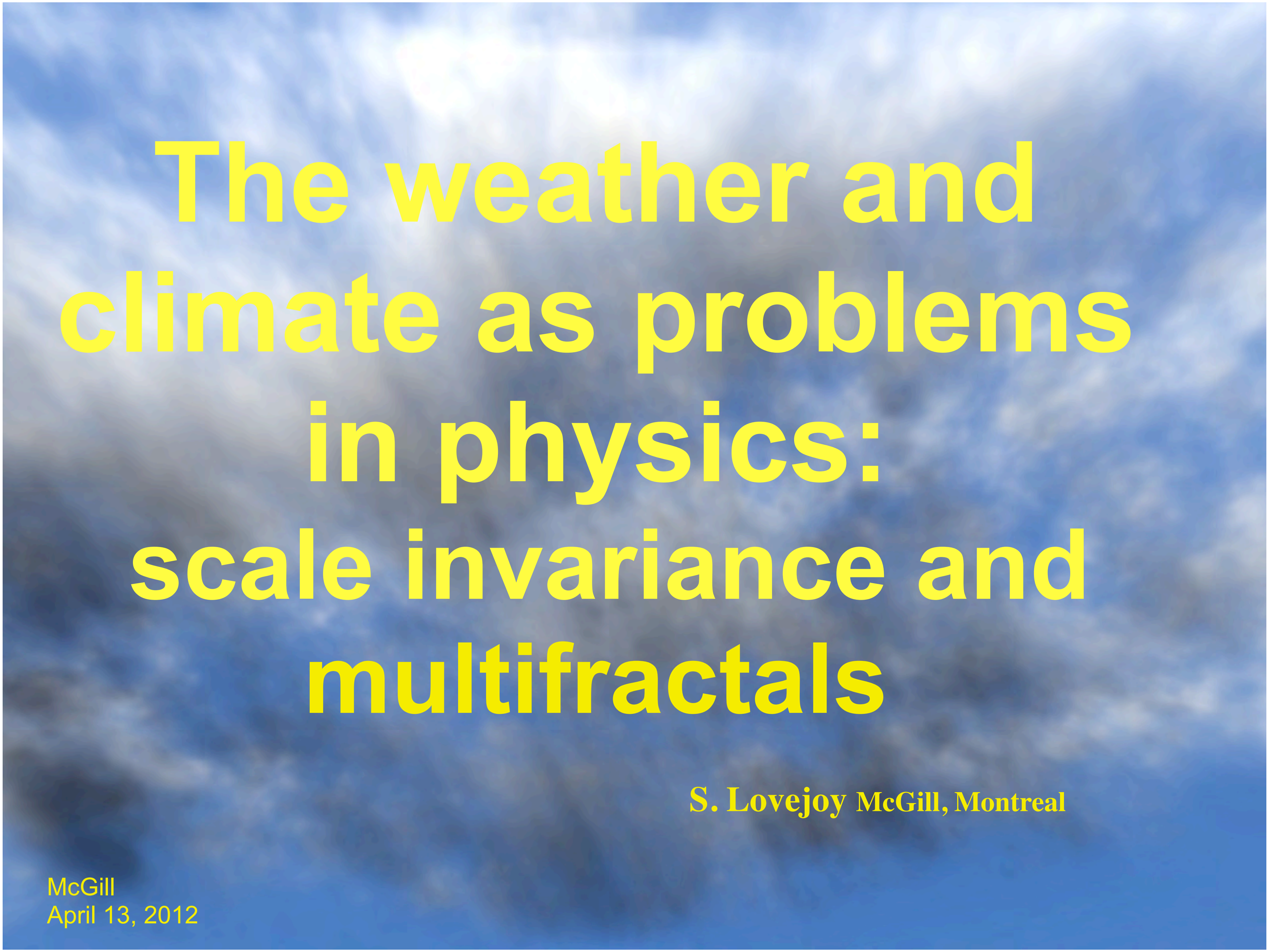


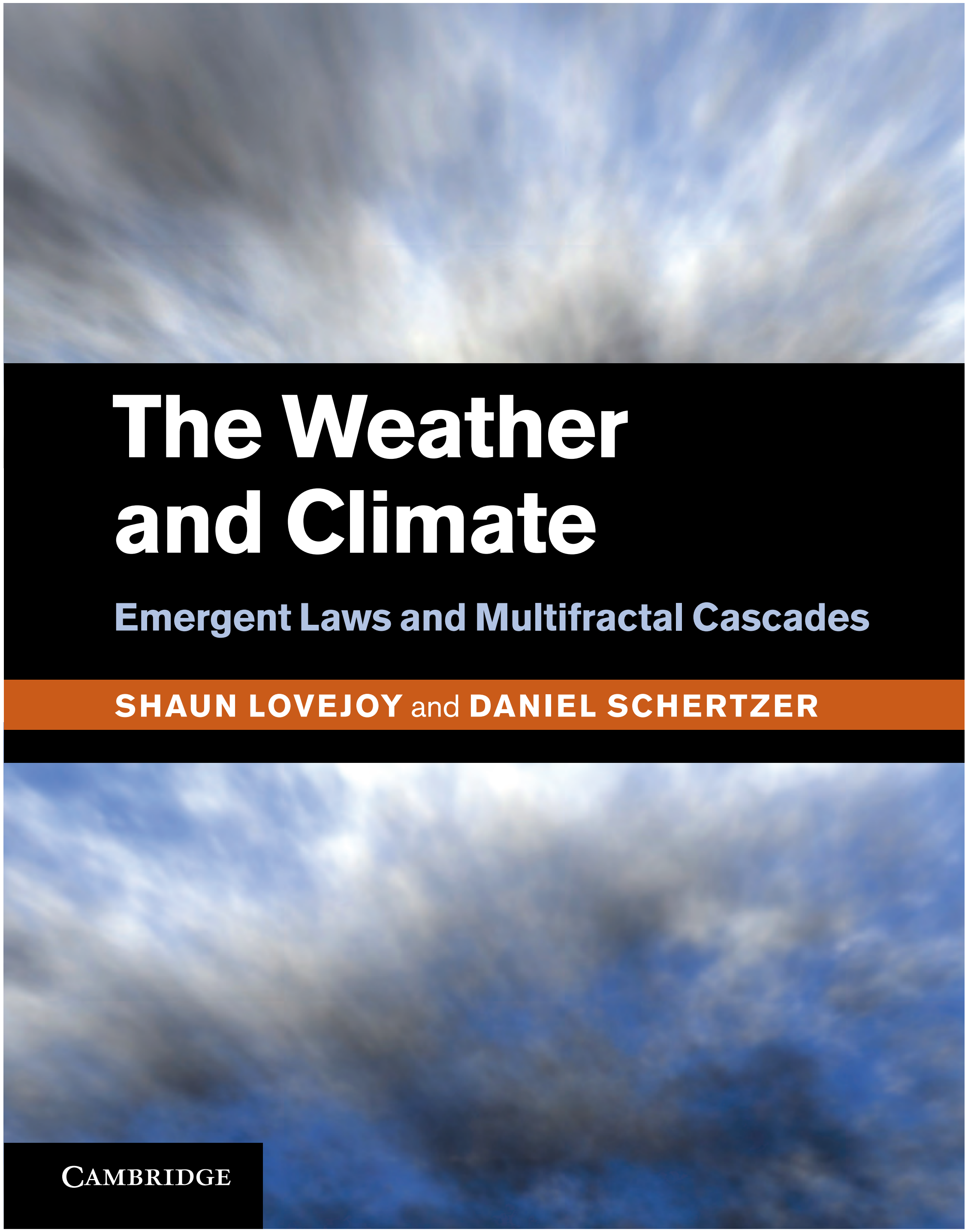
The weather and climate as problems in physics:

scale invariance and multifractals

S. Lovejoy McGill, Montreal

McGill
April 13, 2012

Required reading for this course



The Weather and Climate

Emergent Laws and Multifractal Cascades

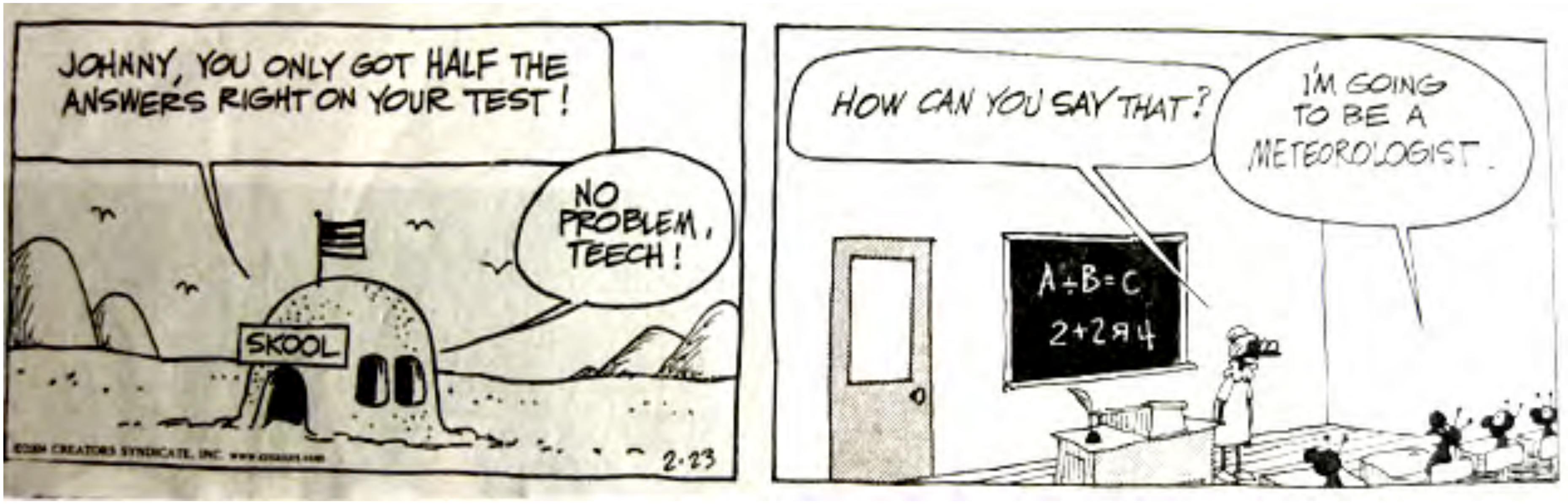
SHAUN LOVEJOY and DANIEL SCHERTZER

CAMBRIDGE

(in press, Oct. 2012)

The Weather

Meteorologists



The Emergence of physical laws

Quantum mechanics → Classical Mechanics

Large scales (usually)

Statistical mechanics → Continuum mechanics, **Fluid mechanics** thermodynamics

Large numbers of particles

General Relativity → Special Relativity

Low energy mass density

Special Relativity → Classical (Galilean) Relativity

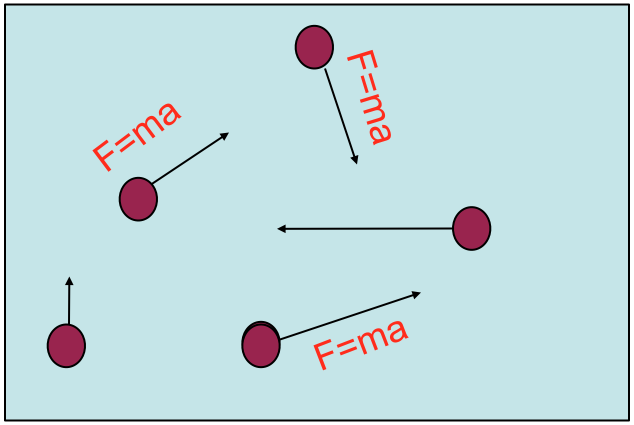
Velocities << speed of light

Low level
(fundamental)

high level
(simpler if applicable)

Example: The emergence of Thermodynamics from Newton's laws

Newton's laws:



Low level, (difficult to handle for many particles)

Large number of particles

Thermodynamics:

First law: conservation of energy

Second law: increase in entropy

ex.: Boyle's law:
(pressure) x (volume) = constant

High level
(Valid when many particles are present)

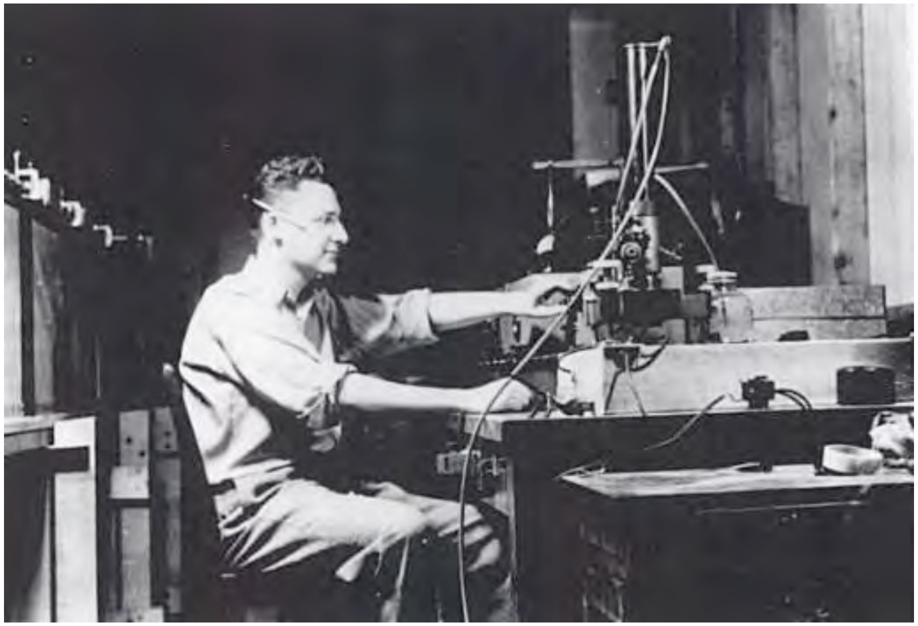
Pioneers of turbulence

Richardson
1881 - 1953

Kolmogorov
1903 – 1987

Corrsin

1920 – 1986



Obukhov

1918 – 1989

Ralph Bolgiano, Jr.

1922 — 2002

The emergence of turbulence dynamics (Classical)

Fluid mechanics

Low level
(fundamental)

Strong stirring
(nonlinearity)

Laws of turbulence

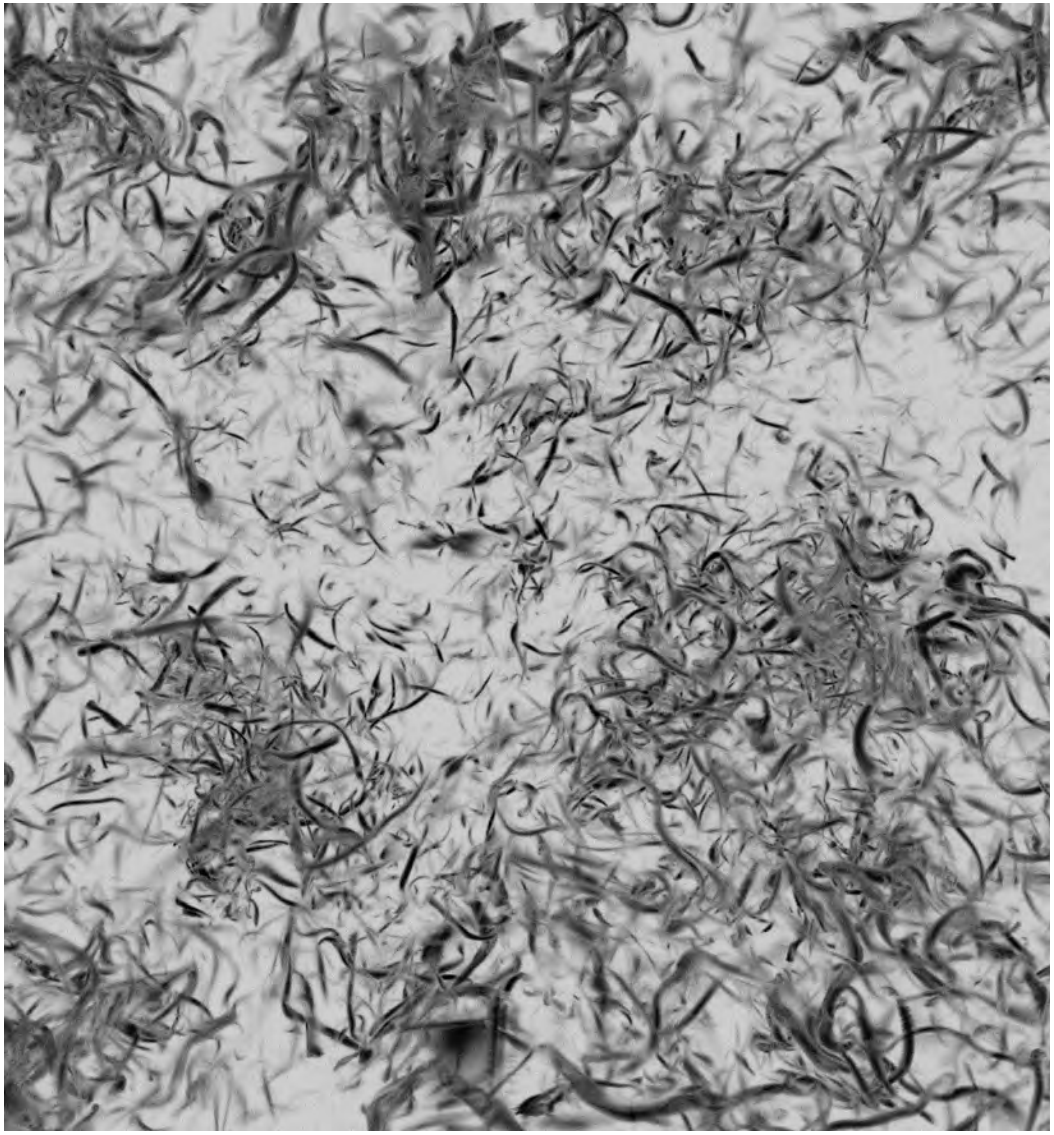
Classical:
Richardson, Kolmogorov,
Corrsin, Obukhov, Bolgiano

High level

Vortices in strongly turbulent fluid

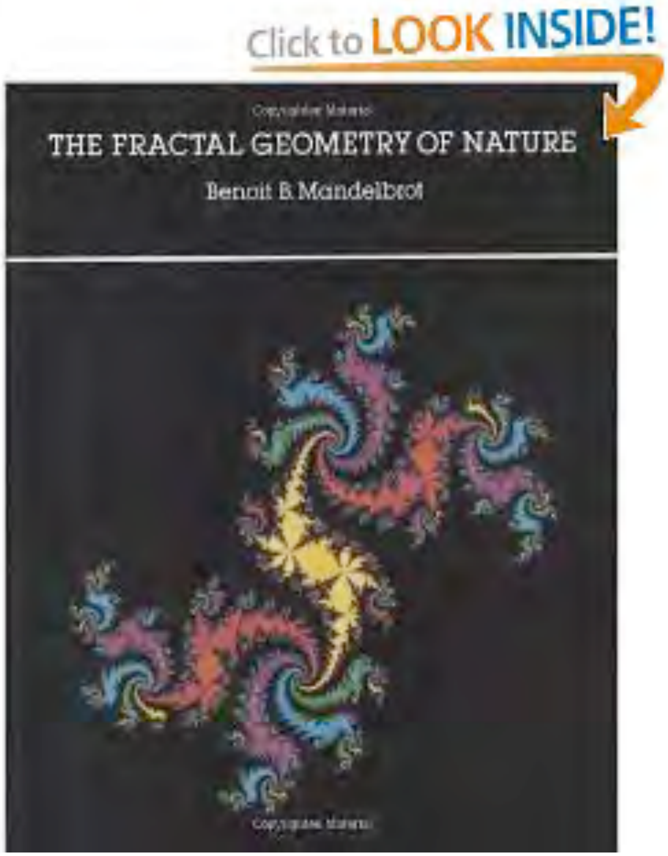
(M. Wiczek, numerical simulation, 2010)

“Spaghetti”



Emergent laws reduce seeming complexity to simplicity at another level

Mandelbrot
1924-2010



THE FRACTAL GEOMETRY OF NATURE

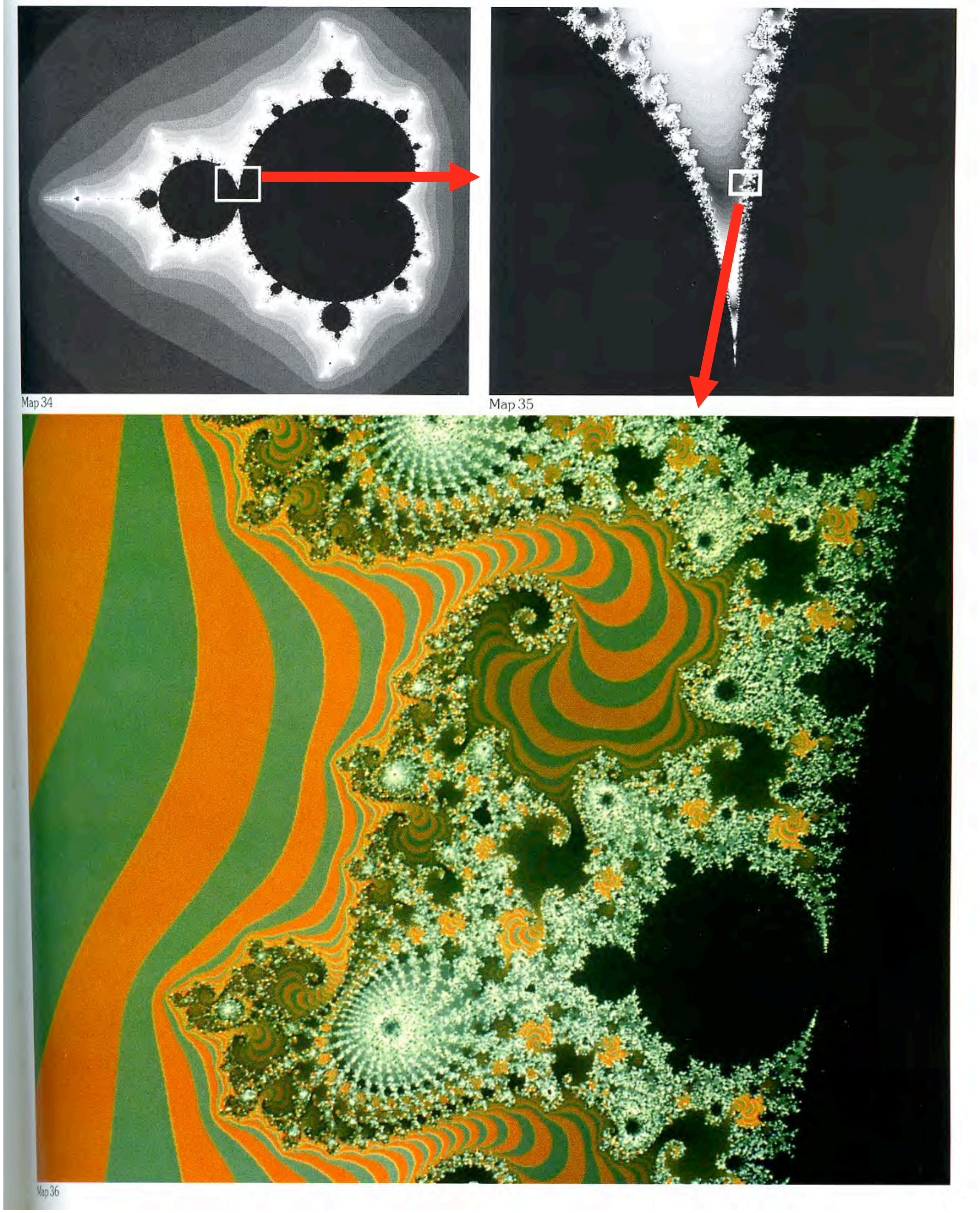
Benoit B. Mandelbrot

Complex?

Blowing up gives the same type of shapes

The Mandelbrot set

(“self-similar”, scale invariant, fractal)



Or simple?

Generating the Mandelbrot set

-Take a number.

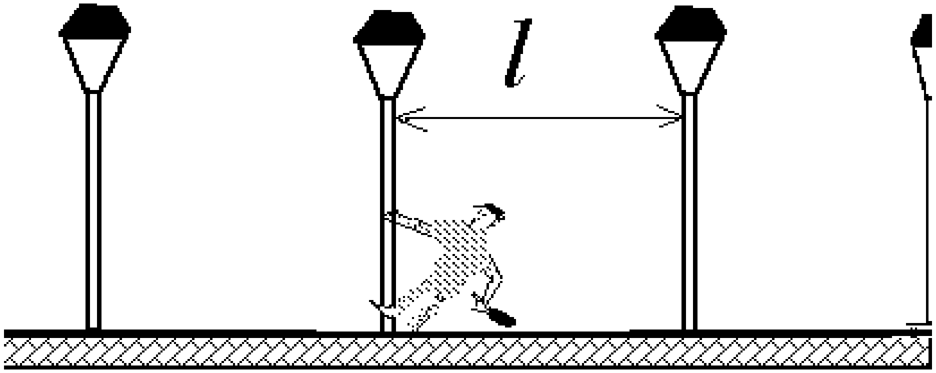
-Multiply it by itself.

-Add a constant.

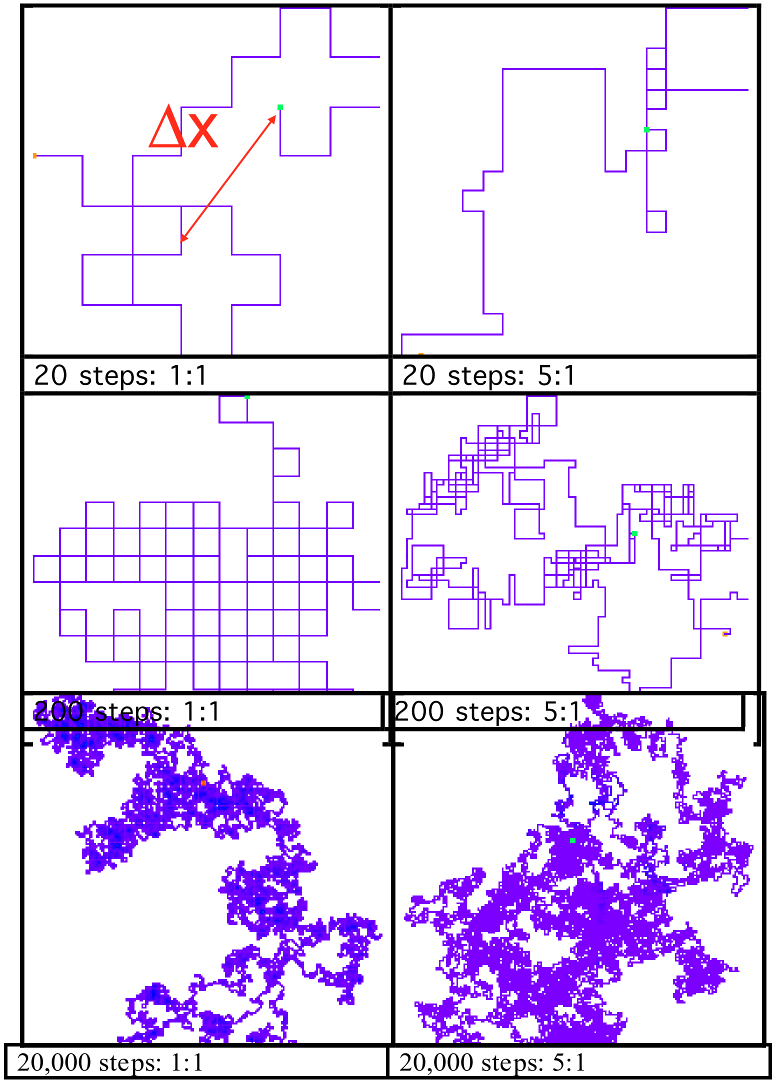
-Repeat.

(I forgot to mention: take a COMPLEX number)

Complex?



Drunkard’s walk



Or simple?

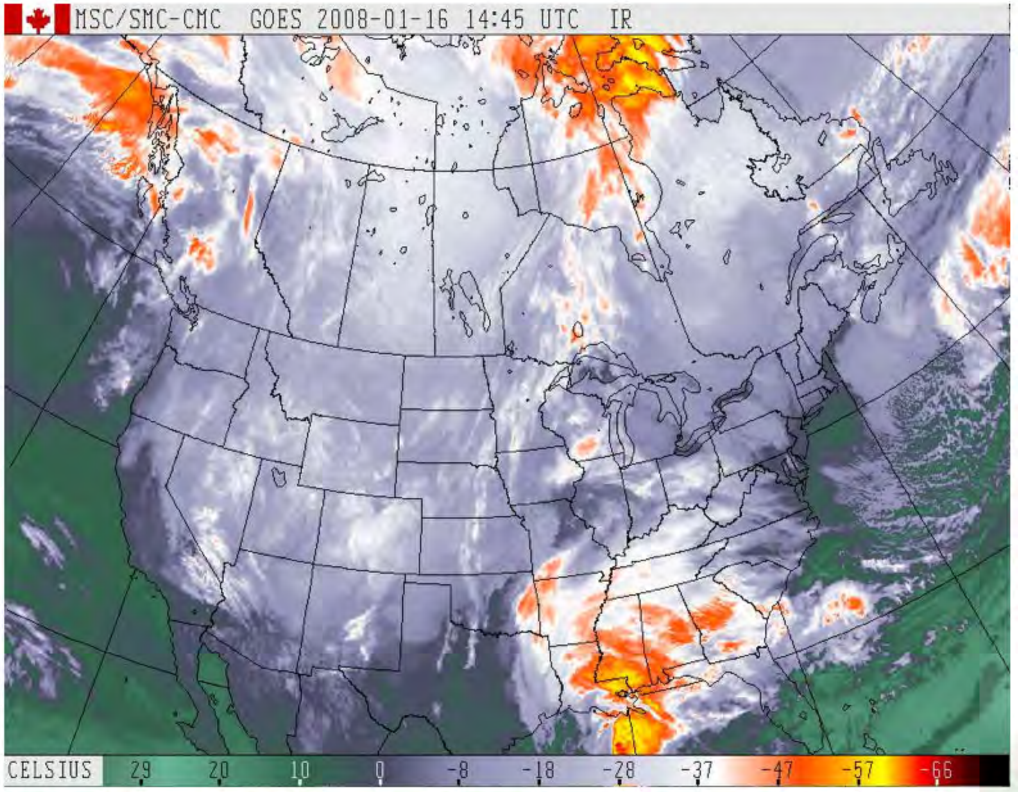
(distance) x (distance) = number of bars visited

From initial bar

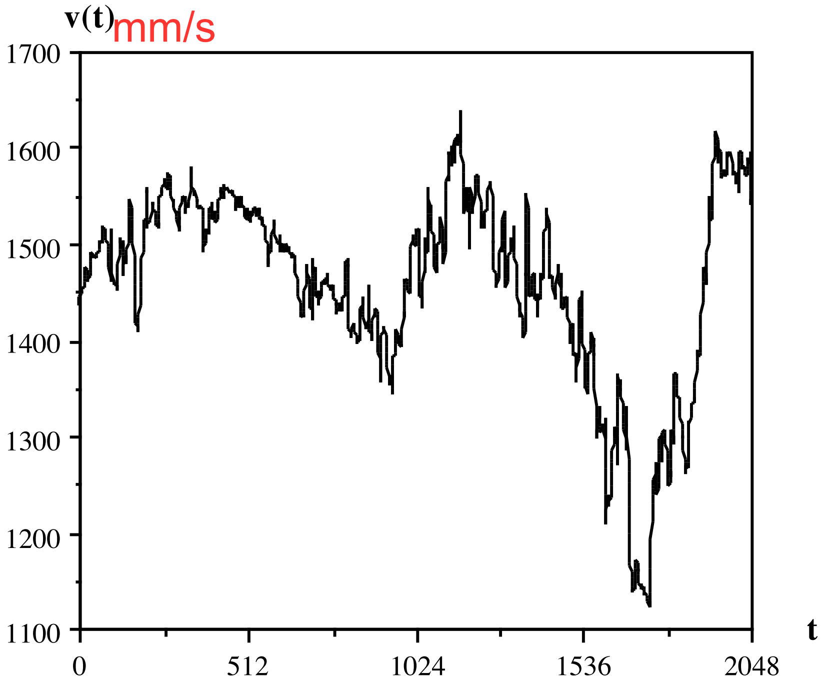
Average number of bars visited
(or displacements made)

(Brownian motion)

Complex?… or simple?



Infra Red satellite effective temperatures, January 16, 2008



1 second of wind data (roof of Rutherford building, McGill)

The Atmosphere

Brute force…

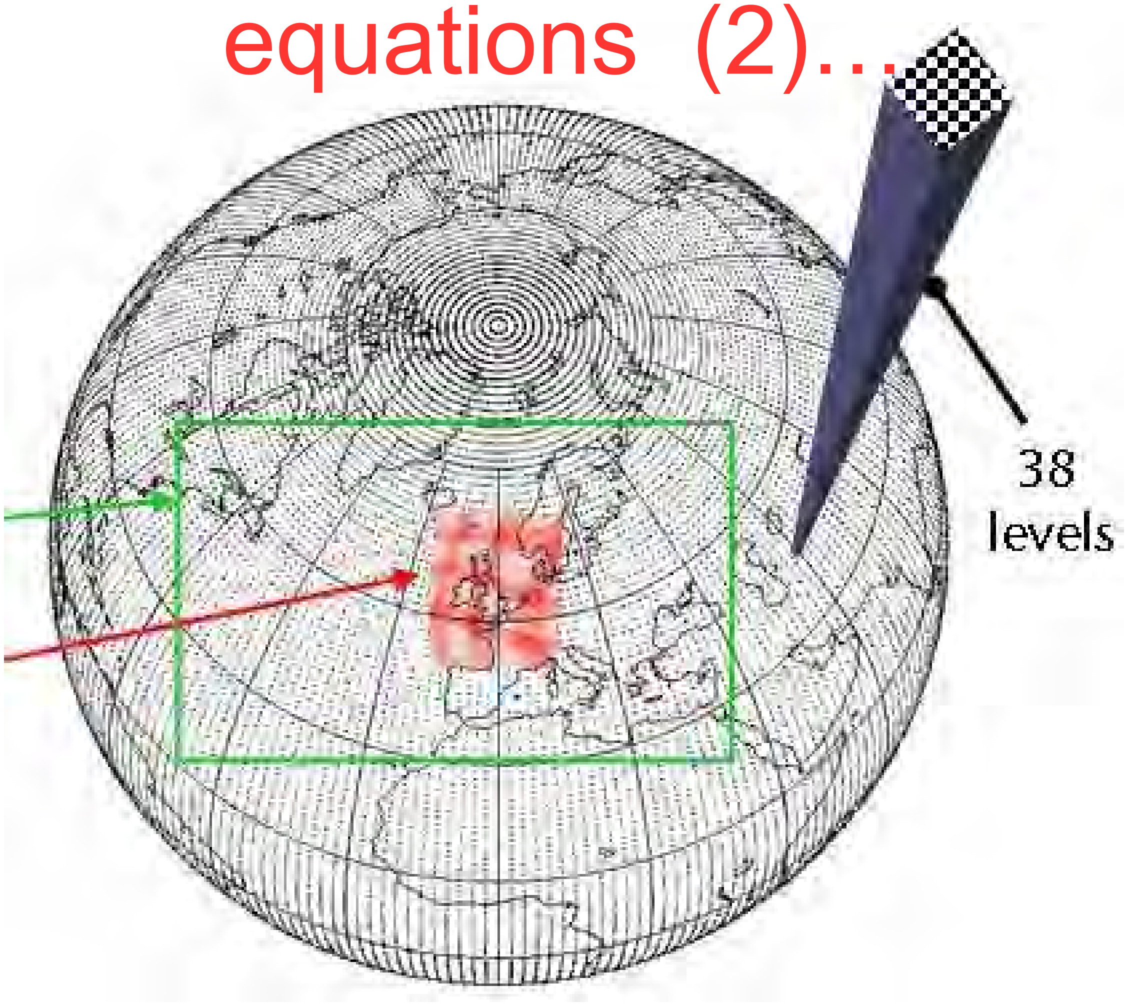
Atmosphere: Laws of Fluid mechanics

(low level)

$$
\begin{aligned}
\frac{\partial \overset{\text{wind}}{\mathbf{u}}}{\partial t} &= -(\mathbf{u}.\,\mathrm{grad})\mathbf{u} - 2\overset{\text{Earth angular velocity}}{\boldsymbol{\Omega}} \times \mathbf{u} - \underset{\text{Specific volume}=1/\rho}{\alpha}\,\mathrm{grad}\,\underset{\text{pressure}}{p} - \mathrm{grad}\,\overset{\text{Gravitational potential}}{\boldsymbol{\Phi}} + \underset{\text{Friction}}{\mathbf{F}} \\
c_v \frac{\partial T}{\partial t} &= -\underset{\text{Specific heat}}{c_v}(\mathbf{u}.\,\mathrm{grad})\underset{\text{temperature}}{T} - \frac{p}{\rho}\,\mathrm{div}\,\mathbf{u} + \underset{\text{Heating rate}}{Q} \\
\frac{\partial \rho}{\partial t} &= -(\mathbf{u}.\,\mathrm{grad})\rho - \rho\,\mathrm{div}\,\mathbf{u} \\
p &= \underset{\text{density}}{\rho}\,\underset{\text{Gas constant}}{R}\,T.
\end{aligned}
$$

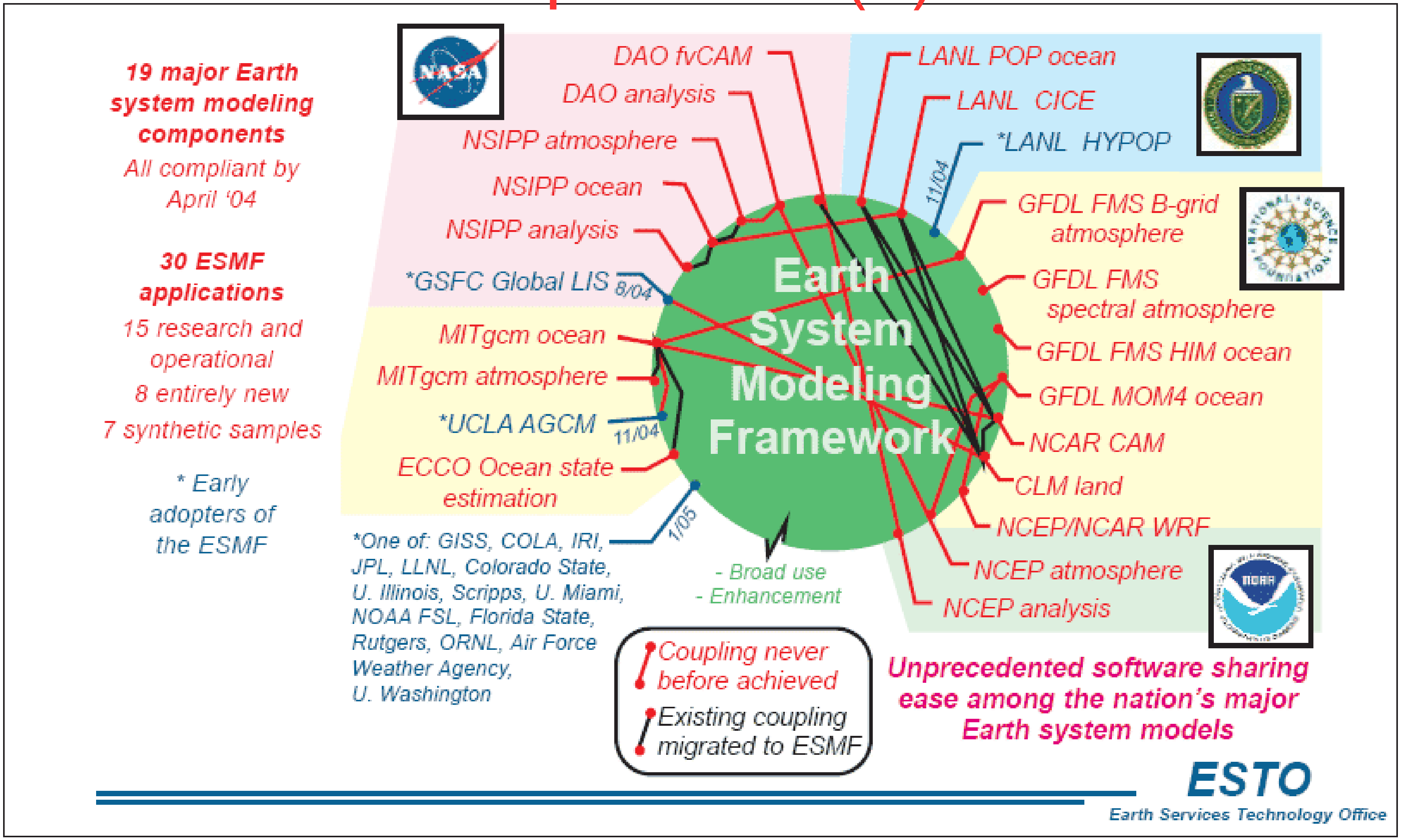
Governing atmospheric laws

Brute force numerical solution of the equations (2)…



Discretization of the equations

Brute force numerical solution of the equations (3)…



19 major Earth system modeling components

All compliant by April '04

30 ESMF applications

15 research and operational

8 entirely new

7 synthetic samples

** Early adopters of the ESMF*

**One of: GISS, COLA, IRI, JPL, LLNL, Colorado State, U. Illinois, Scripps, U. Miami, NOAA FSL, Florida State, Rutgers, ORNL, Air Force Weather Agency, U. Washington*

Coupling never before achieved

Existing coupling migrated to ESMF

Unprecedented software sharing ease among the nation's major Earth system models

ESTO

Earth Services Technology Office

Earth system modelling

Or simplicity?

Atmosphere: Emergent laws

(high level)

Power law

$$\text{Fluctuations} \approx (\text{turbulent flux})\ \text{x}\ (\text{scale})^{H}$$

Differences,
tendencies,
wavelet
coefficients

Cascading
Turbulent flux

Size:
Anisotropic
Space-time
Scale function

Fluctuation
/conservation
exponent

These laws are scale invariant

Fluctuation = change in time and/or space

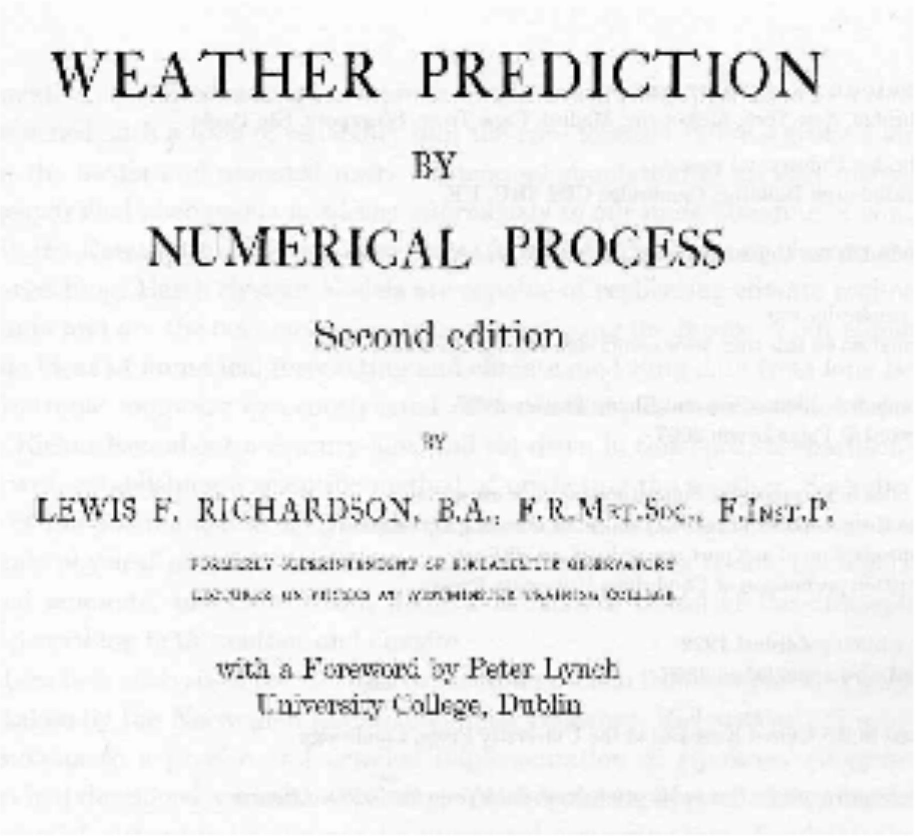
Scale = size

Turbulent flux = strength of stirring

Which Richardson? The father of Numerical Weather Prediction…

L. F. Richardson, 1931

The father of numerical weather prediction



WEATHER PREDICTION

BY

NUMERICAL PROCESS

Second edition

BY

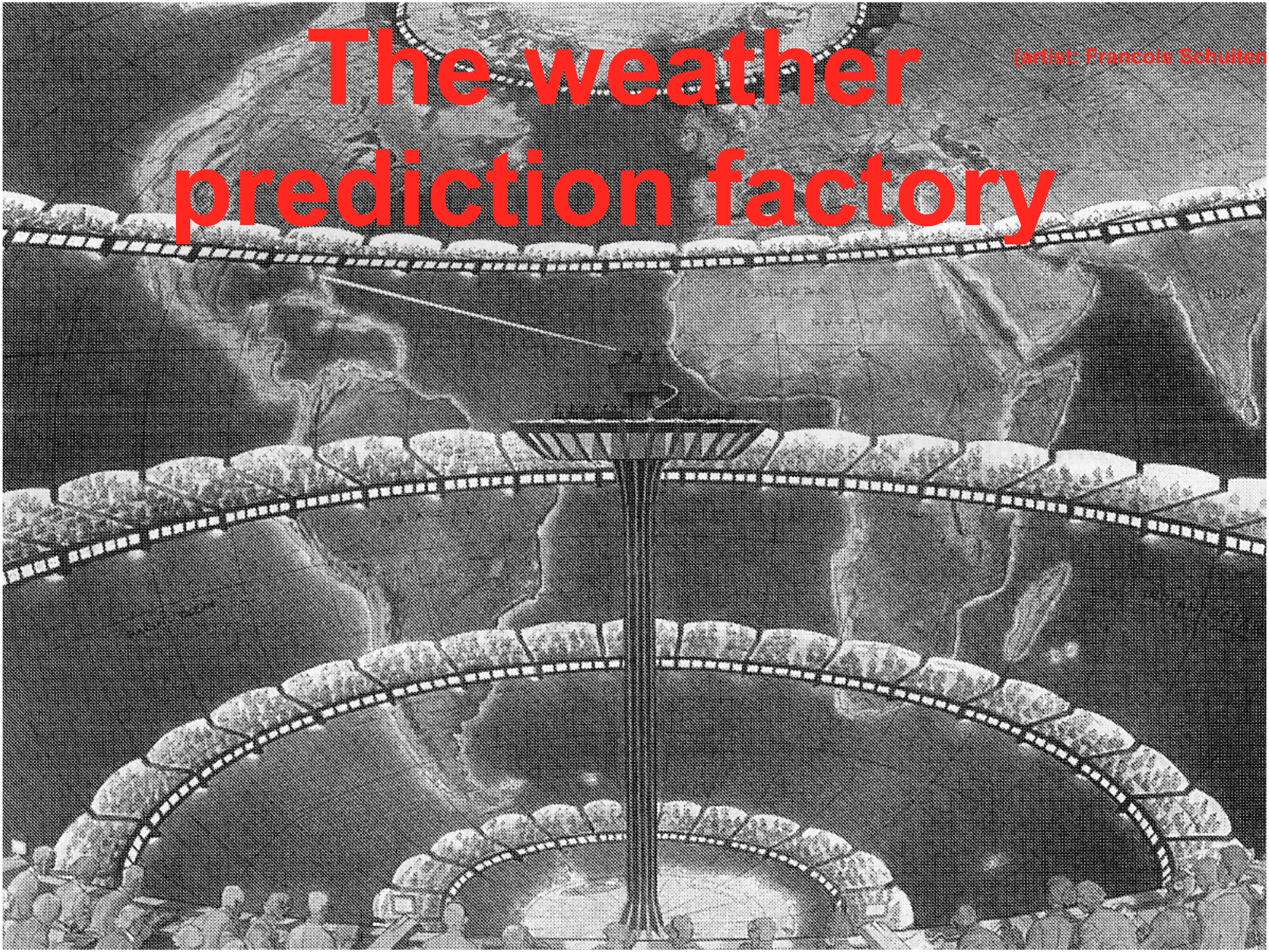
LEWIS F. RICHARDSON, B.A., F.R.Met.Soc., F.Inst.P.

with a Foreword by Peter Lynch
University College, Dublin

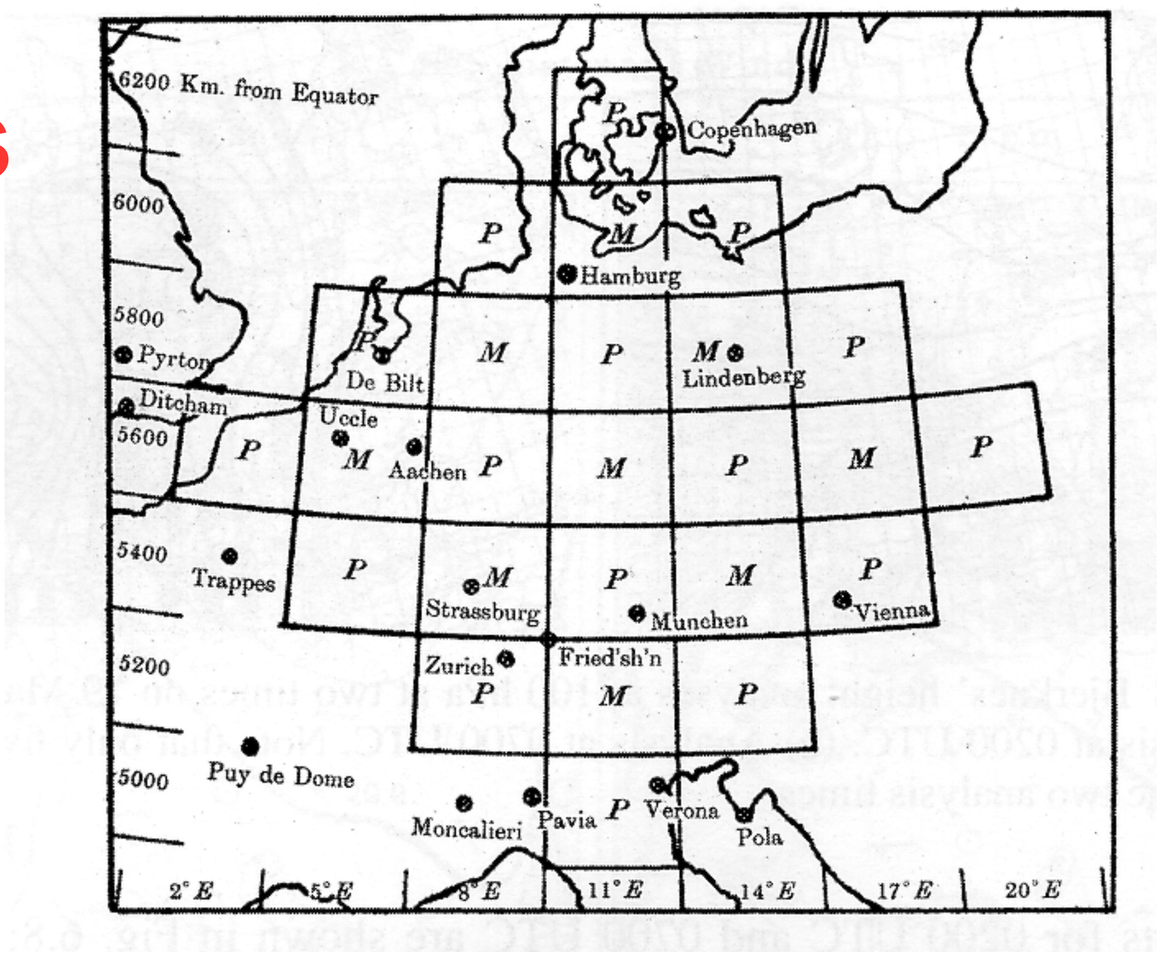
1922

The weather prediction factory

(artist: Francois Schuiten



Richardson's numerical grid for integrating

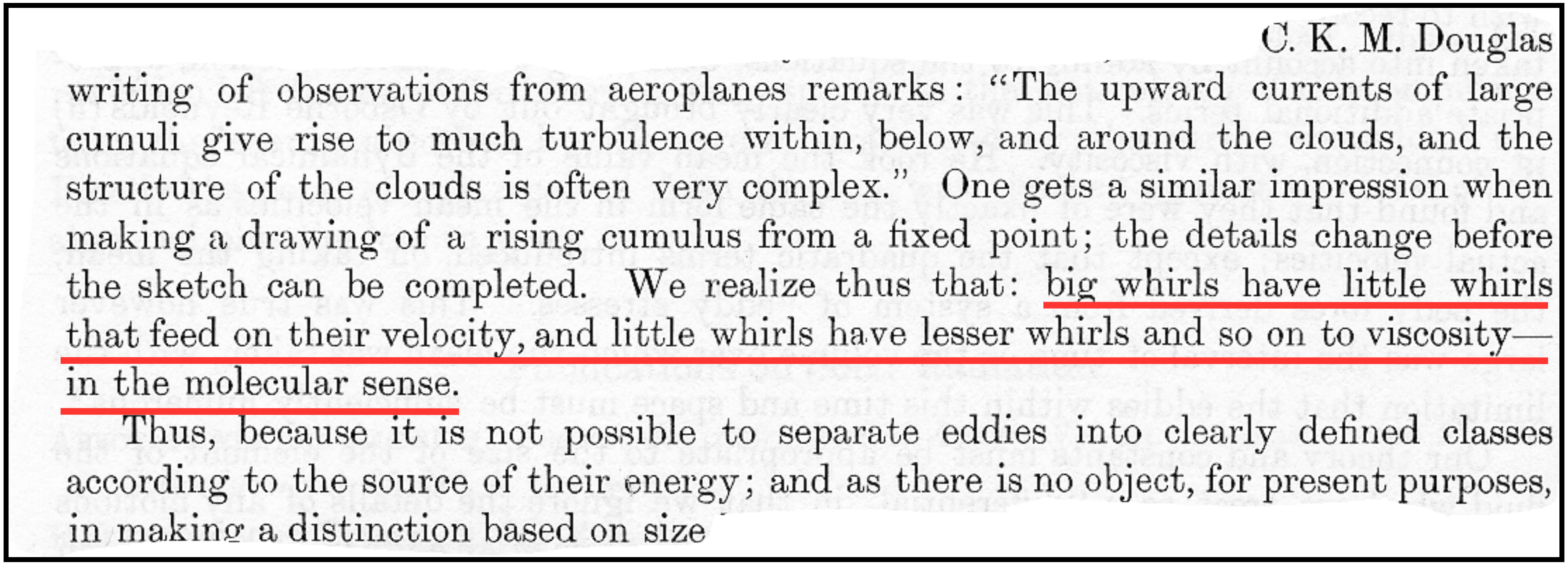


Each column was divided into 5 vertical cells and defined 7 quantities: pressure, temperature, density, water content, 3 velocity components

"It took me the best part of six weeks to draw up the computing forms and to work out the new distribution in two vertical columns for the first time. My office was a heap of hay in cold rest billet. With practice the work of an average computer might go perhaps ten times faster. If the time-step were 3 hours, then 32 individuals could just compute two-points so as to keep up with the weather."

-Richardson 1922

... or the grandfather of cascades?

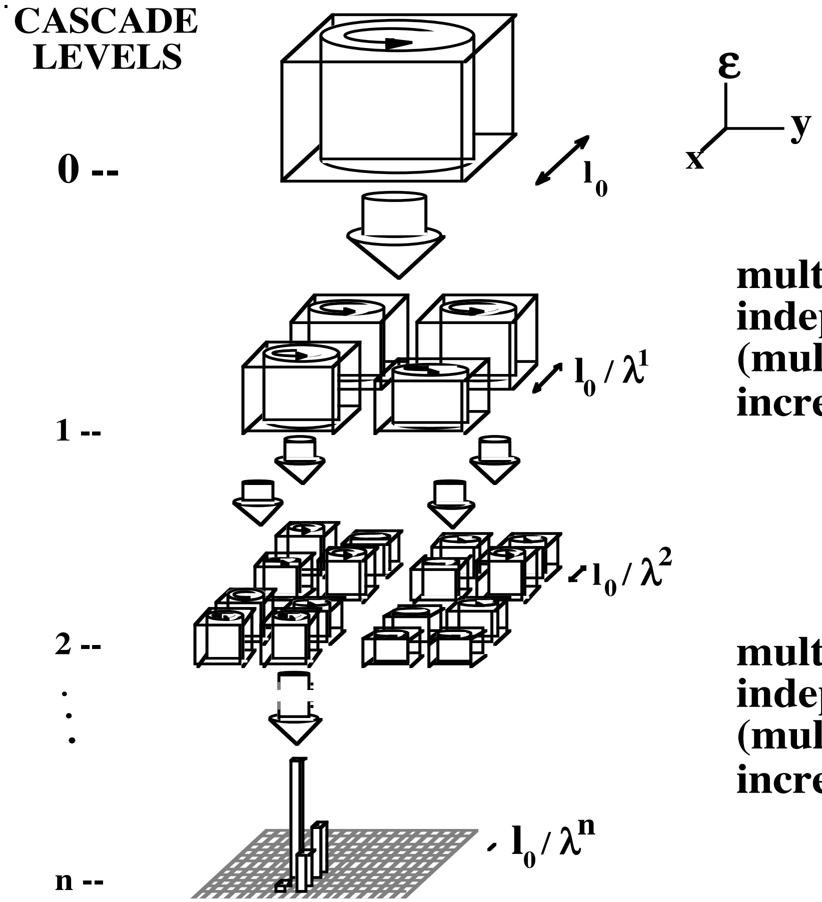


C. K. M. Douglas writing of observations from aeroplanes remarks: "The upward currents of large cumuli give rise to much turbulence within, below, and around the clouds, and the structure of the clouds is often very complex." One gets a similar impression when making a drawing of a rising cumulus from a fixed point; the details change before the sketch can be completed. We realize thus that: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense.

Thus, because it is not possible to separate eddies into clearly defined classes according to the source of their energy; and as there is no object, for present purposes, in making a distinction based on size

Weather prediction by Numerical Process 1922, p.66

Scale by scale simplicity: cascades

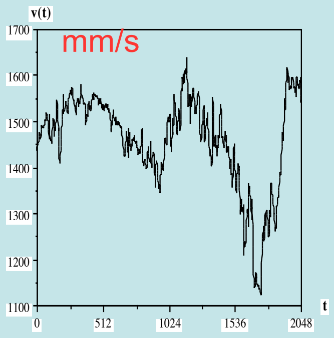


multiplication by 4 independent random (multiplicative) increments

multiplication by 16 independent random (multiplicative) increments

“Does the wind have a velocity?”

“Although at first sight strange, the question grows upon acquaintance…” - Richardson 1926

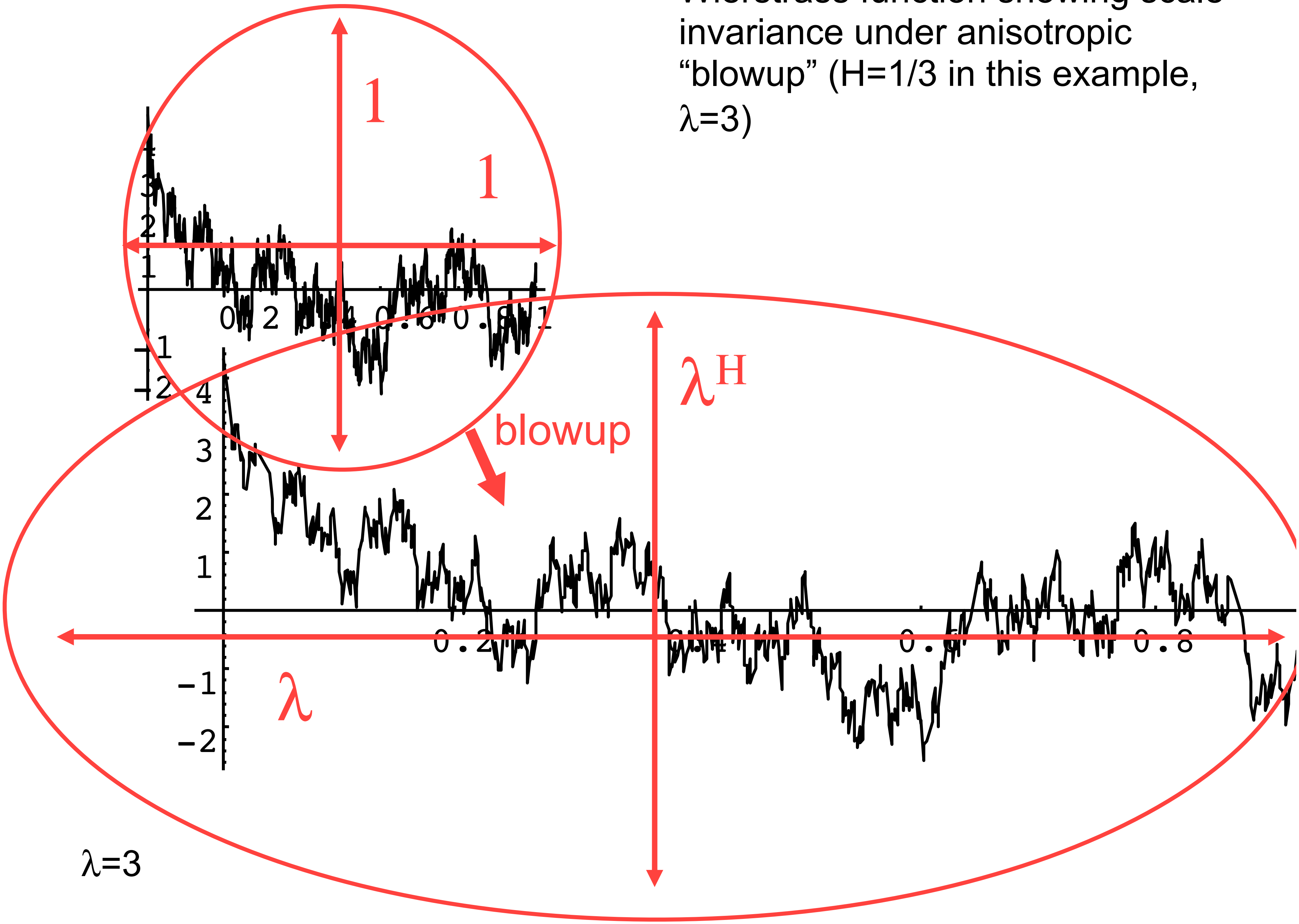


1 second of wind data (roof of Rutherford building, McGill)

Richardson suggested that the trajectory of a particle could be like a Wierstrass function (1872)

Scale invariance and fractals

Wierstrass function showing scale invariance under anisotropic “blowup” (H=1/3 in this example, λ=3)

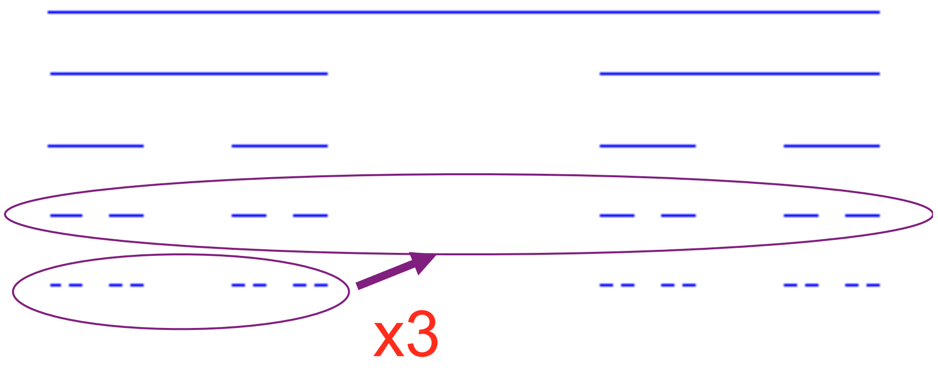


λ=3

Cantor set

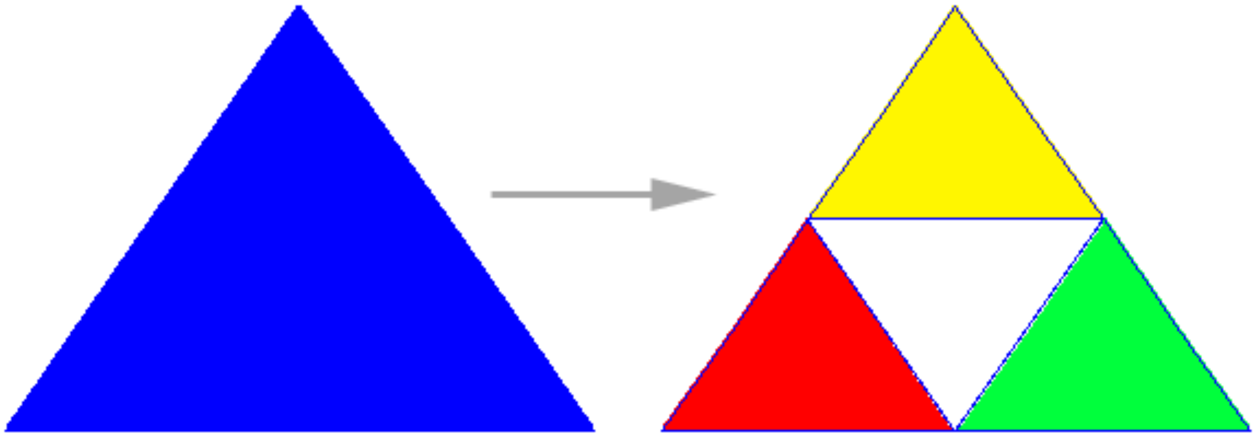
- Let us start with:

and let us iterate:



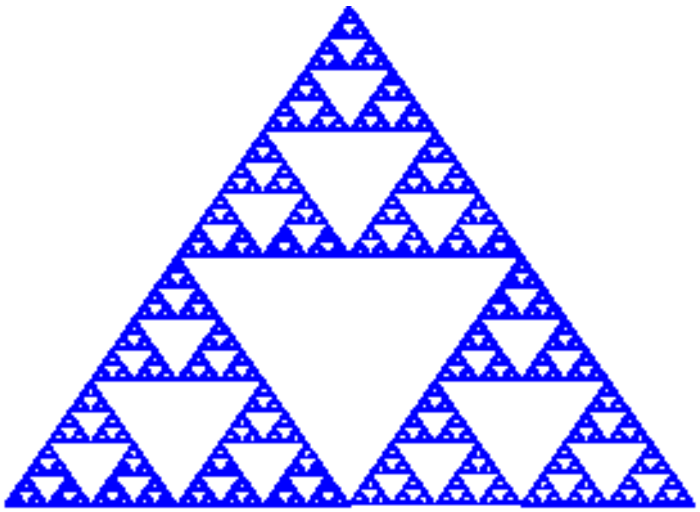
A small part is same as the whole if “blown up” by a factor 3 (“scale invariance”, “self-similarity”)

Sierpinski Triangle



1

2

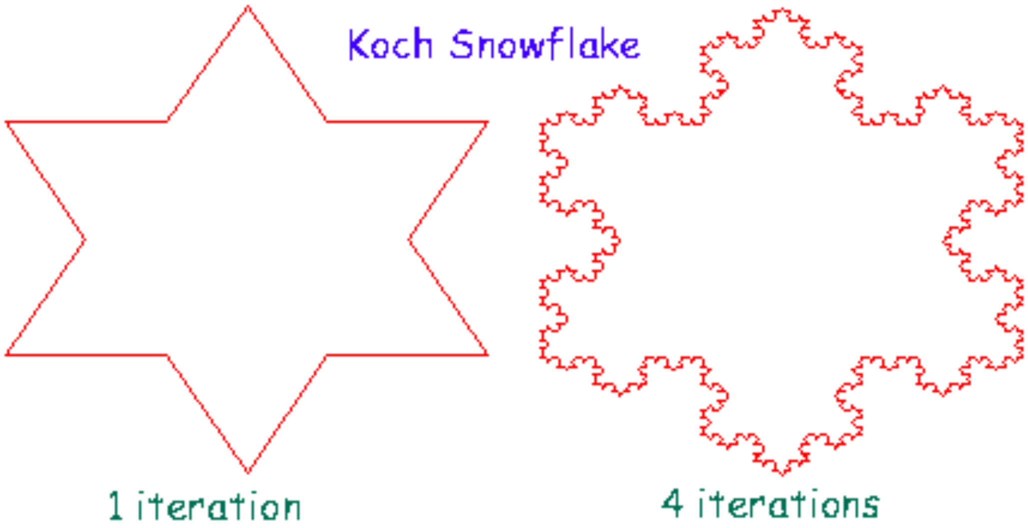


10 iterations

Koch snowflake

Let us start with:

and let us iterate:



Sierpinski Pyramid

- First iteration:

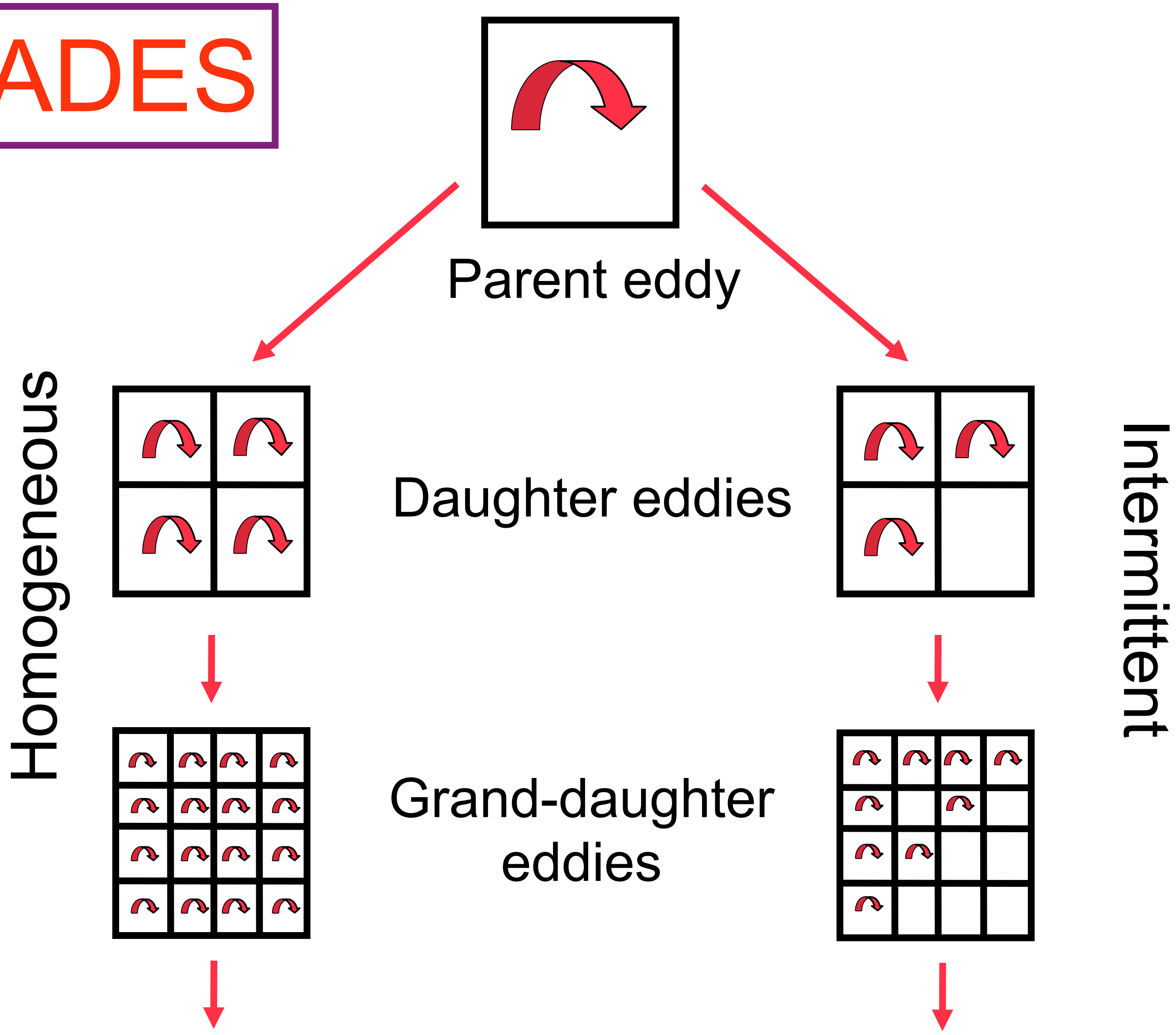
10 th
iteration:

Menger Sponge

- motif:

iterations:

CASCADES



Parent eddy

Homogeneous

Intermittent

Daughter eddies

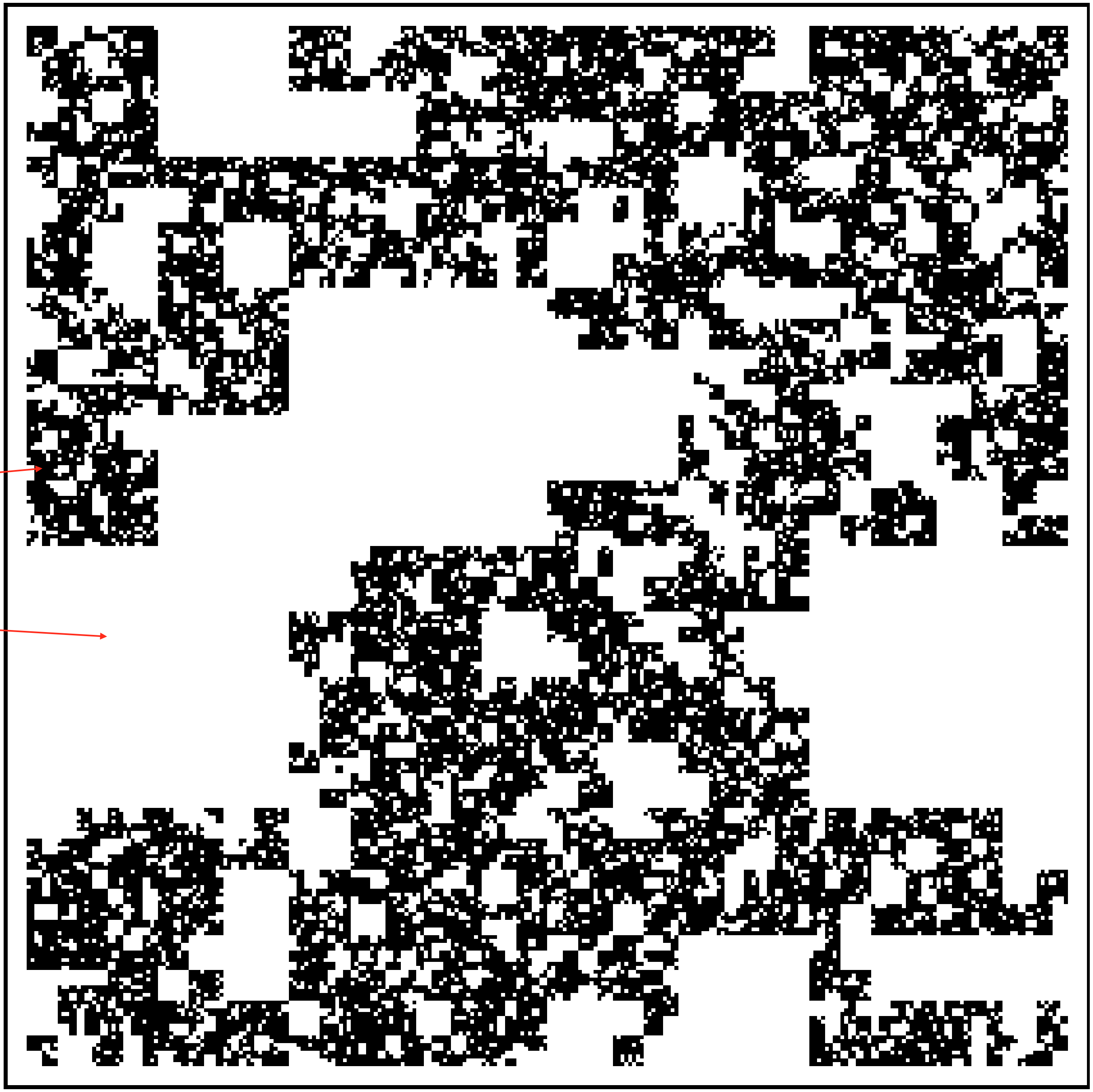
Grand-daughter eddies

β-model

Fractal set

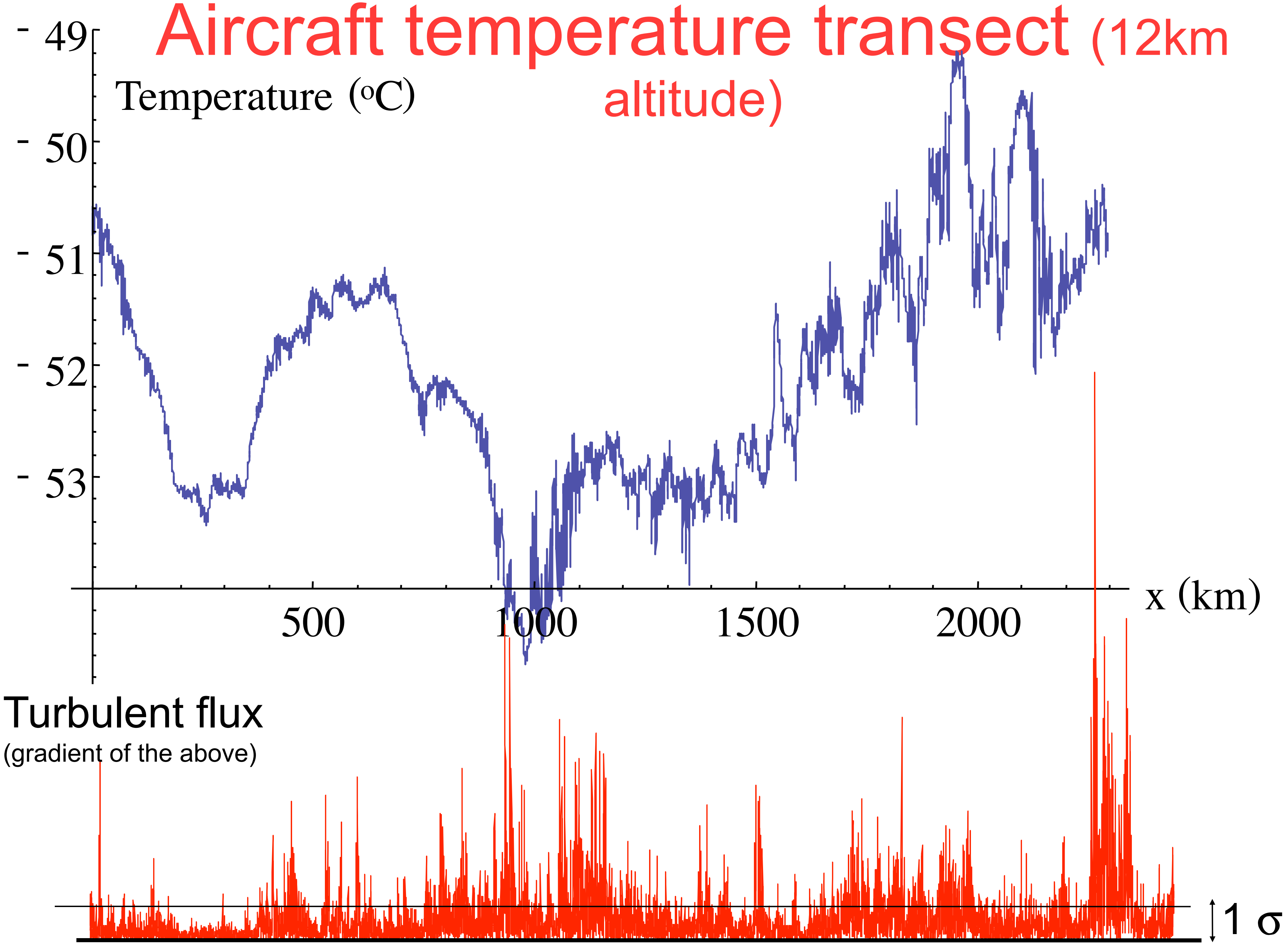
“active”

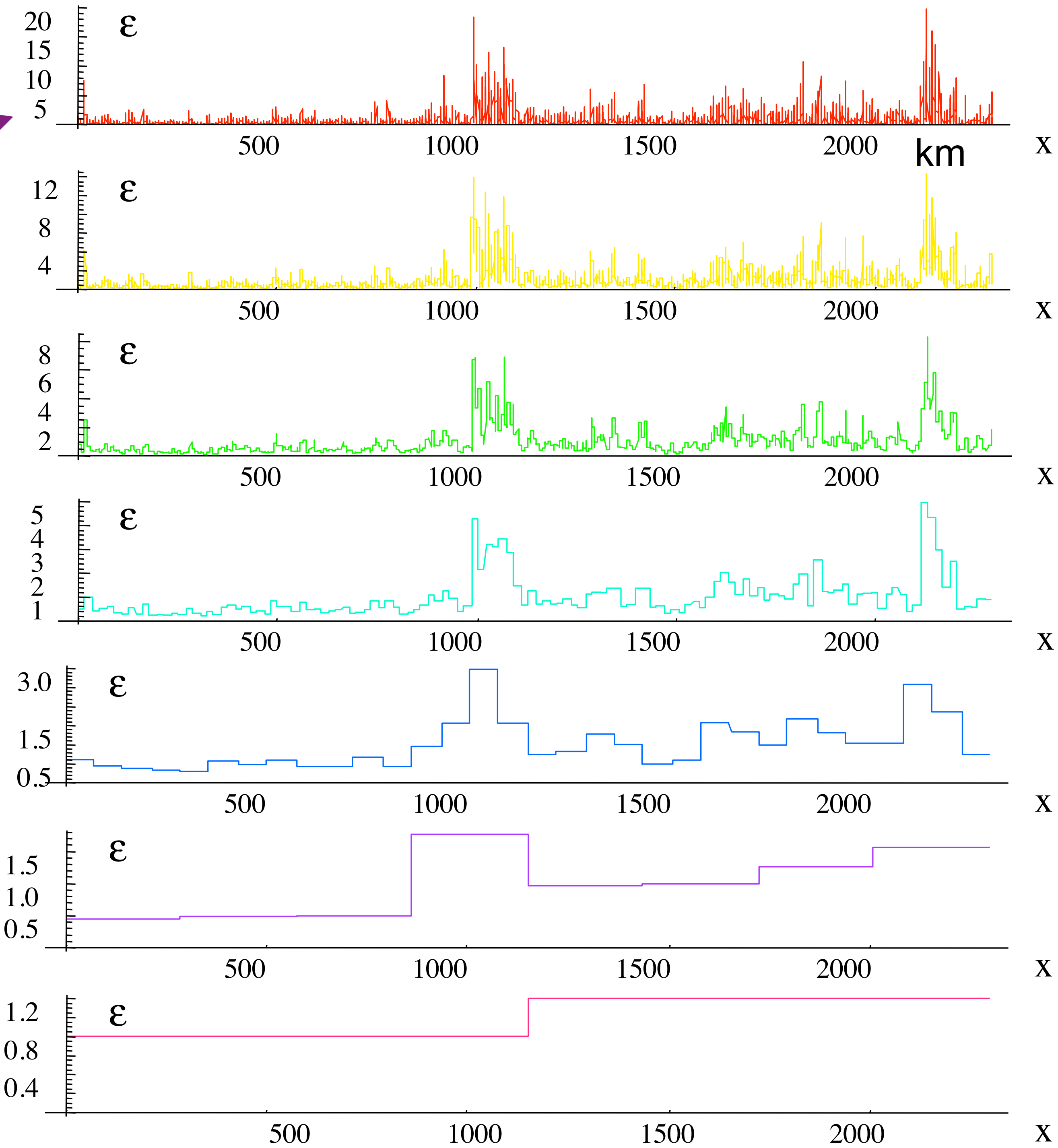
“calm”



Cascades and Multifractals

Aircraft temperature transect (12km altitude)



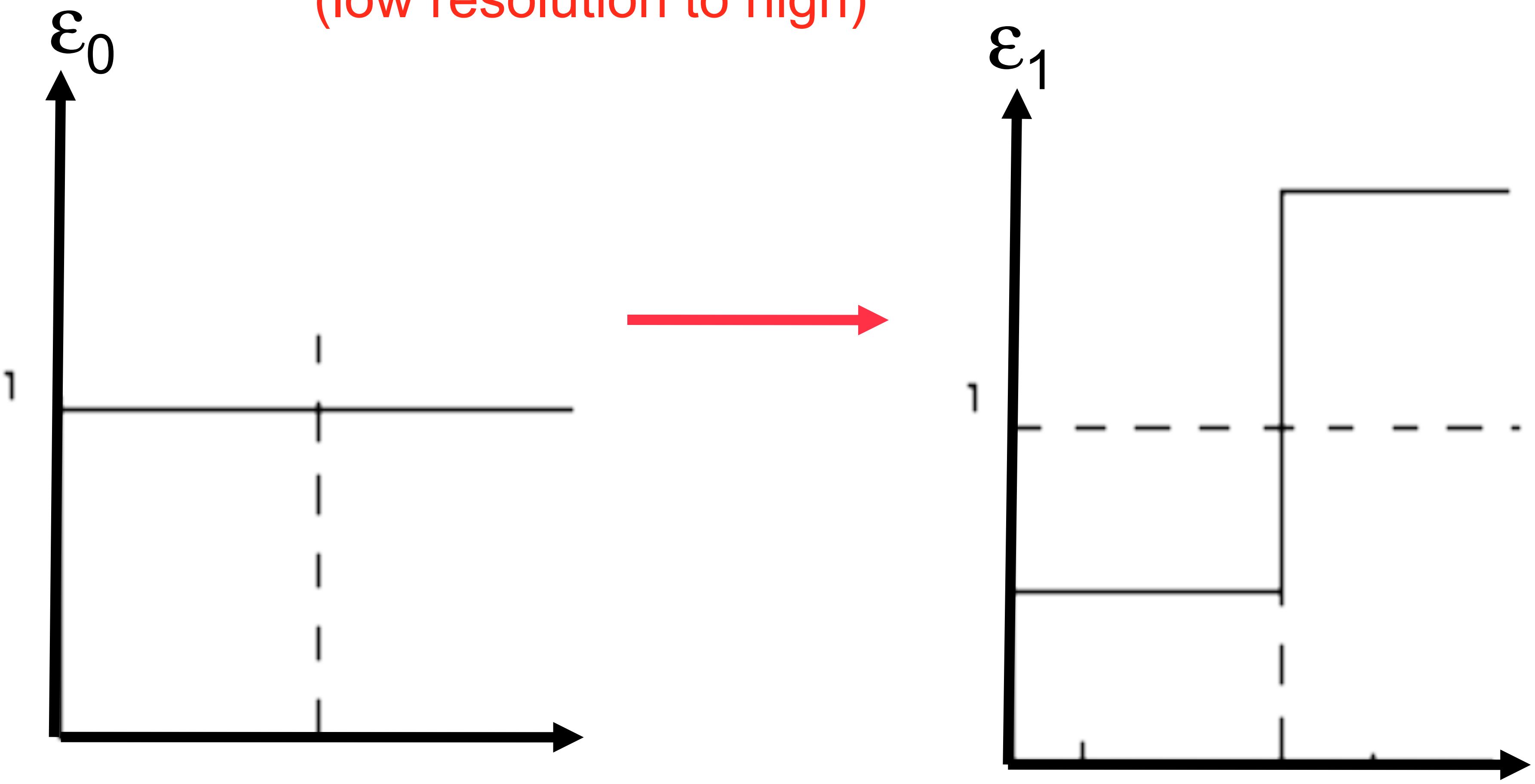


Temperature
turbulent flux ε
at 280m resolution

High to low
Resolution:
degrading by
factors of 4

Cascades and Multifractals

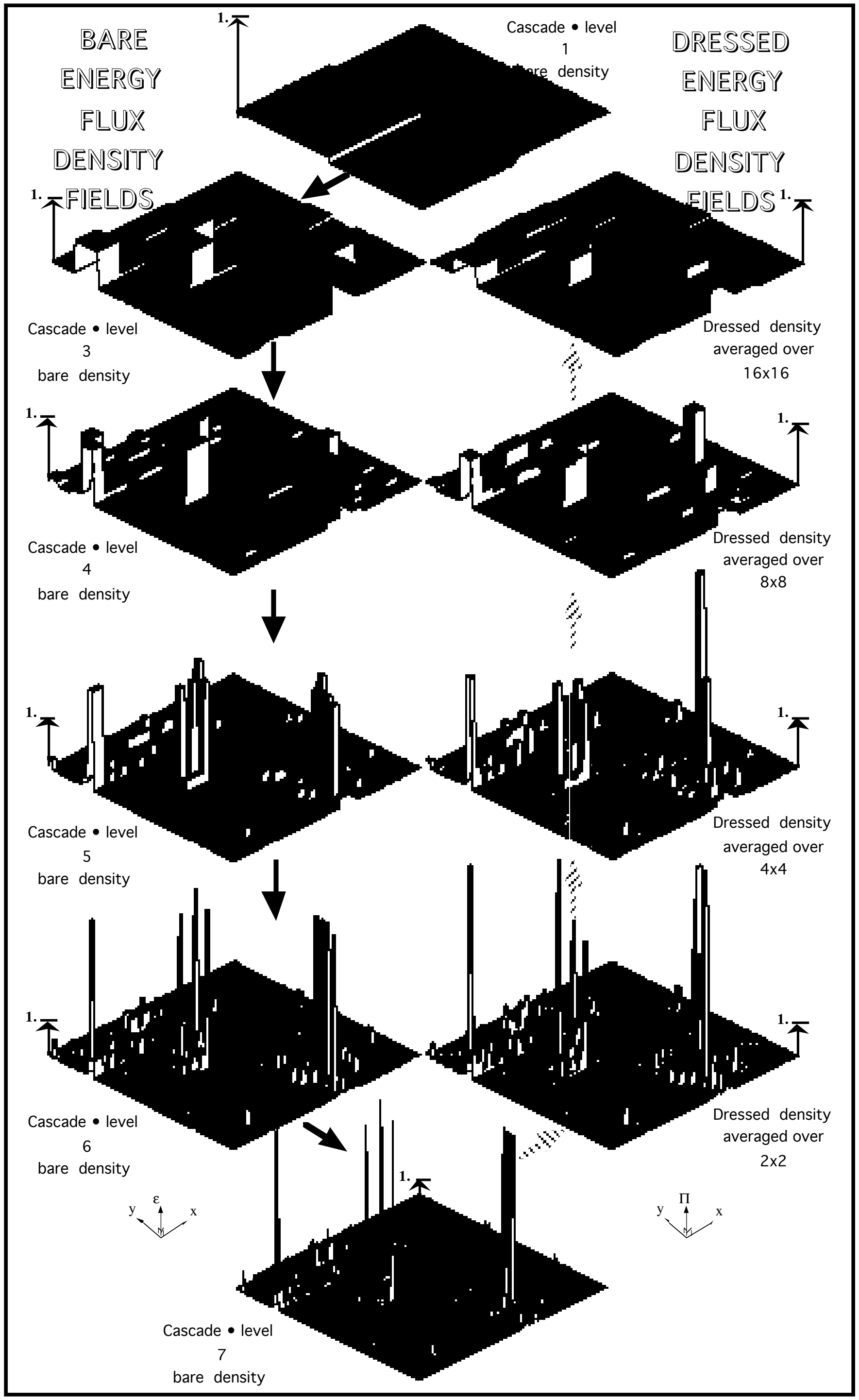
Simulations: adding small scale details
(low resolution to high)



("α model")

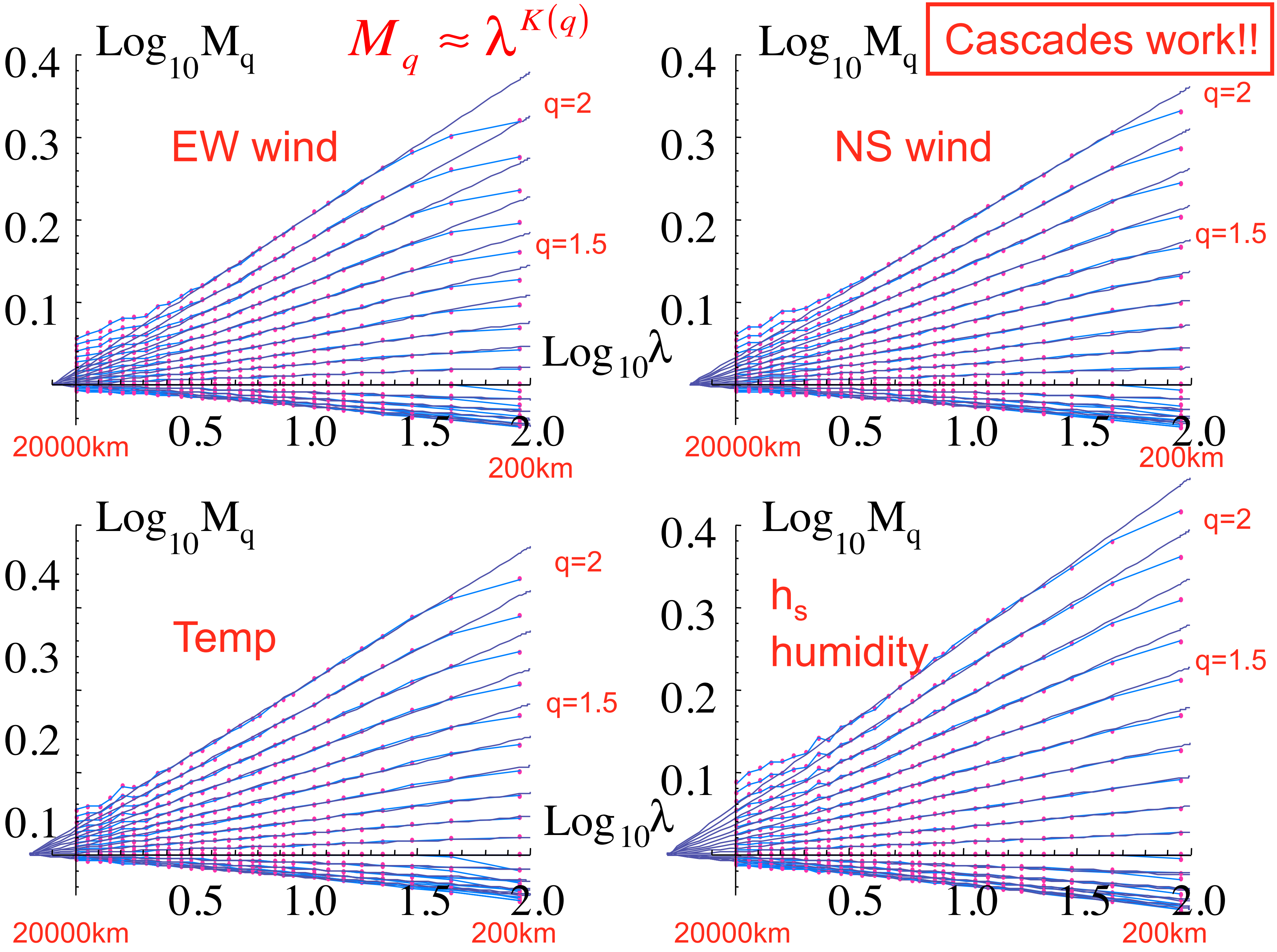
Cascades

Multifractal



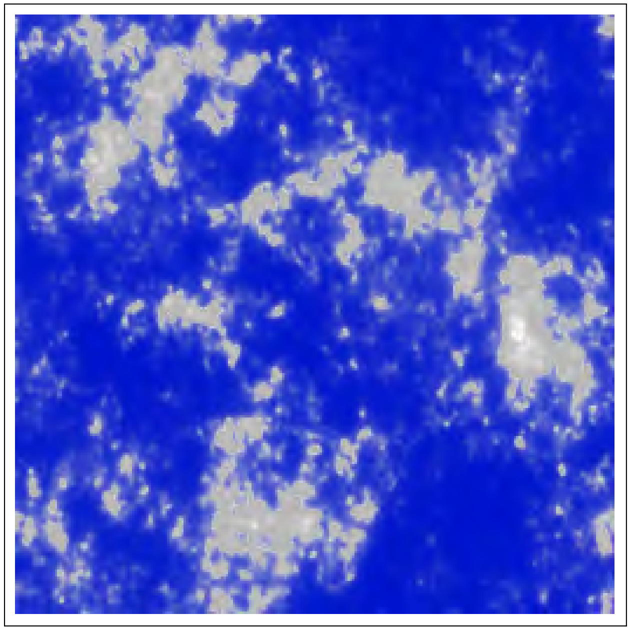
$$M_q \approx \lambda^{K(q)}$$

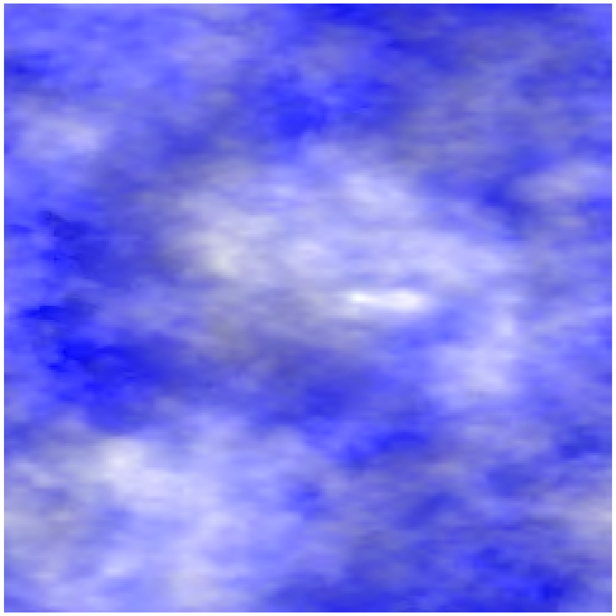
Cascades work!!



Cascade modeling: clouds and radiative transfer

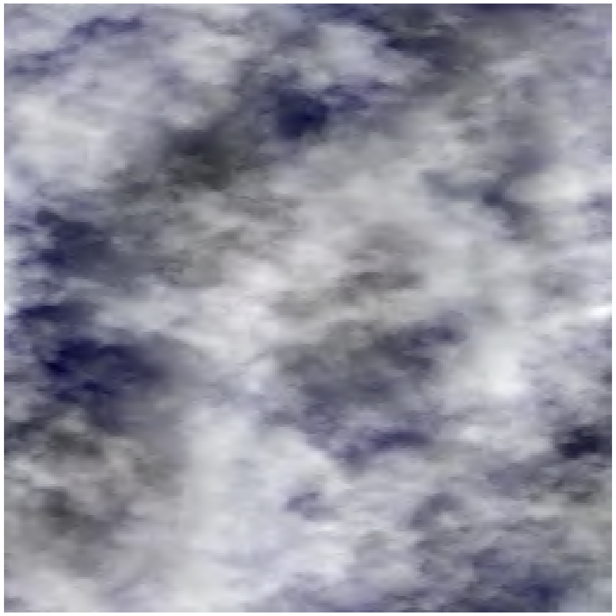
Cloud liquid water (top)

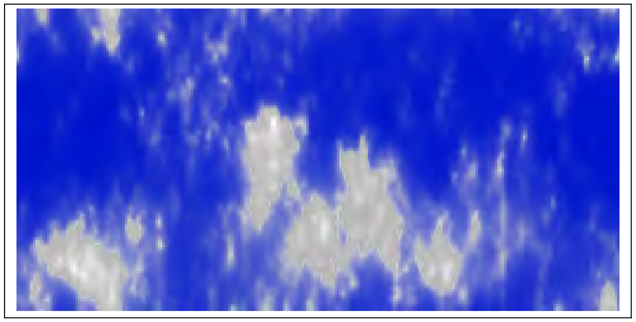




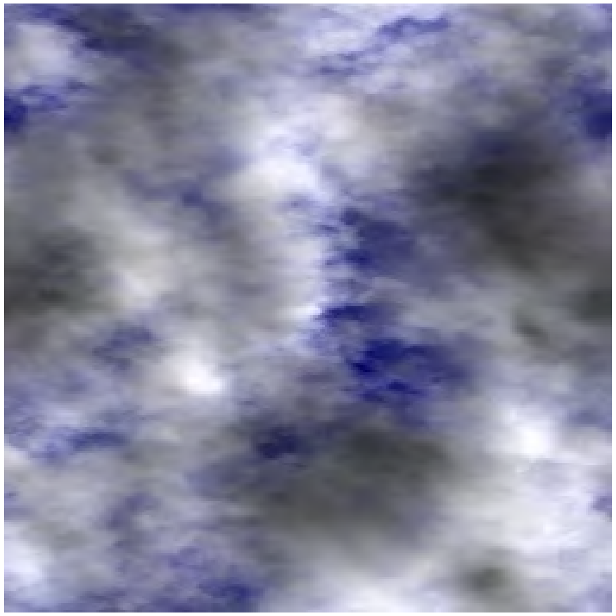
Cloud top, infra red

Cloud top visible





Cloud liquid water (side)

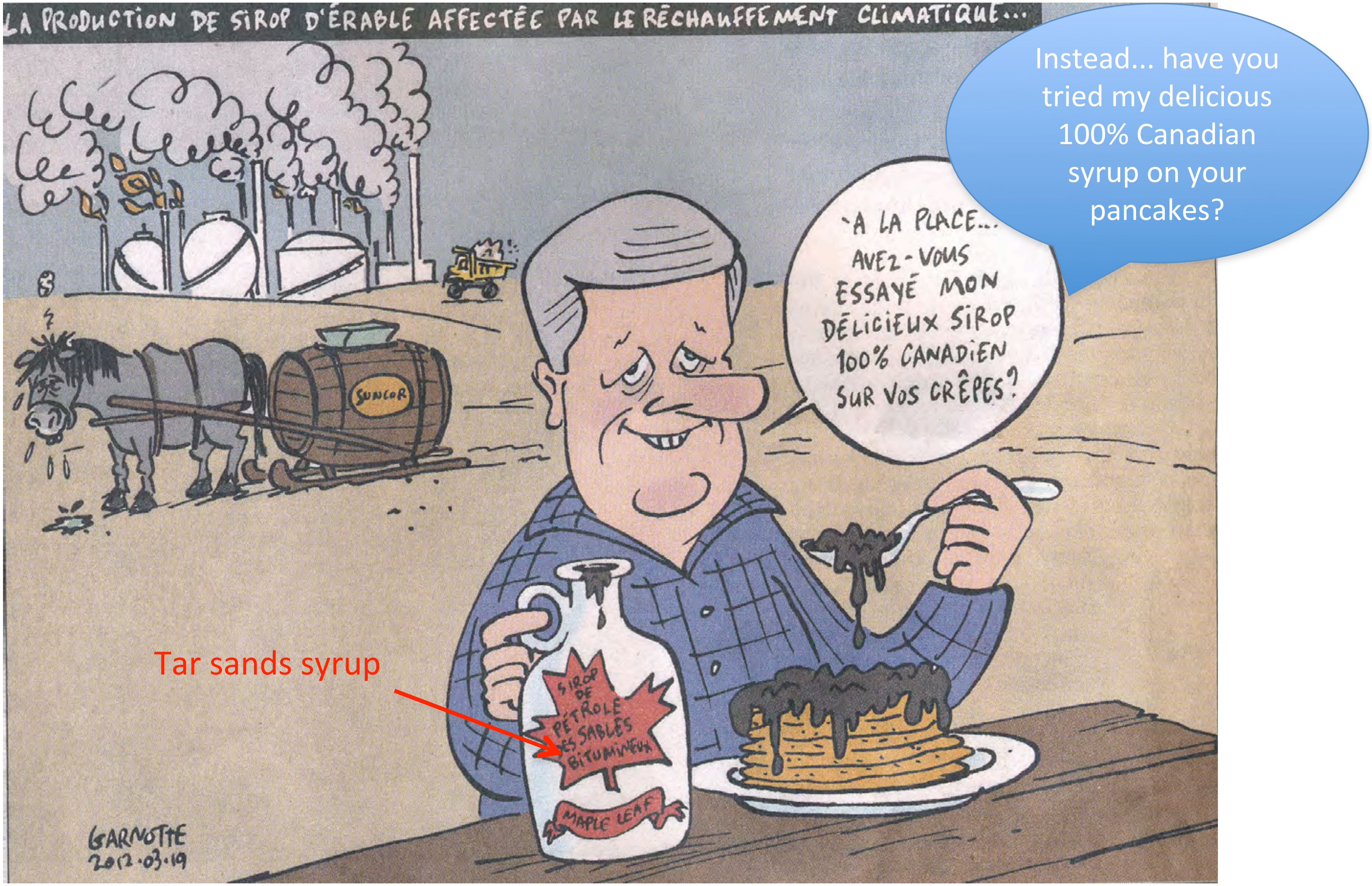


Cloud bottom visible

Cascade Simulations

The Climate

The production of maple syrup is affected by global warming...



Instead... have you tried my delicious 100% Canadian syrup on your pancakes?

Tar sands syrup

What is the climate?

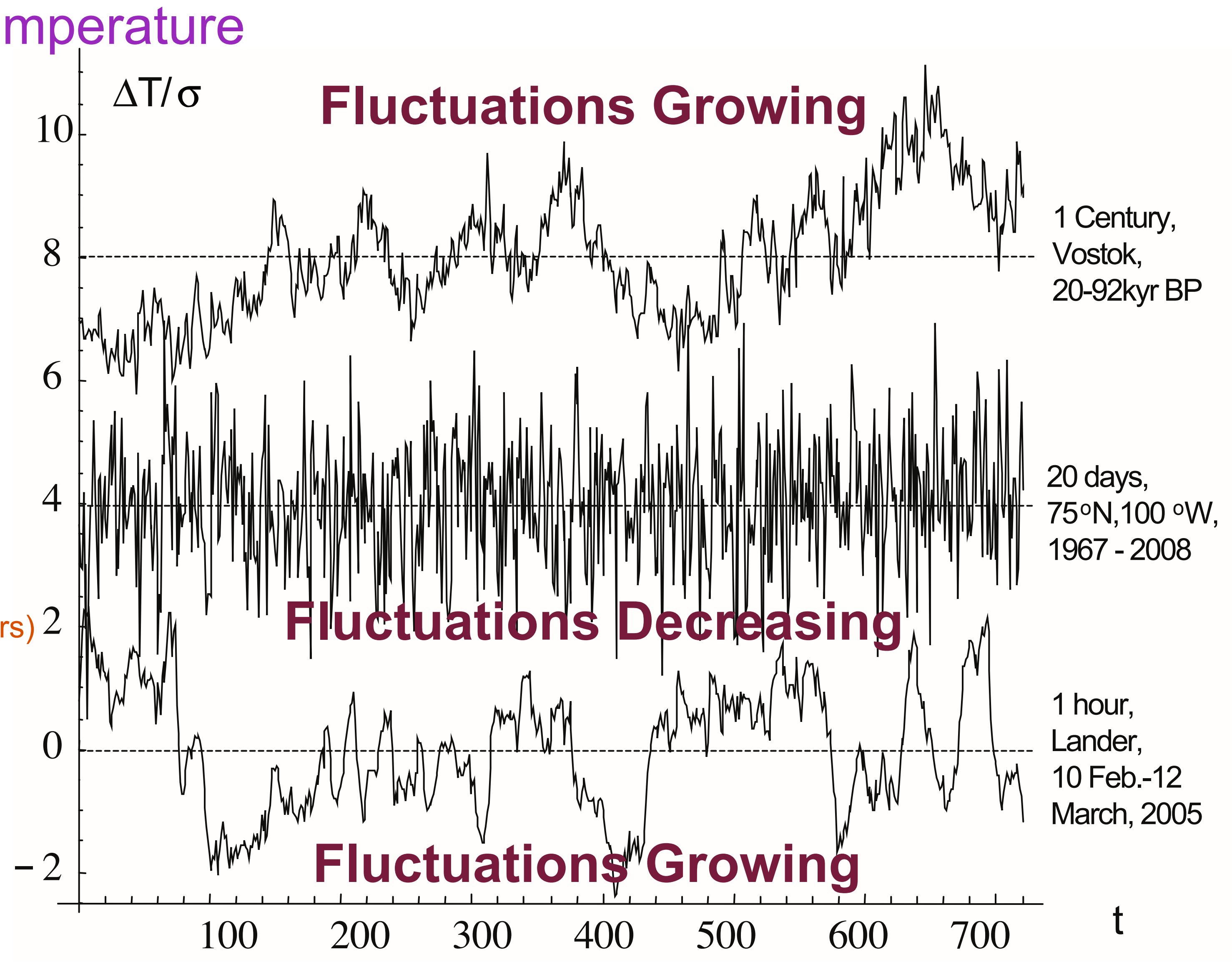
"Climate is what you expect, weather is what you get."

-Farmers Almanac

"Climate is conventionally defined as the long-term statistics of the weather…".

-Committee on Radiative Forcing Effects on Climate, 2005 US National Academy of Science

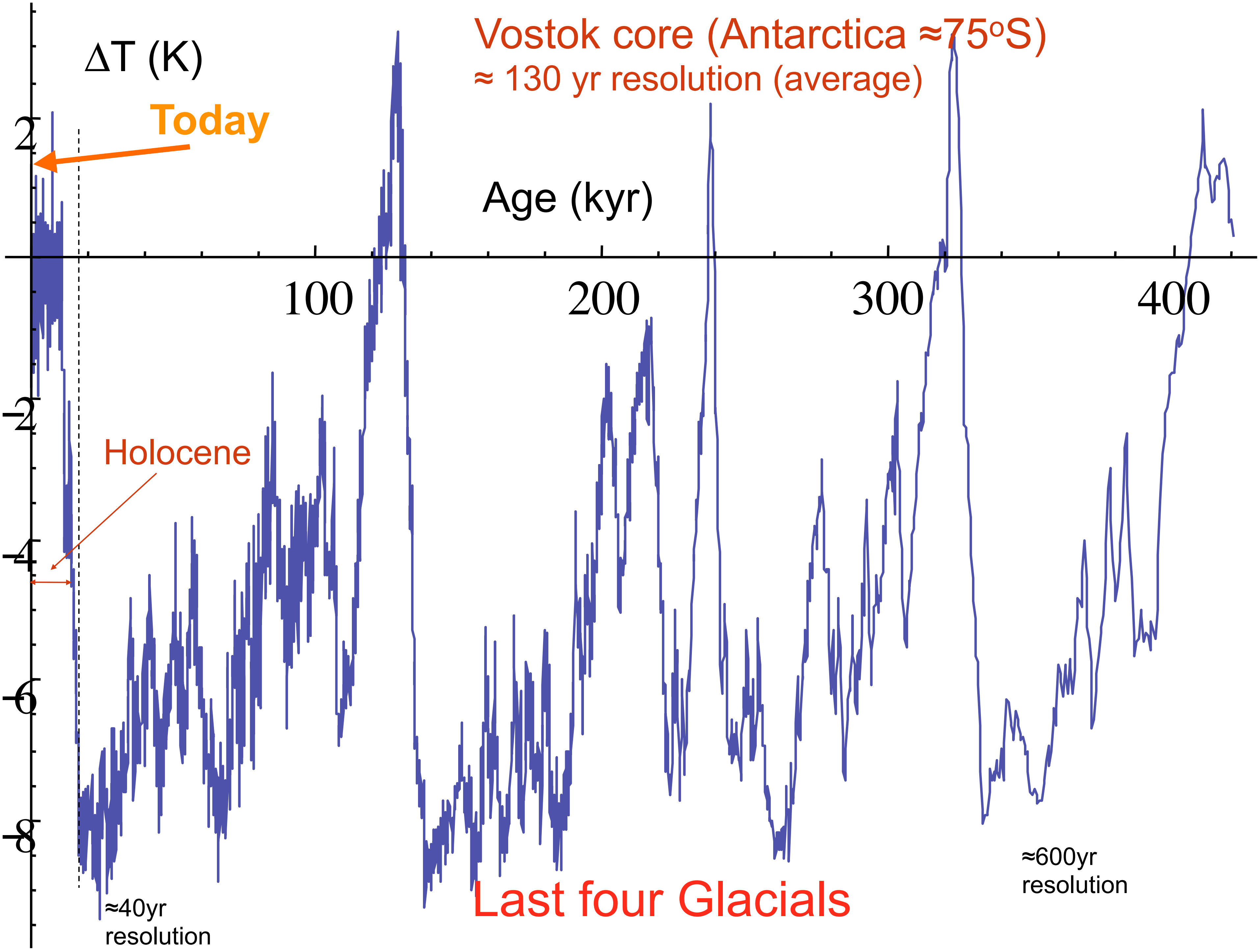
Three regimes:
three types of variability: *not* two!



Climate
(10-30 yrs to 50,000 yrs)

Low frequency weather
(10 days to 10-30 yrs)

Weather
(up to 10 days)



Vostok core (Antarctica ≈75°S)

≈ 130 yr resolution (average)

Today

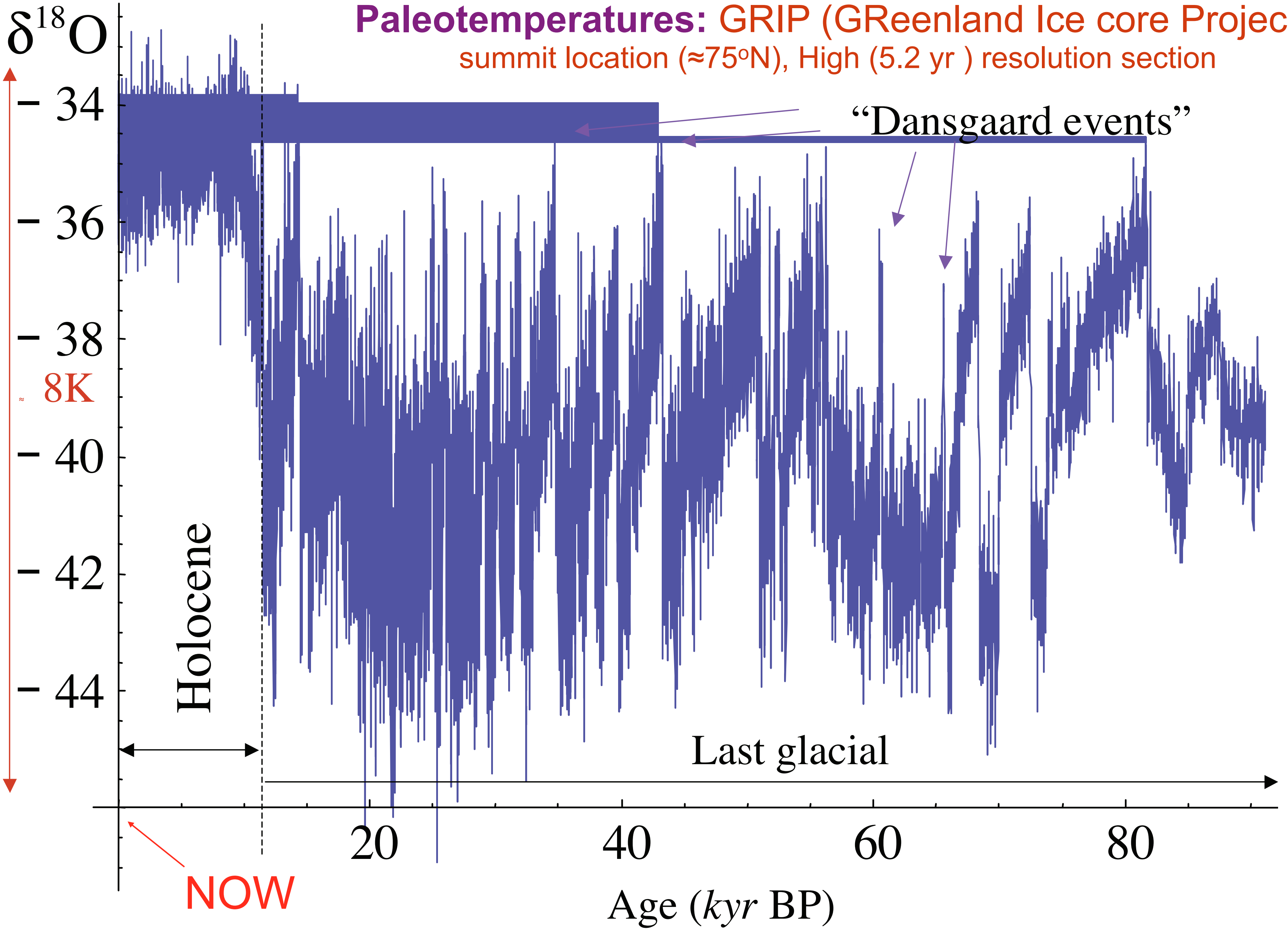
Holocene

≈40yr resolution

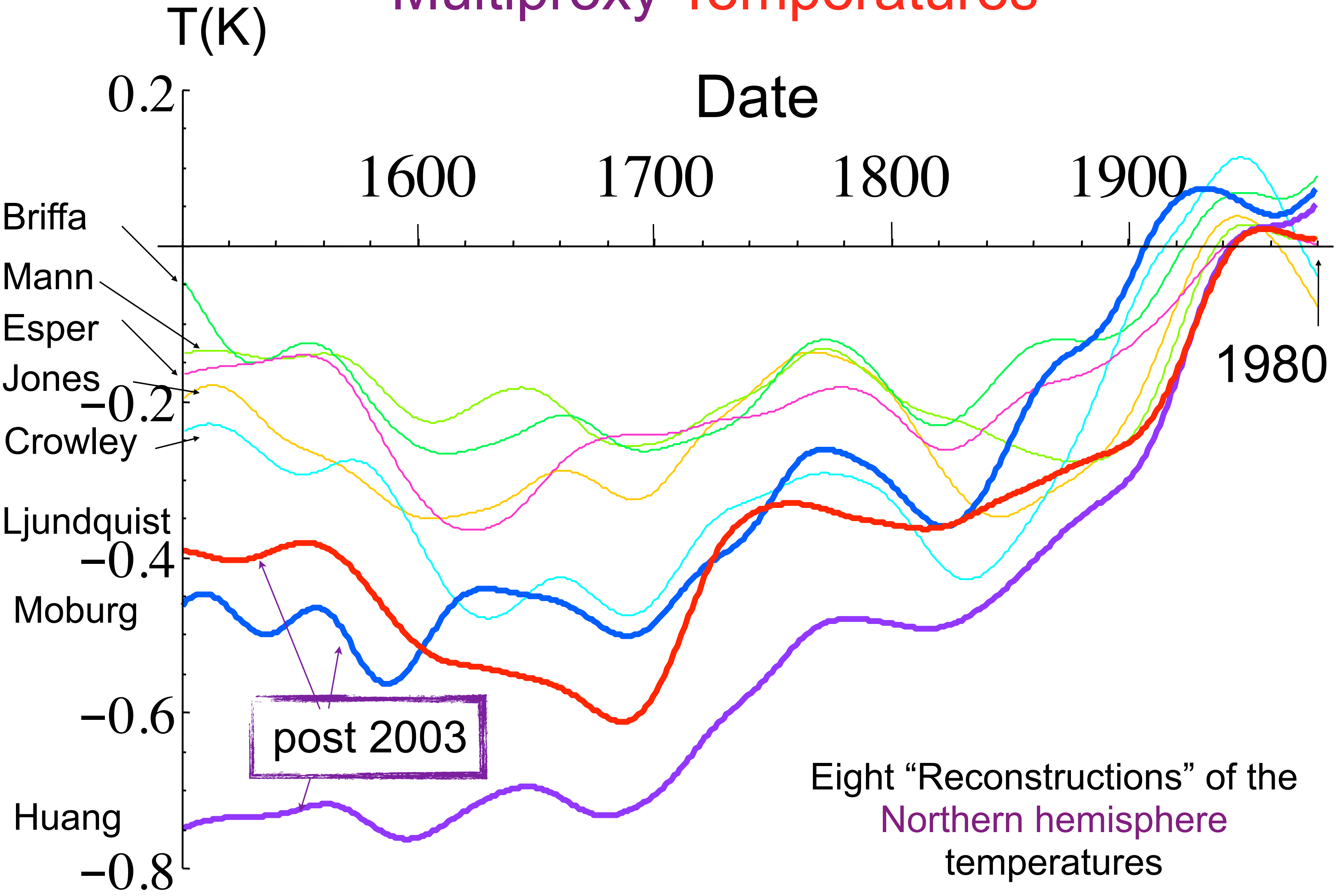
≈600yr resolution

Last four Glacials

Paleotemperatures: GRIP (GReenland Ice core Project), summit location (≈75°N), High (5.2 yr) resolution section

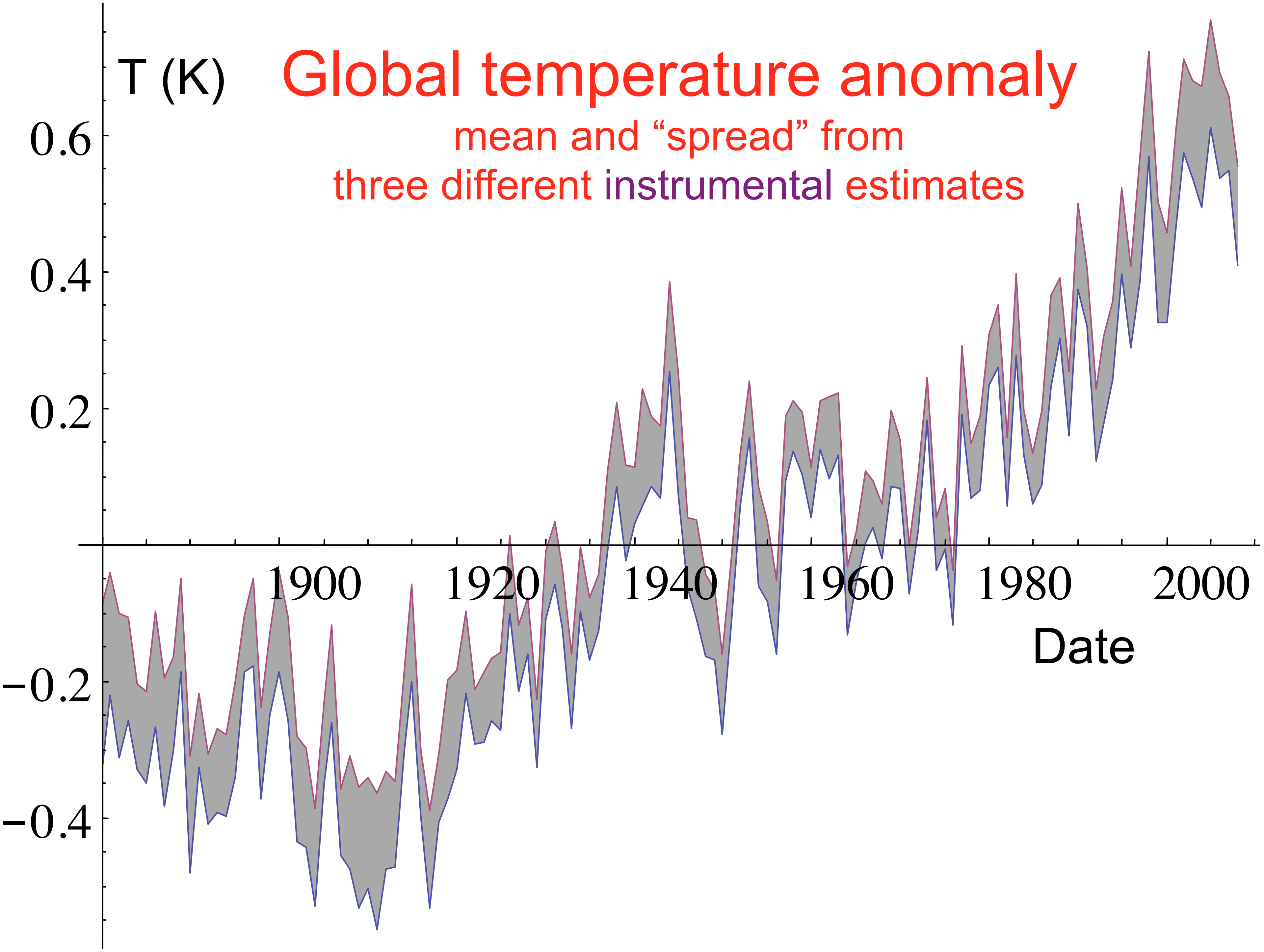


Multiproxy Temperatures



Eight “Reconstructions” of the
Northern hemisphere
temperatures

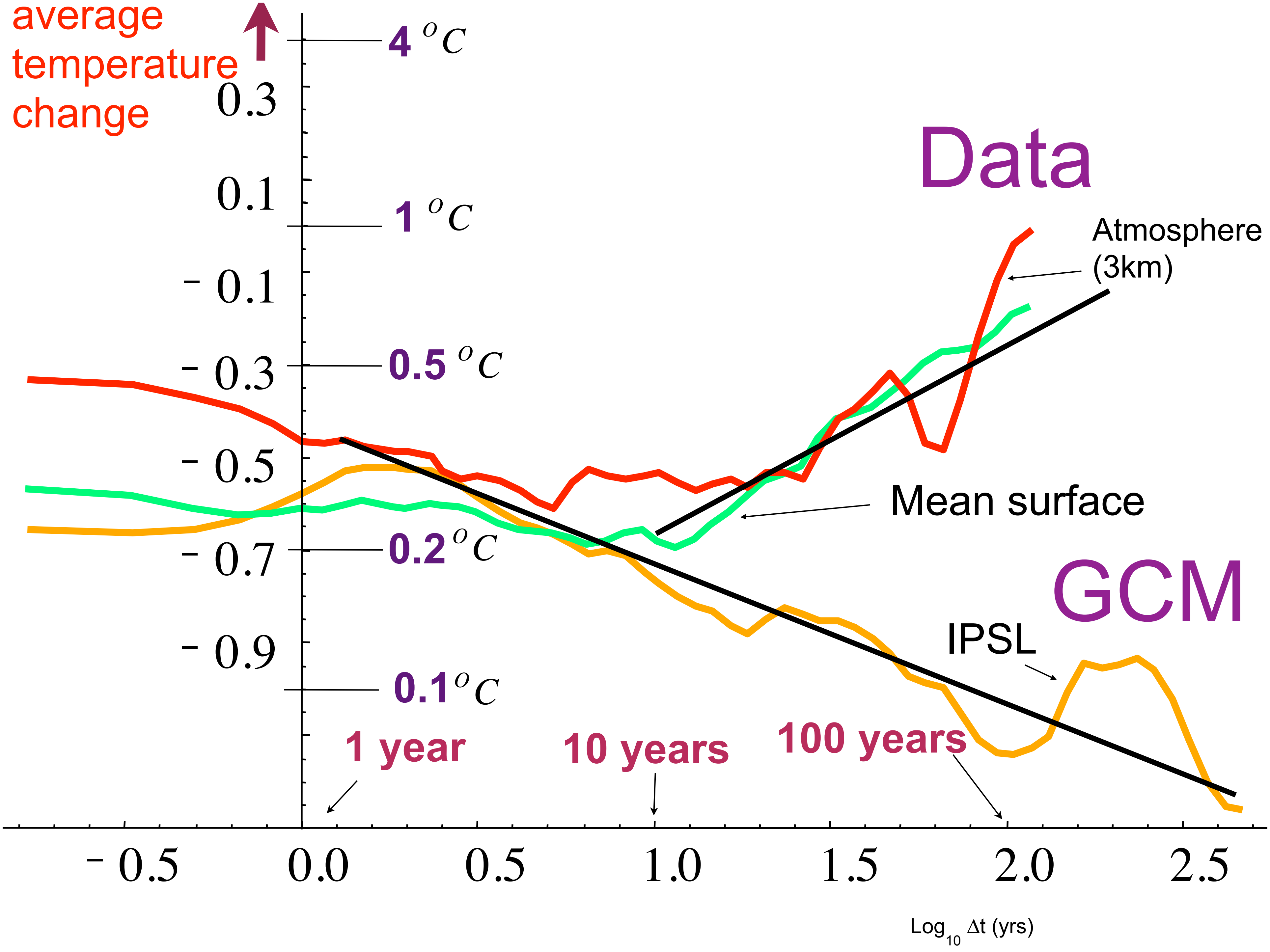
(30 year average applied)


T (K)
Global temperature anomaly
mean and "spread" from
three different instrumental estimates
0.6
0.4
0.2
−0.2
−0.4
1900
1920
1940
1960
1980
2000
Date

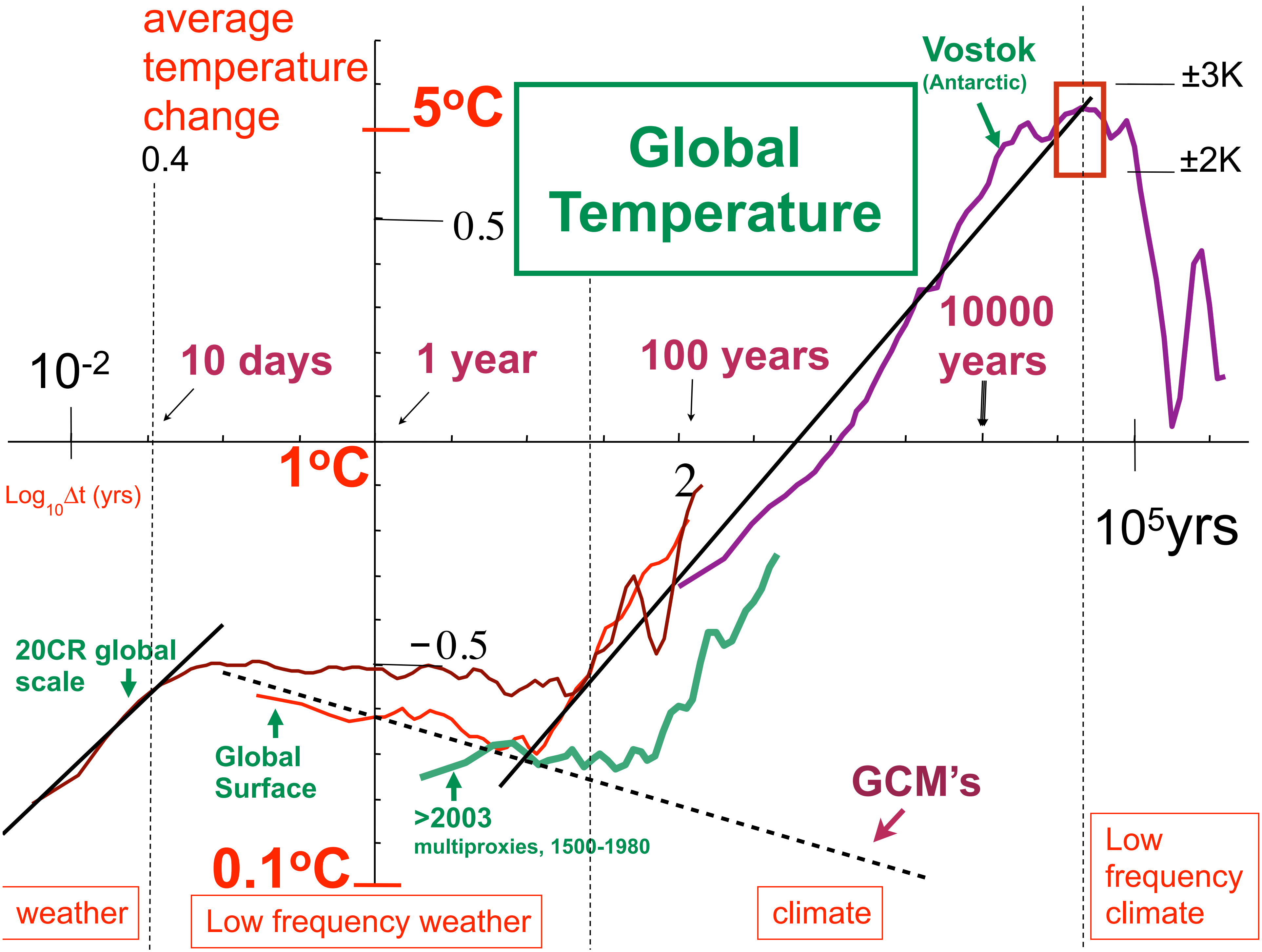
Do Global Climate models predict...

The climate?

...or low frequency weather?


average temperature change
4 °C
0.3
0.1
1 °C
- 0.1
- 0.3
0.5 °C
- 0.5
0.2 °C
- 0.7
- 0.9
0.1 °C
Data
Atmosphere (3km)
Mean surface
GCM
IPSL
1 year
10 years
100 years
- 0.5
0.0
0.5
1.0
1.5
2.0
2.5
$\mathrm{Log}_{10}\ \Delta t$ (yrs)

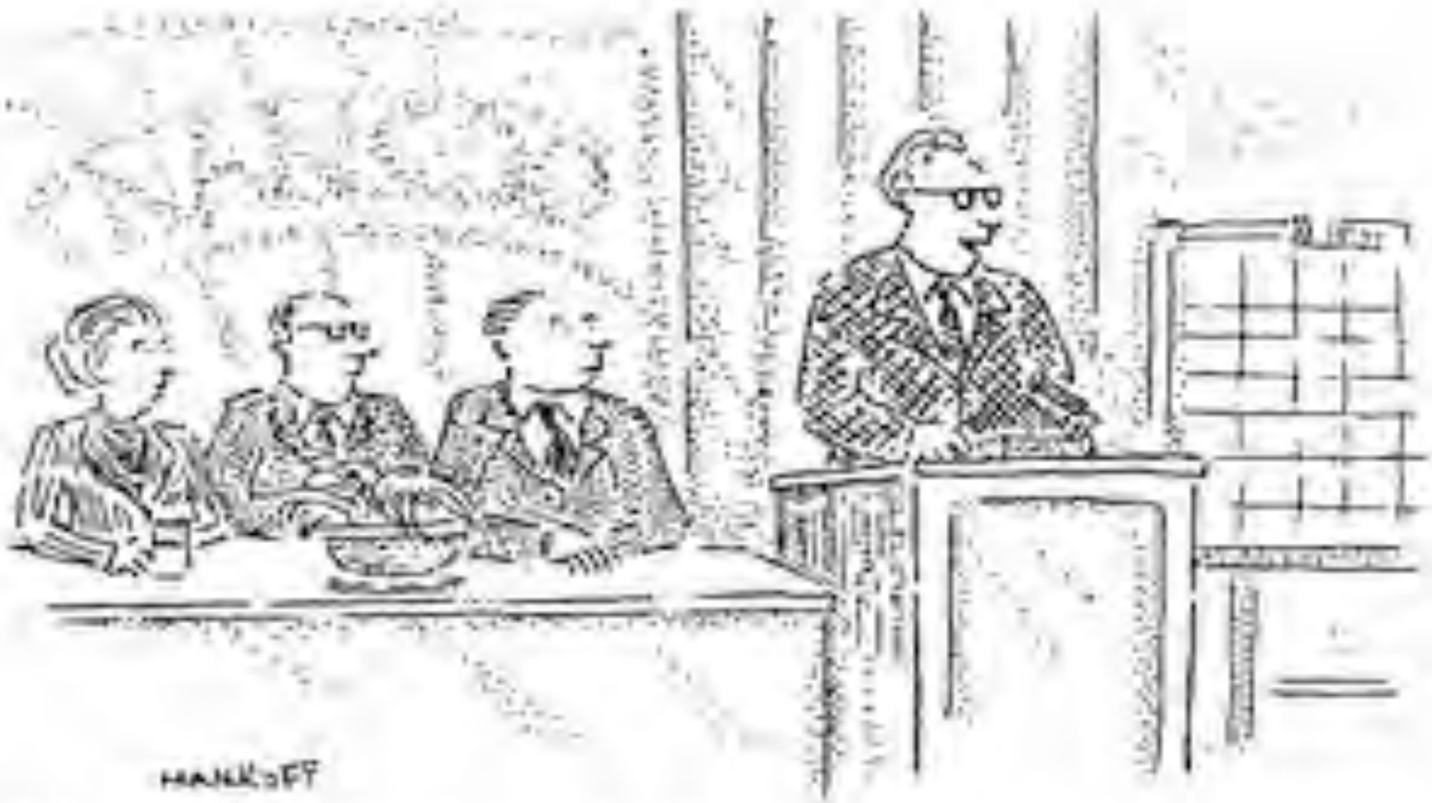
Global Temperature



Implications for global warming

- By comparing model and natural variability, we found that GCM’s seem to be missing a long-time mechanism of internal variability such as land-ice.
- Anthropogenic contributions to 20th warming and 21st C warming scenarios may thus be either over - or under estimated.

© Cartoonbank.com



"And so, while the end-of-the-world scenario will be rife with unimaginable horrors, we believe that the pre-end period will be filled with unprecedented opportunities for profit."

Conclusions

1. Low level laws: complex (Fluid mechanics)
High level laws simplicity (emergent turbulent laws)

2. Emergent Atmospheric laws are power laws
Fluctuations are scaling, their exponents are scale invariant

3. There are three different regimes:
Weather to ≈ 10 days,
Low frequency weather to ≈ 10-30 yrs,
Climate to ≈ 50- 100kyrs.

4. Without special forcing GCM's produce low frequency weather not climate type variability