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Bayesian techniques for the calibration of 21cm global experiments

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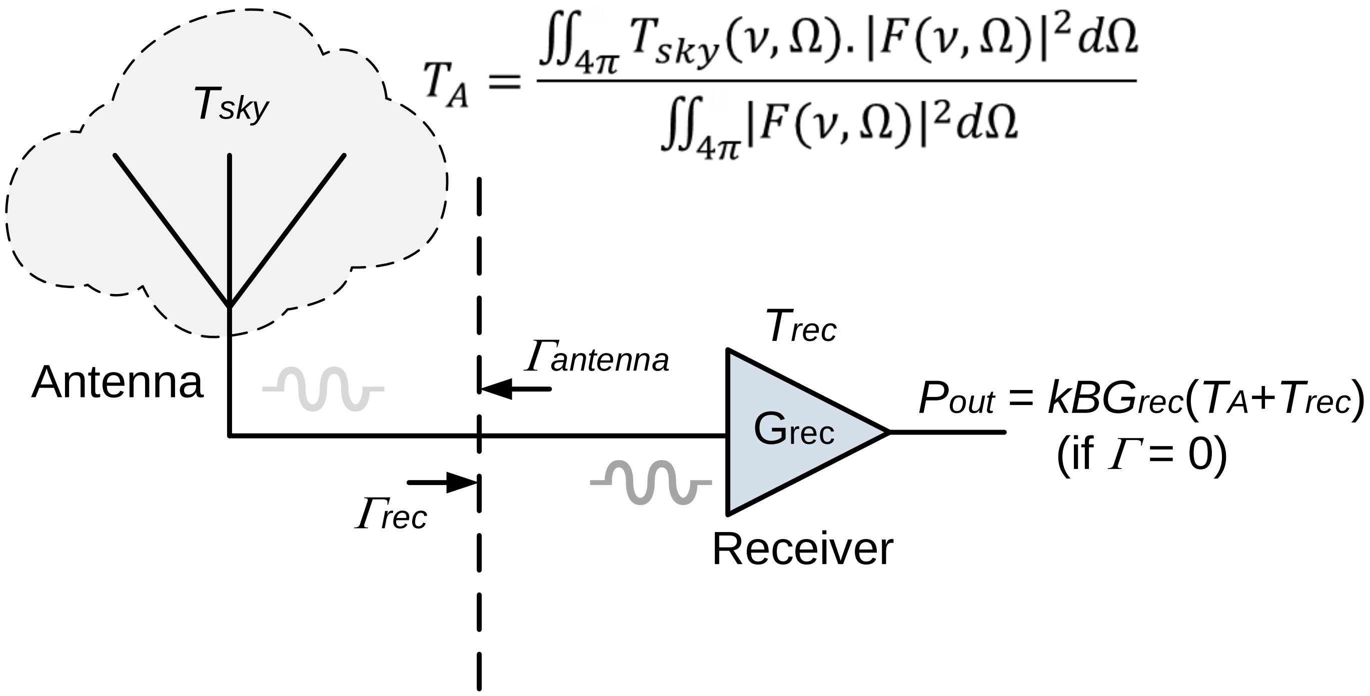
Overview

- The absolute calibration problem
- The EDGES calibration formalism
- A Bayesian framework
- Instrument model inaccuracies
- REACH radiometer
- Future rollout

The absolute calibration problem

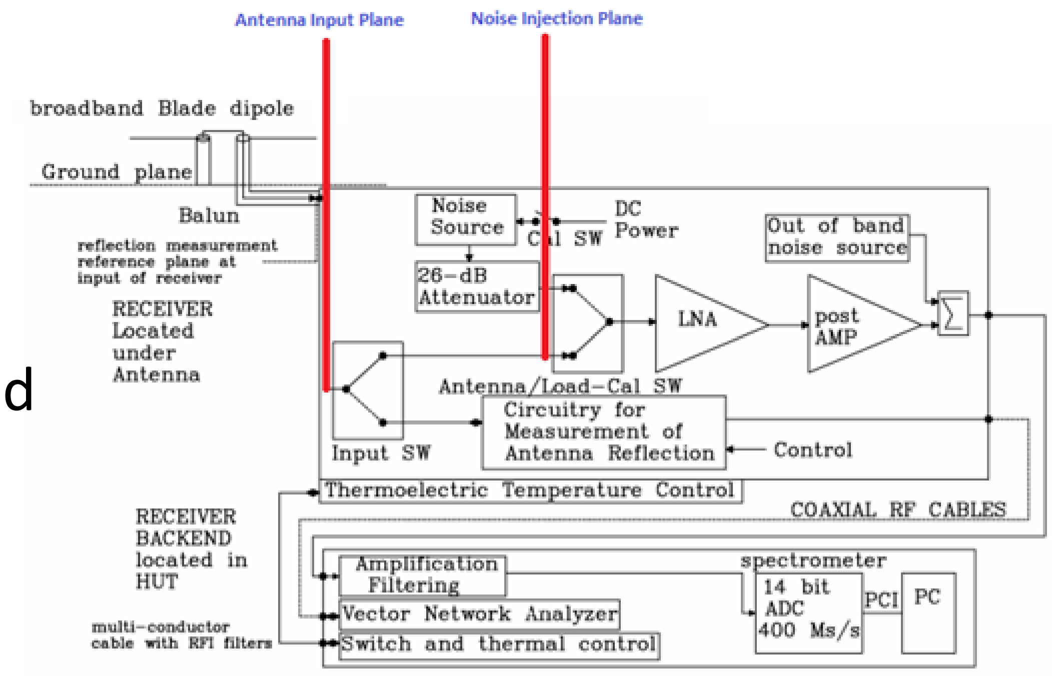
- Important to characterise the noise added by the system taking into account the antenna mismatch
- This not only will have spectral structure but also a magnitude that is many 10s of Kelvin
- It is fundamental to any such experiment to aim to characterise this to sub Kelvin levels

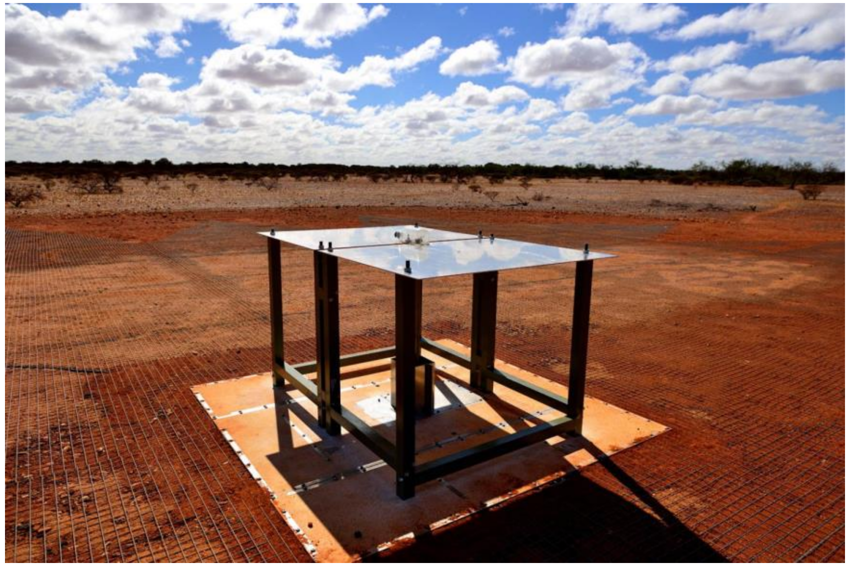
$$T_A = \frac{\iint_{4\pi} T_{sky}(\nu,\Omega).|F(\nu,\Omega)|^2 d\Omega}{\iint_{4\pi} |F(\nu,\Omega)|^2 d\Omega}$$



The EDGES system

- Relies on the noise waves formalism introduced by René Meys in 1978
- This formalism is only fully implemented for the EDGES system.
- Uses calibration sources and Dicke switching to help characterise noise waves resulting from the mismatch
- For EDGES the antenna impedance is measured in the field but most of the necessary parameters are measured in the laboratory
- Out of band noise injected for ADC calibration and temperature control of the receiver system





The calibration formalism (used by EDGES)

- The “uncalibrated” antenna temperature is formed by

$$T^*_{ant} = T_{NS}\frac{P_{ant} - P_L}{P_{NS} - P_L} + T_L$$

- here each spectra can be described as the following instrument responses including the noise wave parameters T_{unc}, T_{cos}, and T_{sin}

$$P_L = g[T_L(1 - |\Gamma_{rec}|^2) + T_0]$$

$$P_{NS} = g[(T_L + T_{NS})(1 - |\Gamma_{rec}|^2) + T_0]$$

$$P_{ant} = g[T_{ant}(1 - |\Gamma_{ant}|^2)|F|^2 + T_{unc}|\Gamma_{ant}|^2|F|^2 + T_{cos}|\Gamma_{ant}||F|\cos\phi + T_{sin}|\Gamma_{ant}||F|\sin\phi + T_0]$$

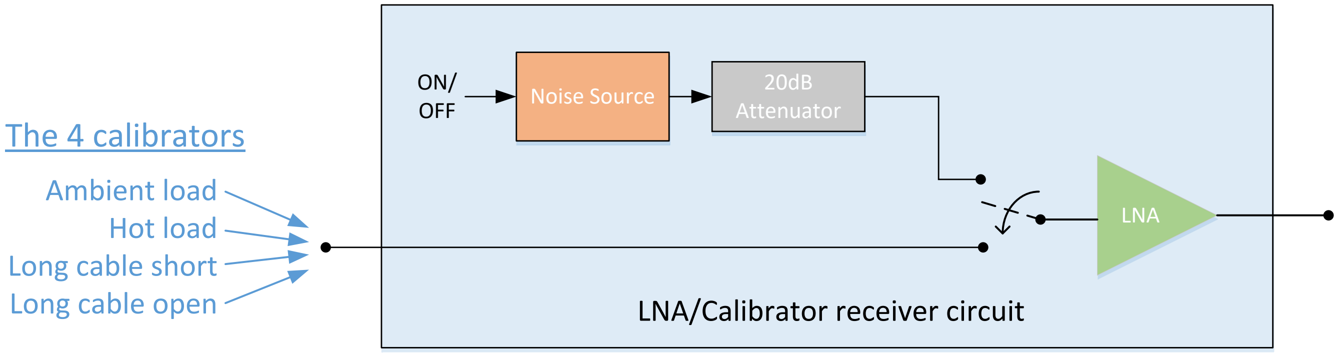
- given

$$F = \frac{\sqrt{1-|\Gamma_{rec}|^2}}{1-\Gamma_{ant}\Gamma_{rec}} \qquad \phi = \arg(\Gamma_{ant}F)$$

The calibration equation (EDGES)

$$(T^*_{\rm ant} - T_{\rm L})C_1 + (T_{\rm L} - C_2) = T_{\rm ant}\left[\frac{(1-|\Gamma_{\rm ant}|^2)\,|F|^2}{(1-|\Gamma_{\rm rec}|^2)}\right] + T_{\rm unc}\left[\frac{|\Gamma_{\rm ant}|^2|F|^2}{(1-|\Gamma_{\rm rec}|^2)}\right] + T_{\rm cos}\left[\frac{|\Gamma_{\rm ant}||F|}{(1-|\Gamma_{\rm rec}|^2)}\cos\alpha\right] + T_{\rm sin}\left[\frac{|\Gamma_{\rm ant}||F|}{(1-|\Gamma_{\rm rec}|^2)}\sin\alpha\right]$$

- Hot and cold standards are used to determine scale (C1) and offset temperature (C2)
- Cable open and short to determine frequency behaviour
- EDGES computes these calibration parameters in an iterative way starting with C1 and C2, then Tsin, Tcos and Tunc
- Contributions from noise waves should be minimal if the system is matched.



Our definitions

$$K_1 \equiv \frac{(1 - |\Gamma_{ant}|^2)|F|^2}{(1 - |\Gamma_{rec}|^2)}$$

$$K_2 \equiv \frac{|\Gamma_{ant}|^2|F|^2}{(1 - |\Gamma_{rec}|^2)}$$

$$K_3 \equiv \frac{|\Gamma_{ant}||F|}{(1 - |\Gamma_{rec}|^2)} cos\alpha$$

$$K_4 \equiv \frac{|\Gamma_{ant}||F|}{(1 - |\Gamma_{rec}|^2)} sin\alpha$$

$$K_5 \equiv \frac{P_{ant} - P_L}{P_{NS} - P_L}$$

(K1-4 based on S11, K5 based on PSDs)

$$A \equiv \begin{pmatrix} -\frac{K_2}{K_1} \\ -\frac{K_3}{K_1} \\ -\frac{K_4}{K_1} \\ \frac{K_5}{K_1} \\ \frac{1}{K_1} \end{pmatrix}$$

$$\theta \equiv \begin{pmatrix} T_{unc} \\ T_{cos} \\ T_{sin} \\ T_{NS} \\ T_L \end{pmatrix}$$

Note we replace C1 and C2 with T_{NS} and T_L

Our calibration

$$T_{ant} = A^T\theta + \sigma$$

$$\chi^2_\nu = \frac{(T_{ant} - A^T\theta)^2}{2\sigma^2}$$

$$M \equiv \sum_\nu A\,A^T \qquad b \equiv \sum_\nu T_{ant} A$$

$$P(D|\theta) = \prod_\nu \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\chi^2}$$

$$\theta = M^{-1} b$$

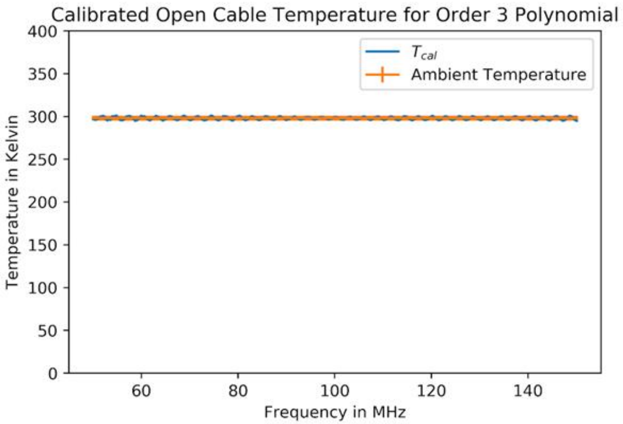
$$\sigma = \left(\frac{1}{N}\left(\sum_\nu T_{ant}^2 - bM^{-1}b\right)\right)^{\frac{1}{2}}$$

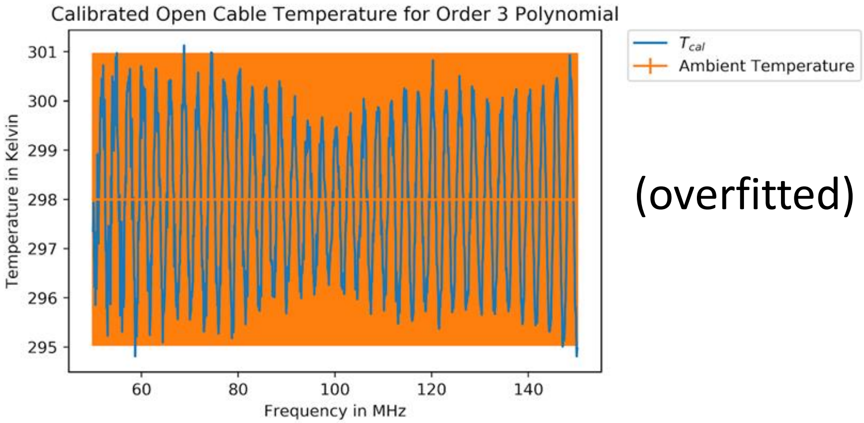
Optimisation of noise wave parameters

- E.g. estimating polynomial order using Bayes

$$\mathrm{P}(\underline{\mathbf{n}}|\mathrm{D}) = \frac{\mathrm{P}(\mathrm{D}|\underline{\mathbf{n}})\ \mathrm{p}(\underline{\mathbf{n}})}{\mathrm{P}(\mathrm{D})}$$

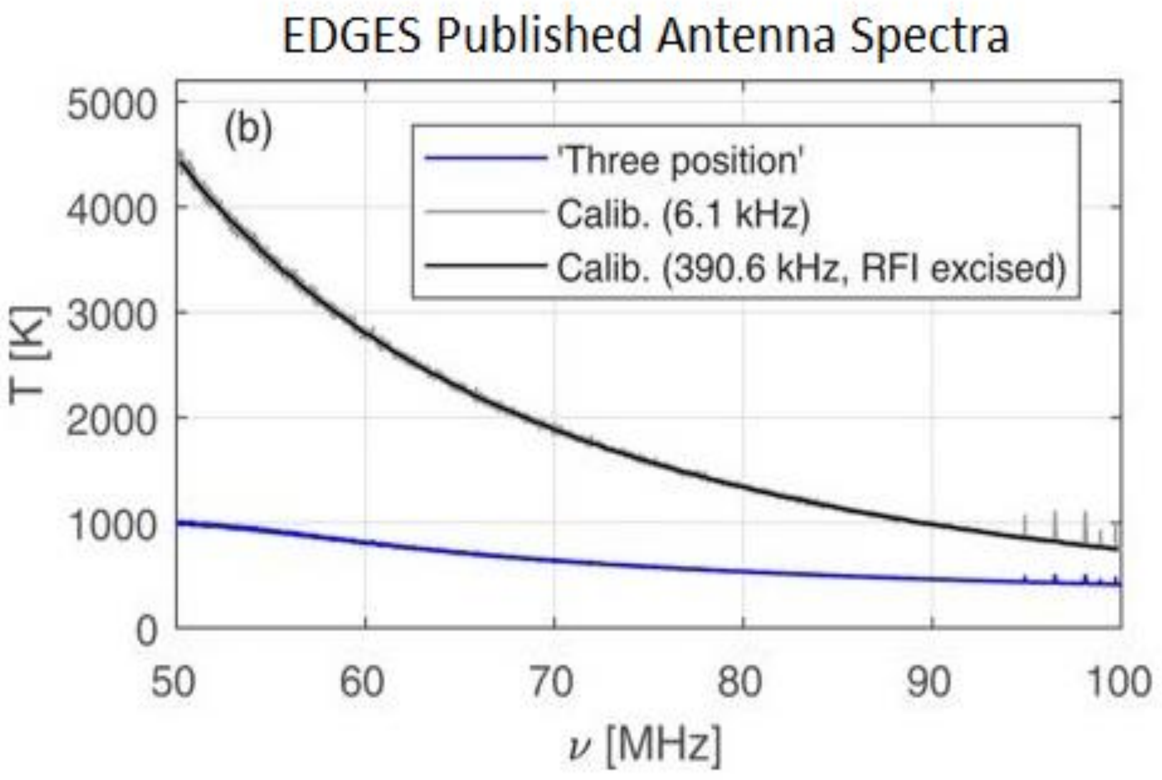
$$\mathrm{P}(\mathrm{D}|\underline{\mathbf{n}}) = \mathrm{P}(\underline{\Theta}, \sigma)\frac{\left|\frac{2\pi\sigma^2}{\mathrm{M}}\right|^{1/2}}{\sqrt{2\pi\sigma^2}^{\mathrm{N}}} 2^{\frac{\mathrm{k}-3}{2}} \mathrm{Y}^{\frac{1-\mathrm{k}}{2}} \Gamma\left(\frac{\mathrm{k}-1}{2}\right)$$

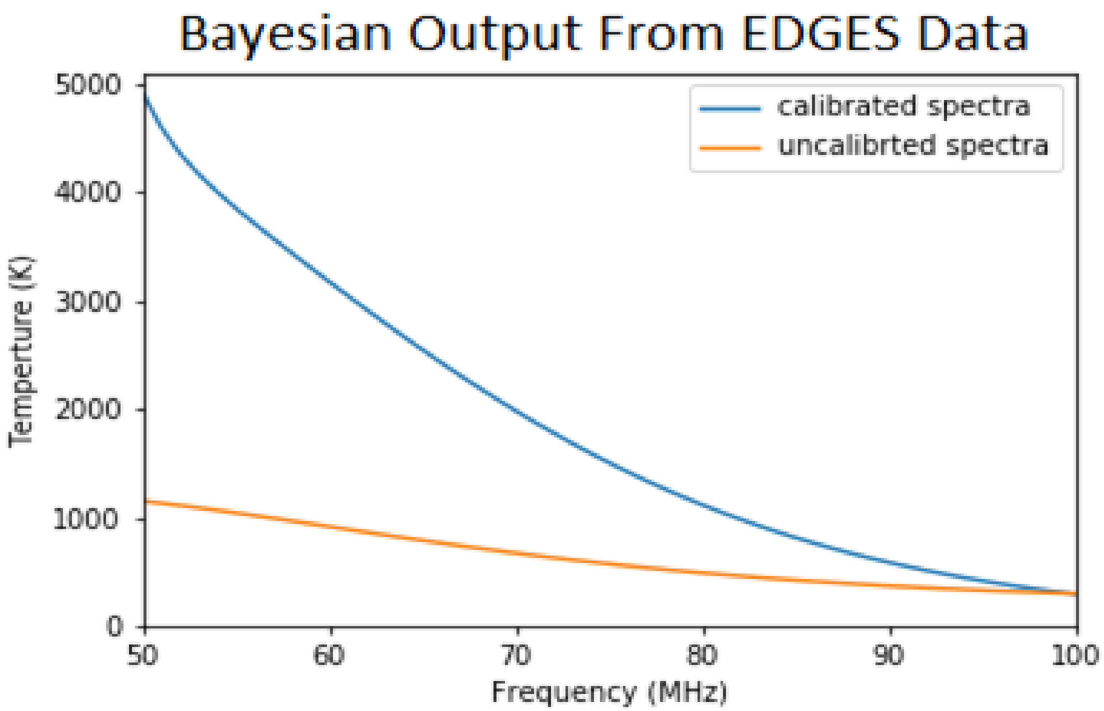




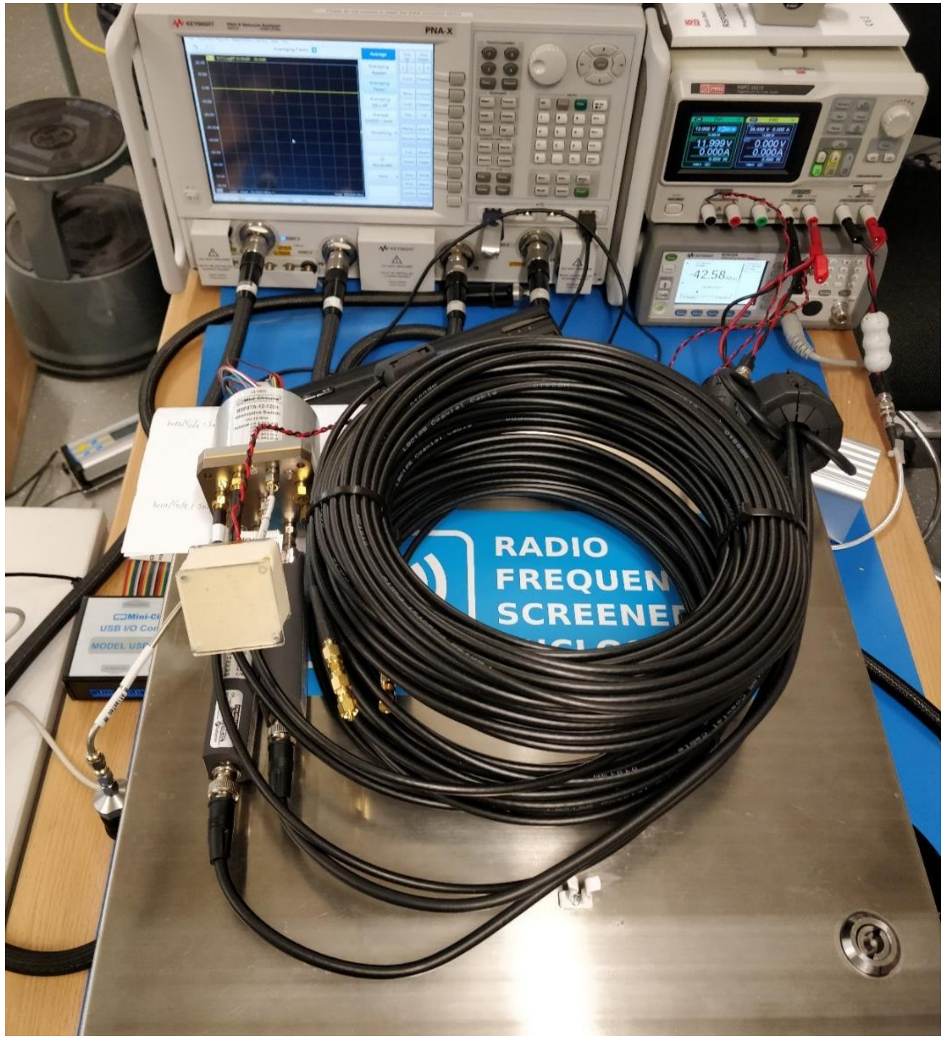
(overfitted)

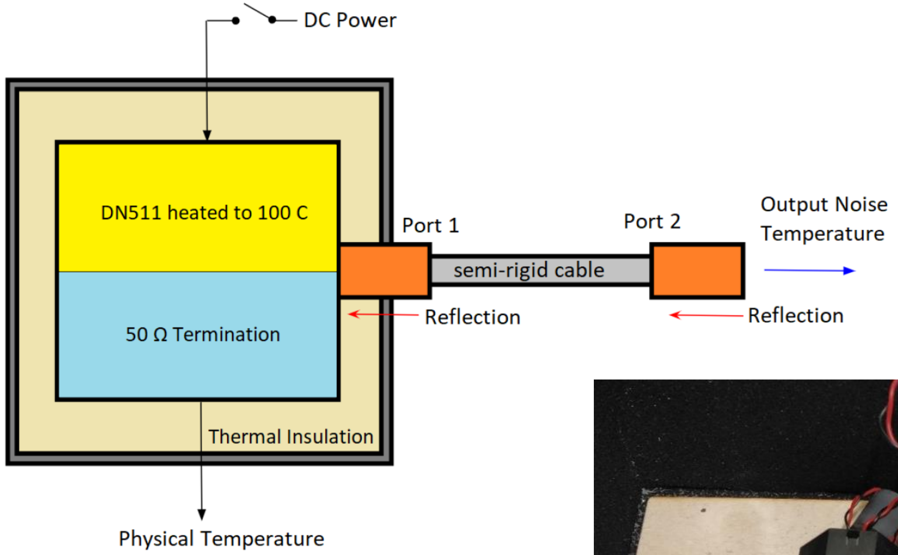
Application (using partial EDGES estimated data)





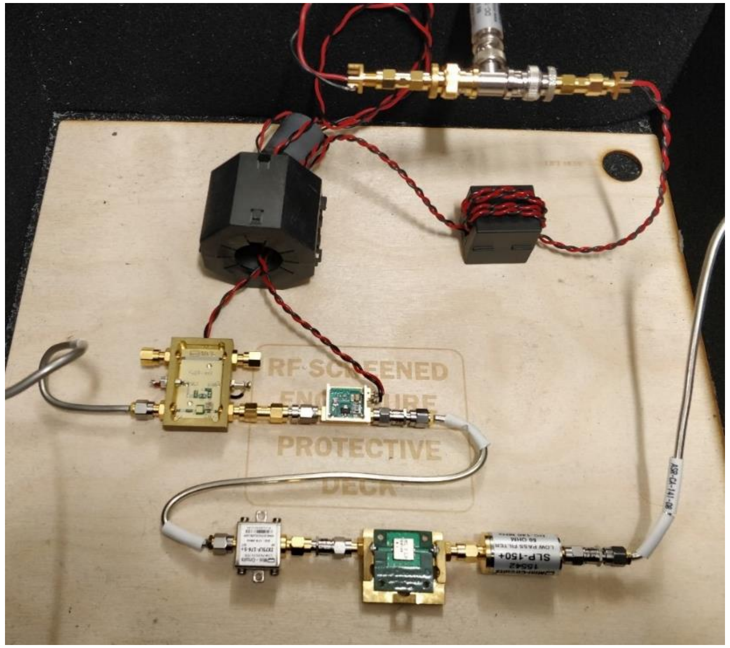
Test system used to generate calibration data



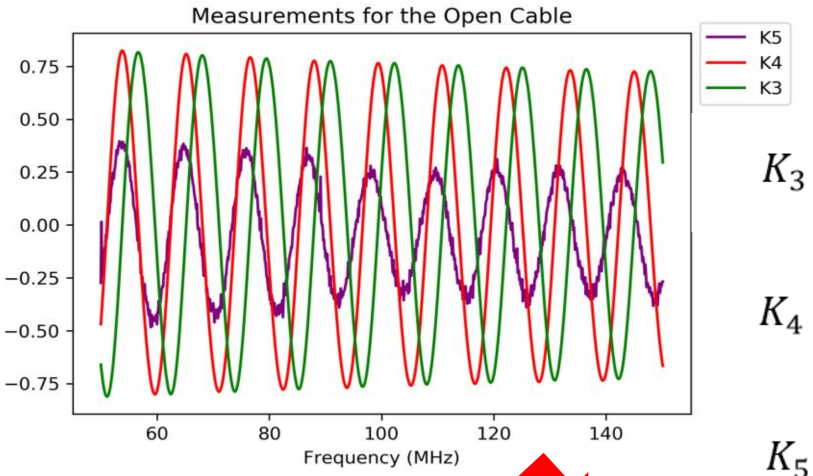


Input sources

- Noise source
- Load/Cold
- Hot (373K)
- 10m cable (open & short)
- 45m cable (open and short)



Potential model inaccuracies (shift in PSD)



$$K_3 \equiv \frac{|\Gamma_{ant}||F|}{(1-|\Gamma_{rec}|^2)} cos\alpha$$

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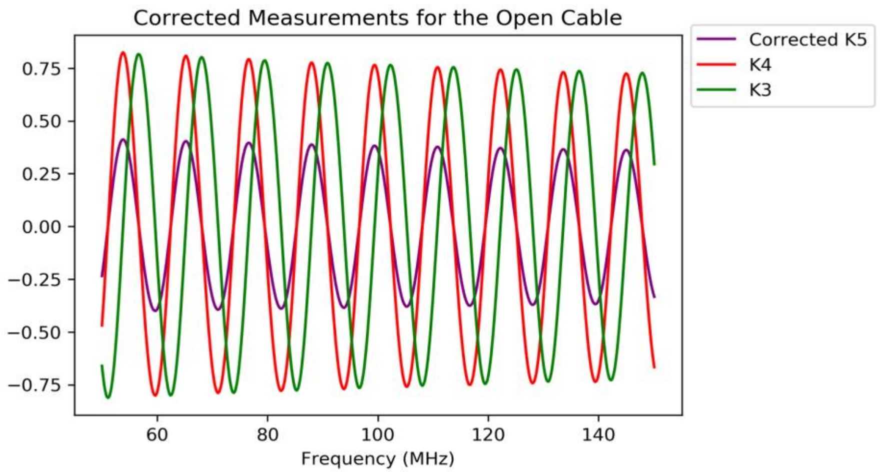
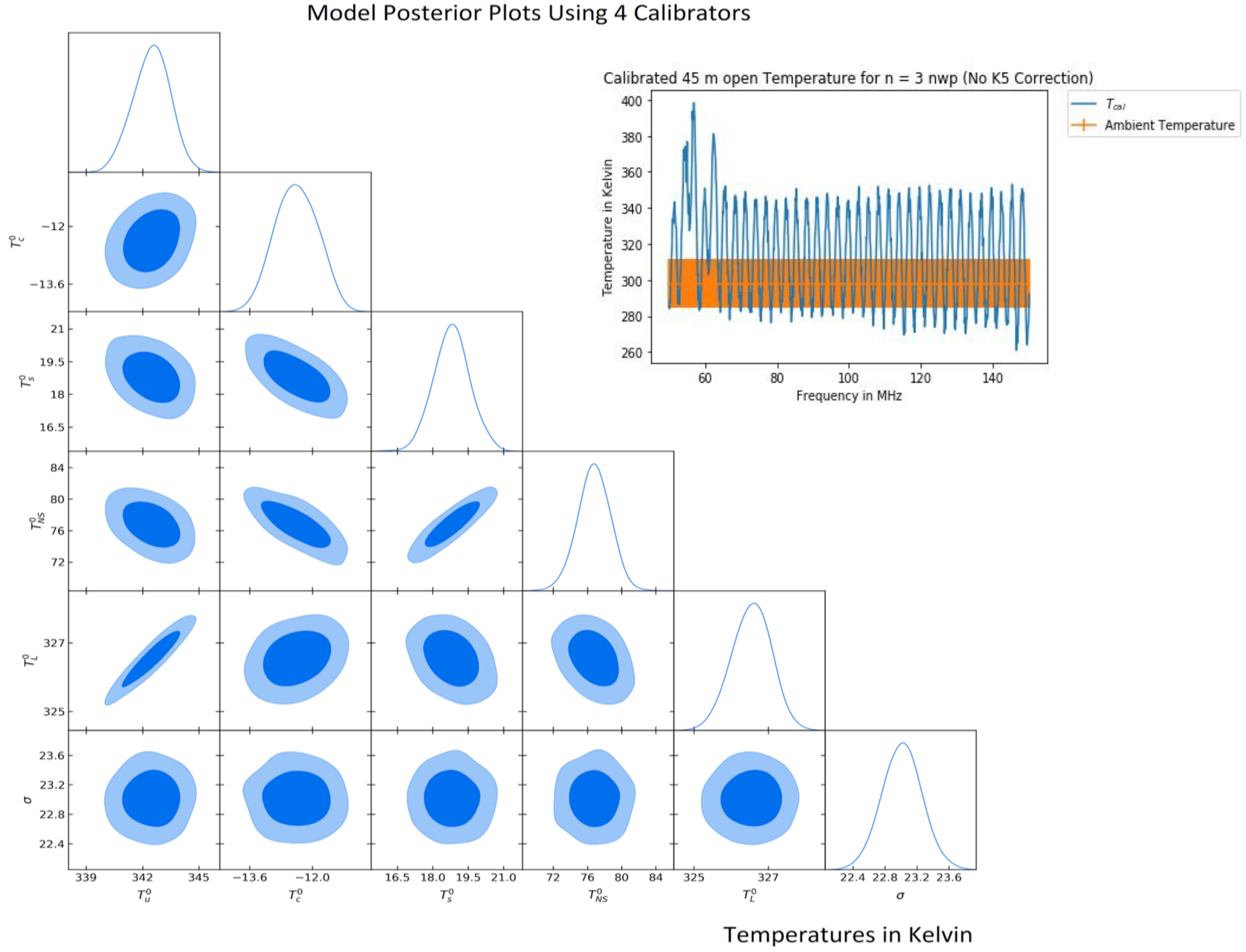
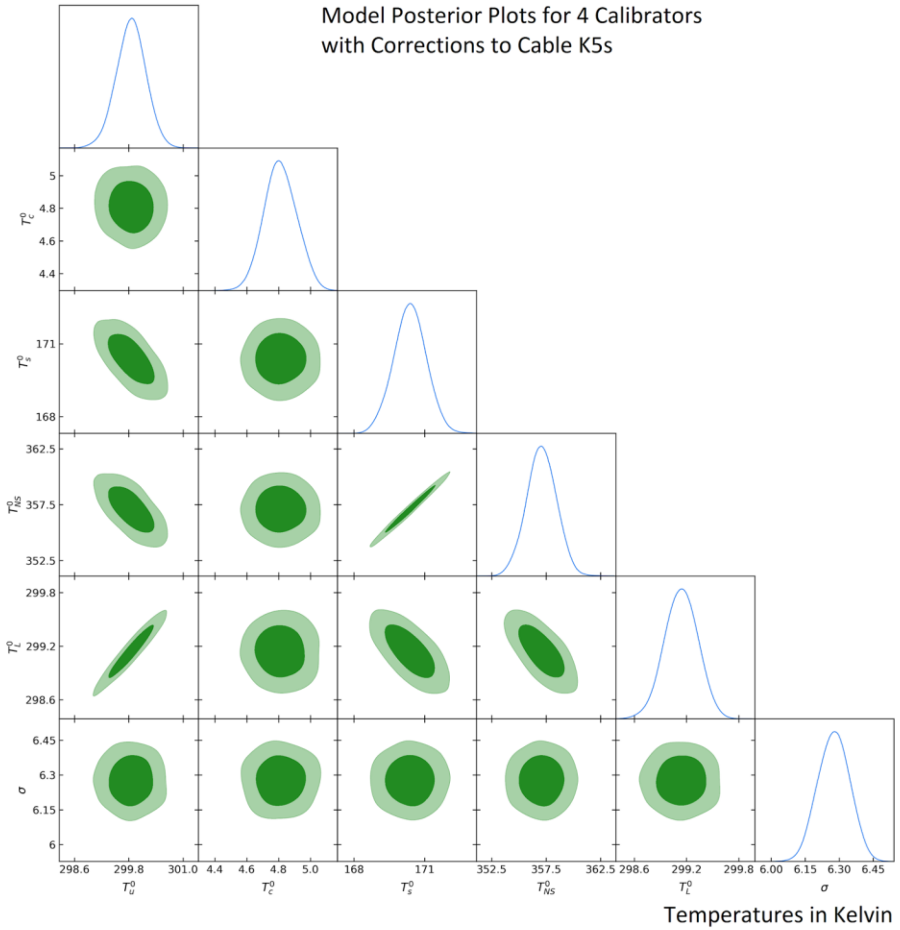


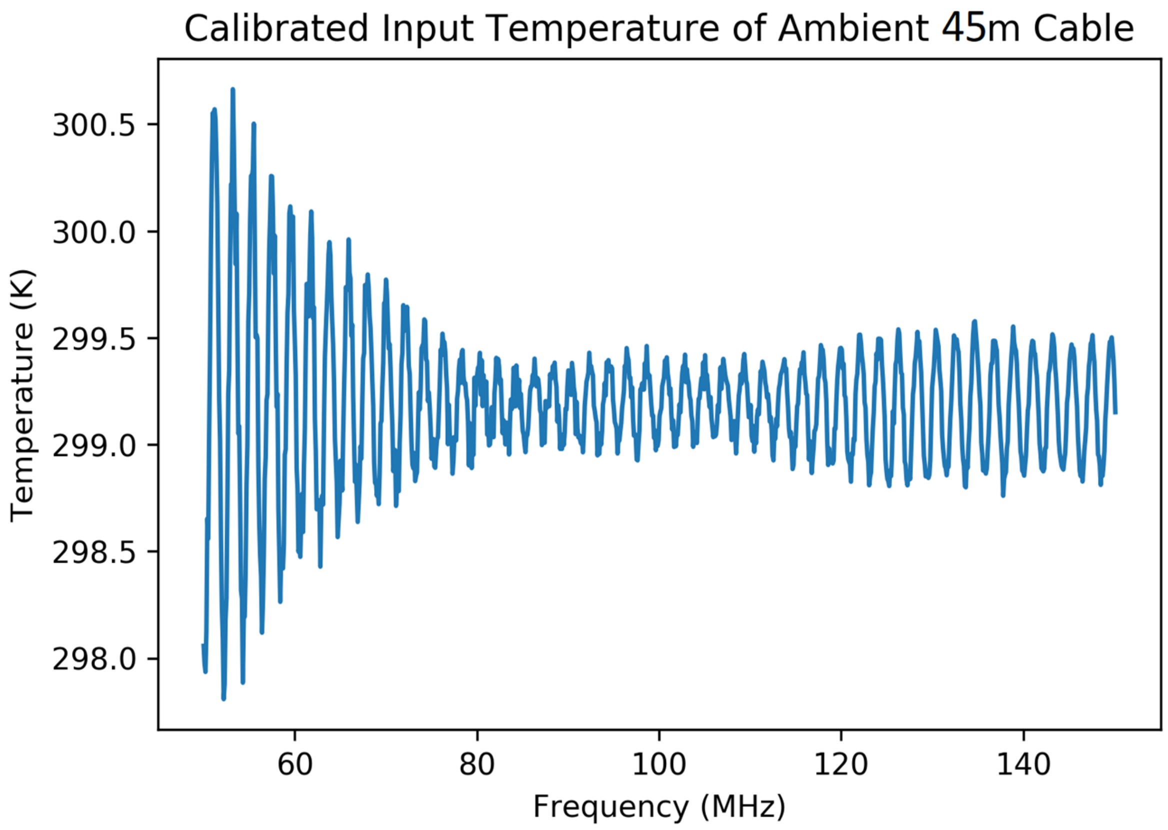
PSD frequency shift mainly encountered with using 10m cables



Corrected results

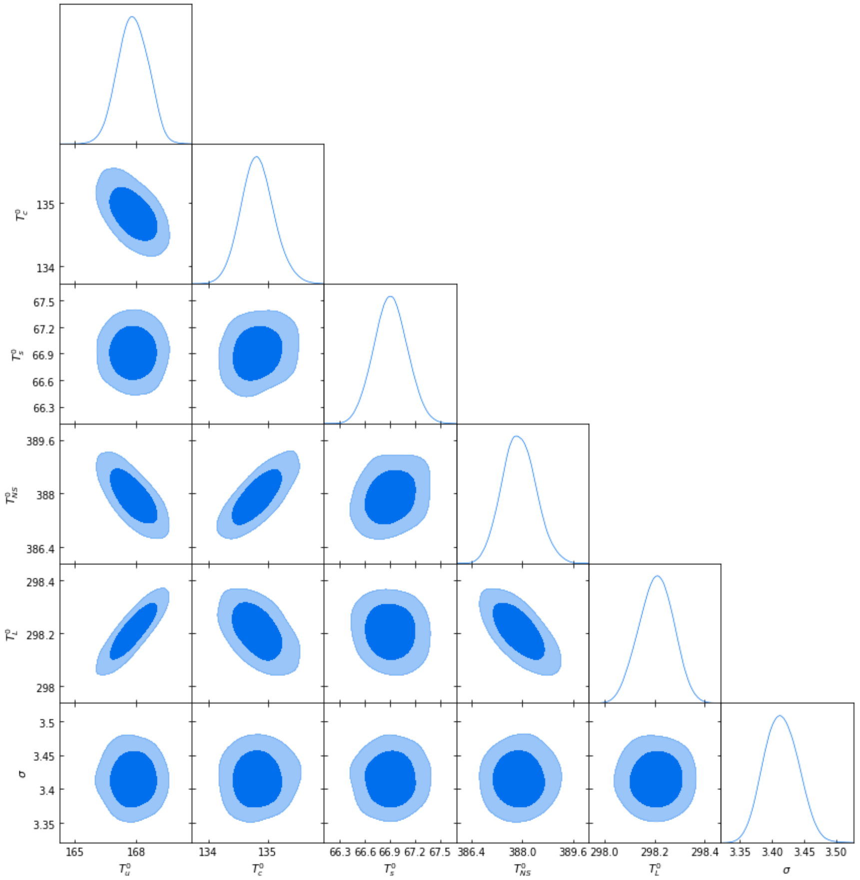


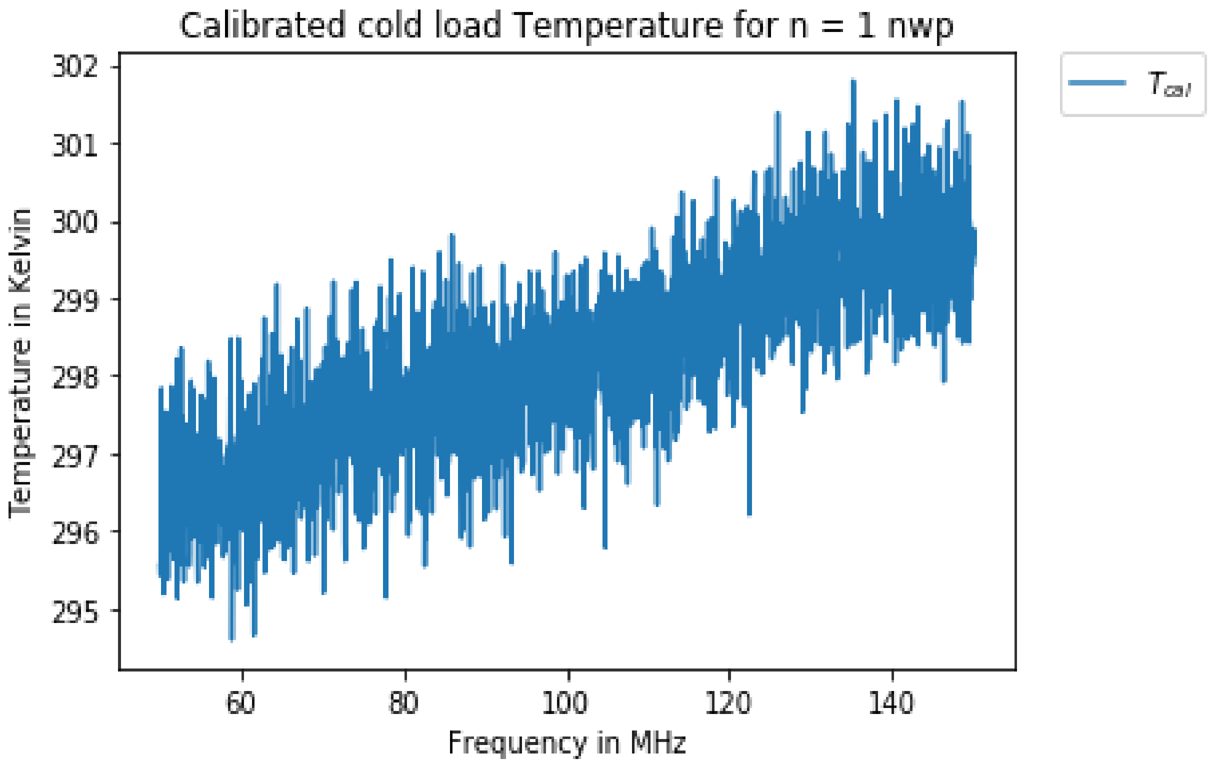
Much better fit to expected response



Calibration using longer cables (45m)

Results are more reasonable with the use of 45m open and short cables

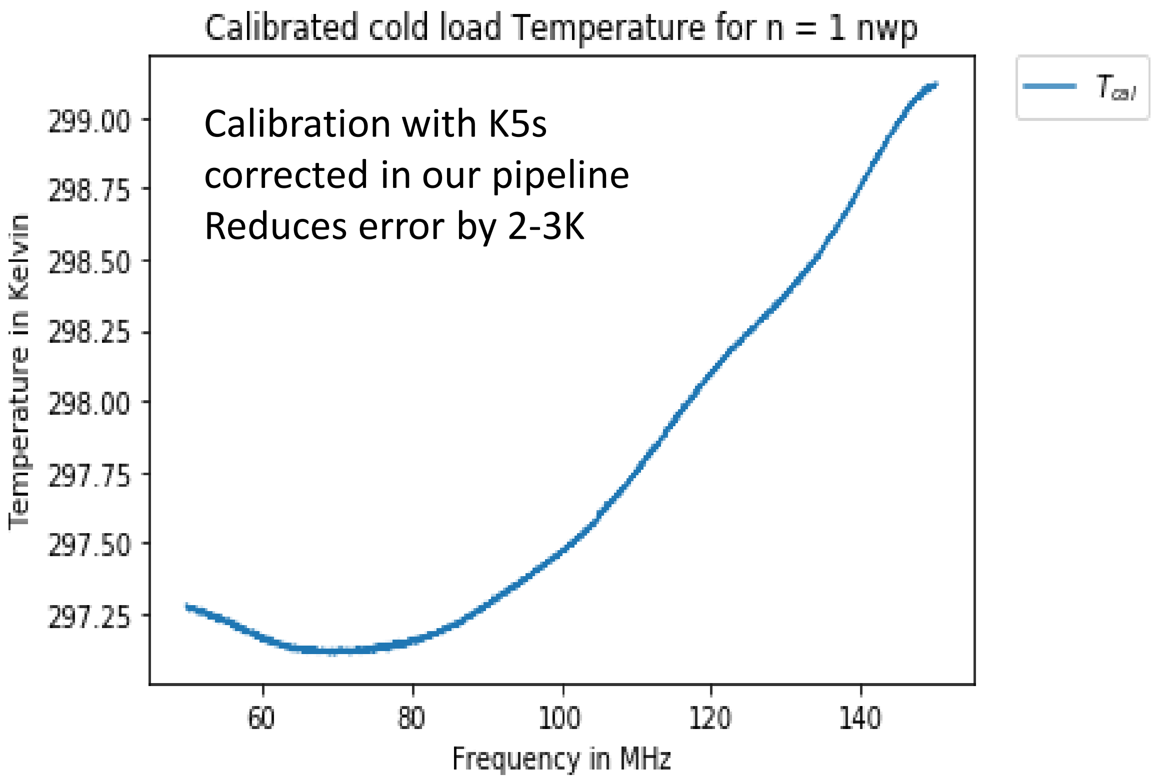




PSD shift corrections (in K5)

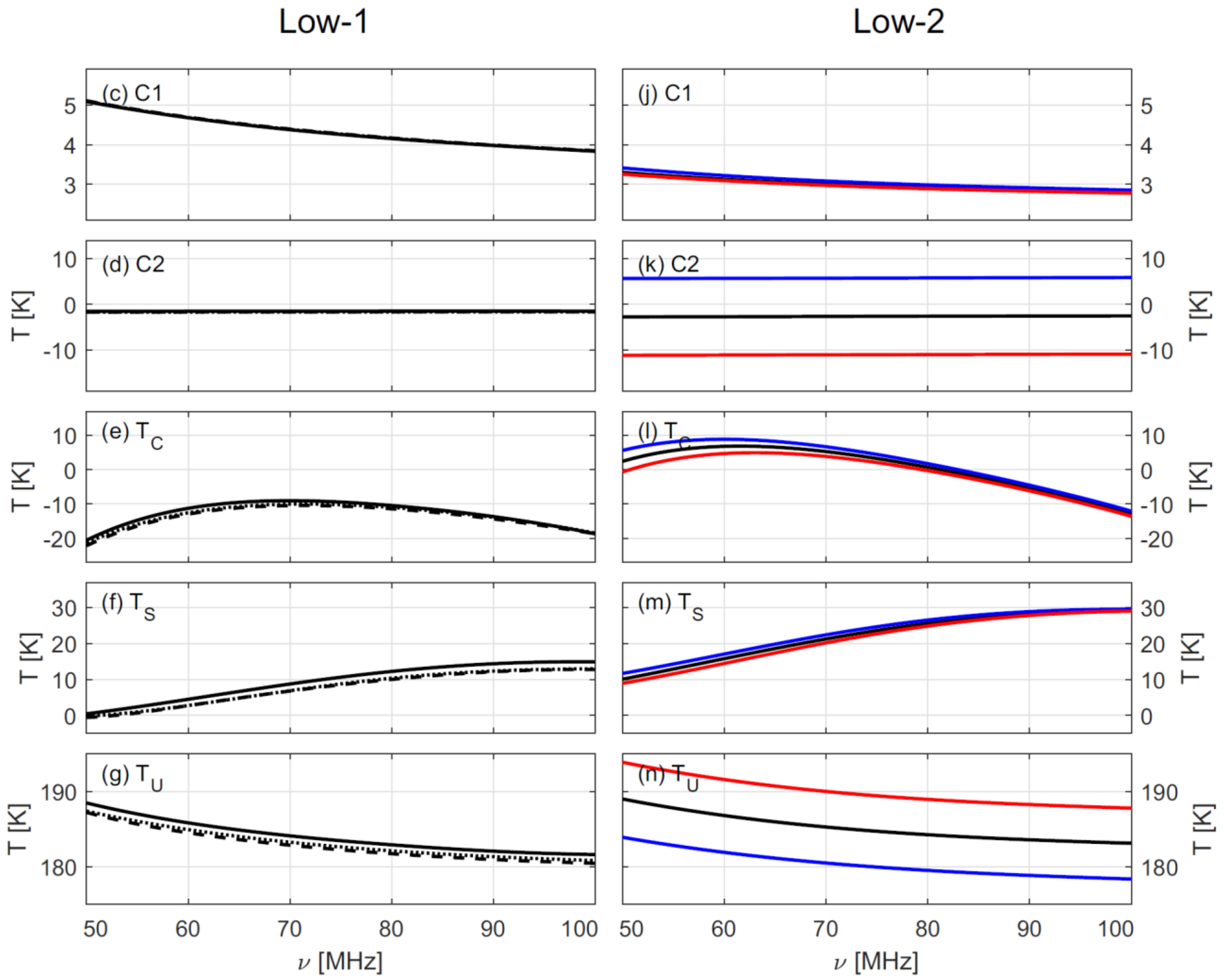
Even though the posteriors are better with the use of 45m cables, shift corrections to K5 make the analysis better so this phenomenon still needs investigation.

Possible causes of this problem
- LNA non-linearities
- Missing cable calibration
- Measurement issues
(e.g. slower sweep needed on the VNA)
- Model inaccuracies



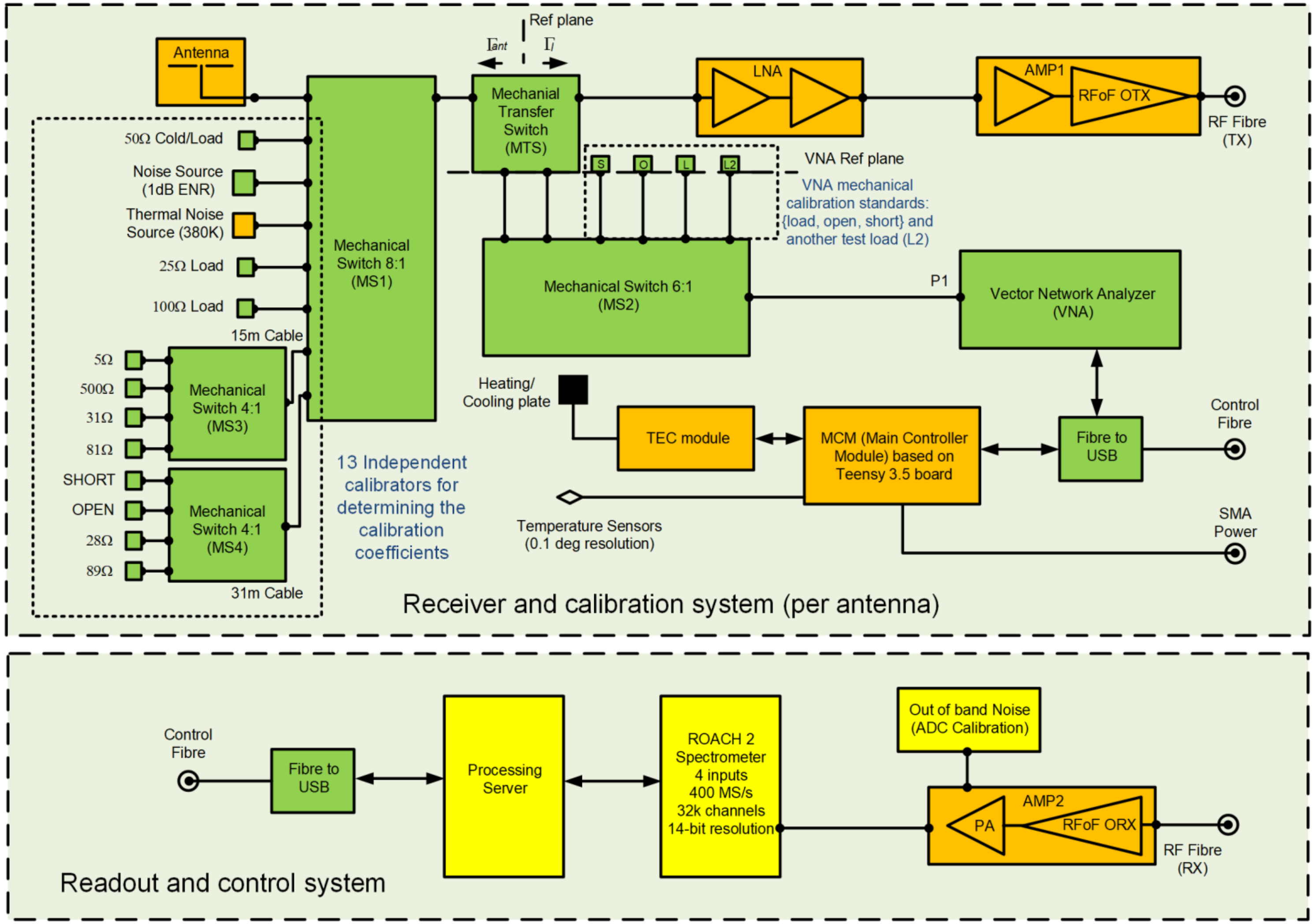
Interesting takeaways on the EDGES calibration

- Based on preliminary measurements, the calibration equation is not doing a good enough job of fitting for the receiver parameter for us (measured using different VNAs, and different LNAs)
- If match is good/reasonable, C1 (scale) ~ 1, C2 (offset) ~ 0, or in our case TL ~ 300K, TNS ~ 370K (excess)
- Potential for a residual reflections in the calibrated data which has similar structure to the open/short cables used (e.g. 8m cable ~ 12.5MHz)



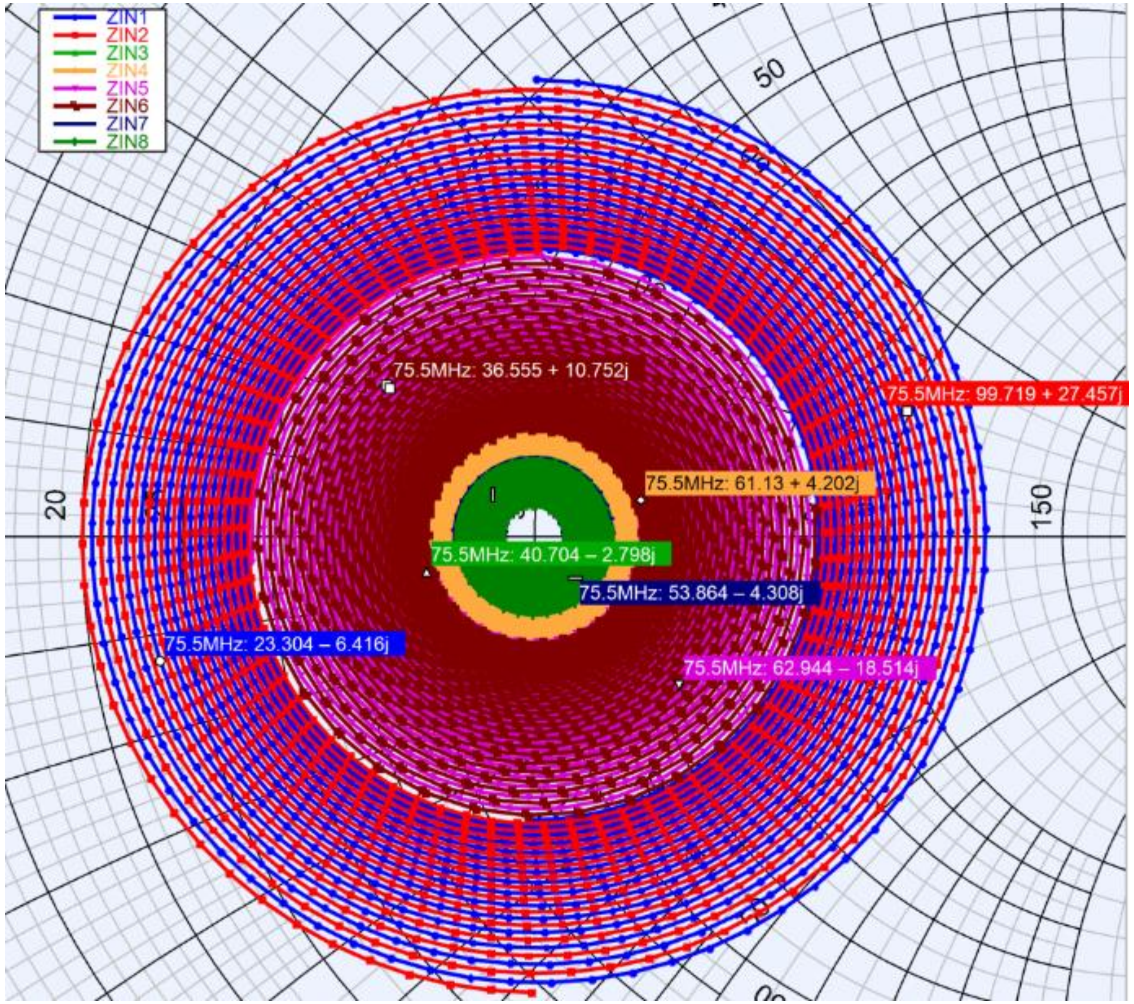
REACH radiometer

- All calibration to be done in the field and relying on no lab data
- Antenna, LNA and 13 source s-parameters measured with a highly accurate VNA
- Input reference plane the same for antenna and other sources
- Signal and control via fibre. The only galvanic connection to the receiver is power
- Calibration pipeline evaluates all parameters at once
- PID controller for constant temperature
- RX box 50 x 50 x 20 cm

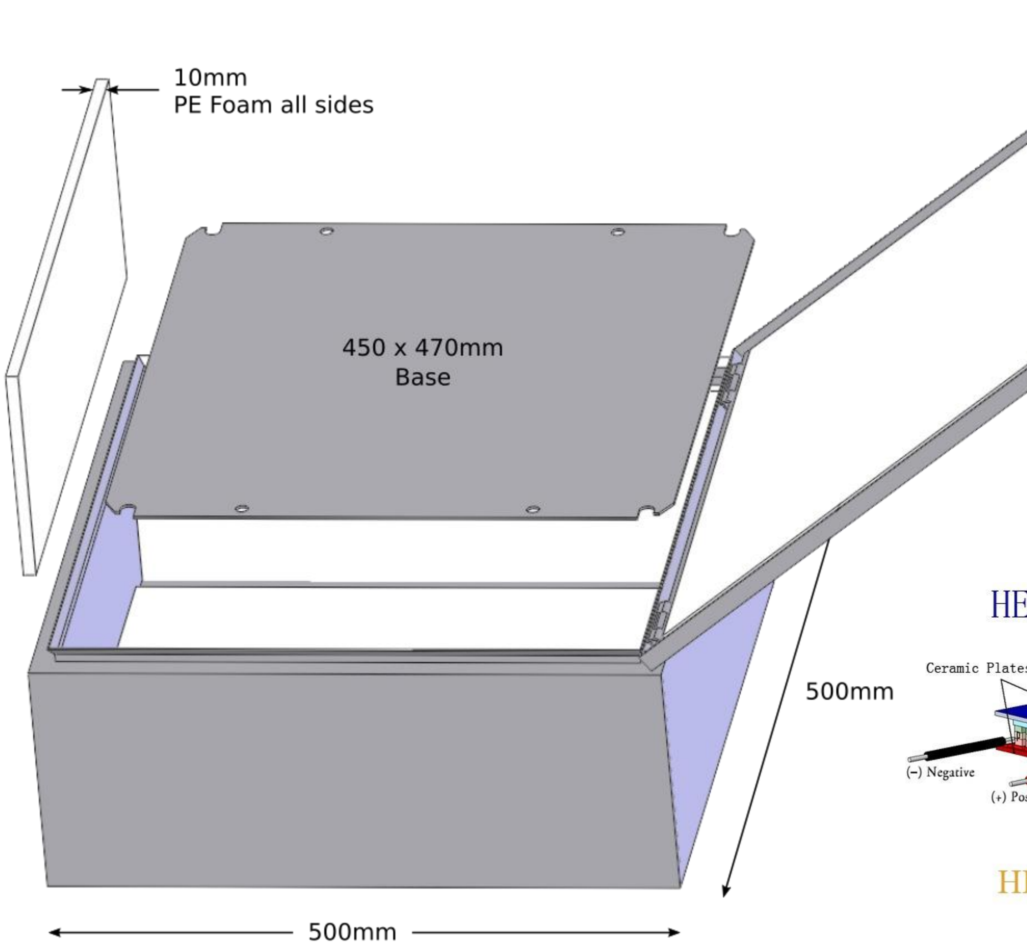


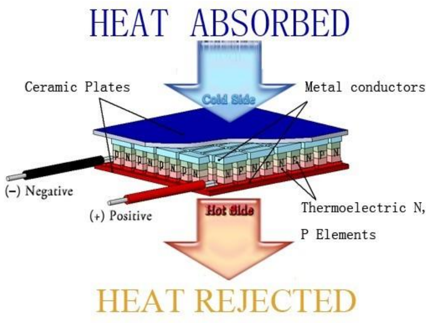
Useful to use many input sources

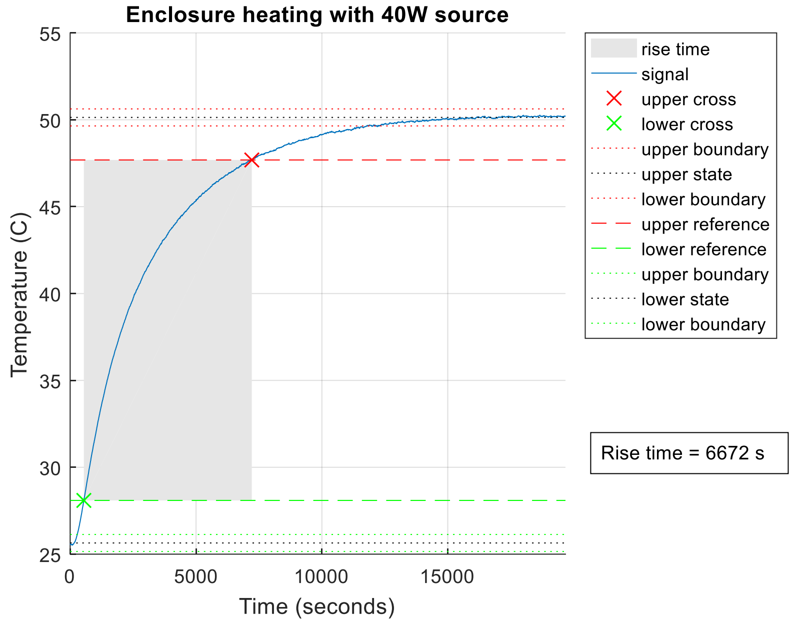
- Try to reduce impedance presented to LNA
- Cover a wide range of possible complex antenna impedances as a function of frequency (20Ω-130Ω)
- Reduce effects of poorly modelled LNA noise

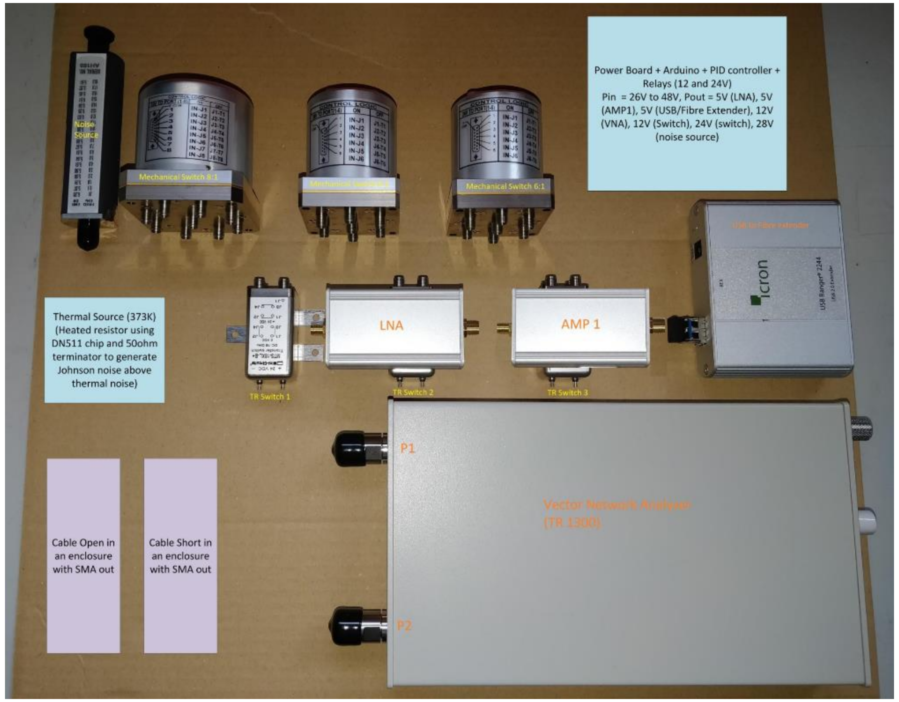


REACH receiver layout

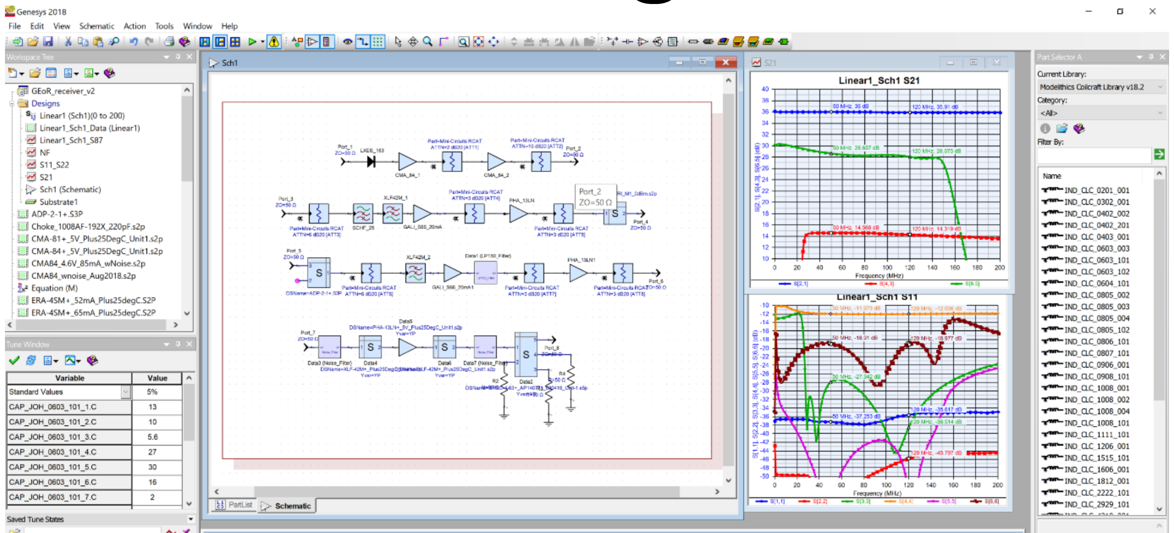


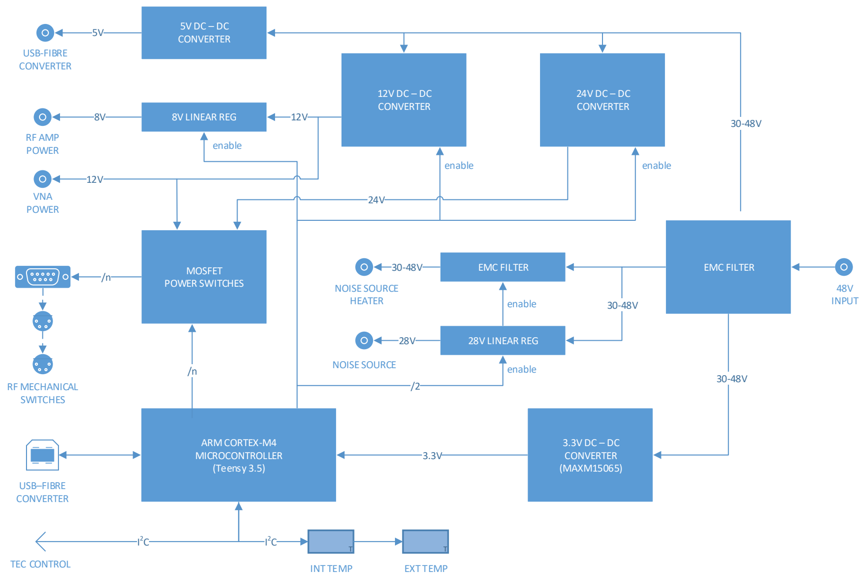


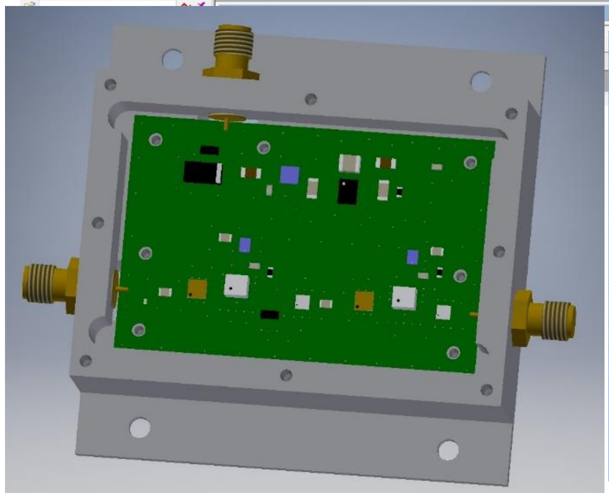


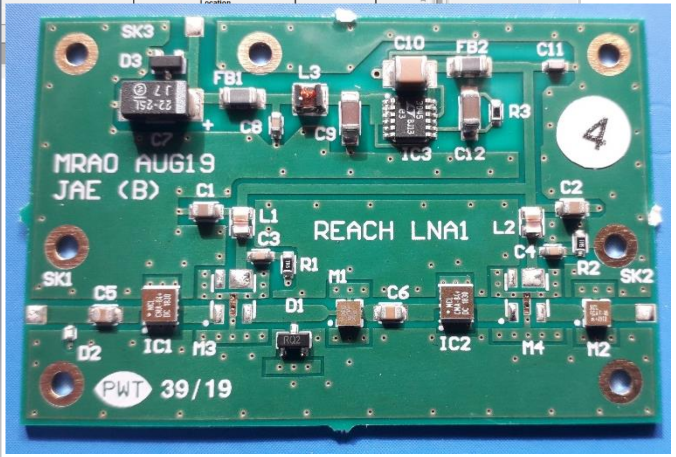


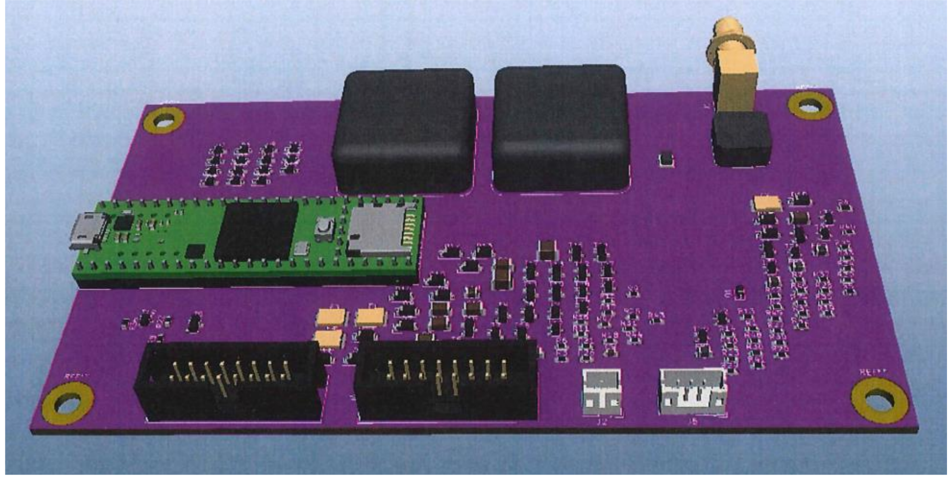
Receiver design and control



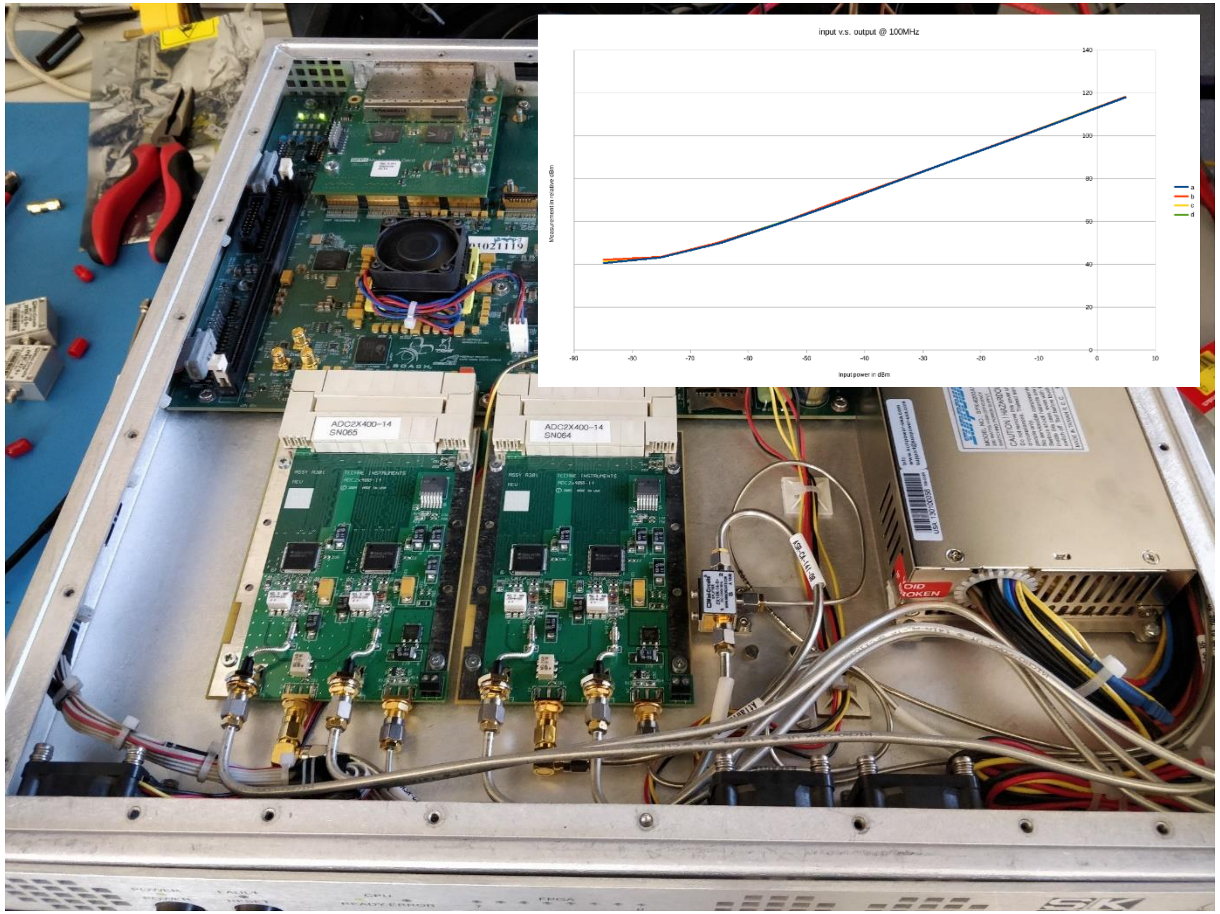


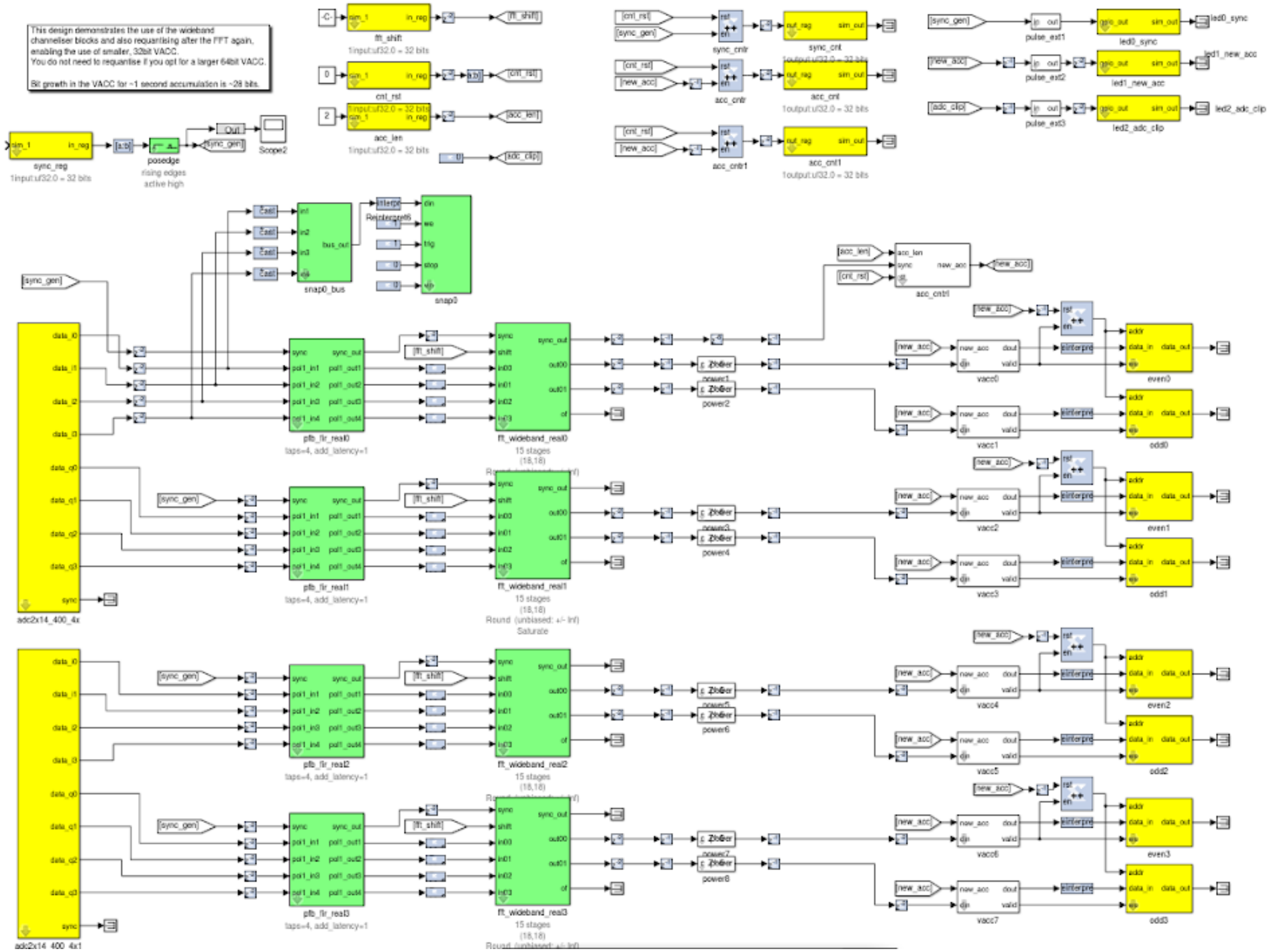






16/32k Spectrometer (ROACH2 version)





4-input 16k channel (DC-200MHz) 14-bit readout system working, control system being developed

Calibration and observing

Calibrate VNA and measure test load

Measure calibration S11 of all sources

Measure calibration spectra of all sources

Determine calibration coefficients

Loop i = 1:num_period

    Set MS1-1

    SPECTROMETER (30 second integration) measure as “obs_ant”

    Set MS1-2

    SPECTROMETER (30 second integration) measure as “obs_load”

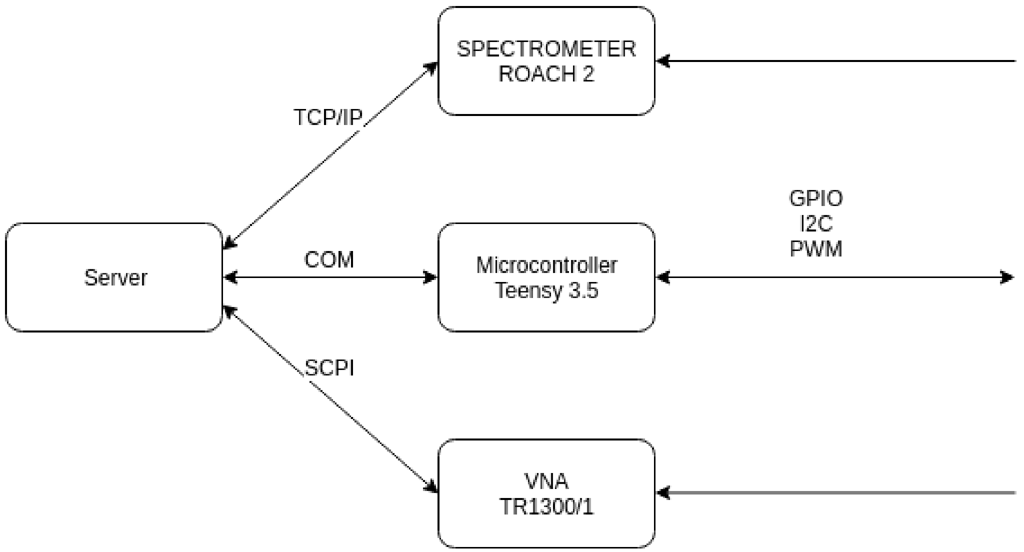
    Set MS1-3

    SPECTROMETER (30 second integration) measure as “obs_ns”

End

Set MS2-6 (test load position)

VNA measure S11 as “test_load”



REACH data products

- VNA calibration states
- Calibration data: measured test load S11
- Calibration data: measured source and LNA S11
- Calibration data: integrated spectra (5min) per source
- Calibration data: coefficients recorded for pipeline
- Observation data (antenna)
- Observation data (noise source)
- Observation data (load)
- Flagged channel index for all calibration and observation spectra
- Timestamps and LST

Rollout

- Understand issues with our calibration possibly related to non linear effects (lacking a physics rooted model of the LNA)
- Develop prototype receiver hardware for research purposes by Q4 2019
- Develop first REACH receiver system, possibly using a new spectrometer (TPM) by Q1 2020
- Module status: LNA (built), rest of RF chain (mostly designed), MCM (built), TEC module (being designed), mechanics and wiring (to do), spectrometer software (mostly written), control software (partly written), VNA controller (written), scheduling software (to write), RFI flagger (to write), calibration data class (to write)