Beam Chromaticity with Gaussian Random Fields

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Foregrounds, man…

- Nobody likes them.
- Is the GSM good enough to get rid of them? Well, no.
- Can we use it to get *some* estimate of leakage? Maybe.
- Chromatic beams will fold spatial structure into spectral structure.
- One way to estimate is assuming sky is a Gaussian random field, described by power spectrum.
- This is not close to reality! But maybe still useful?

Power of GRF Assumption

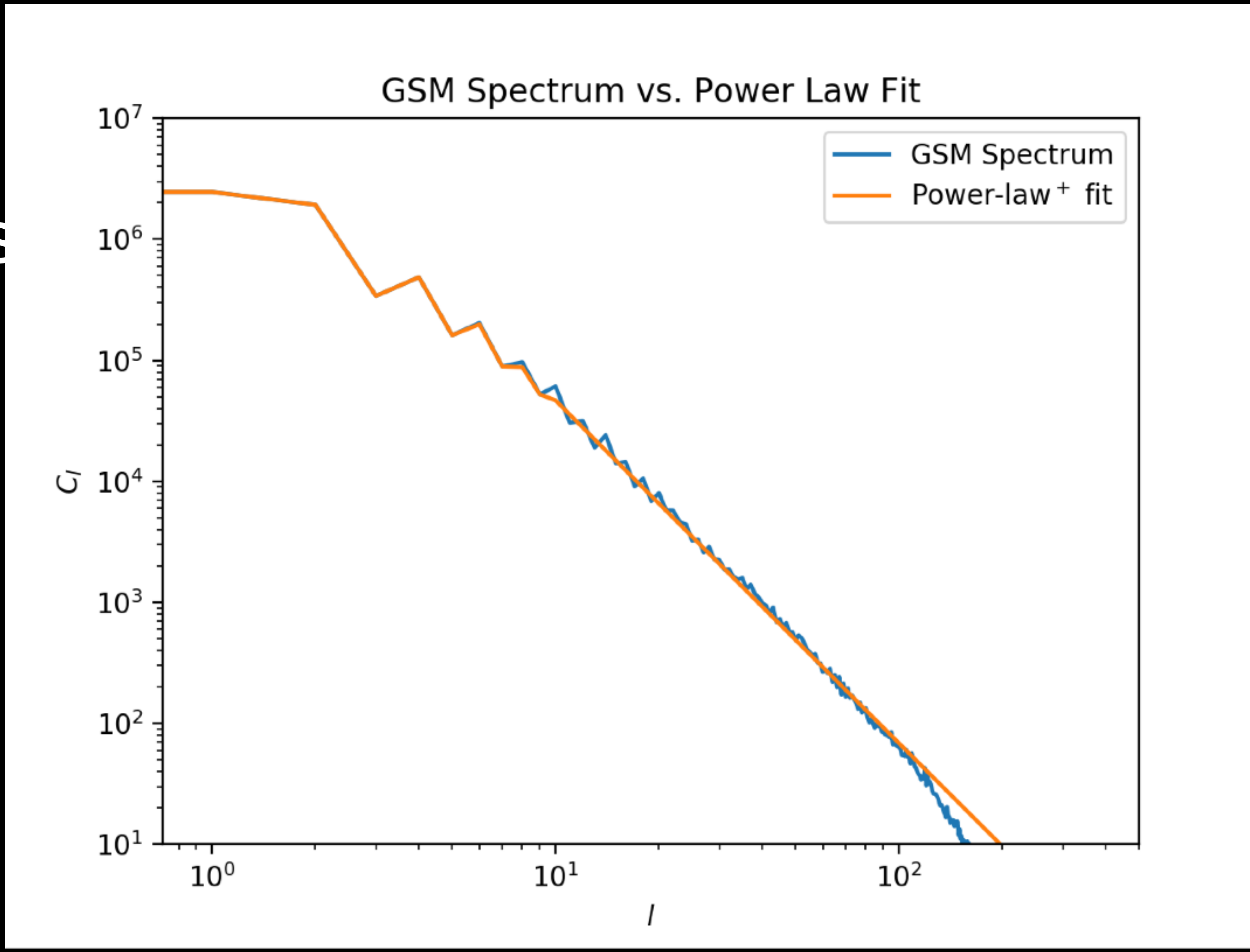
- Gaussian random fields have well understood statistical properties.
- Almost any question I can ask, I can write down a (semi-)analytic answer to.
- Often calculations end up ignoring the phase correlations we see as dominant deviations from Gaussianity.
- Maybe we could have done better with phase correlations, but answers can be far less wrong than it might seem.

Beam Chromaticity

- Leading term will be beam chromaticity coupling spatial fluctuations into spectral signal. Sky assumed to have zero spectral structure.
- We can calculate what this looks like.
- Calculate covariance of signal($\nu\nu'$) under assumption sky is described by a power spectrum.
- $Cov(\nu\nu')=\sum a_{lm}(\nu)a^*_{lm}(\nu')C_l$ where $a_{lm}(\nu)$ is beam SPH transform and C_l is foreground power spectrum.
- Normalize so that diagonal is 1. Fluctuations now completely encoded by eigenvalues/eigenvectors.

GSM Power Spectrum

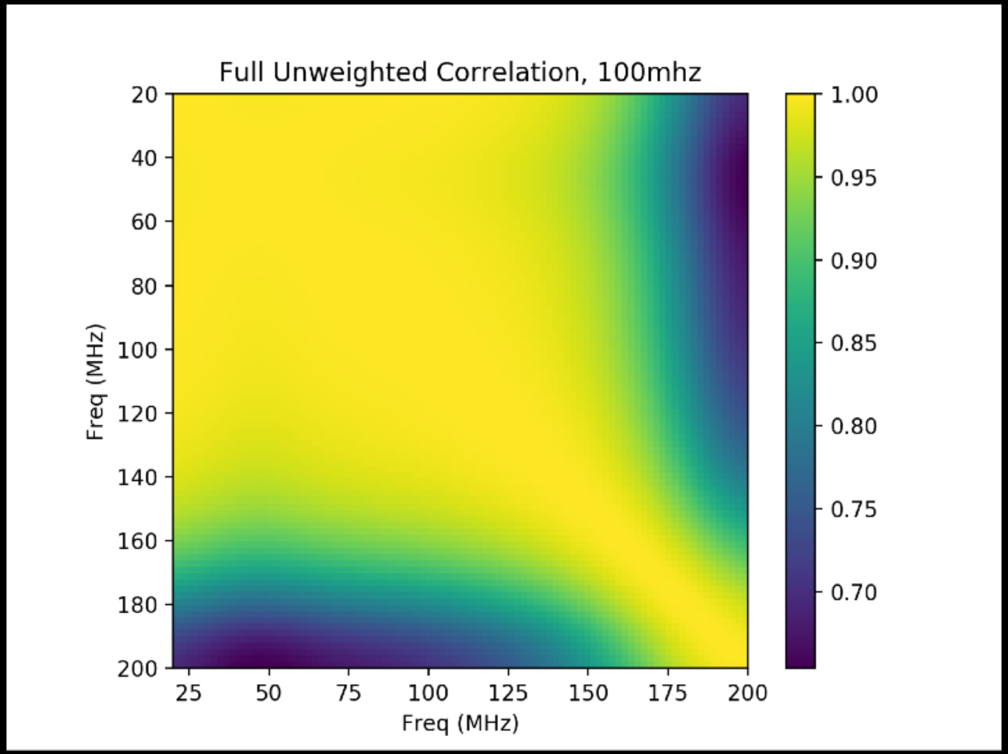
- Take GSM 80 MHz map power spectrum. Differences small if yo try to mask/cap brightest patches
- Fit power law over lowish ell range.
- Keep l=0 value from GSM.
- Where GSM < fit, use GSM for lowest ells. Avoids potential to be overly optimistic.

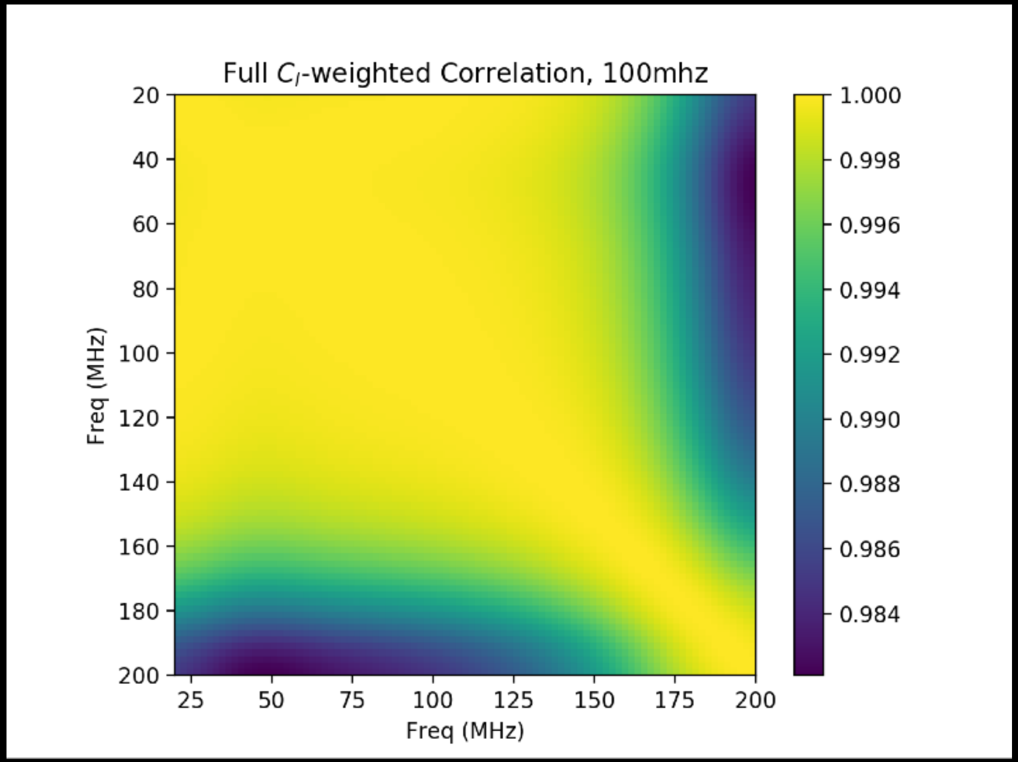


Thresholded at 10x mean

Correlation Matrix

- Spatial smoothness of sky drives frequencies to be more strongly correlated.
- Effectively - if sky isn't changing, matters a lot less if beam is.
- Right: unweighted (i.e. point-source weighted) vs. C_l-weighted covariance for Hibiscus 100 MHz beams.
- Shapes look similar, but color bars differ by factor of 20.



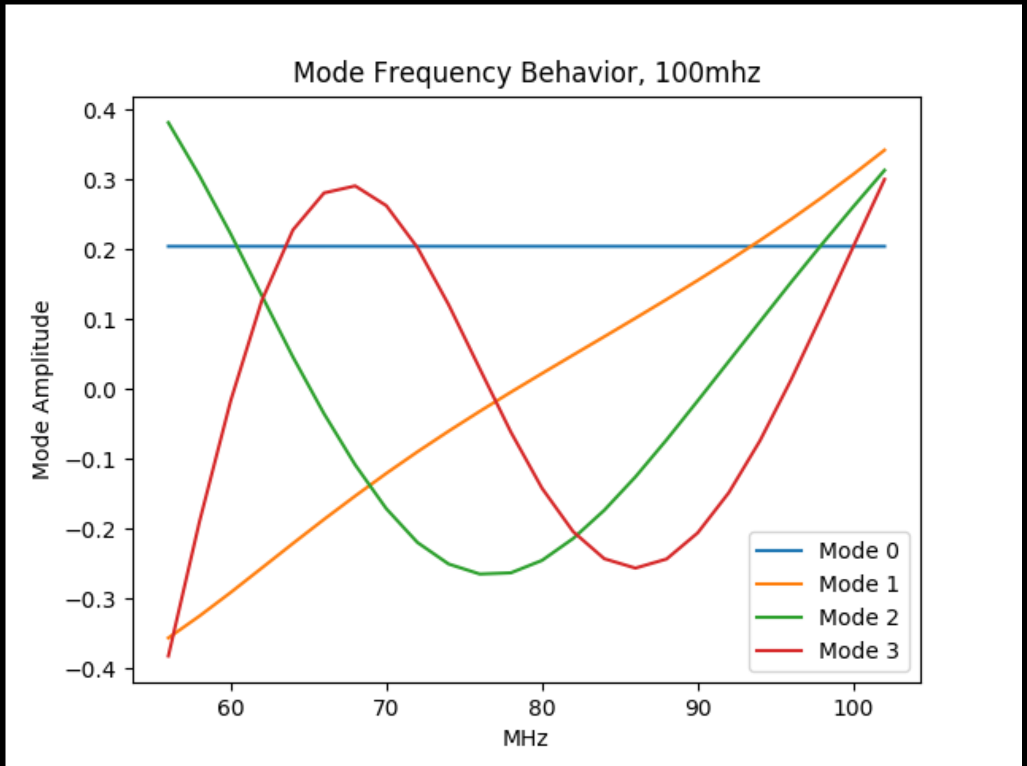


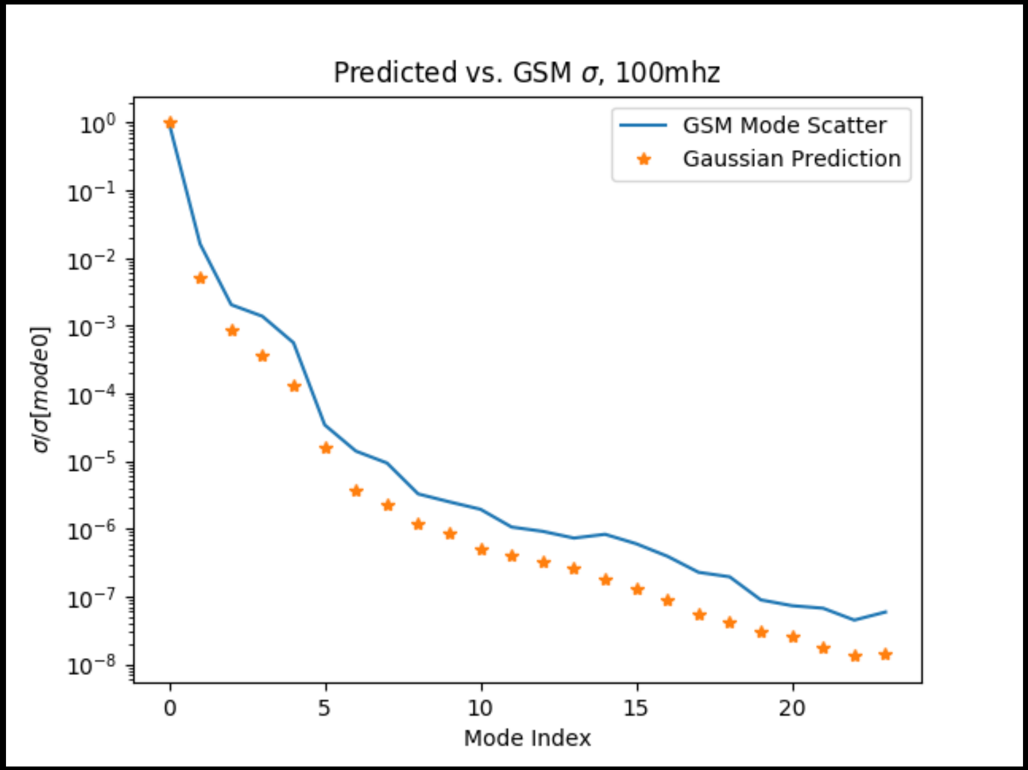
Frequency Modes

- Eigenmodes tell us the types and amplitudes of frequency structures we expect to see.
- Reminder - assumption is sky is perfect, uniform power-law. Induced behavior solely from beam chromaticity coupling to spatial structure.
- Unsurprisingly, modes look roughly like you'd expect. Offset, slope (hard to distinguish from different spectral index), higher-order terms…

Hibiscus 100 MHz @Marion

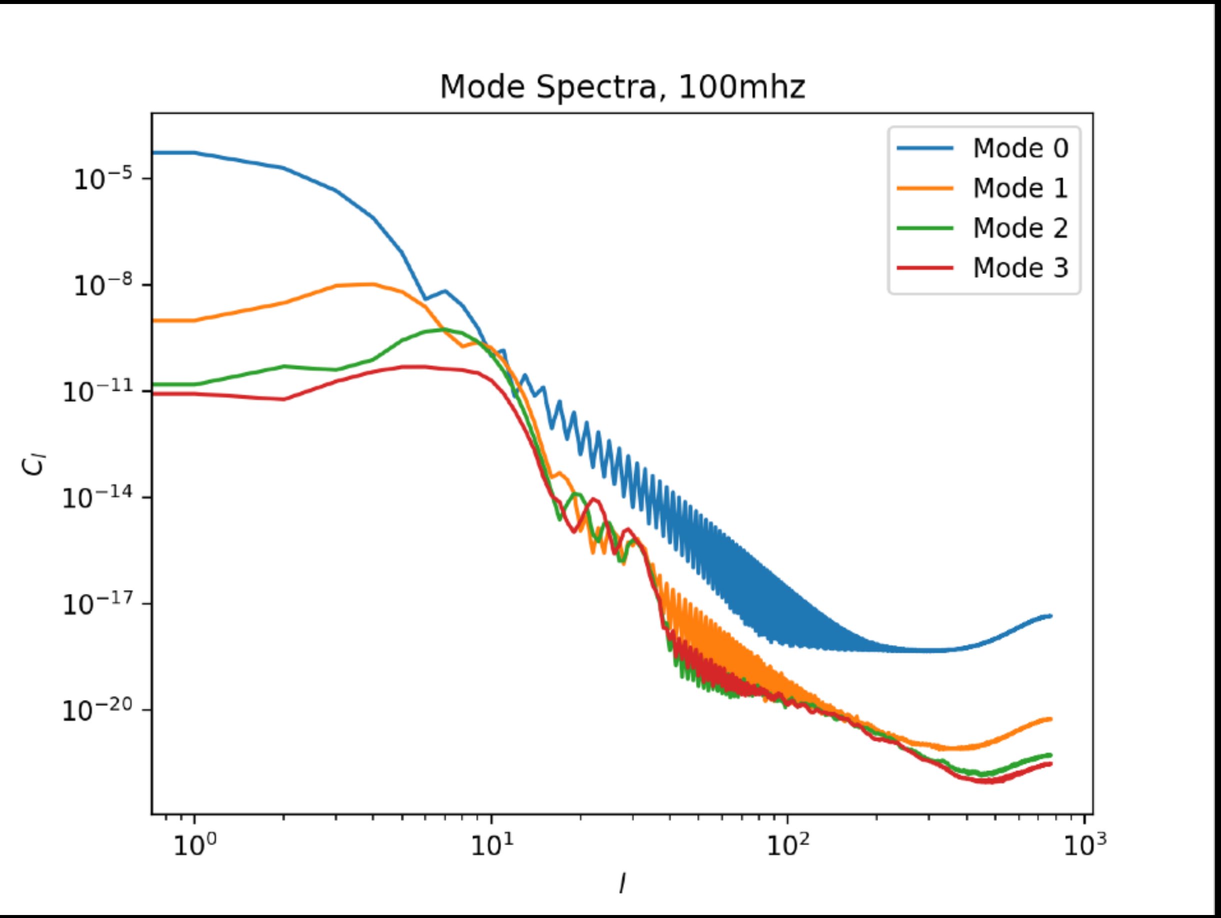
- Top: frequency behavior of eigenmodes after taking 55-105 MHz.
- Bottom: standard deviation of those modes projected onto GSM vs. eigen prediction - not bad for lowest modes.
- Amplitudes: sqrt(e_i/e_0)=5.0e-03, 9e-04, 3.6e-04.
- Median @80 MHz = 1400K, so mode 2 signal ~ 1.3K RMS, mode 3 ~0.5K predicted (mode 1 goes into spectral index).
- Can also calculate σ(mean)/σ = 0.44/0.87, so LST-averaged variance will be 0.6/0.4K for modes 2,3.



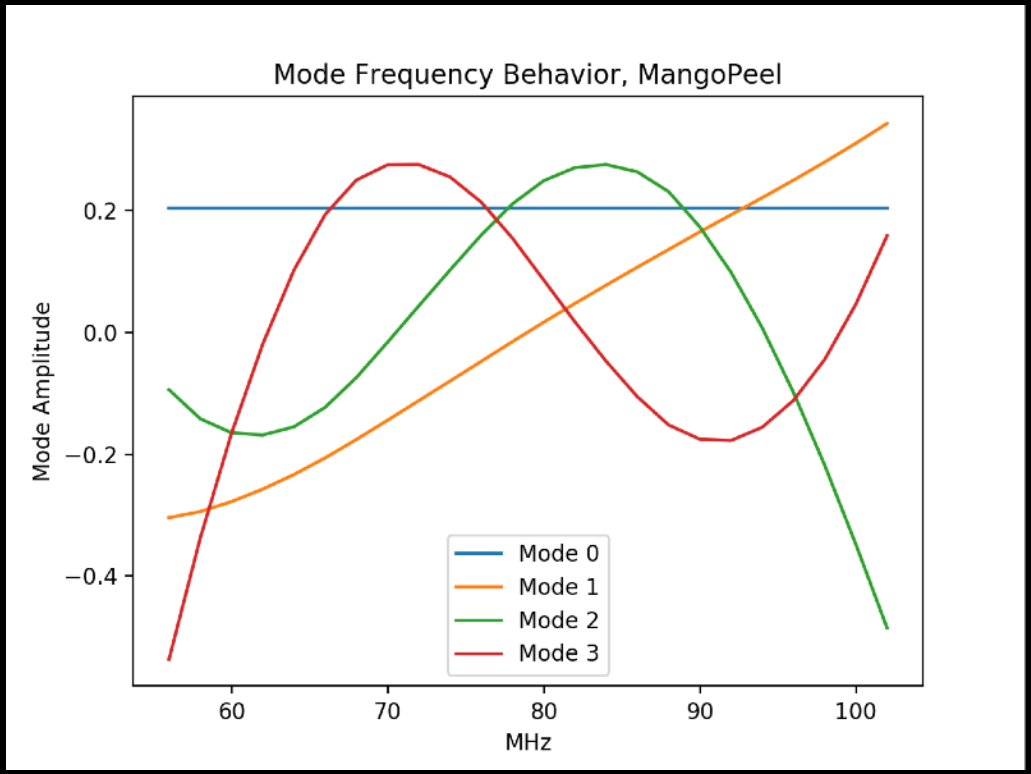


Implications

- We predict coupling of sky to spectral structure through beam chromaticity @~0.5K level, averaged over LST.
- Can look at effective beam of modes projected onto sky. Sits at *l*<20 or so.
- (Semi-)naive prediction: we will need to map sky to *l*=20 to ~10% accuracy for foreground removal.

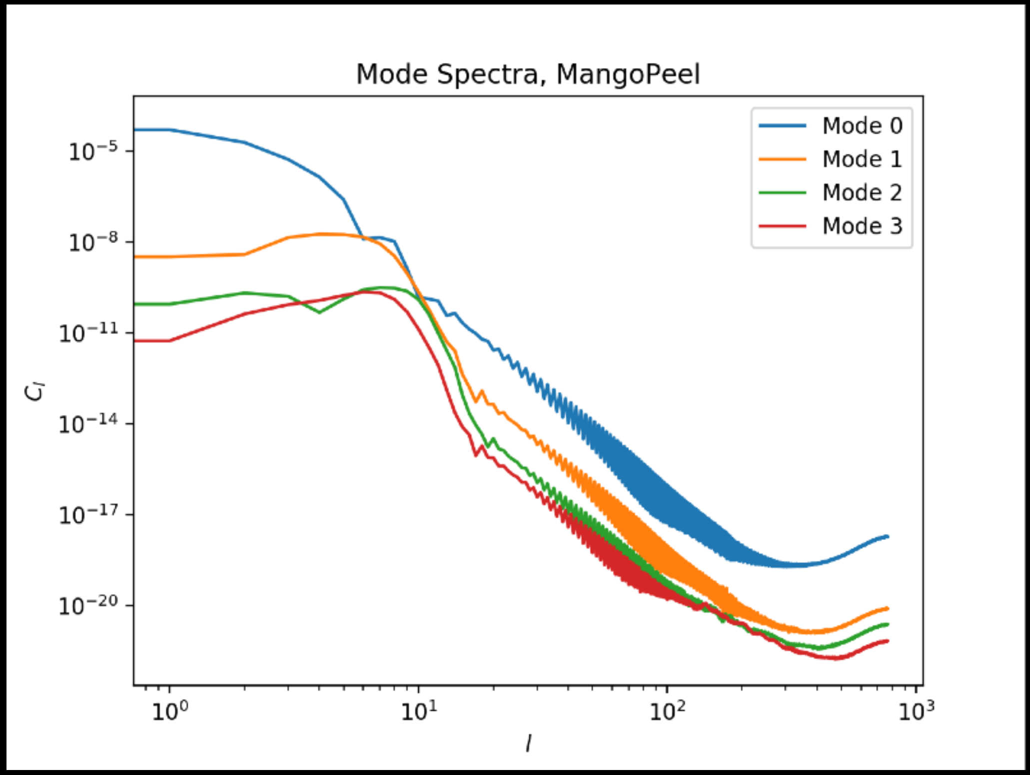


Mango Peel



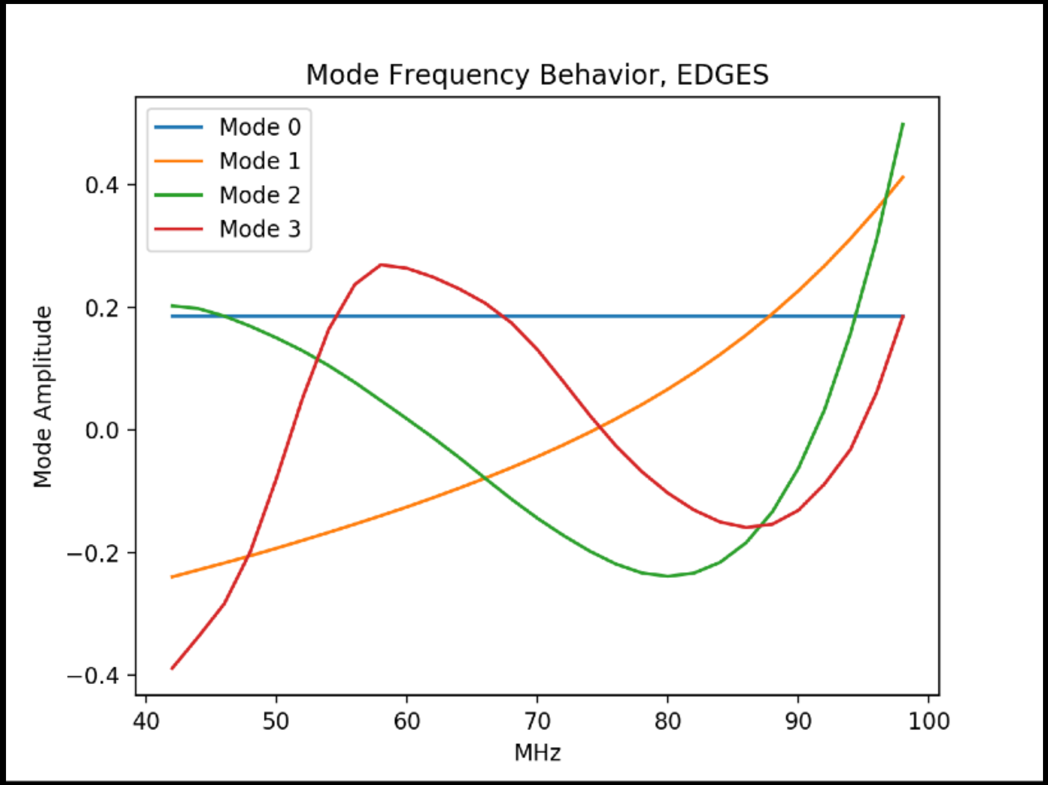
- Mango peel antenna looks similar, perhaps slightly worse - amplitudes 7.6e-3, 9e-4, 7e-4.

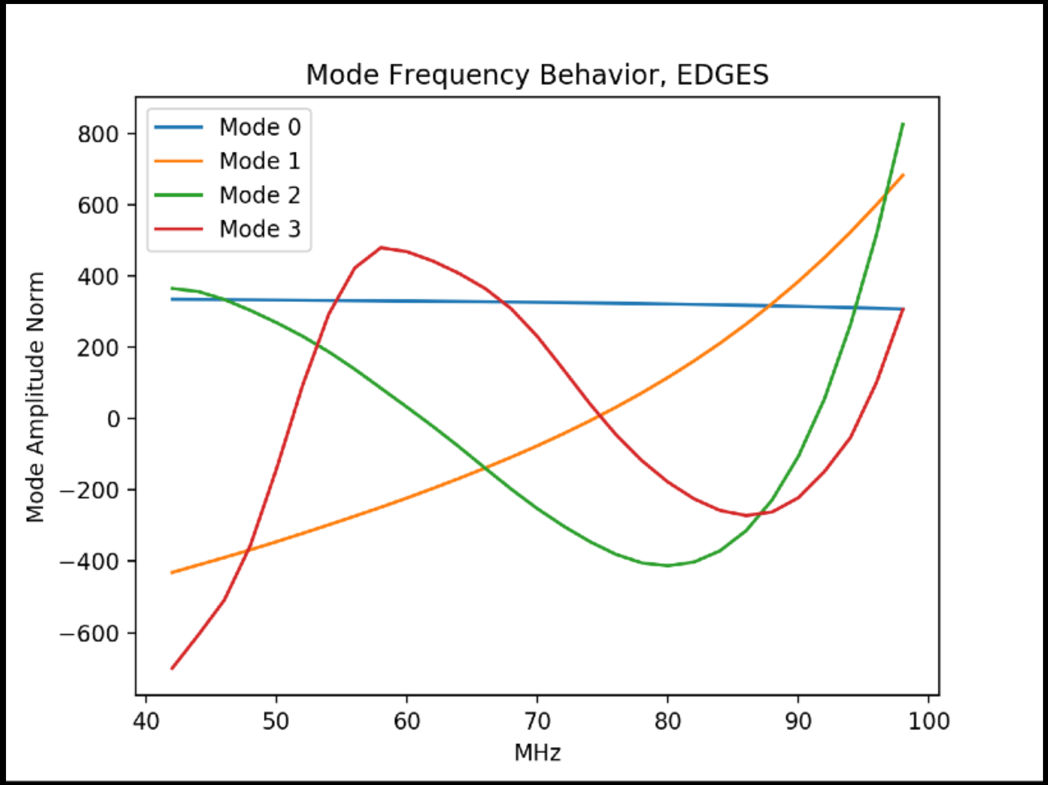
- σ(mean)/σ=0.75/0.7 for 1.0/0.7K RMS on mean for modes 2,3.



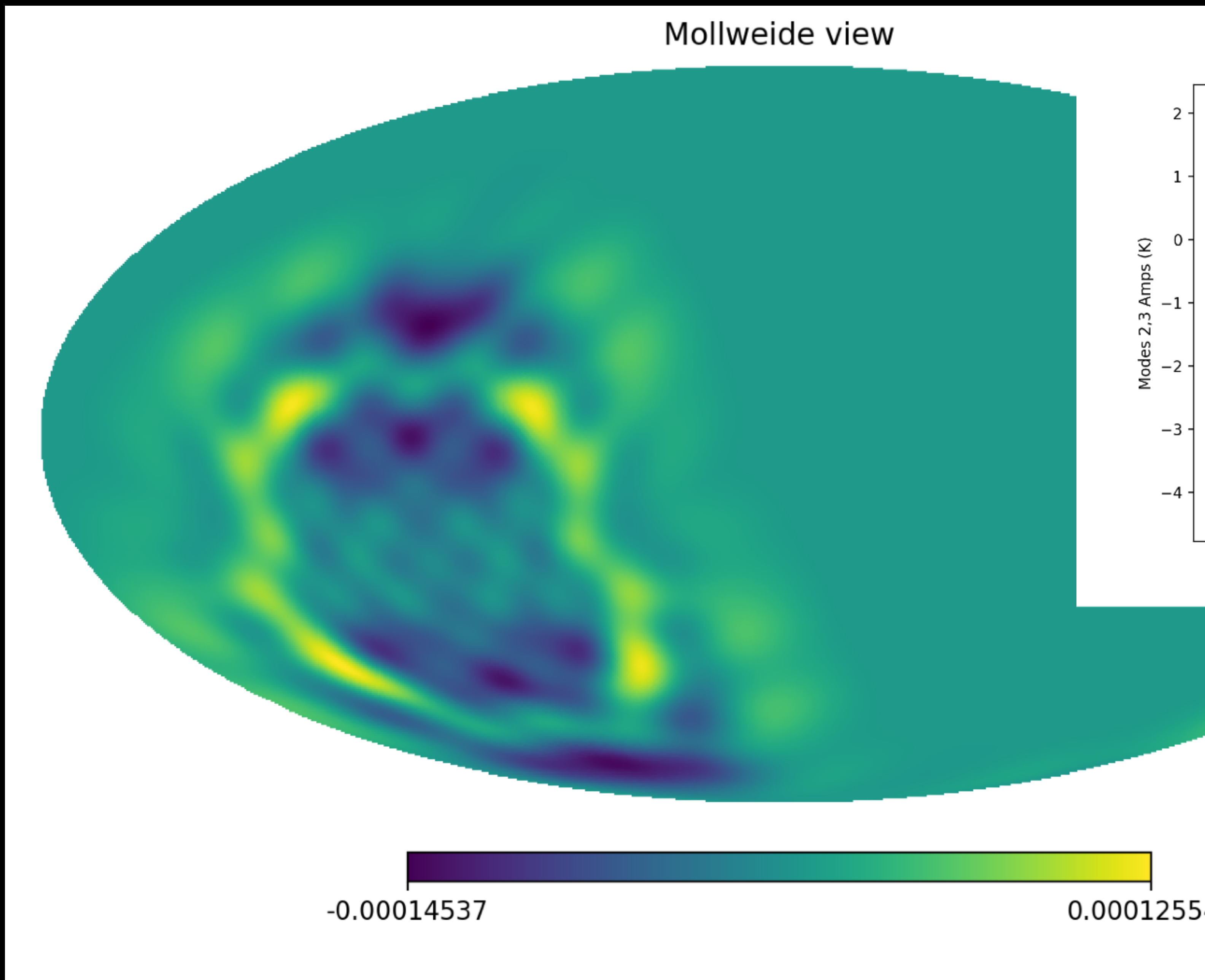
EDGES @-26

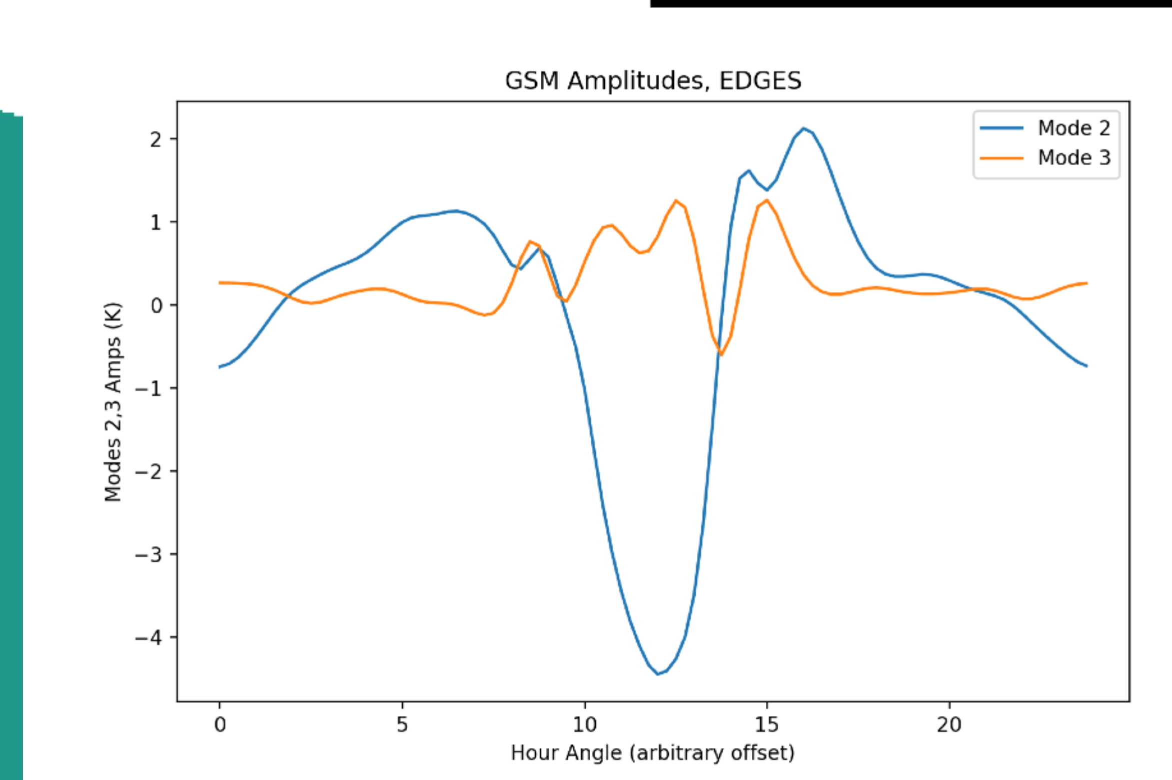
- EDGES team have kindly provided beam models for EDGES-low.
- Behavior qualitatively similar - mode amplitudes 1.4e-2, 5.8e-4, 1.7e-4.
- σ(mean)/σ = 0.35/0.73 for modes 2,3, for total uncertainty ~ 0.3K/0.18K when LST-averaged using latitude of -26 degrees.
- Not sensitive to normalization - bottom is after beam area put back in.
- If this is accurate, I would generically expect more LST variation than seen. Still, suggestive.
- NB - have not run modes through usual foreground filtering to see what would survive.

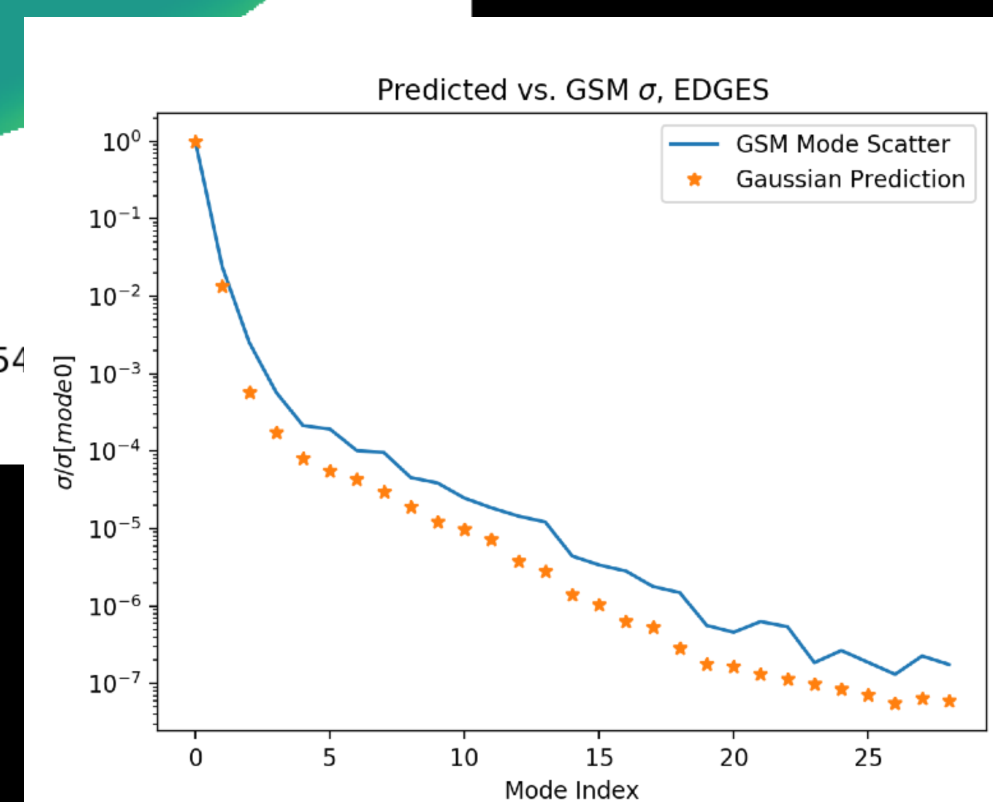




EDGES Mode 2







Visualization of mode 2 on the sky.
Amplitudes of modes 2,3 vs. HA
Clearly not right in the details…

Summary

- Beam chromaticity matters (no surprise)
- Gaussian random fields give us a way to estimate chromaticity errors w/out details of sky
- Stationary GRF model seems way closer to GSM predictions than it has any right to be.
- Resonant antennas seem to introduce noise at the ~0.5K level from modes after first two (which get absorbed into spectral index/ curvature).
- If this treatment valid, suggests we’ll need ~10% accurate maps of sky to *l*~20 for resonant antennas.