Pion Induced Pion Production in Heavy-Baryon Chiral Perturbation Theory ($\text{HB}\chi\text{PT}$)

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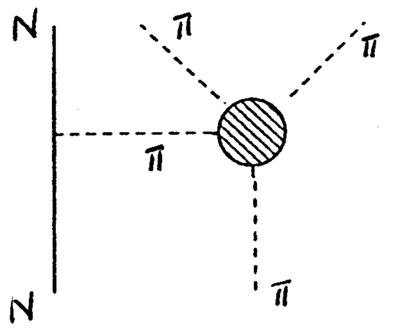
MOTIVES

- Aspects of chiral symmetry and its spontaneous breaking
  - $\pi\pi$ scattering
  - Non-linear realization of chiral symmetry



: HBχPT at $O(q^3)$

- How well does HBχPT reproduce the experimental data for the reaction $N(\pi, 2\pi)N$?

- How fast does HBχPT converge?
  - How large are the Low Energy Constants (LECs) of the theory?
  - How significant are the loop effects (unitarity corrections)?

- How high in energy can one go before HBχPT breaks down?

BUILDING BLOCKS

Meson Sector

The building blocks involve:

$$
\begin{aligned}
U &= u^2 = e^{i\vec{\tau}\cdot\vec{\pi}/F} \\
D_\mu U &= \partial_\mu U - iUr_\mu + il_\mu U \longrightarrow \partial_\mu U \\
\chi &= 2B_0(S + iP) = (m_u + m_d)B_0 \longrightarrow m_\pi^2 \\
\chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u \longrightarrow m_\pi^2(U^\dagger \pm U) \\
u_\mu &= i\Big(u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\Big) \longrightarrow i(u^\dagger \partial_\mu U u^\dagger)
\end{aligned}
$$

Nucleon Sector

The heavy-field transformation is defined as:

$$
\begin{aligned}
N_v &= e^{iMv_\mu x^\mu} P_v^+ \Psi \\
P_v^+ &= \frac{1}{2}(1 + \not{v}), \quad v^2 = 1.
\end{aligned}
$$

The building blocks involve U, u_μ, and

$$\nabla_\mu = \partial_\mu + \Gamma_\mu - iv_\mu^{(\mathrm{s})} \longrightarrow \partial_\mu + \frac{1}{2}\left\{u^\dagger \partial_\mu u + u\partial_\mu u^\dagger\right\}$$

$$S^\mu = \frac{i}{2}\gamma_5 \sigma^{\mu\nu} v_\nu$$

The Lagrangian

- The chiral expansion of the effective Lagrangian reads:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \quad \cdots \quad + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \cdots$$

$$\Downarrow$$

Transition amplitudes of $\mathcal{O}(q)$, $\mathcal{O}(q^2)$, $\mathcal{O}(q^3)$ or higher

- In HBχPT the chiral loops first appear at $\mathcal{O}(q^3)$.

Meson Sector

Lagrangian	Number of LECs	$N(\pi, 2\pi)N$	Status of LECs
$\mathcal{L}_{\pi}^{(2)}$	2	2	known (F_π and m_π)
$\mathcal{L}_{\pi}^{(4)}$	7	4	known (l_i)

The renormalized scale-independent LECs of $\mathcal{O}(q^4)$ are defined as:

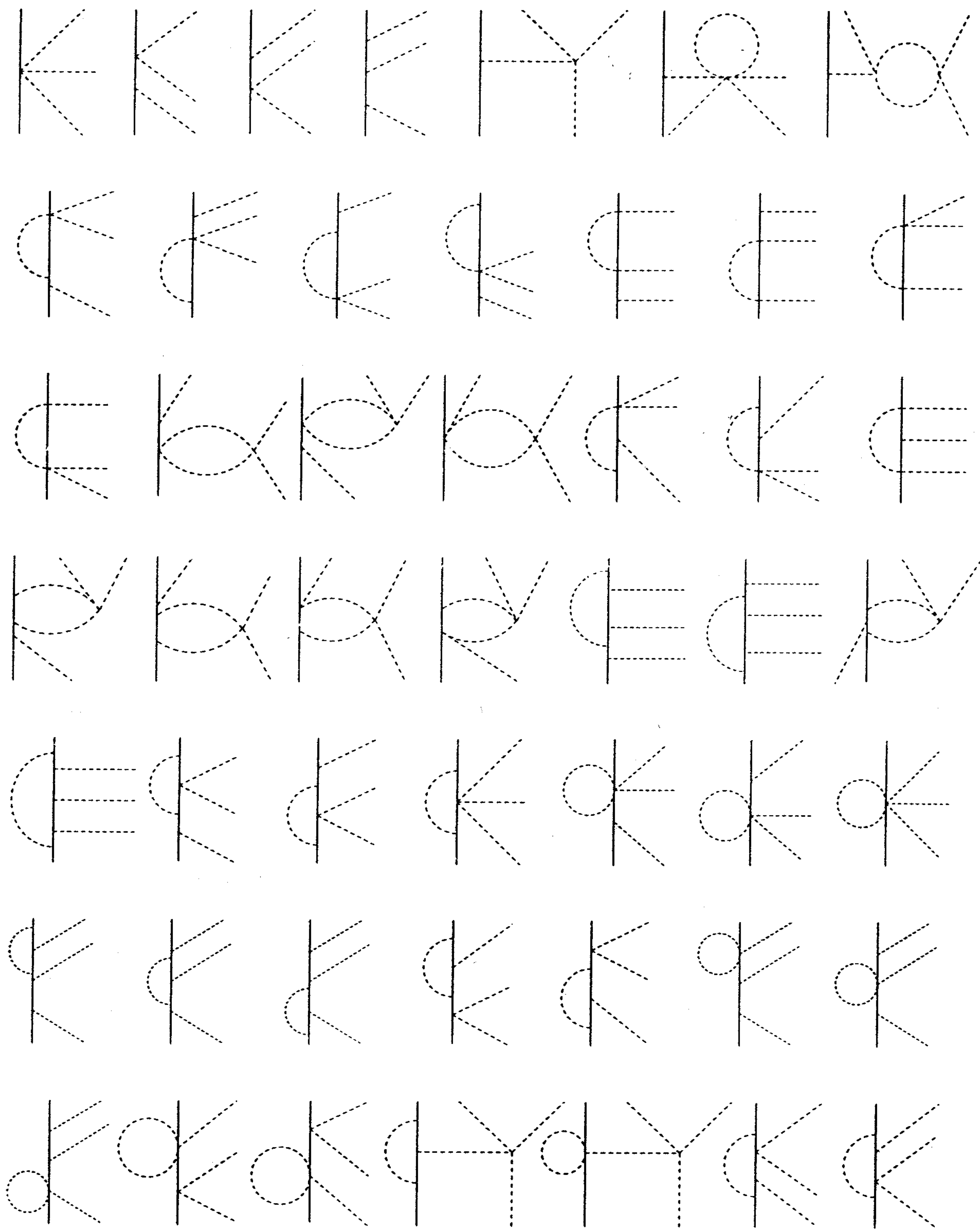
$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i - 2\ln\frac{m_\pi}{\mu} - \left\{\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + 1 + \Gamma'(1)]\right\}$$

Baryon Sector

Lagrangian	Number of LECs	$N(\pi, 2\pi)N$	Status of LECs
$\mathcal{L}_{\pi N}^{(1)}$	2	2	known (g_A and m_N)
$\mathcal{L}_{\pi N}^{(2)}$	7	5	known (a_i)
$\mathcal{L}_{\pi N}^{(3)}$	[23]	13	7 known (b_i) 6 unknown (b_i)

The scale-independent renormalized LECs of $\mathcal{O}(q^3)$ are defined as:

$$\tilde{b}_i = b_i - \beta_i\left\{\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + 1 + \Gamma'(1)] + \ln\frac{m_\pi}{\mu}\right\}$$



CALCULATIONS

There are five experimentally accessible channels for the reaction $N(\pi, 2\pi)N$.

$$\begin{aligned} \pi^- p &\longrightarrow \pi^0 \pi^0 n \\ \pi^+ p &\longrightarrow \pi^+ \pi^+ n \\ \pi^- p &\longrightarrow \pi^+ \pi^- n \\ \pi^- p &\longrightarrow \pi^- \pi^0 p \\ \pi^+ p &\longrightarrow \pi^+ \pi^0 p \end{aligned}$$

The unknown LECs in $\mathcal{L}_{\pi N}^{(3)}$ were determined by fitting the available data for:

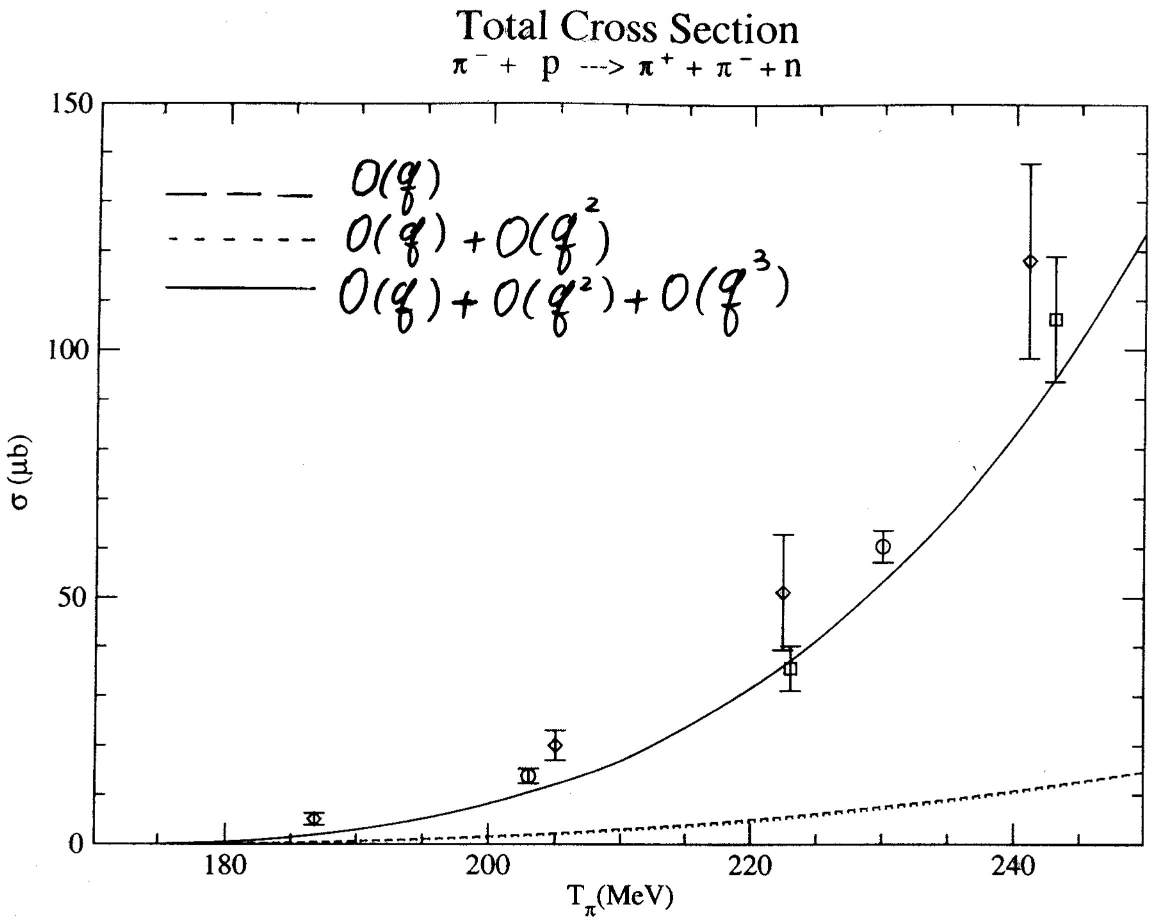
- $\sigma_{N(\pi,2\pi)N}$
- $\dfrac{d\sigma_{N(\pi,2\pi)N}}{dT_\pi d\Omega_\pi}$

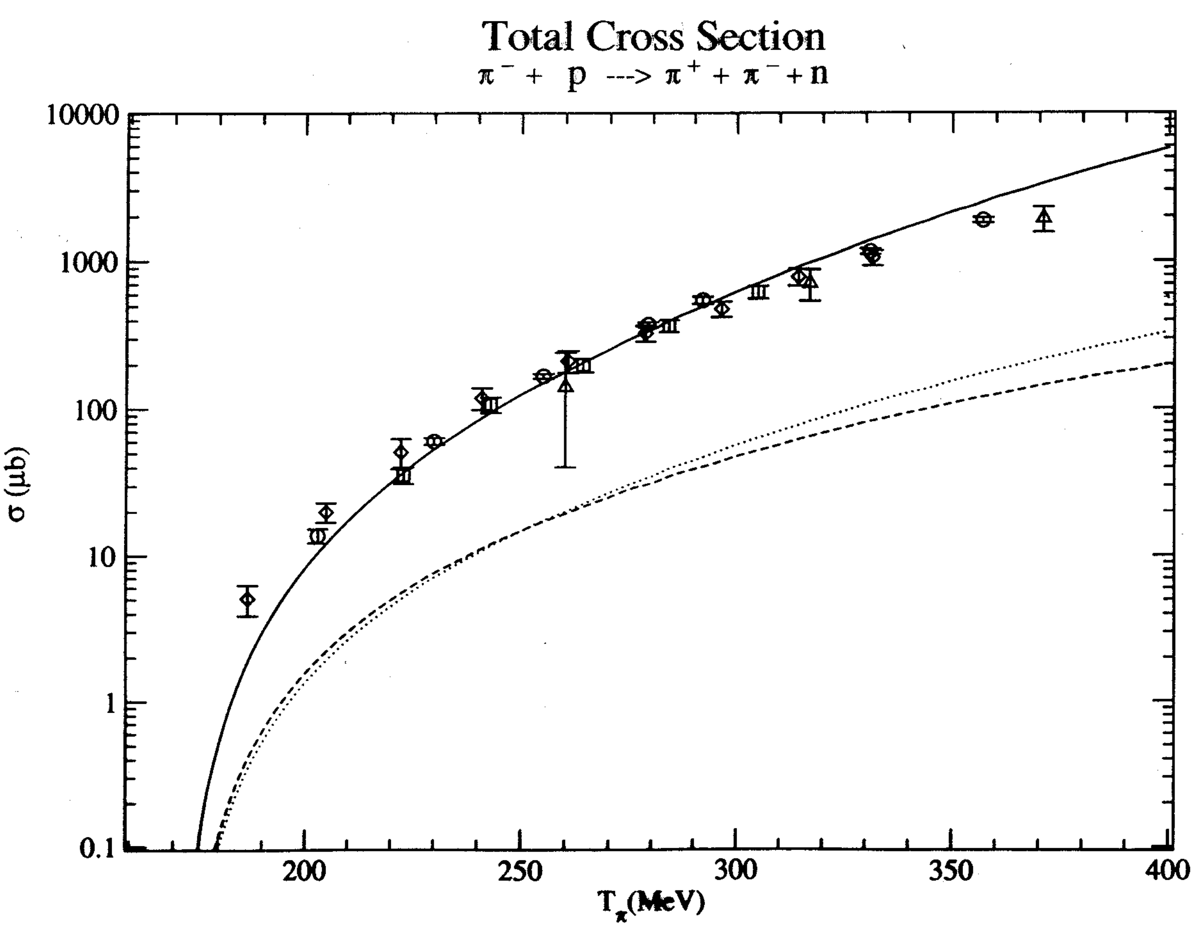
in the energy range 170 MeV $\leq (T_\pi)_{\text{lab}} \leq$ 260 MeV.

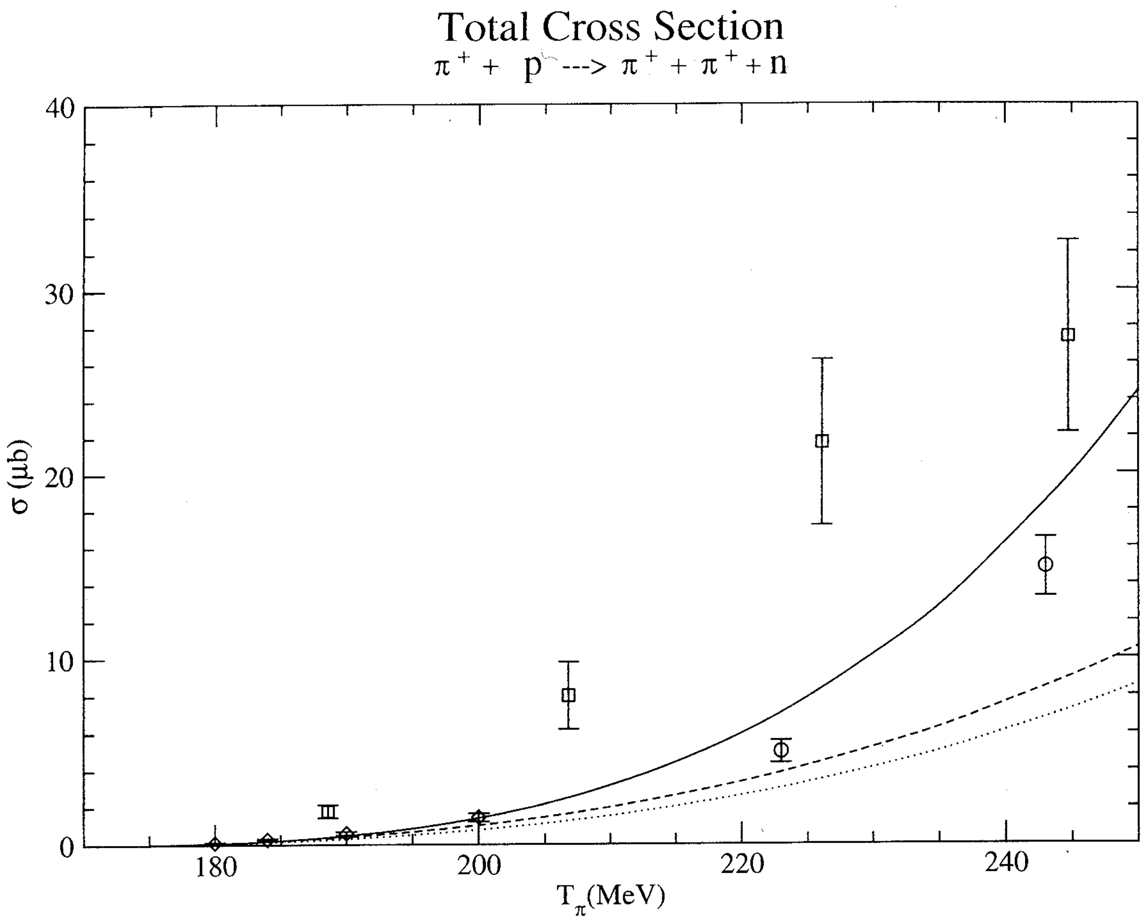
LECs of $\mathcal{O}(q^3)$ determined in this work*. The χ^2/dof of the fit is 3.4.

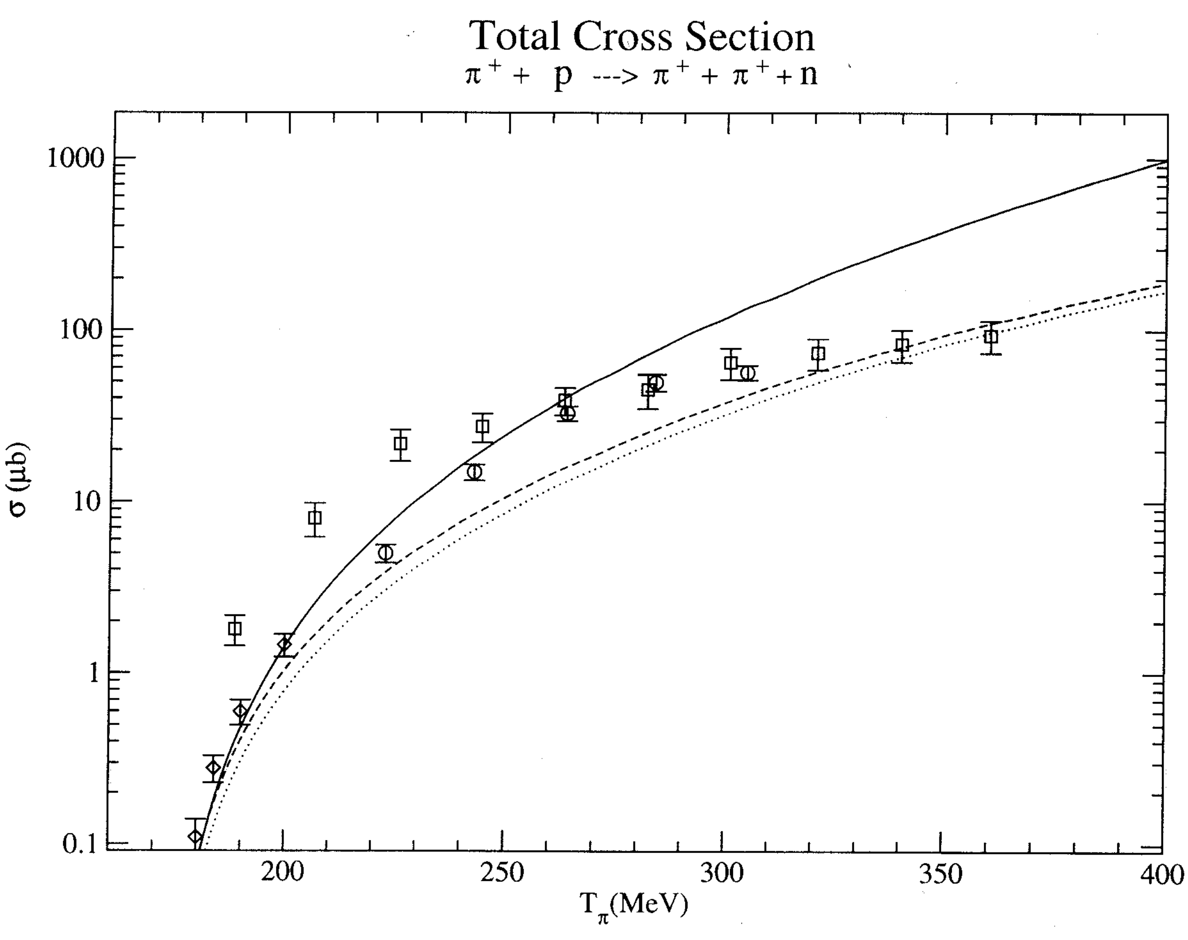
$\tilde{b}_5$	$\tilde{b}_{11}$	$\tilde{b}_{12}$	$\tilde{b}_{13}$	$\tilde{b}_{14}$	$\tilde{b}_{17}$
2.6 ± 3.3	-25.1 ± 6.2	-10.2 ± 4.7	22.3 ± 5.7	-8.4 ± 2.7	-5.3 ± 1.1

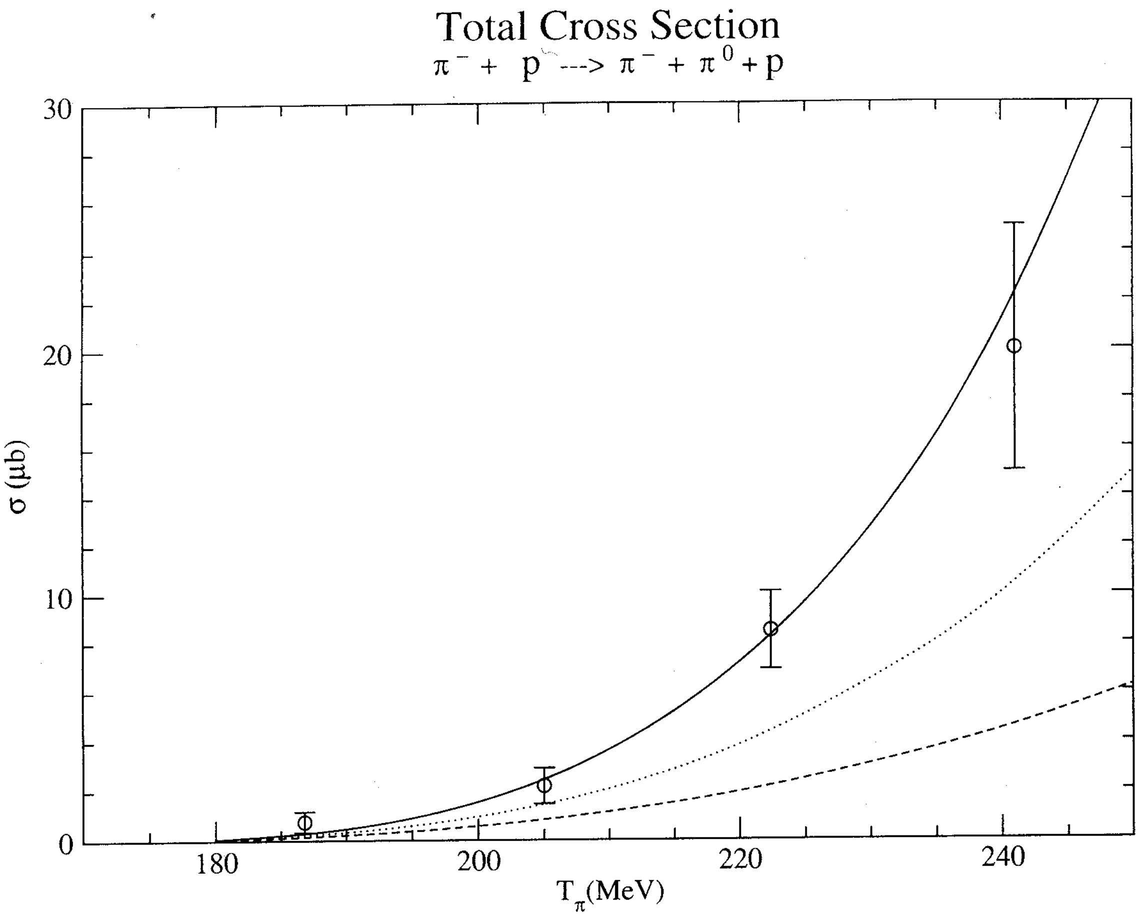
* The renormalization group equations were employed to arrive at the initial inputs to the fitting routine.


Total Cross Section
π⁻ + p ---> π⁺ + π⁻ + n
O(q)
O(q) + O(q²)
O(q) + O(q²) + O(q³)
σ (μb)
T_π(MeV)
150
100
50
0
180
200
220
240

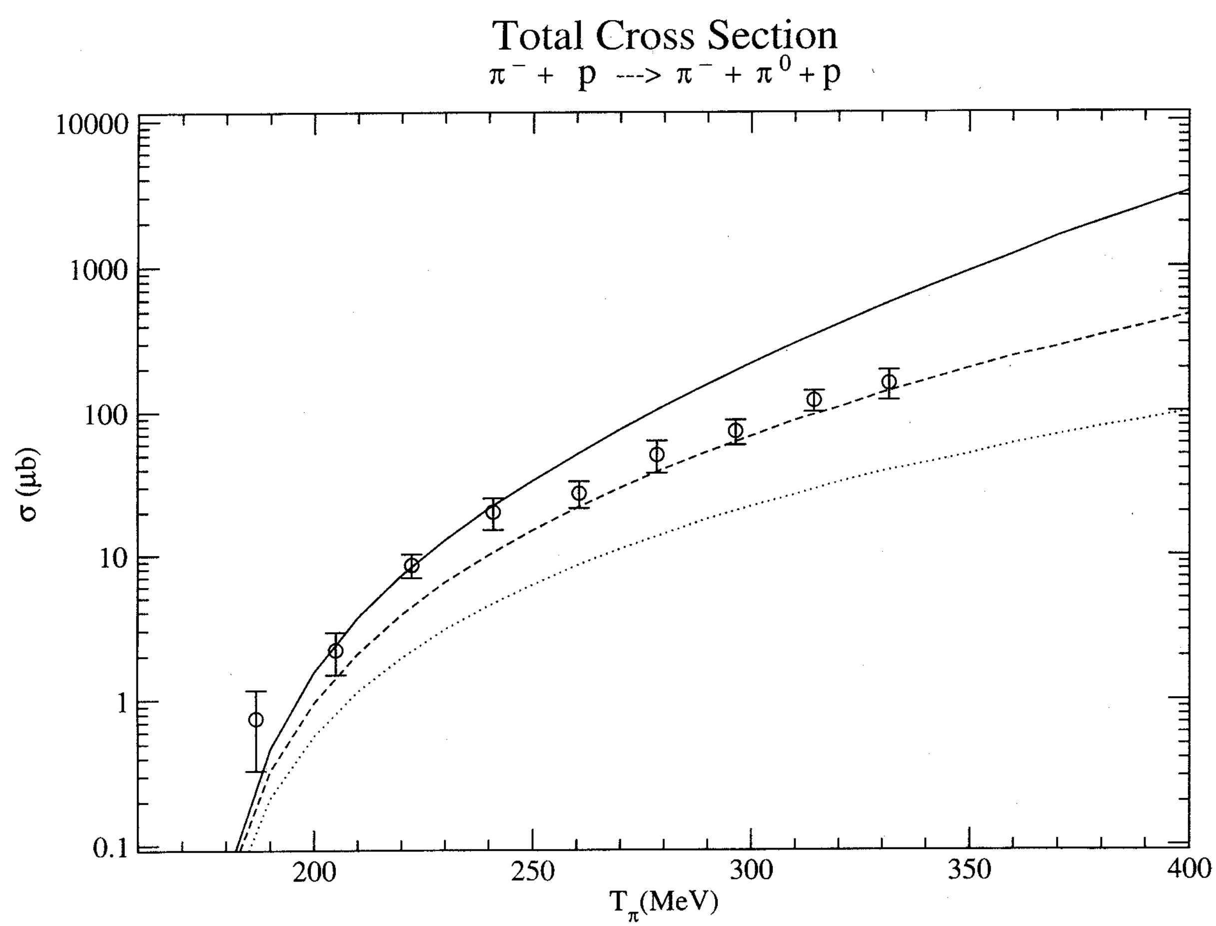

Total Cross Section
π⁻ + p ---> π⁺ + π⁻ + n
σ (μb)
T_π(MeV)
10000
1000
100
10
1
0.1
200
250
300
350
400


Total Cross Section
π⁺ + p ---> π⁺ + π⁺ + n
σ (μb)
T_π(MeV)
0
10
20
30
40
180
200
220
240

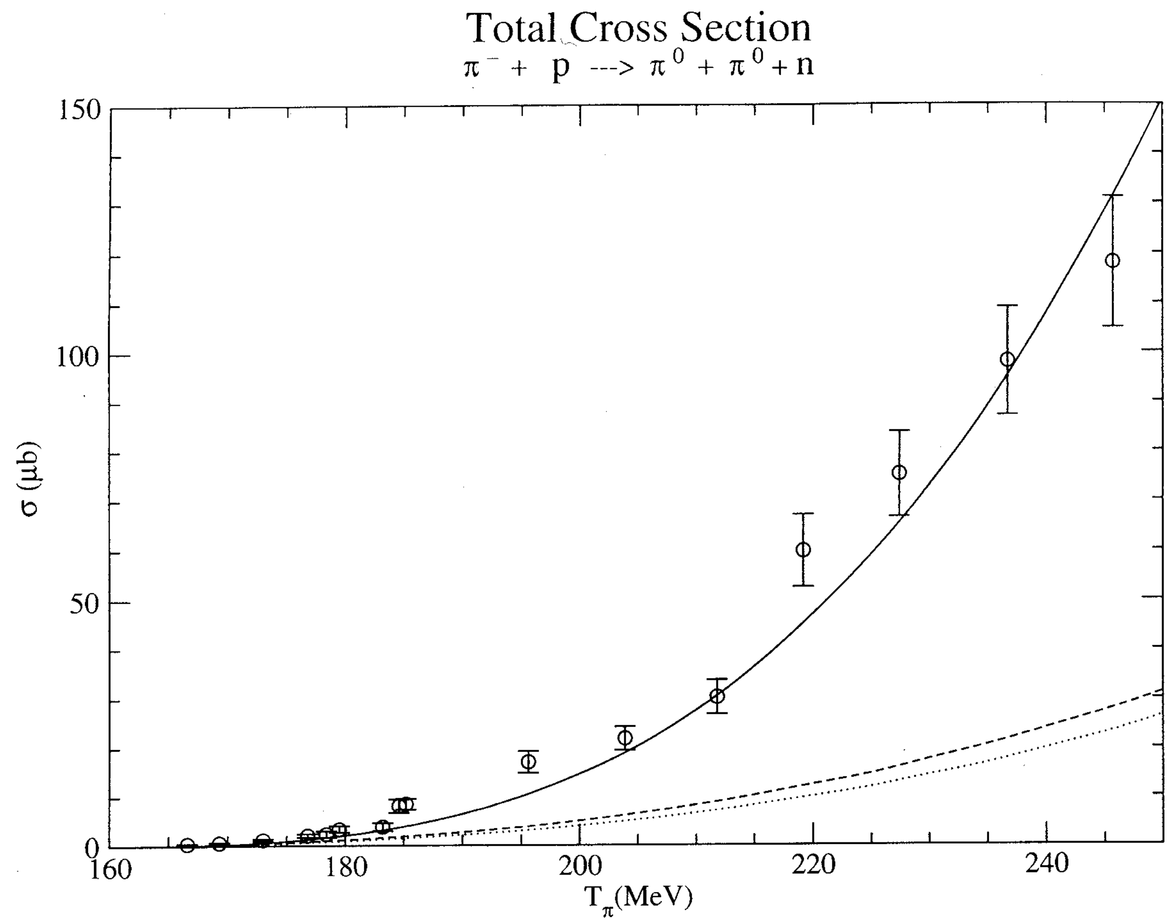

Total Cross Section
π⁺ + p ---> π⁺ + π⁺ + n
σ (μb)
T_π(MeV)
0.1
1
10
100
1000
200
250
300
350
400

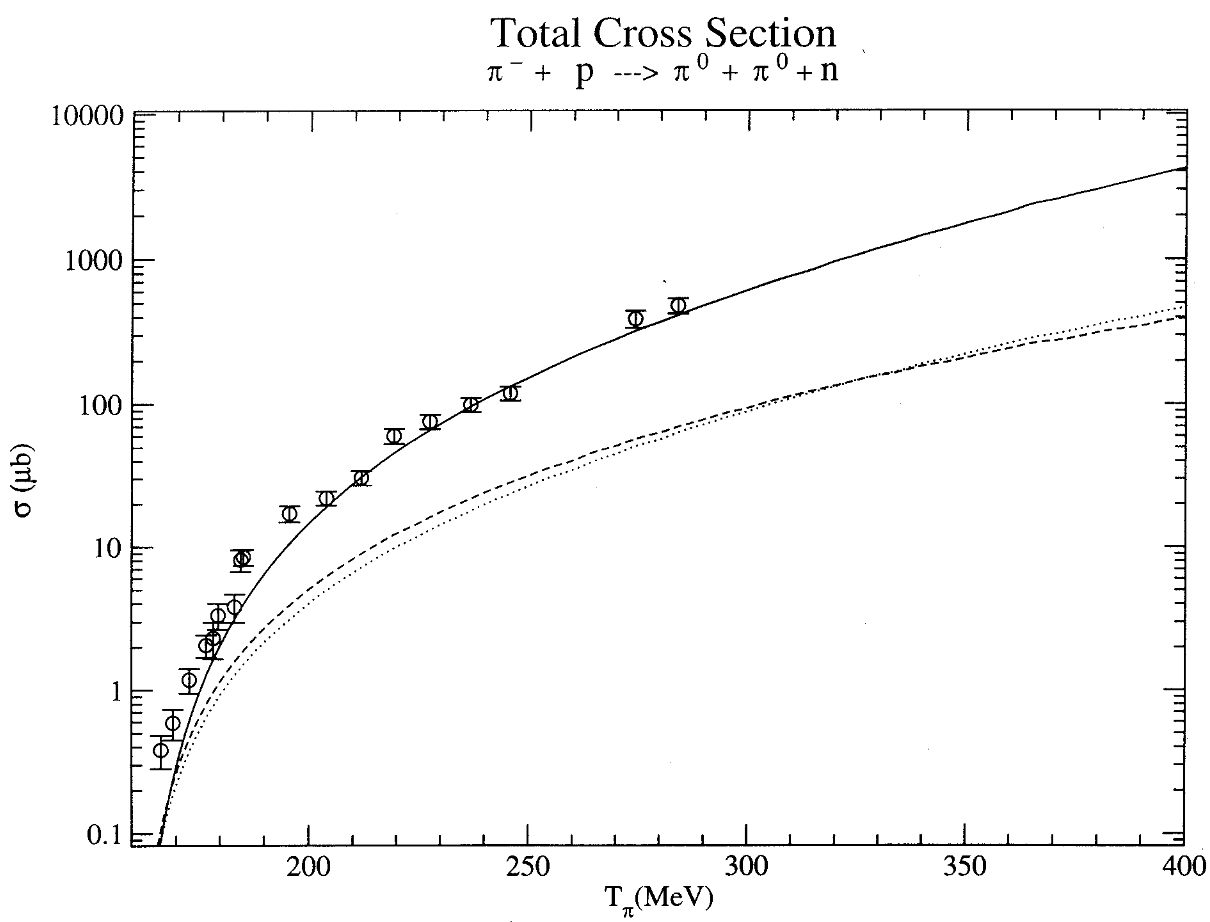


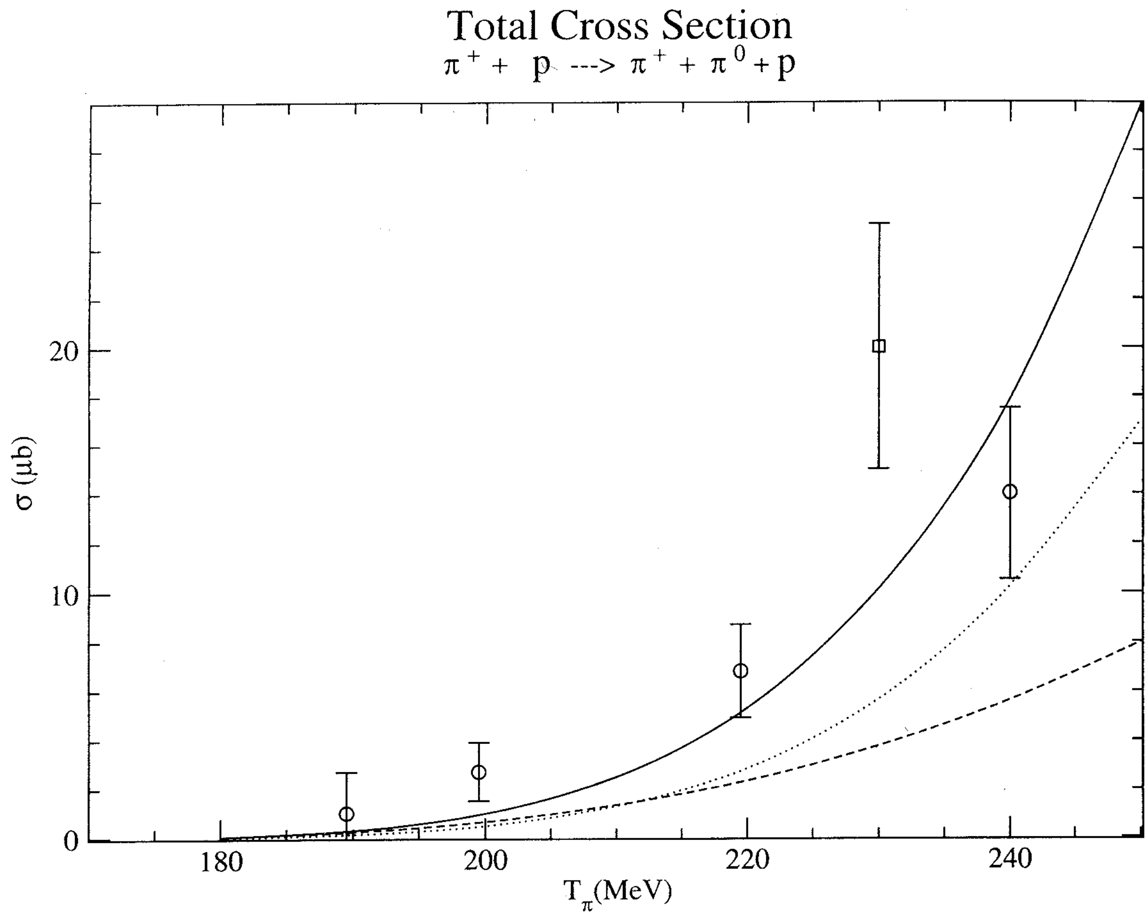
Total Cross Section
$\pi^- + p \to \pi^- + \pi^0 + p$
σ (μb)
T_π(MeV)
0
10
20
30
180
200
220
240

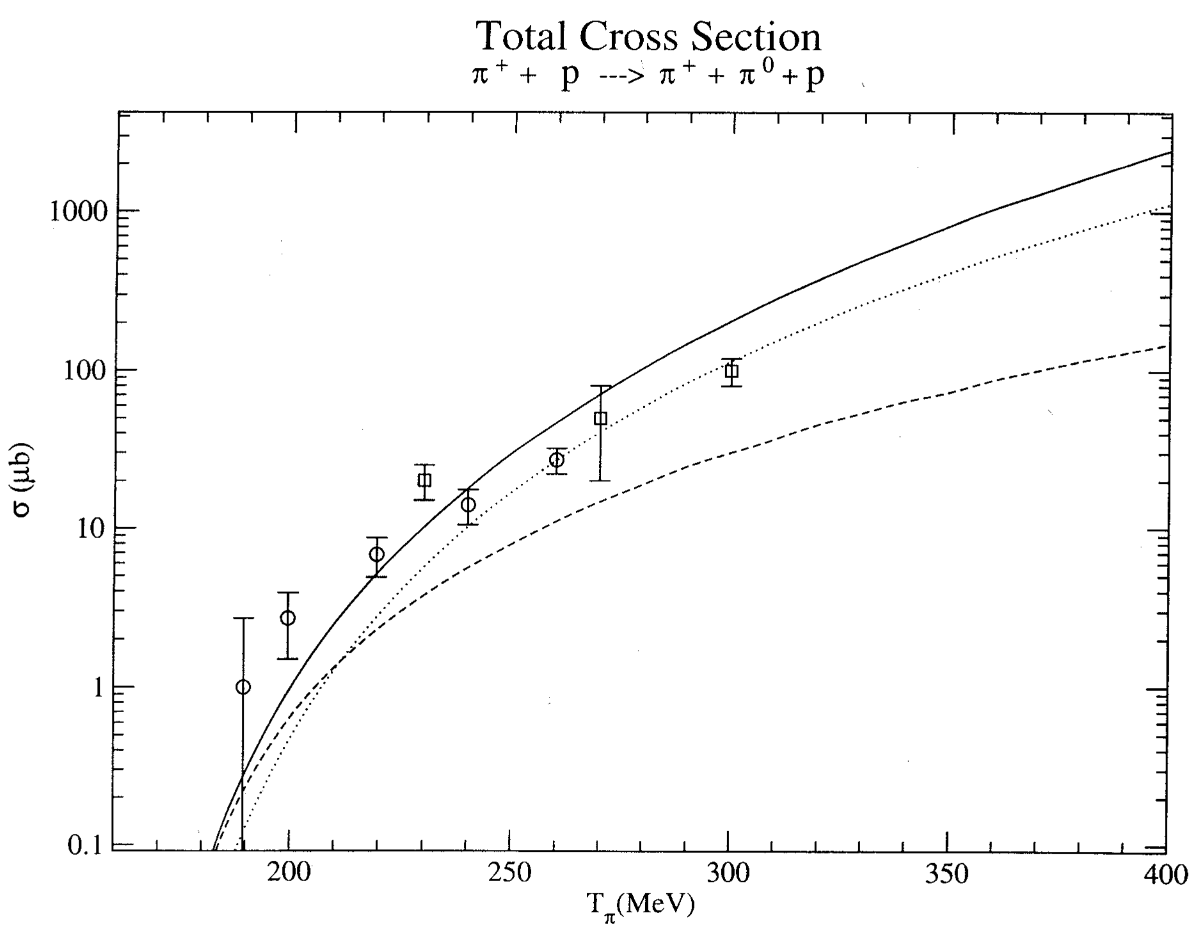


Total Cross Section
$\pi^- + p \to \pi^- + \pi^0 + p$
σ (μb)
T_π(MeV)
0.1
1
10
100
1000
10000
200
250
300
350
400


Total Cross Section
$\pi^- + p \rightarrow \pi^0 + \pi^0 + n$
σ (μb)
T_π(MeV)
150
100
50
0
160
180
200
220
240

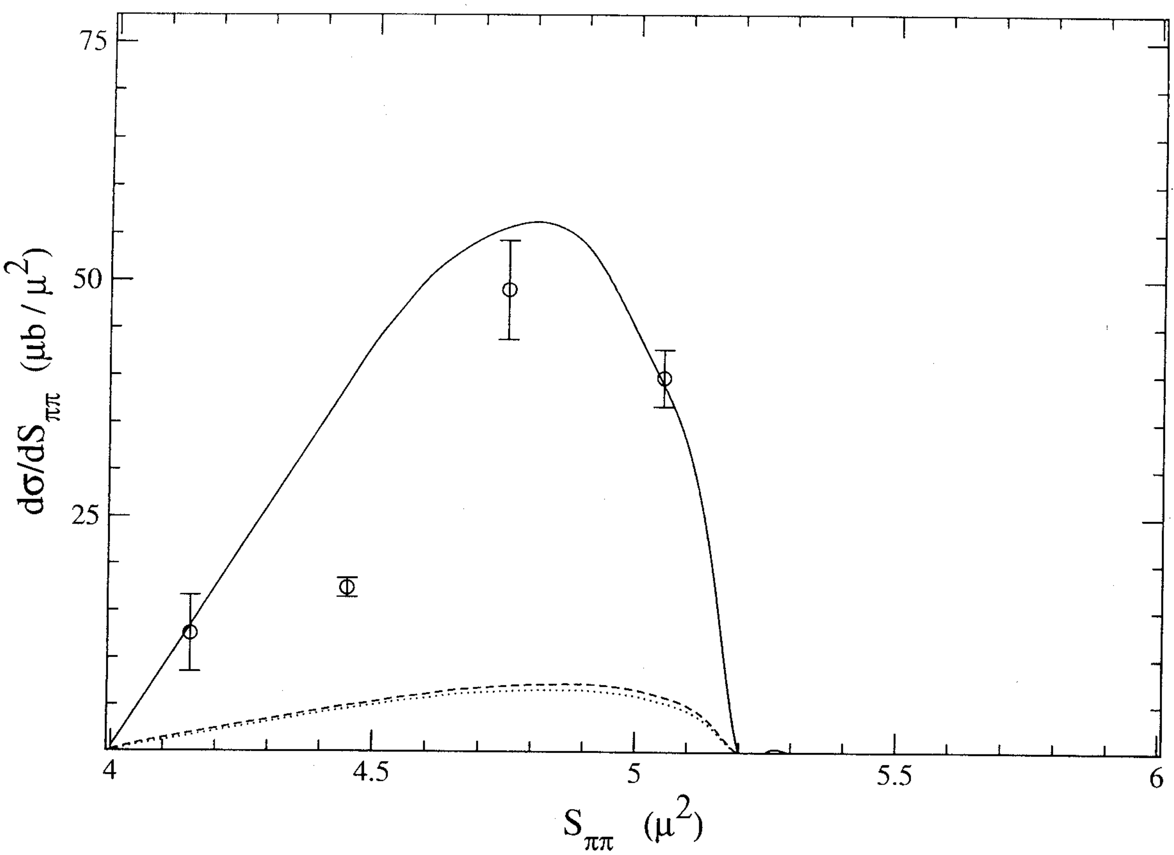

Total Cross Section
$\pi^- + p \rightarrow \pi^0 + \pi^0 + n$
σ (μb)
T_π(MeV)
10000
1000
100
10
1
0.1
200
250
300
350
400


Total Cross Section
π⁺ + p ---> π⁺ + π⁰ + p
σ (μb)
0
10
20
180
200
220
240
T_π(MeV)


Total Cross Section
π⁺ + p ---> π⁺ + π⁰ + p
σ (μb)
0.1
1
10
100
1000
200
250
300
350
400
T_π(MeV)

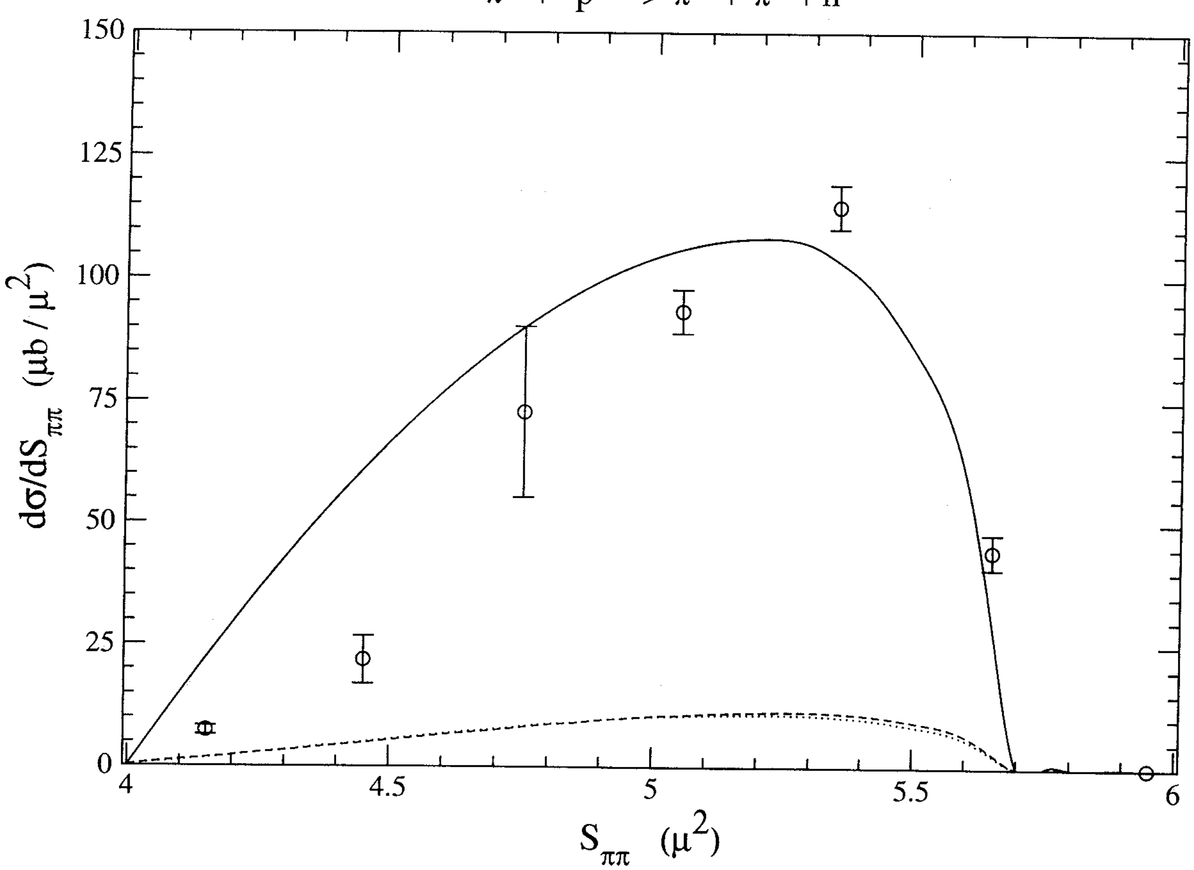
Differential Cross Section: $T_\pi = 223$ MeV

$\pi^- + p \dashrightarrow \pi^+ + \pi^- + n$



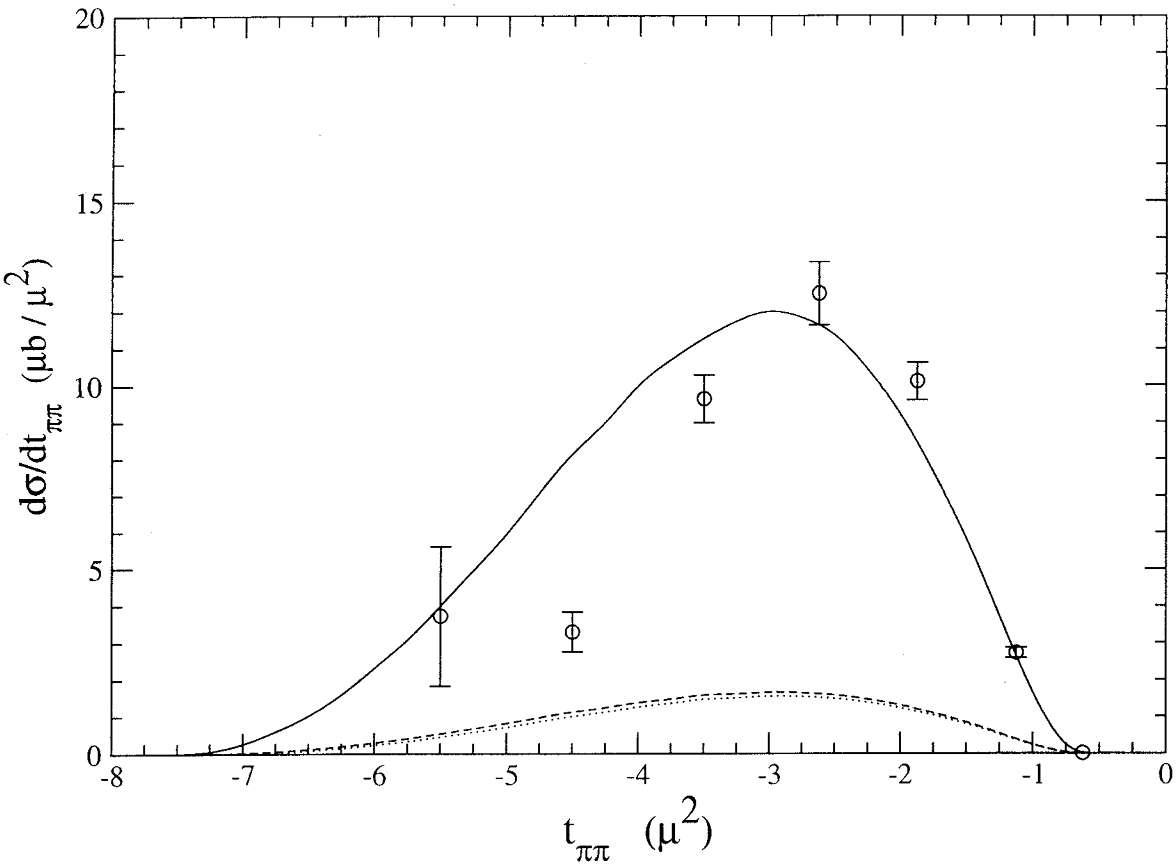
Differential Cross Section: $T_\pi = 243$ MeV

$\pi^- + p \dashrightarrow \pi^+ + \pi^- + n$



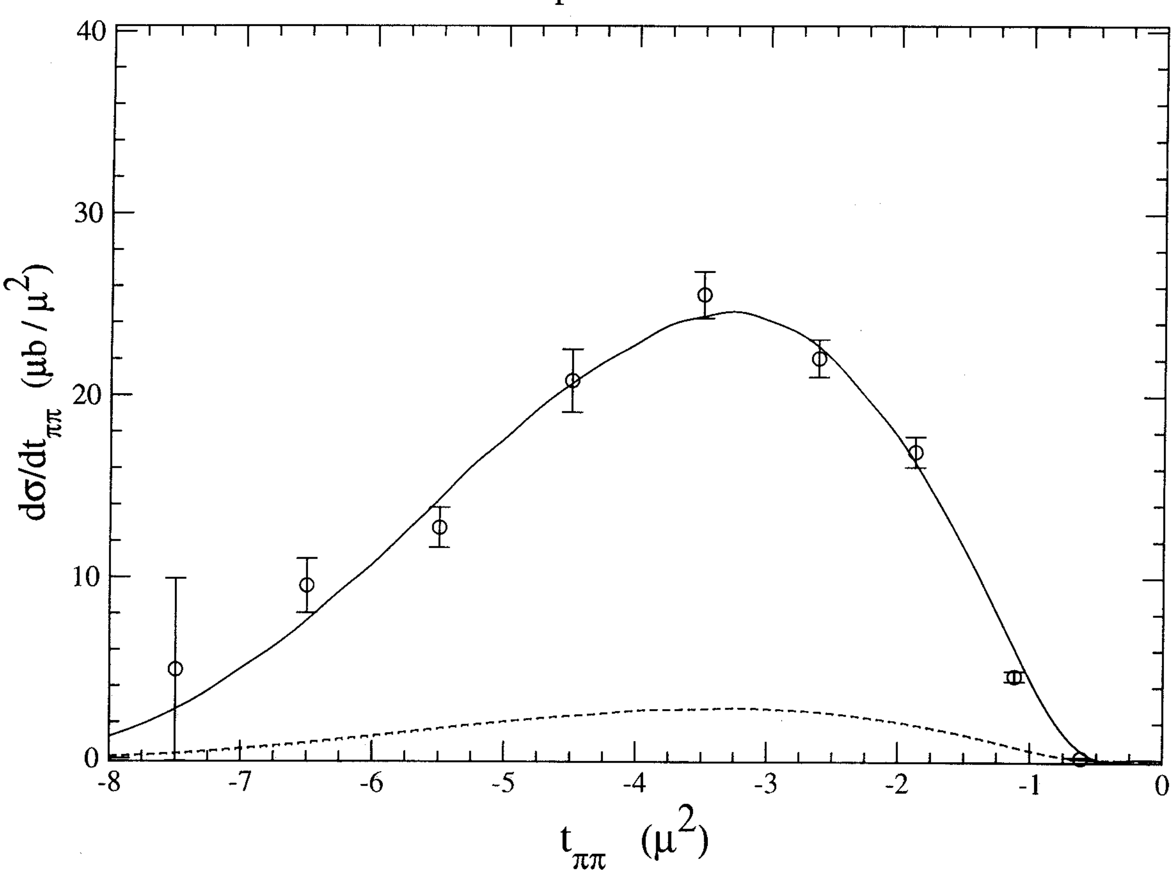
Differential Cross Section: $T_\pi = 223$ MeV

$\pi^- + p \rightarrow \pi^+ + \pi^- + n$



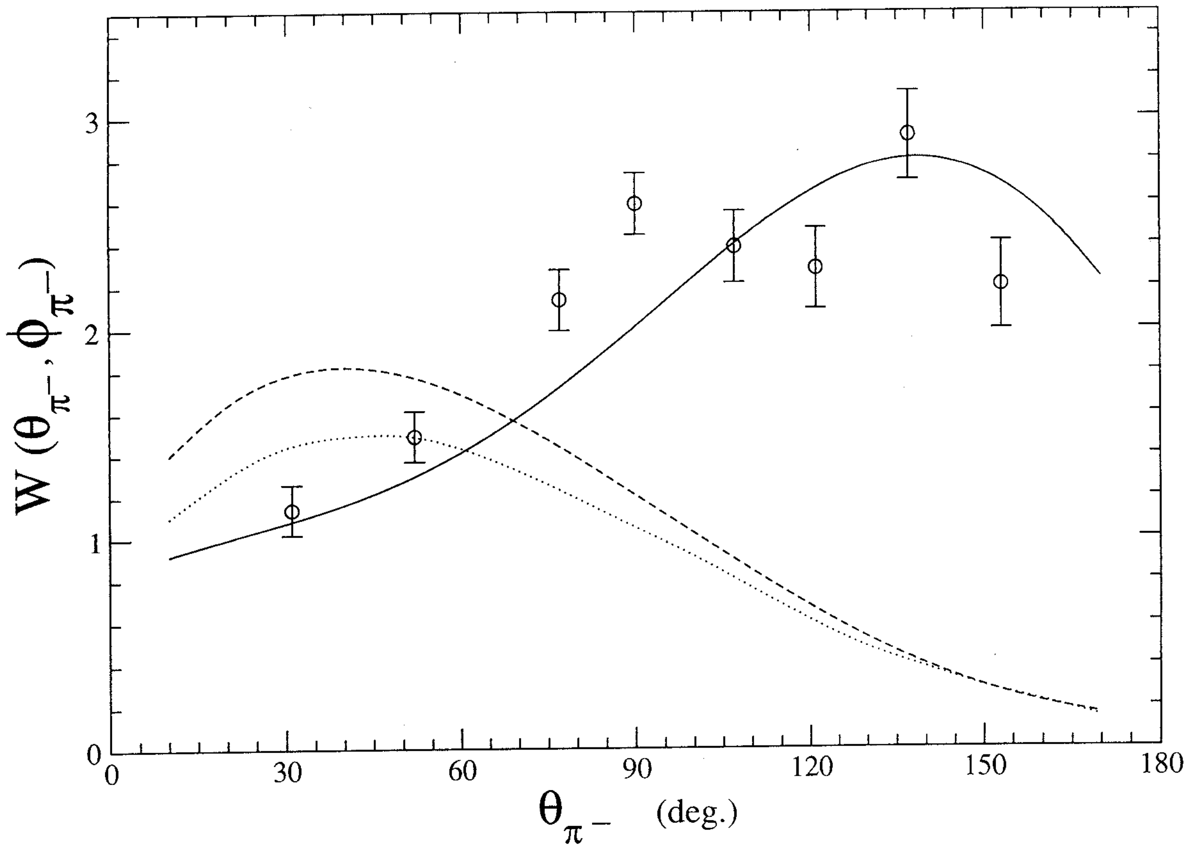
Differential Cross Section: $T_\pi = 243$ MeV

$\pi^- + p \rightarrow \pi^+ + \pi^- + n$



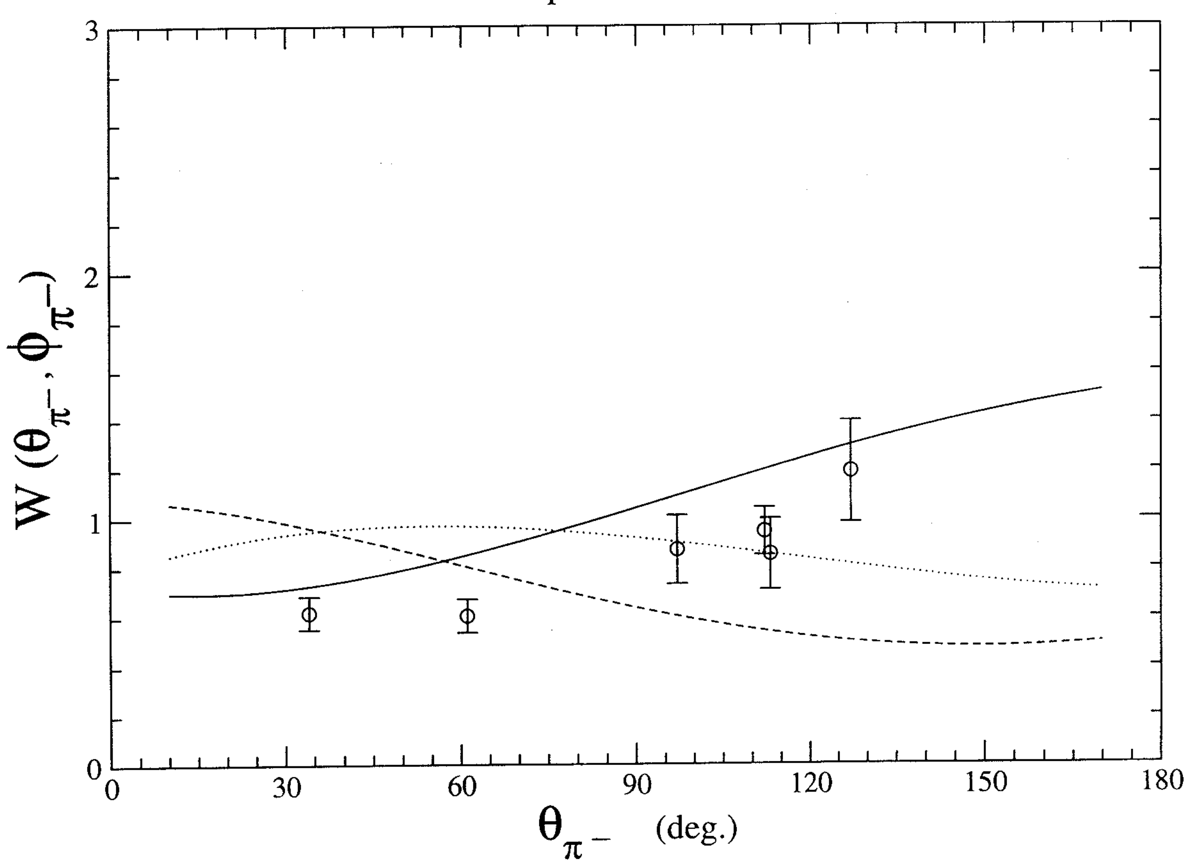
Angular Correlation: $T_{\pi^-} = 283$ MeV, $\theta_{\pi^+} = 91.0^{O}$, $\phi_{\pi^-} = 155.6^{O}$

$\pi^- + p \dashrightarrow \pi^+ + \pi^- + n$



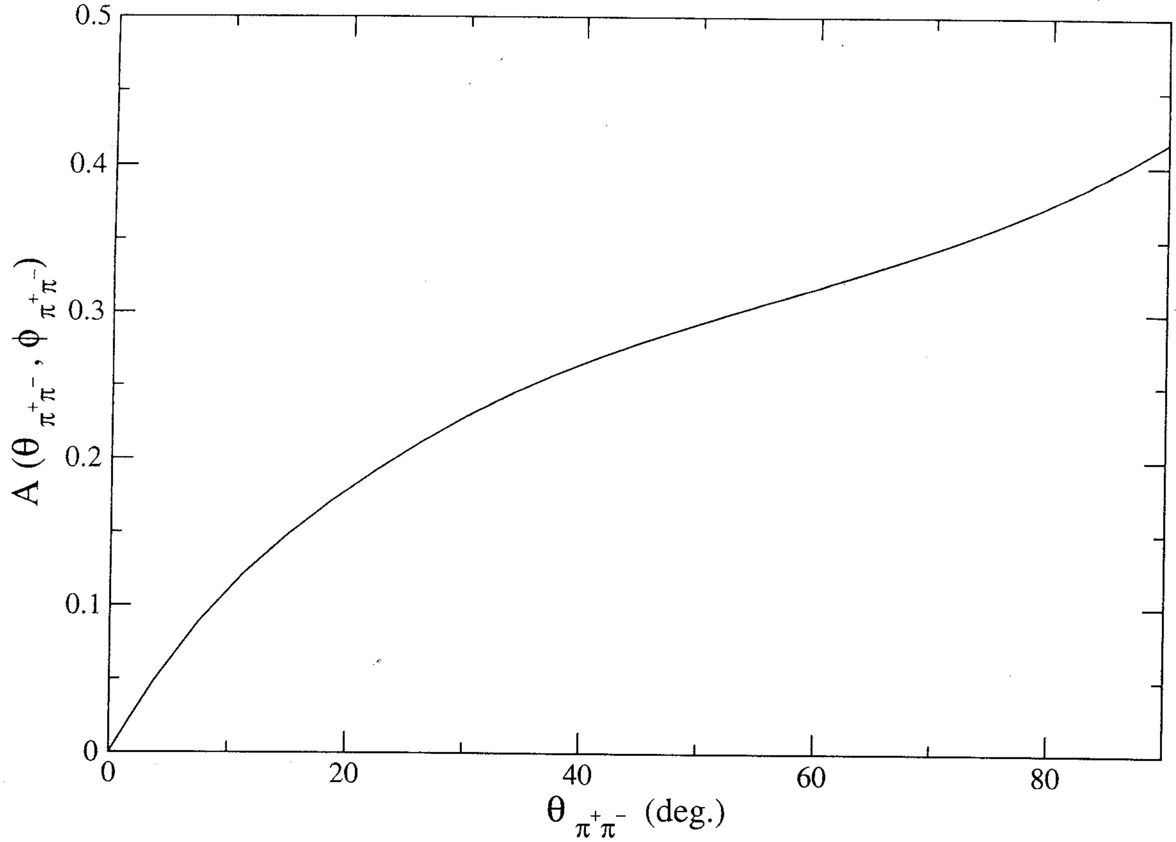
Angular Correlation: $T_{\pi^-} = 283$ MeV, $\theta_{\pi^+} = 91.0^{O}$, $\phi_{\pi^-} = 91.3^{O}$

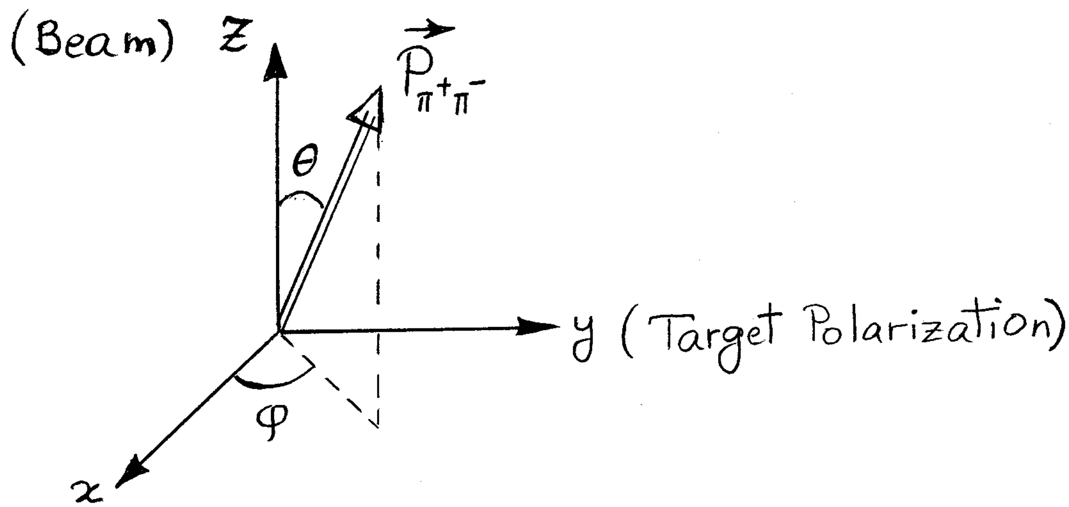
$\pi^- + p \dashrightarrow \pi^+ + \pi^- + n$



Asymmetry for T_{π^-} = 280 MeV, at $\phi_{\pi^+\pi^-} = 0°$

$\pi^- + p \rightarrow \pi^+ + \pi^- + n$





THRESHOLD AMPLITUDES

The information regarding S-wave $\pi\pi$ scattering lengths is contained in the Threshold Amplitudes (TA) for the reactions:

$$\mathrm{TA} \equiv a^0 : \quad \pi^- p \longrightarrow \pi^0\pi^0 n$$
$$\mathrm{TA} \equiv a^+ : \quad \pi^+ p \longrightarrow \pi^+\pi^+ n$$

The results can be expressed in terms of the threshold isospin amplitudes:

$$D_1 = \frac{1}{2\sqrt{2}} a^+, \qquad D_2 = -\frac{1}{\sqrt{2}} a^0$$

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	Experiment [1]
$D_1(\mathrm{fm}^3)$	2.50	2.14	1.90 ± 0.43	2.26 ± 0.12
$D_2(\mathrm{fm}^3)$	-7.70	-6.96	-11.44 ± 2.60	-9.05 ± 0.36

[1] H. Burkhardt, J. Lowe, Phys. Rev. Lett. 72 (1991) 2622

Unitarity Corrections

The amplitude for the reaction $N(\pi, 2\pi)N$ can be written as

$$\mathcal{M}_{fi} = \bar{u}(p_f)\gamma_5[\, f_1 + f_2 \not{q}_1 + f_3 \not{q}_2 + f_4 \not{q}_1 \not{q}_2] u(p_i) \xrightarrow{\text{Threshold}} a\, \chi_f^\dagger\, \vec{\sigma}\cdot\vec{q}\, \chi_i$$

where

$$a = \frac{1}{2M}[m_\pi(f_2 + f_3) - f_1 - m_\pi^2 f_4].$$

The invariant amplitudes f_1, f_2, f_3, and f_4 can be calculated in HBχPT.

The imaginary parts of the invariant amplitudes originate entirely form loops and are hence parameter free quantities.

Threshold amplitudes for the reaction $\pi^- p \longrightarrow \pi^0 \pi^0 n$

$\mathcal{O}(q^n)$	$f_1 \times m_\pi^{-2}$	$f_2 \times m_\pi^{-3}$	$f_3 \times m_\pi^{-3}$	$f_4 \times m_\pi^{-4}$
$\mathcal{O}(q)$	-44.30	3.90	3.90	0
$\mathcal{O}(q^2)$	-39.51	3.81	3.81	0
$\mathcal{O}(q^3)$	$-38.69 + i47.88$	$3.20 - i5.22$	$3.20 - i5.22$	0

Threshold amplitudes for the reaction $\pi^+ p \longrightarrow \pi^+ \pi^+ n$

$\mathcal{O}(q^n)$	$f_1 \times m_\pi^{-2}$	$f_2 \times m_\pi^{-3}$	$f_3 \times m_\pi^{-3}$	$f_4 \times m_\pi^{-4}$
$\mathcal{O}(q)$	30.75	-1.44	-1.44	0
$\mathcal{O}(q^2)$	25.36	-1.71	-1.71	0
$\mathcal{O}(q^3)$	$-23.71 - i3.89$	$1.67 + i0.03$	$1.67 + i0.03$	0

SUMMARY AND CONCLUSIONS

Comparison with Data

- The calculated values of different observables are in reasonable agreement with experimental data. In most cases contributions of $\mathcal{O}(q^3)$ are essential in order to reproduce the data.

Issues

- It appears to be difficult to arrive at a unique (or reasonably constrained) set of LECs.
- It appears that some of the LECs of $\mathcal{O}(q^3)$ are larger than their expected "Natural Size".
- In some cases the unitarity corrections appear to be sizable.
- Is HBχPT a (rapidly) converging series?

Possible Future Directions

- Calculate amplitudes of $\mathcal{O}(q^4)$ in HBχPT for the reaction $N(\pi, 2\pi)N$ (in order to complete the one-loop analysis of the reaction).
- Calculate the process $N(\pi, 2\pi)N$ within the framework of the infrared regularization. (Possible improved convergence rate of the chiral series.)

  T. Becher and H. Leutwyler, Eur. Phys. J. C9 (1999) 643; JHEP 0106 (2001) 017

  M. R. Schindler, J. Gegelia, S. Scherer Phys.Lett. B586 (2004) 258
- The results of the analysis of polarization measurements will provide additional constraints.