Bulk viscosity of QCD matter near the critical temperature

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Nuclei as heavy as bulls through collision generate new states of matter

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3. Since N=4 SUSY YM is exactly conformally invariant the corresponding matter has vanishing bulk viscosity $\zeta=0$. However, this is not necessarily true for QCD matter which conformal invariance is broken by quantum fluctuations.

4. Fortunately, we can determine a non-perturbative QCD contribution to the bulk viscosity ζ without invoking any exotic theories.

Kubo formula for bulk viscosity

$$\eta(\omega)\left(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}-\frac{2}{3}\delta_{ik}\delta_{lm}\right)+\zeta(\omega)\delta_{ik}\delta_{lm}=\frac{1}{\omega}\lim_{\mathbf{k}\to\mathbf{0}}\int\int_0^\infty e^{i(\omega t-\mathbf{kr})}\langle[\theta_{ik}(t,\mathbf{r}),\theta_{lm}(0)]\rangle dt d^3x$$

where $\theta_{ik}(x)$ is the operator of the stress tensor.

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Contracting *i,k* and *l,m* (*i=1,2,3)* we get in the static limit

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Indeed $\langle[\int d^3x\,\theta_{00},O]\rangle_{\rm eq}=\langle[H,O]\rangle_{\rm eq}=i\left\langle\frac{\partial O}{\partial t}\right\rangle_{\rm eq}=0$

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We will calculate this object in QCD

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Low-energy theorems (LET) in vacuum

Novikov, Shifman, Vainshetein, Zakharov 1981

- Conformal symmetry of QCD (@ m=0) is broken by vacuum fluctuations. However, there is still a certain unbroken symmetry which manifests itself as a set of LET. Consider an operator of canonical dimension d:

$$\langle O \rangle_v \sim \left[M_0 e^{-\frac{8\pi}{b\, g^2(\mu)}} \right]^d \qquad \text{(by RGE)}$$

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LET in vacuum (cont.)

$$i \lim_{q \to 0} \int dx\, e^{iqx} \langle T[O(x), \frac{\beta(\alpha_s)}{4\alpha_s} F^2(0)] \rangle = (-d) \langle O \rangle_v$$

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- Differentiating n times we can derive LET for Green's function of n'th order.

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Note: Coupling constant does not explicitly appear in LET →
LET contain a non-perturbative information about the correlation functions.

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Effective Dilaton Lagrangian

- LET can be saturated by a single scalar field χ

Migdal, Shifman, 1982

$$L = \frac{|\epsilon_v|}{m^2}\,\frac{1}{2}\,e^{\chi/2}\,(\partial_\mu\chi)^2 \;+\; |\epsilon_v|\,e^{\chi}\,(1-\chi)$$

$$\theta^\mu_\mu \;=\; -\,4\,|\epsilon_v|\,e^{\chi}$$

- The field χ is referred to as the *dilaton.* In gluodynamics it corresponds to the scalar glueball. In the real world, it mixes up with light quarks to produce the σ-meson.

LET at finite temperature

Ellis, Kapusta, Tang 1998
Shushpanov, Kapusta, Ellis 1999

- At finite temperature there is an additional dimensional parameter T. The grand potential Ω per unit volume can be written in imaginary time formalism as

$$\Omega = -T \ln Z = -T \ln \int \mathcal{D}\tilde{A}_a^\mu \exp\left(-\frac{1}{4g^2} \int_0^{1/T} d\tau \int d^3x \, \tilde{F}_{\mu\nu}^2 \tilde{F}^{a\mu\nu} \right)$$

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- Differentiating with respect to (-1/4 g^2) we obtain

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$$\mathcal{E} - 3P$$

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Sum rule for the spectral density

Kharzeev, KT 2007

- ✳ Note, that on the lattice one computes not $<\theta_{\mu\mu}>_T$ but, $<\theta_{\mu\mu}>_T$ - $<\theta_{\mu\mu}>_0$ (subtracting the vacuum expectation value), i.e.

$$(\mathcal{E} - 3P)_{\rm LAT} = \langle \theta^\mu_\mu \rangle_T - \langle \theta^\mu_\mu \rangle_0$$

- ✳Denote $\langle \theta^\mu_\mu \rangle_0 = -4|\epsilon_v|$

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- The following exact sum rule holds

$$2\int_0^\infty \frac{\rho(u,\vec{0})}{u}\,du = -\left(4 - T\frac{\partial}{\partial T}\right)\langle \theta \rangle_T = T^5 \frac{\partial}{\partial T}\frac{(\mathcal{E} - 3P)_{\mathrm{LAT}}}{T^4} + 16|\epsilon_v|$$

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Extracting the bulk viscosity

- In order to extract bulk viscosity we need an ansatz for the spectral density ρ

  ✳ In pQCD (high frequencies) $\rho(\omega) \sim \alpha_s^2 \omega^4$. This divergent part is subtracted on both sides of the sum rule.

  ✳ At small frequencies we assume the following functional form which is odd in ω and has correct ω→0 limit:

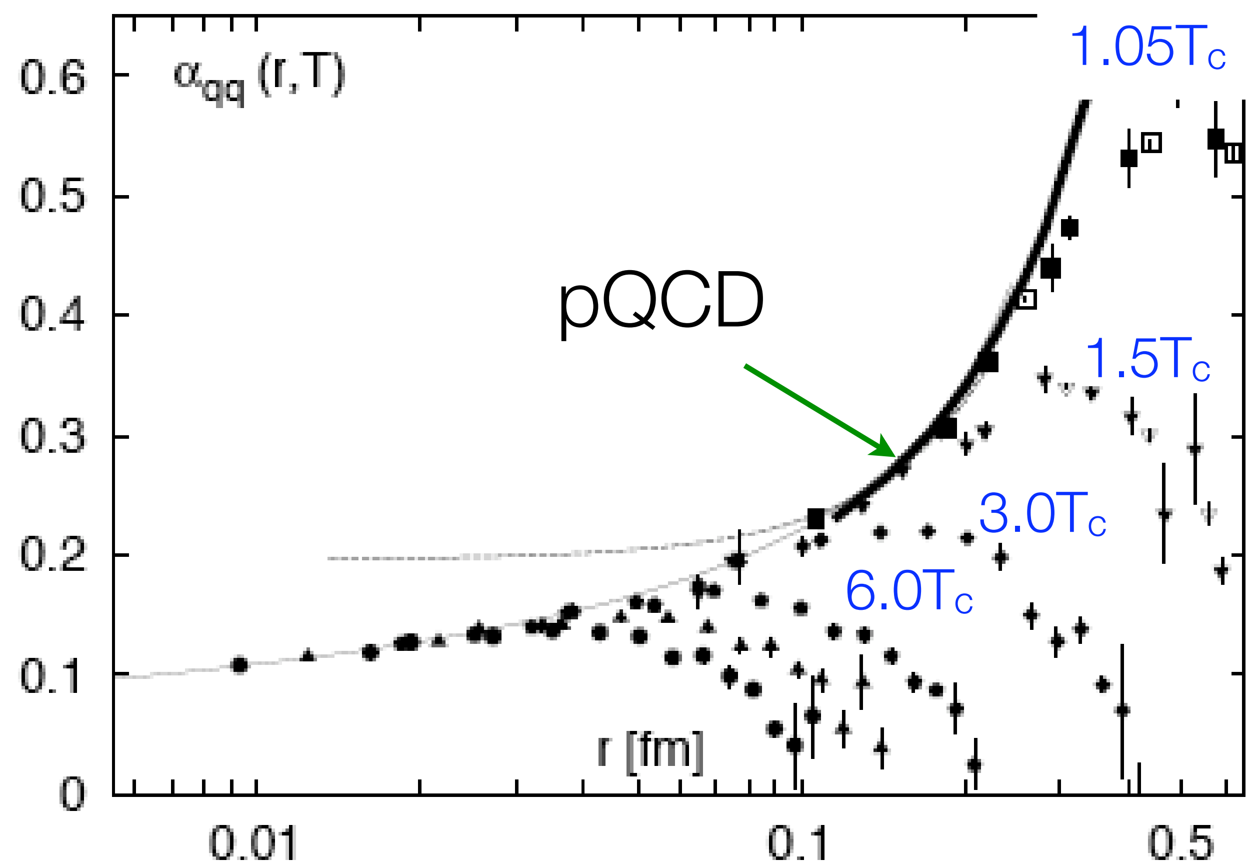
$$\frac{\rho(\omega,\vec{0})}{\omega} = \frac{9\,\zeta}{\pi}\frac{\omega_0^2}{\omega_0^2+\omega^2}$$

- We have

$$\zeta = \frac{1}{9\,\omega_0}\left\{T^5\frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\rm LAT}}{T^4} + 16|\epsilon_v|\right\}$$

Extracting the bulk viscosity (cont.)

- Parameter ω_0 is a scale at which the perturbation theory becomes valid.

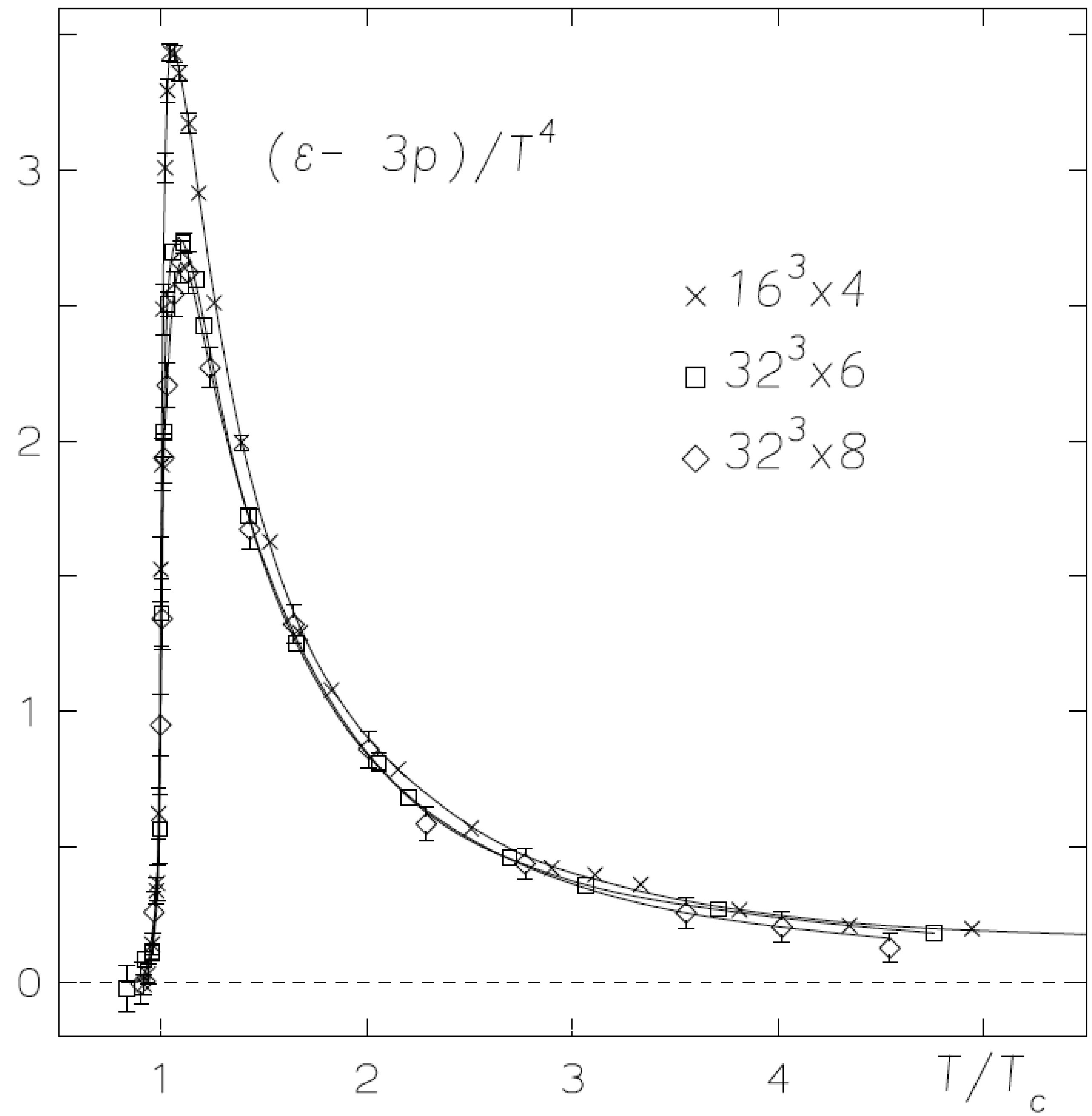


Kaczmarek, Karsch, Zantow, Petreczky, 2004

- In the region $1 < T/T_c < 3$ we find $\omega_0 \approx (T/T_c)$ 1.4 GeV
- T_c=0.28 GeV; $|\epsilon_v|=0.62\ T_c^4$.

Lattice data

Boyd et al (Bielefeld) , 1996



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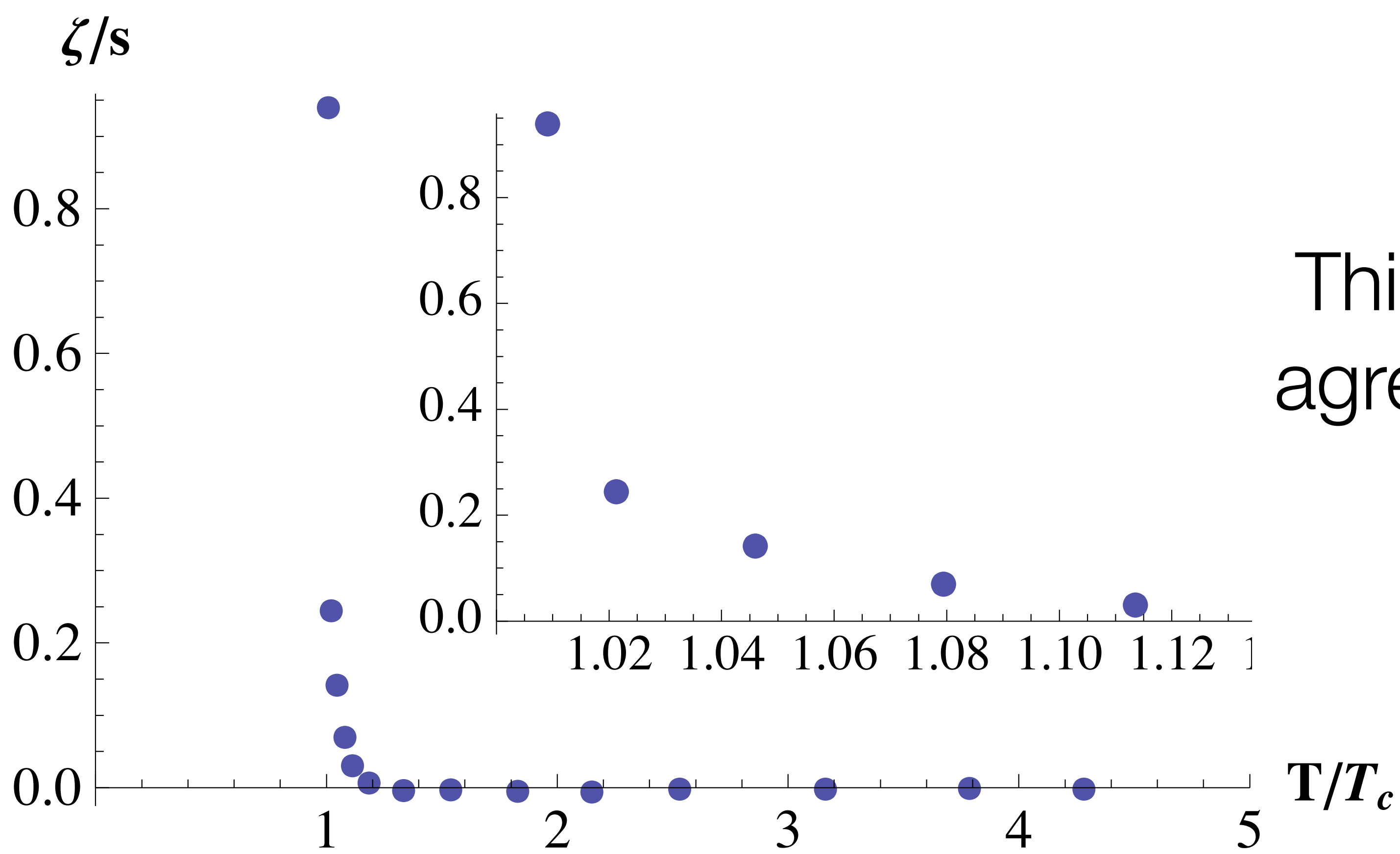
Bulk viscosity from the lattice



This result in a qualitative agreement with the recent lattice calculation.

Meyer, 2007

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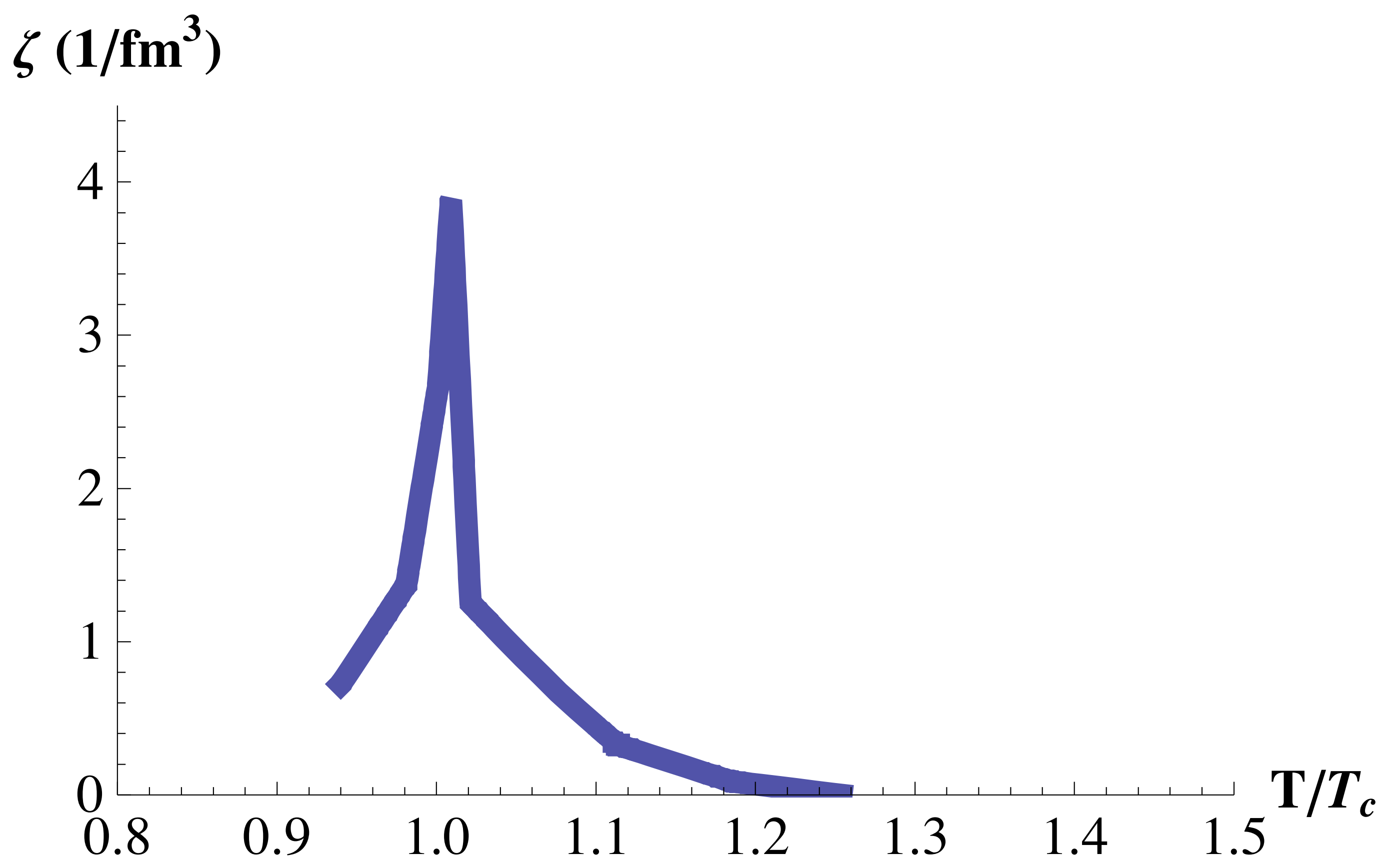

Bulk viscosity is

- small at T>>T$_c$ in accord with expectations from pQCD.
- small at T<<T$_c$ due to a derivative interactions

$$\theta^\mu_\mu = -\partial_\mu \pi^a \, \partial^\mu \pi^a + 2 m_\pi^2 \pi^a \pi^a + \cdots$$

- large at $T \approx T_c$ where it becomes the dominant correction to the ideal hydrodynamics.

see also Paech, Pratt, 2006

Implications

Bulk viscosity and relaxation processes

- In general, pressure in a moving gas or liquid P is different from the one in a static case P_0. Assuming that the deviation is small and noting that P is scalar we can write

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- Large deviation from equilibrium implies generation of a large amount of entropy: energy is dissipated in the relaxation process.

Relaxation time τ

- All relaxation processes are characterized by a common asymptotic form of time-dependence

$$\frac{dN}{dt} = \frac{N_0 - N}{\tau} \qquad \Rightarrow \qquad N(t) = N_{\rm in}\, e^{-t/\tau} + N_0(1 - e^{-t/\tau})$$

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- It follows that $\zeta = \dfrac{\tau\mathcal{E}}{1 - i\omega\tau}\,(c_\infty^2 - c_0^2)$

Sound propagation

- Consider propagation of a sound wave of frequency ω and wave vector k=ω/c, where $c^2=(\partial P/\partial \rho)$ and P=P(ρ;ω,τ).

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★ In relativistic medium $c_\infty=1/\sqrt{3}$ (no interactions)

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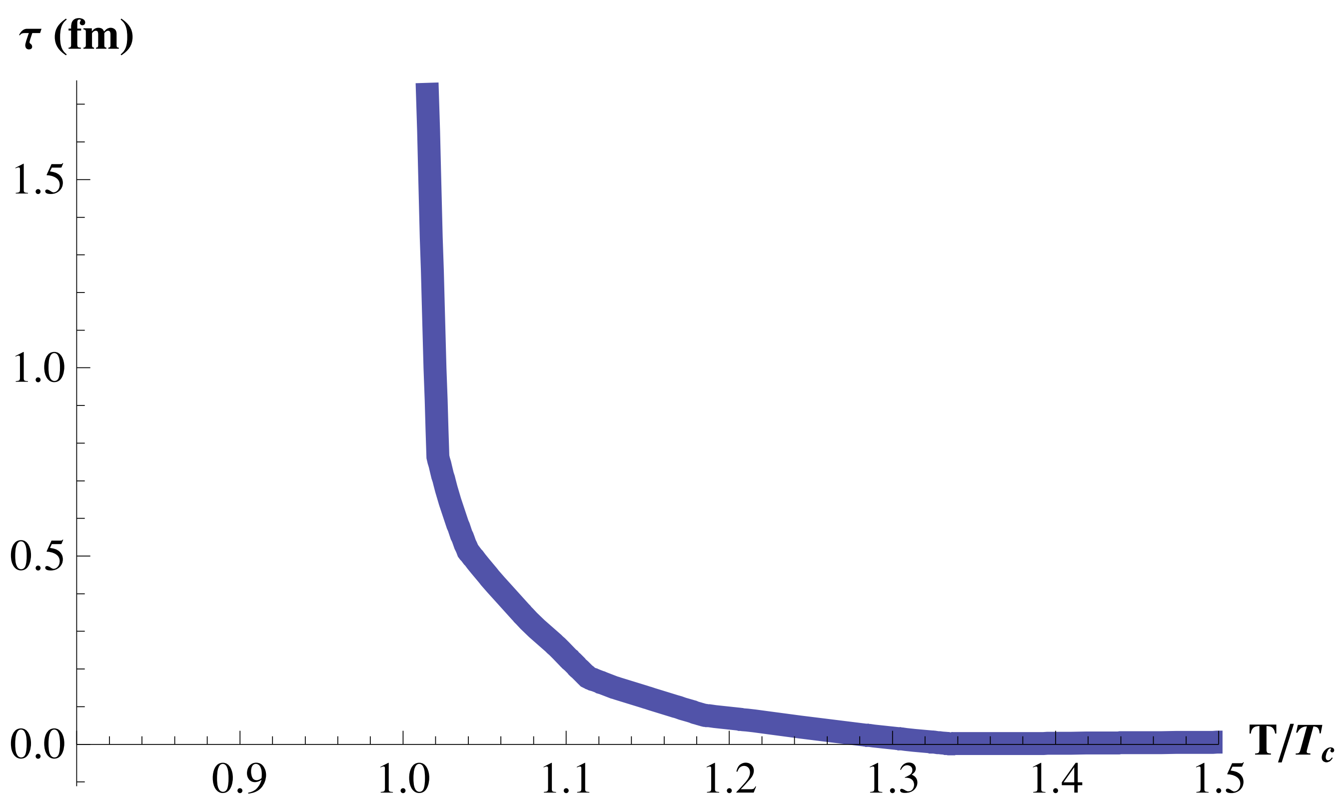
Relaxation time



- At $\omega \to 0$ (static, adiabatic case) we can use the lattice data to determine the relaxation time.

- <u>Lessons:</u>
  1. At $T \approx T_c$ relaxation processes are very slow.
  2. The system is far from equilibrium.
  3. Speed of sound is $c \approx c_\infty = 1/\sqrt{3} >> c_0$.

Dilaton excitations in QGP

1. We have demonstrated that existence of a colorless scalar excitation of the trace of energy-momentum tensor (dilaton) is a very important feature of QGP near T_c.

2. Unlike in vacuum where the dilaton is massive (it is a part of the scalar glueball), at finite T it becomes massless.

Propagation of a jet through QGP (*A toy model*)

- A jet propagating through the medium generates a dilaton sound wave in its wake. This is a shock wave of finite thickness ~ $\tau c_\infty = \tau/\sqrt{3}$.

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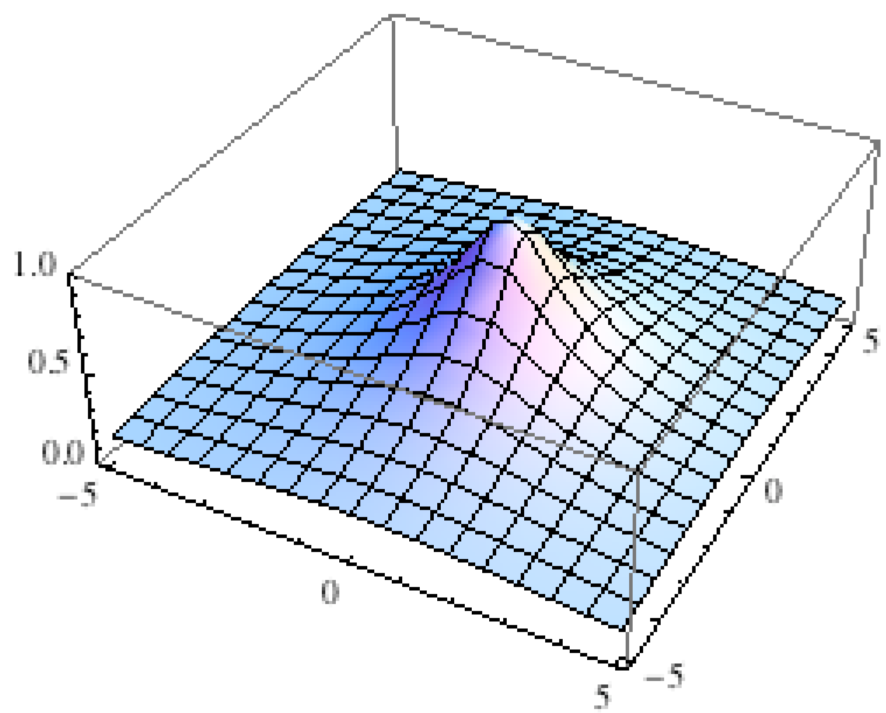

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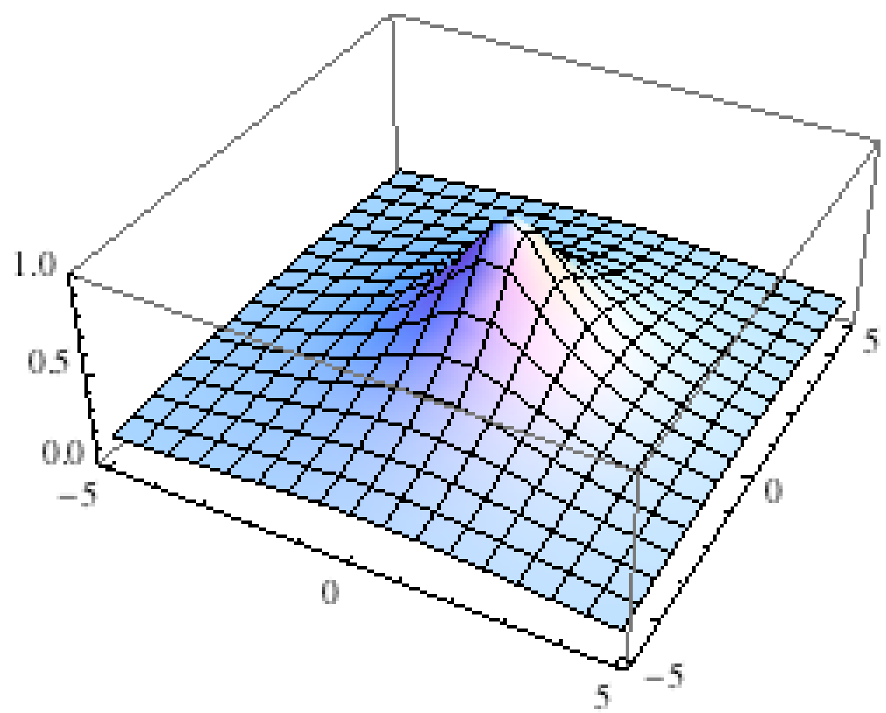

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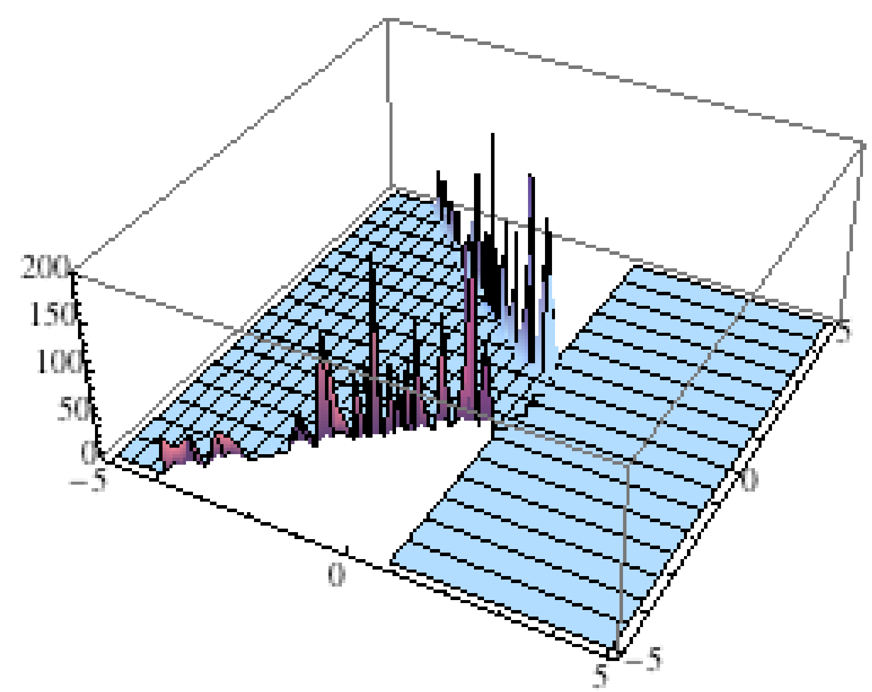

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Summary

- We derived an exact sum rule for the spectral density of $\theta_{\mu\mu}$ correlator which relates it to E-3P computed on the lattice.
- We used it to estimate the bulk viscosity in gluodynamics and found it to be large near $T=T_c$.
- A (small) contribution from light quarks will soon be calculated.
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Work in progress!