correlations in HIJING
  - Particles are shifted in φ direction to obtain v_2. PRC58 (1998) 1671

Standard HIJING
With φ correlations
No Flow



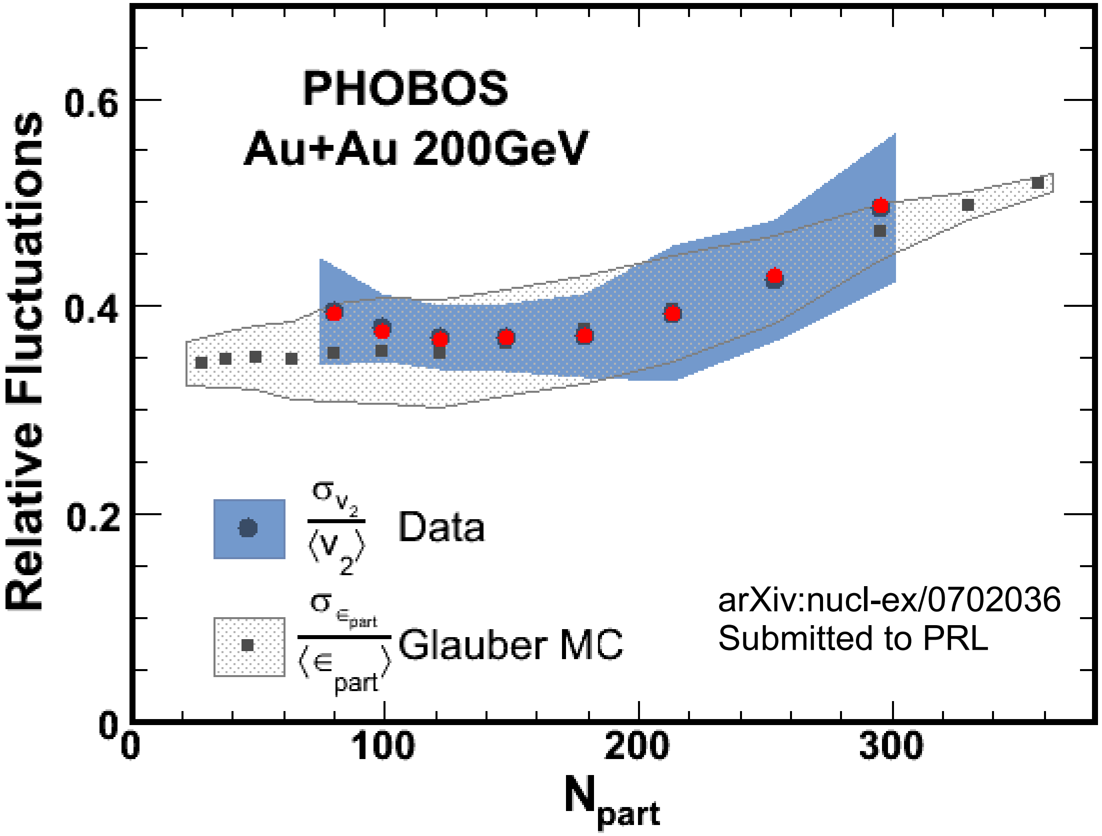
Modified HIJING
With φ correlations
With Flow



Results

- Measuring response function on HIJING with correlations
- Results move by at most 2%
  - Included in systematic errors

Data – kernel with correlations



arXiv:nucl-ex/0702036
Submitted to PRL

A MC particle generator

- HIJING might not be an accurate description for correlations
- To study effect of non-flow correlation on fluctuation measurement as a function of the correlation strength:
  - Need a simple MC event generator with correlations
- Inspired by cluster like correlations in data

Nucl Phys B85, 61 (1975)
PRC75 054913 (2007)

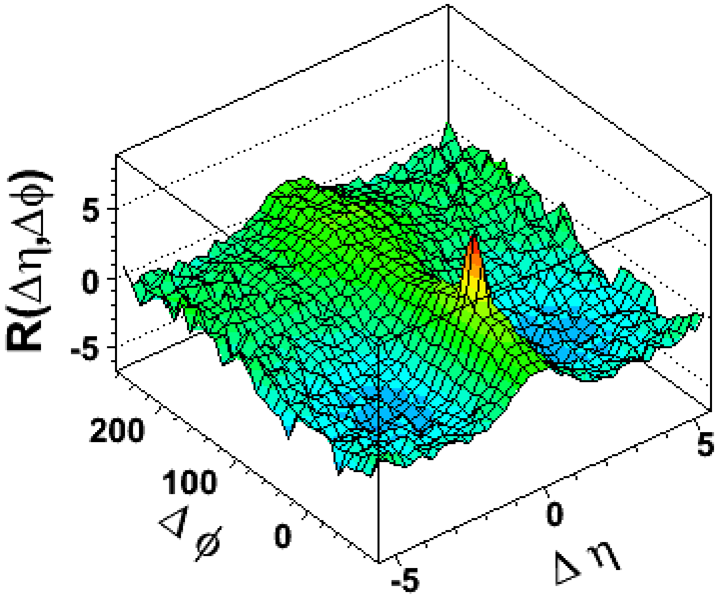
- "Clusters" of different cluster sizes and masses are produced independently, and decay into "particles".

Note: PHOBOS only sees hits.
Only need to reproduce dN/dη and 2-particle correlations

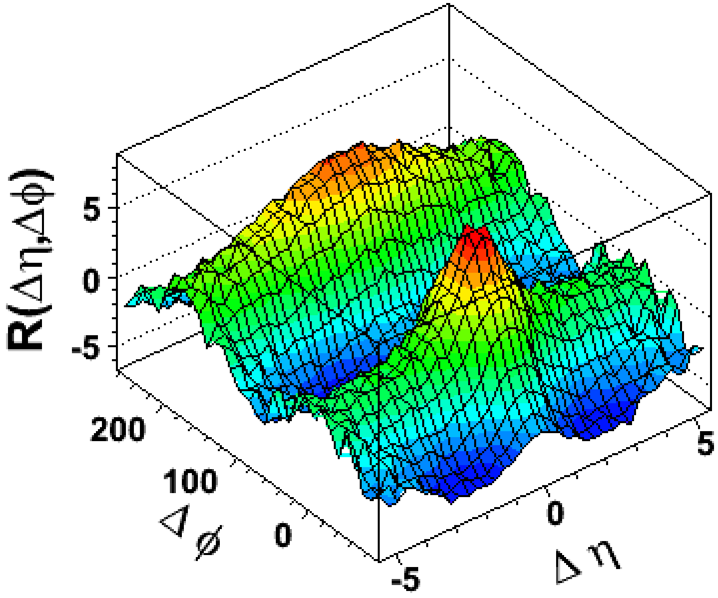
Correlation Function in Sample Model

- Clusters can be given some $v_{2.}$
  - Particles will also have v_2

Cluster Model
With φ correlations
No Flow



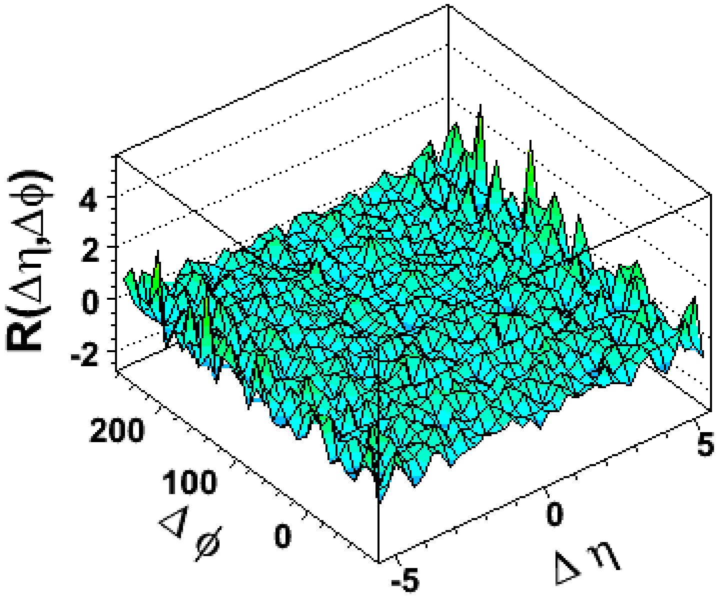
Cluster Model
With φ correlations
With Flow



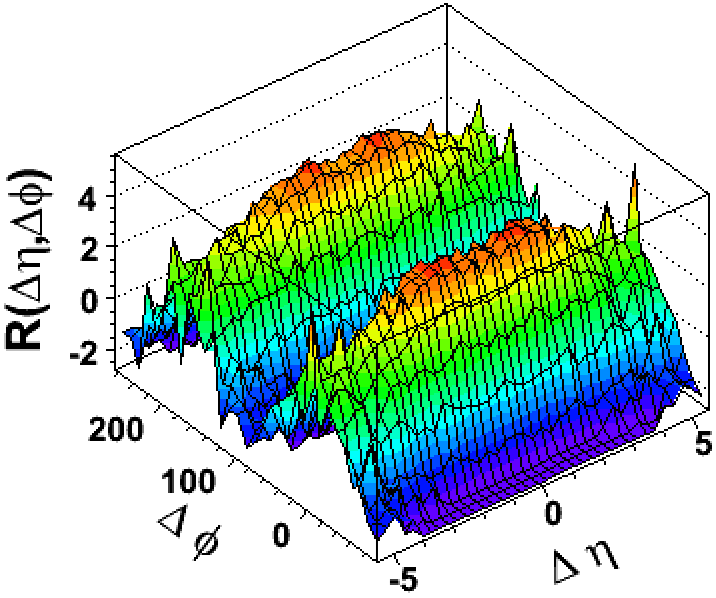
Correlation Function in Sample Model

- Choosing one particle from each cluster, events with no correlations can be produced

Cluster Model
No φ correlations
No Flow



Cluster Model
No φ correlations
With Flow



Elliptic Flow Fluctuations

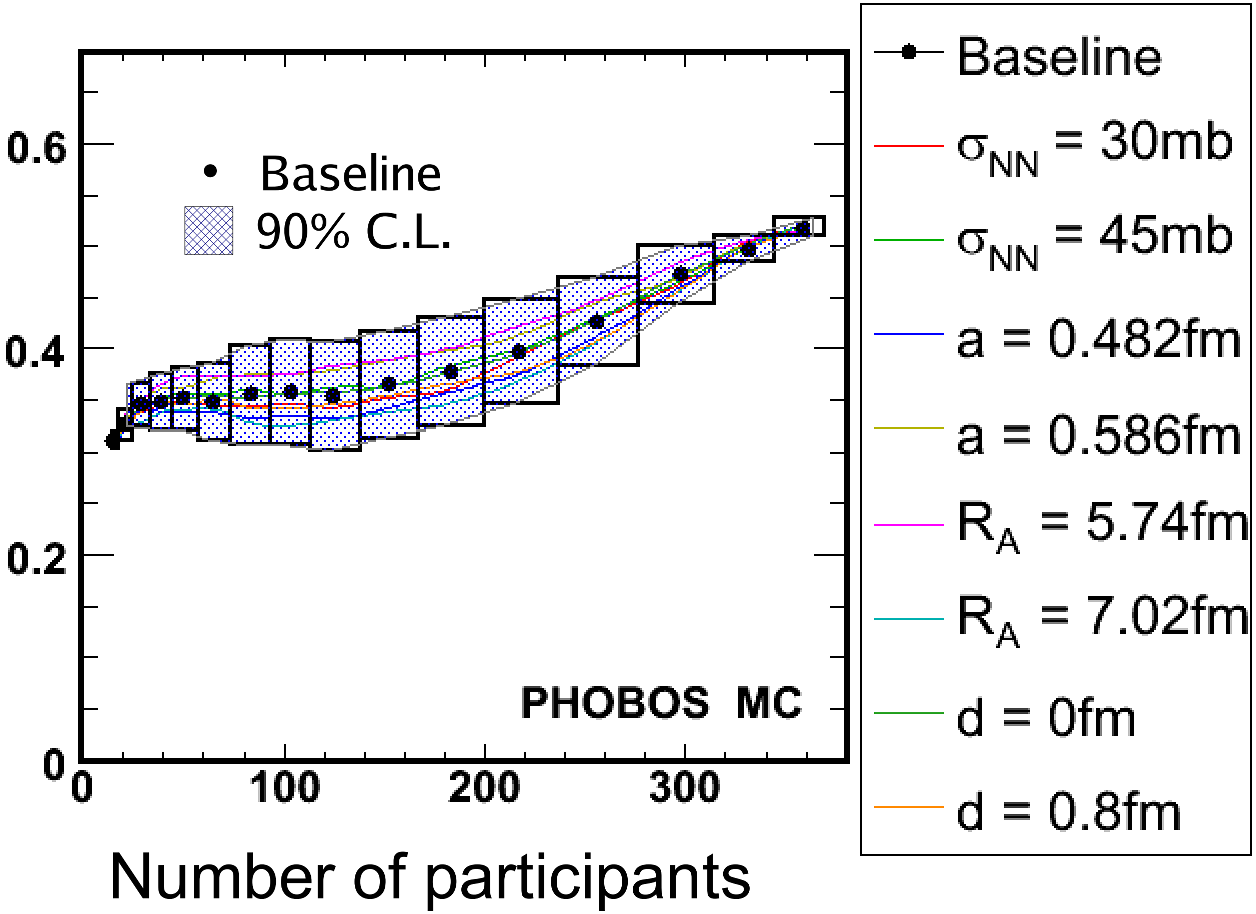
- Input flow fluctuations at the cluster level
- Compare measurement for samples w/ and w/o correlations
- We find evidence that the effect of non-flow correlation is small for relative fluctuations of 40%
- Ongoing work to quantify the dependence of the effect on correlation function
  - Possible to study various correlation structures

Summary

- PHOBOS has measured elliptic flow fluctuations in peripheral to semi-central Au+Au collisions at 200 GeV
  - Absolute fluctuations (σ_{v2}) are about 0.02
  - Relative fluctuations ($\sigma_{v2}/<v_2>$) are about 40%
  - The participant eccentricity predictions for the magnitude of the relative fluctuations are in striking agreement with the measurement
- Nonflow correlations have a small effect on the observed fluctuations signal.
  - Included in systematic errors
  - Working on better understanding the effect as a function of correlation strength.

Backup

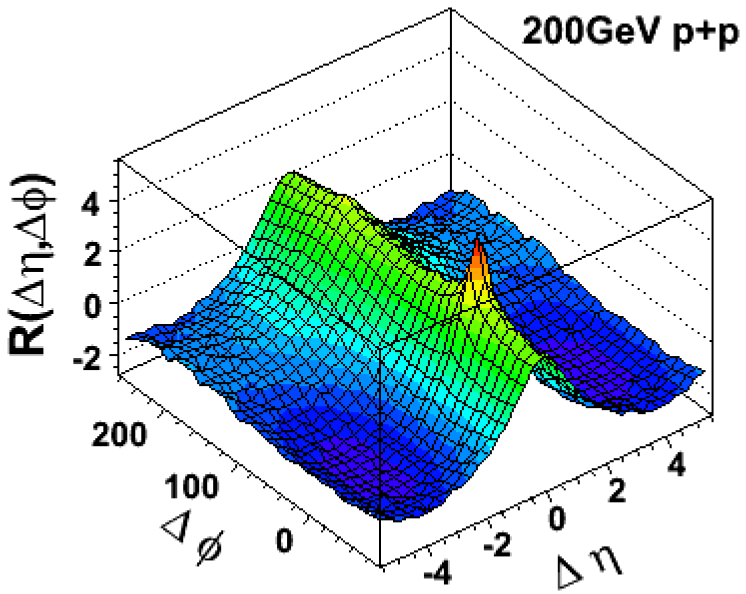
Expected elliptic flow fluctuations



Baseline parameters:

- Nucleon-nucleon cross section: σ_{NN}=41mb
- Inter-nucleon separation distance: d=0.4fm
- Wood-saxon radius: R_A=6.38fm
- Skin depth: a=0.535fm

Correlation Function in pp collisions



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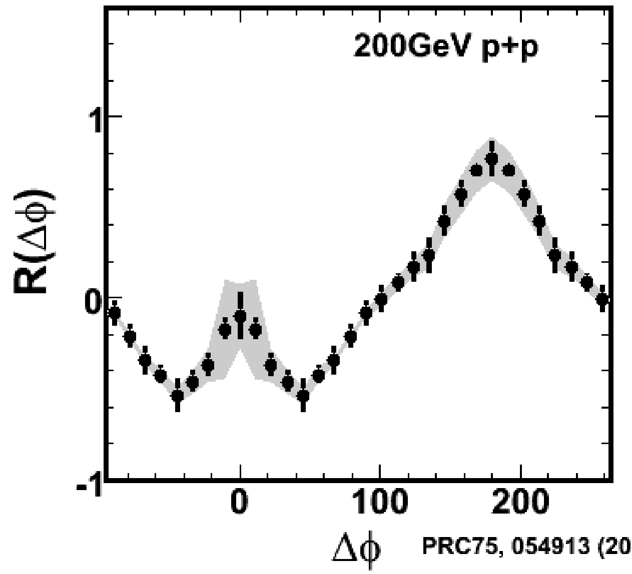

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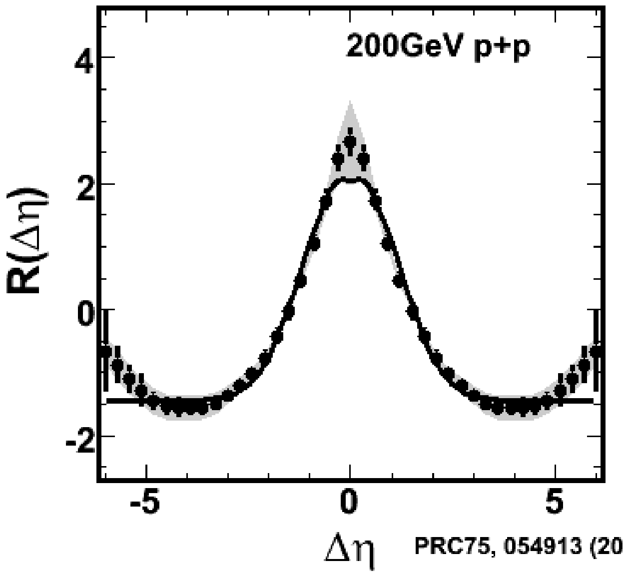

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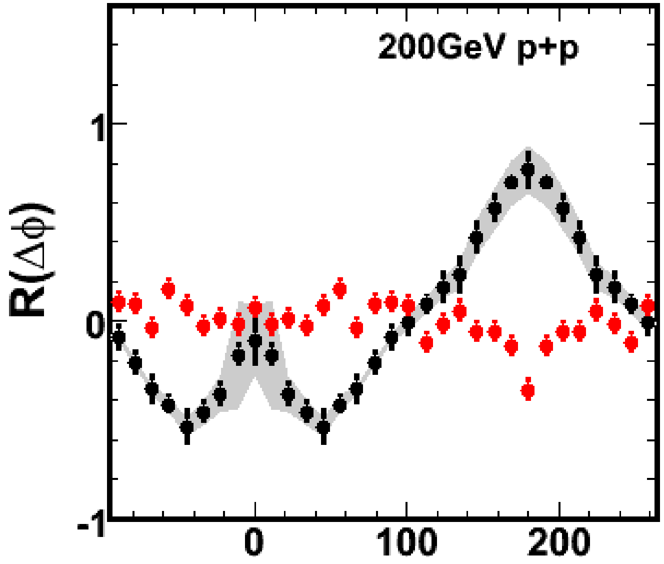
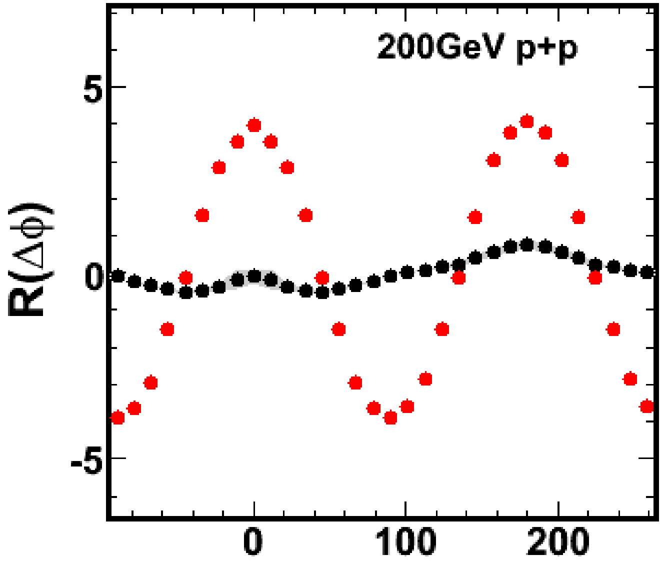
1D correlation function HIJING

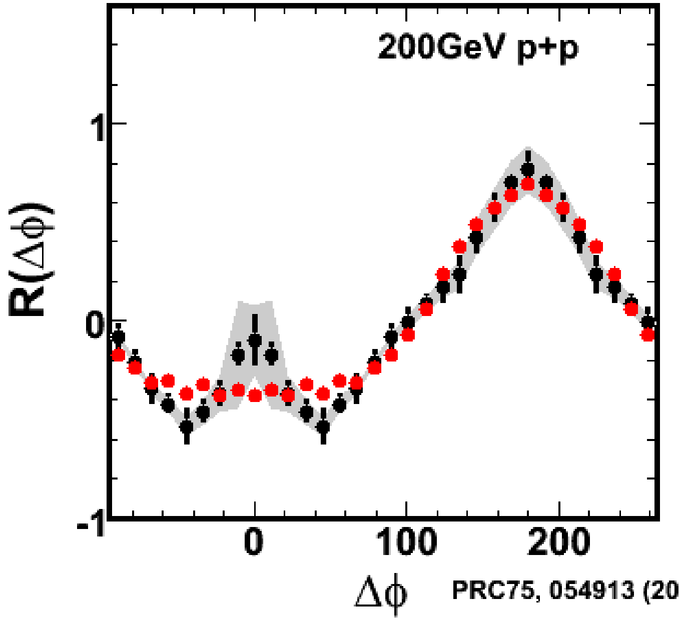
Hijing AuAu w/o corr.



Hijing AuAu w/o corr.



Hijing AuAu w/ corr.



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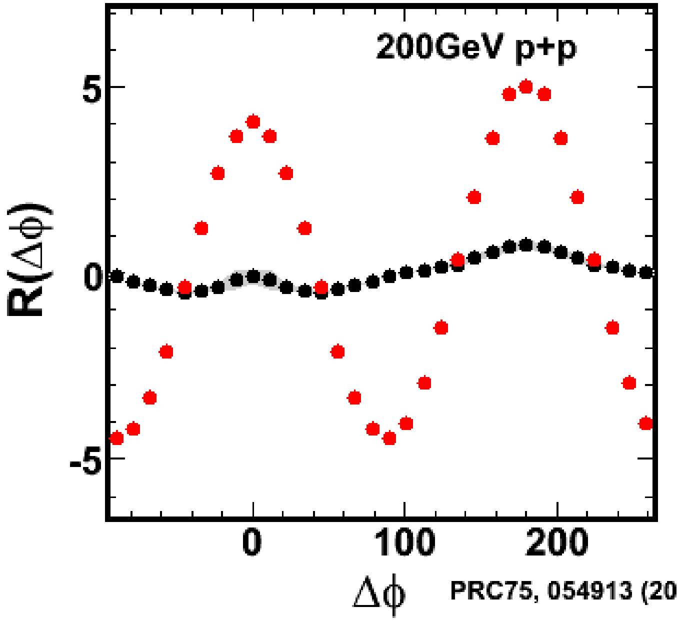
Hijing AuAu w/ corr.



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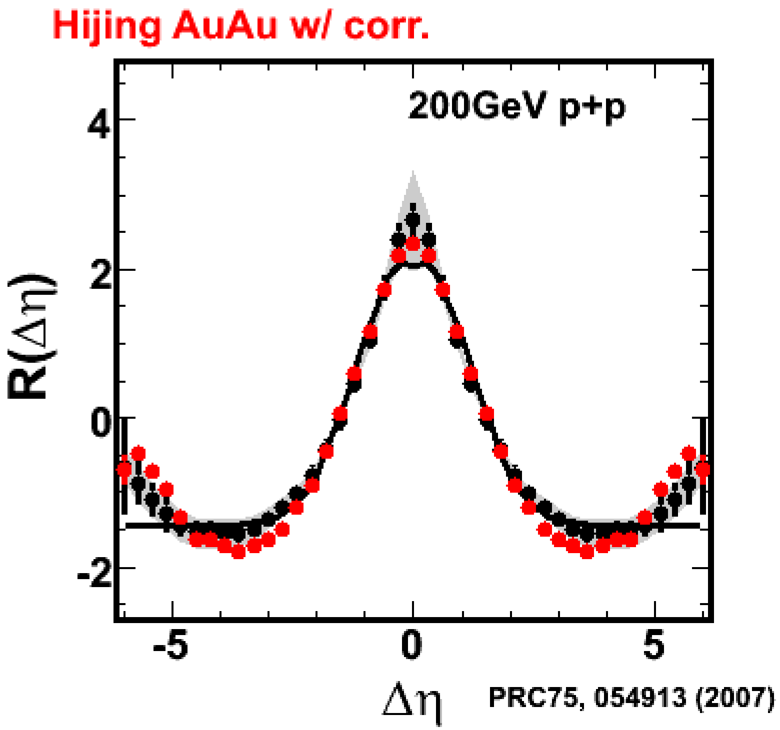
1D correlation function HIJING



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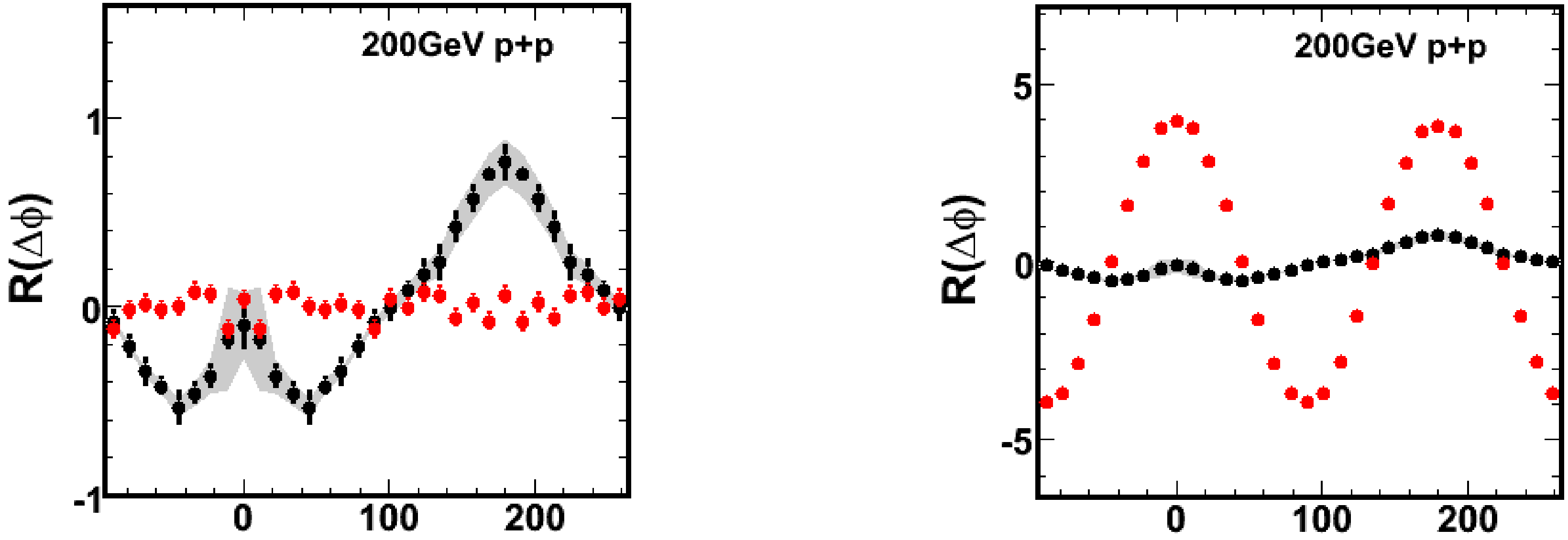
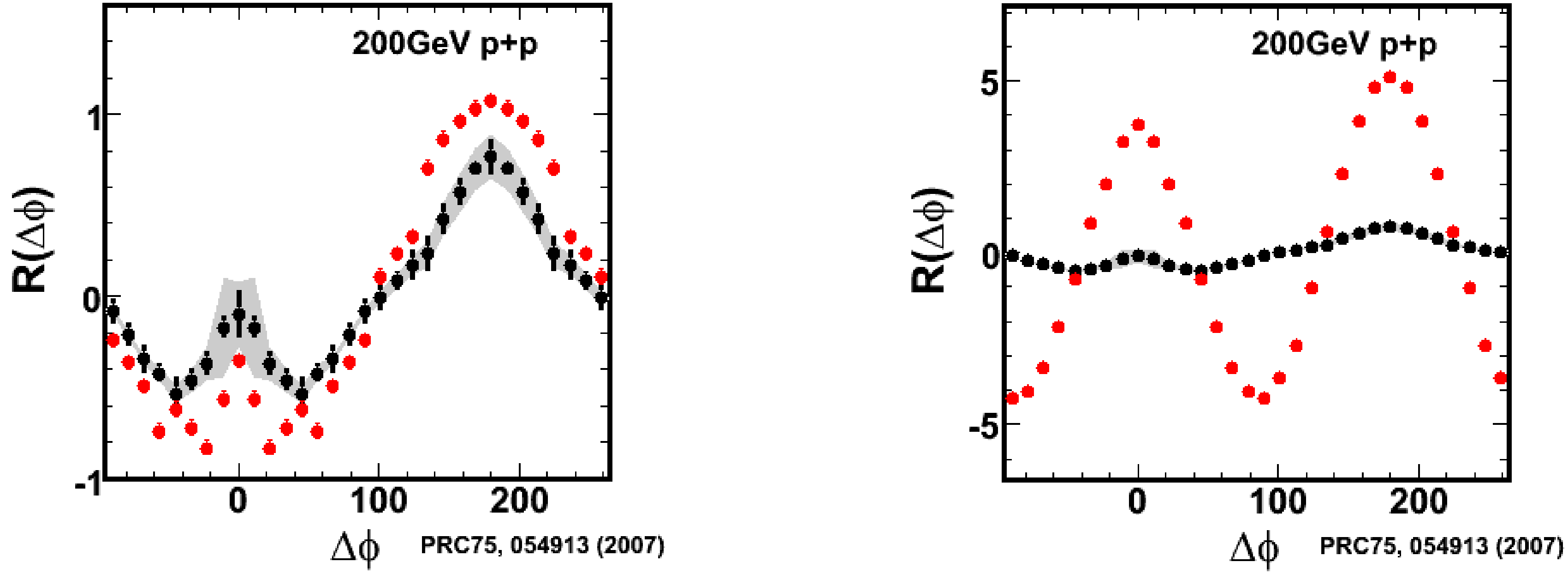
1D correlation function Cluster Model

Cluster MC w/o corr.   Cluster MC w/o corr.



Cluster MC w/ corr.   Cluster MC w/ corr.



PRC75, 054913 (2007)   PRC75, 054913 (2007)

Properties of the Sample Model shown

- Clusters produced independently
  - $<p_T>$=0.9GeV
  - Type 1 produced with probability = 0.31, Mass=0.9, Number of decay products=3
  - Type 2 produced with probability = 0.31, Mass=0.7, Number of decay products=2
  - Type 3 produced with probability = 0.31, No decay
  - Type 4 produced with probability = 0.07, Mass=0.3, Number of decay products=2
  - p_z distribution adjusted to match $dN/d\eta$ in data.
  - Note : This is a sample model. These properties can be changed to obtain different correlation structures.
- Clusters decay to a certain kinematics with a probability proportional to the available phase space
- Global Momentum Conservation
  - Moving to the center of mass frame of the clusters