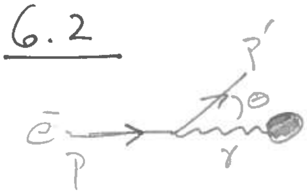
6.2



$$q = p' - p$$

$$\Rightarrow q^2 = 2m_e^2 - 2p'\cdot p$$
$$= 2m_e^2 - 2(E'E - |\vec{p}||\vec{p}'|\cos\theta)$$
$$|\vec{p}|,|\vec{p}'| \gg m_e \simeq -2EE'(1-\cos\theta)$$

- In the limit of forward scattering $\theta \simeq 0$ the momentum of ~~the~~ virtual photon approaches $q^2 \simeq 0$
- In what sense is the virtual photon real in this lim?

a) $i\mathcal{M} = -ie\,\bar{u}(p')\gamma^\mu u(p) \frac{-ig_{\mu\nu}}{q^2} \tilde{\mathcal{M}}^\nu(q)$

Let $q = (q^0, \vec{q})$ and $\tilde{q} = (q^0, -\vec{q})$

$f^\mu = \bar{u}(p')\gamma^\mu u(p) = A q^\mu + B\tilde{q}^\mu + C\varepsilon_1^\mu + D\varepsilon_2^\mu$

$$q_\mu f^\mu = A \underbrace{q\cdot q}_{\simeq 0} + B q\cdot\tilde{q} + C \cancelto{0}{q\cdot\varepsilon_1} + D \cancelto{0}{\varepsilon_2\cdot q}$$
$$\simeq B\, q\cdot\tilde{q}$$
$$= B(q^{0\,2} + \vec{q}\cdot\vec{q})$$
$$= 2B(q^{0\,2} + EE'(1-\cos\theta))$$

$$q^2 = q^{0\,2} - \vec{q}\cdot\vec{q} = -2EE'(1-\cos\theta)$$
$$\Rightarrow \vec{q}\cdot\vec{q} = q^{0\,2} + 2EE'(1-\cos\theta)$$

now we are in the limit of forward scattering

$$\Rightarrow \theta \simeq 0$$
$$\Rightarrow E' \simeq E \Rightarrow q^0 \simeq 0$$
$$\Rightarrow q_\mu f^\mu \simeq 2B\, EE'\left(1 - 1 + \frac{\theta^2}{2}\right)$$
$$= B(EE'\theta^2)$$
$$= \mathcal{O}(\theta^2)$$

b) $p=(E,0,0,E)$ Let $p'=(E', E'\sin\theta, 0, -E'\cos\theta)$

$\bar{u}(p'), u(p) \rightarrow$ take to be massless.

From P&S 3.3 the massless spinors are:

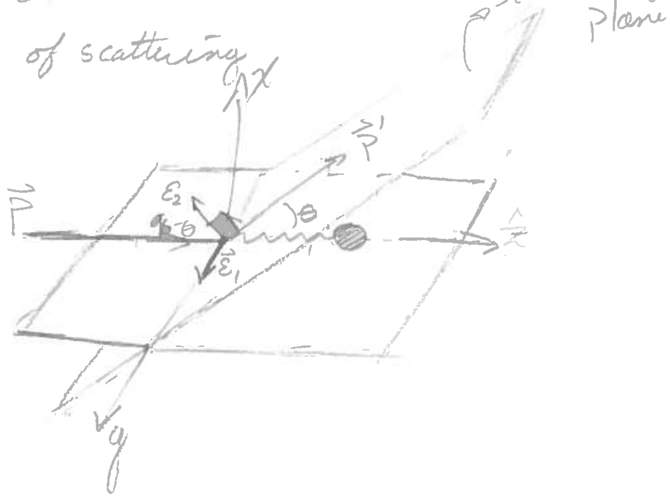
$$u_L(p) = \frac{1}{\sqrt{2E}}\begin{pmatrix}0\\2E\\0\\0\end{pmatrix} = \sqrt{2E}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$u_R(p) = \frac{1}{\sqrt{2E}}\begin{pmatrix}0\\0\\2E\\0\end{pmatrix} = \sqrt{2E}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\bar{u}_L(p') = E'\frac{(-\sin\theta\ \ (1+\cos\theta)\ \ 0\ \ 0)}{\sqrt{E'\cdot E'\cos\theta}}\begin{pmatrix}0 & 1_{2\times2}\\ 1_{2\times2} & 0\end{pmatrix} = \sqrt{E'(1-\cos\theta)}\left(0\ \ 0\ \ \frac{-\sin\theta}{\sqrt{1+\cos\theta}}\ \ 1\right)$$

$$\bar{u}_R(p') = \frac{E'(0\ \ 0\ \ (1+\cos\theta)\ \ \sin\theta)}{\sqrt{E'(1-\cos\theta)}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \sqrt{E'(1-\cos\theta)}\left(1\ \ \frac{\sin\theta}{\sqrt{1+\cos\theta}}\ \ 0\ \ 0\right)$$

take $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$ // and $\perp$ to plane of scattering



$p' = (E', E'\sin\theta, 0, -E'\cos\theta)$

$\vec{\varepsilon}_1 = (0, 1, 0)$

$\vec{\varepsilon}_2 = (\sin(\frac{\pi}{2}-\theta), 0, -\cos(\frac{\pi}{2}-\theta))$
$= (\cos\theta, 0, -\sin\theta)$

$$\vec{\gamma}\cdot\vec{\varepsilon}_1 = \gamma^2 = \begin{pmatrix}0&0&0&i\\0&0&-i&0\\0&-i&0&0\\i&0&0&0\end{pmatrix} = i\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}$$

$$\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_L(p) = \sqrt{2E}\, i\,(-1)\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = -i\sqrt{2E}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_R(p) = -i\sqrt{2E}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$\bar{u}_L(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_R(p) = -i\sqrt{2E}\ \sqrt{E'(1+\cos\theta)}\left(0\ \ 0\ \ \frac{-\sin\theta}{\sqrt{1+\cos\theta}}\ \ 1\right)\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = 0$$

$$\bar{u}_R(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_R(p) = -i\sqrt{2E}\ \sqrt{E'(1+\cos\theta)}\left(1\ \ \frac{\sin\theta}{\sqrt{1+\cos\theta}}\ \ 0\ \ 0\right)\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = -i\sqrt{2EE'}\sin\theta$$

$$\bar{u}_R(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_L(p) = 0$$

$$\bar{u}_L(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_L(p) = +i\sqrt{2EE'}\sin\theta$$

$$\vec{\gamma}\cdot\vec{\varepsilon}_2 = \cos\theta\,\gamma^1 - \sin\theta\,\gamma^3$$

$$= \begin{pmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\cos\theta - \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\sin\theta$$

$$= \begin{pmatrix} 0 & 0 & -\sin\theta & \cos\theta \\ 0 & 0 & \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta & 0 & 0 \\ -\cos\theta & -\sin\theta & 0 & 0 \end{pmatrix}$$

$$\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ -\cos\theta \\ -\sin\theta \end{pmatrix} = -\sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_R(p) = \sqrt{2E}\begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{u}_L(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_R(p) = \sqrt{2E}\sqrt{E'(1-\cos\theta)}\begin{pmatrix} 0 & 0 & \frac{-\sin\theta}{\sqrt{1-\cos\theta}} & 1 \end{pmatrix}\begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\bar{u}_R(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_R(p) = \sqrt{2EE'(1-\cos\theta)}\begin{pmatrix} 1 & \frac{\sin\theta}{\sqrt{1-\cos\theta}} & 0 & 0 \end{pmatrix}\begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \\ 0 \end{pmatrix}$$

$$= \sqrt{2EE'(1-\cos\theta)}\left(-\sin\theta + \frac{\sin\theta\cos\theta}{\sqrt{1-\cos\theta}}\right)$$

$$\bar{u}_R(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_L(p) = -\sqrt{2EE'(1-\cos\theta)}\begin{pmatrix} 1 & \frac{\sin\theta}{\sqrt{1-\cos\theta}} & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \cos\theta \\ \sin\theta \end{pmatrix} = 0$$

$$\bar{u}_L(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_L(p) = -\sqrt{2EE'(1-\cos\theta)}\begin{pmatrix} 0 & 0 & \frac{-\sin\theta}{\sqrt{1-\cos\theta}} & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= -\sqrt{2EE'(1-\cos\theta)}\left(\frac{-\sin\theta\cos\theta}{\sqrt{1-\cos\theta}} + \sin\theta\right)$$

to $\mathcal{O}(\theta)$ we have:

$$\bar{u}_L(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_L(p) = i\sqrt{2EE'}\,\theta$$

$$\bar{u}_R(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_1\, u_R(p) = -i\sqrt{2EE'}\,\theta$$

$$\bar{u}_L(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_L(p) = -\sqrt{2EE'}\,\frac{\theta}{\sqrt{2}}\left(-\frac{\theta\left(1-\frac{\theta^2}{2}\right)}{\theta/\sqrt{2}} + \theta\right) = +\sqrt{2EE'}\,\theta$$

$$\bar{u}_R(p')\,\vec{\gamma}\cdot\vec{\varepsilon}_2\, u_R(p) = \sqrt{2EE'}\,\frac{\theta}{\sqrt{2}}\left(-\theta + \frac{\theta\left(1-\frac{\theta^2}{2}\right)}{\theta/\sqrt{2}}\right) = \sqrt{2EE'}\,\theta$$

c) $d\sigma = \frac{1}{2E_{p'}\,2E_q\,|\Delta v|} \quad \frac{d^3p'}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{p'}}\frac{1}{2E_q}\,|\mathcal{M}|^2 (2\pi)^4 \delta^4(p-p'-q)$

$$|\mathcal{M}|^2 = \left(-ie\,\bar{u}(p')\gamma^\mu u(p)\right)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\hat{\mathcal{M}}^\nu(q)$$

$$\times\, \hat{\mathcal{M}}^{\alpha *}(q)\left(\frac{ig_{\alpha\beta}}{q^2}\right)\left(-ie\,\bar{u}(p')\gamma^\beta u(p)\right)^\dagger$$

$$= e^2\frac{g_{\mu\nu}g_{\alpha\beta}}{q^4}\underbrace{\hat{\mathcal{M}}^\nu\hat{\mathcal{M}}^{*}_\alpha}_{\hat{\mathcal{M}}^{\nu\alpha}}\left(\bar{u}(p')\gamma^\mu u(p)\right)\left(\bar{u}(p')\gamma^\beta u(p)\right)^*$$

$$= \frac{e^2\hat{\mathcal{M}}_{\mu\beta}}{q^2}\left(Aq^\mu + B\hat{q}^\mu + C\varepsilon_1^\mu + D\varepsilon_2^\mu\right)\left(Aq^\beta + B\hat{q}^\beta + C\varepsilon_1^\beta + D\varepsilon_2^\beta\right)^*$$

can ignore A b/c of ward identity $q_\mu \mathcal{M}^\mu = 0$
B can also be ignored b/c $B \sim \mathcal{O}(\theta^2)$

$$\simeq \frac{e^2\hat{\mathcal{M}}_{\mu\beta}}{q^2}\left(C\varepsilon_1^\mu + D\varepsilon_2^\mu\right)\left(C\varepsilon_1^\beta + D\varepsilon_2^\beta\right)^*$$

What are C and D?

$$\varepsilon_{i\mu}\,\bar{u}(p')\gamma^\mu u(p) = A\,\cancel{\varepsilon_i\cdot q}^{\,0} + B\,\cancel{\varepsilon_i\cdot\hat{q}}^{\,0} + C\,\varepsilon_i\cdot\varepsilon_1 + D\,\varepsilon_i\cdot\varepsilon_2$$

$$= C\,\delta_{i1} + D\,\delta_{i2}$$

$\Rightarrow \quad C = \bar{u}(p')\gamma^\mu\varepsilon_{1\mu}u(p) \qquad\qquad D = \bar{u}(p')(-\vec{\gamma}\cdot\vec{\varepsilon}_2)u(p)$

$\qquad\quad = \bar{u}(p')(-\vec{\gamma}\cdot\vec{\varepsilon}_1)u(p)$

6.3 Exotic Contributions to g − 2

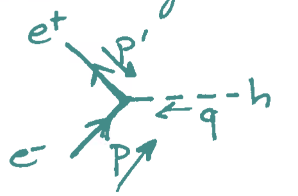
– any particle that couples to the e^- can produce a correction to the e^--photon form factors → in particular to g−2

a) Electroweak theory contains the scalar Higgs field, h, which couples to the $\bar{e}$ according to:

$$H_{int} = \int d^3x \, \frac{\lambda}{\sqrt{2}} h \bar{\psi}\psi$$

compute the contribution of a virtual Higgs to the e^- g−2

The interaction hamiltonian gives the vertex structure



$= \frac{-i\lambda}{\sqrt{2}}$

Recall that to obtain the rule we want to evaluate:

$${}_{out}\langle \vec{p}_1 \vec{p}_2 \cdots \vec{p}_n | k_A k_B \rangle_{in} \equiv \langle \vec{p}_1 \vec{p}_2 \cdots | S | \bar{k}_A \bar{k}_B \rangle = \lim_{T\to\infty} \langle \vec{p}_1 \vec{p}_2 \cdots | e^{-iH(2T)} | \bar{k}_A \bar{k}_B \rangle$$

$$S = 1 + iT$$

$$\langle \vec{p}_1 \vec{p}_2 \cdots | iT | \bar{k}_A \bar{k}_B \rangle = (2\pi)^4 \delta^{(4)}(k_A + k_B - \Sigma p_f) \; i\mathcal{M}(k_A, k_B \to p_f)$$

$$= \lim_{T \to \infty(1-i\epsilon)} \left\{ {}_0\langle \vec{p}_1 \cdots \vec{p}_n | T(\exp[-i\textstyle\int_{-T}^{T} dt H(t)]) | \vec{p}_A \vec{p}_B \rangle \right\}_{\substack{\text{connected} \\ \text{amputated}}}$$

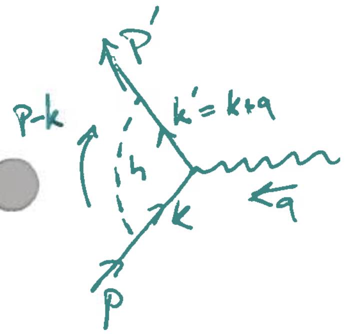
$$i\mathcal{M} (2\pi)^4 \delta^{(4)}(p_A + p_B - \Sigma p_f) = \Sigma \text{ all connected, amputated FD with } p_A p_B \text{ incoming and } p_f \text{ outgoing}$$

take the process: $e^+e^- \to h$

$$\langle \bar{q} | T\left(-i \int d^4x \, \frac{\lambda}{\sqrt{2}} h \bar{\psi}\psi\right) | \bar{p}_1 \bar{p}_2 \rangle$$

$$= \frac{-i\lambda}{\sqrt{2}} \int d^4x \left\{ \langle q | h \, \bar{\psi}\psi | \bar{p}_1 \bar{p}_2 \rangle \right\}$$

$$e^{\pm i q\cdot x} \, \bar{u}(p_2) u(p_1) e^{\pm i(p_1 \pm p_2)\cdot x}$$



$= i\mathcal{M}$

$i^3(-i)^3 = -i^6 = -(-1)^3 = 1$

$$= \int\frac{d^4k}{(2\pi)^4}\,\bar{u}(p')\left[\left(\frac{-i\lambda}{\sqrt{2}}\right)\frac{i}{\not{k}'-m_e^2}(-ie\gamma^\mu)\frac{i}{\not{k}-m_e^2}\left(\frac{-i\lambda}{\sqrt{2}}\right)\frac{i}{(p-k)^2-m_h^2}\right]u(p)$$

$$= \int\frac{d^4k}{(2\pi)^4}\,\frac{\lambda_e^2}{2}\,\bar{u}(p')\left[\frac{(\not{k}'+m_e)\gamma^\mu(\not{k}+m_e)}{(k'^2-m_e^2)(k^2-m_e^2)((p-k)^2-m_h^2)}\right]u(p)$$

Combine denominator using Feynman parameter.

$$\frac{1}{ABC} = \int dx\,dy\,dz\;\delta(x+y+z-1)\frac{(3-1)!}{(xA+yB+zC)^3}$$

$$\frac{1}{(k'^2-m_e^2)(k^2-m_e^2)((p-k)^2-m_h^2)} = \int dx\,dy\,dz\;\delta(x+y+z-1)\frac{2}{[x(k'^2-m_e^2)+y(k^2-m_e^2)+z((p-k)^2-m_h^2)]^3}$$

$$\equiv \int dx\,dy\,dz\;\delta(x+y+z-1)\frac{2}{D^3}$$

where $D = xk'^2 + yk^2 + z(p-k)^2 - (x+y)m_e^2 - zm_h^2$

$= x(k+q)^2 + yk^2 + z(p^2 - 2p\cdot k + k^2) - (x+y)m_e^2 - zm_h^2$

$= x(k^2 + 2k\cdot q + q^2) + yk^2 + zp^2 - 2zp\cdot k + zk^2 - (x+y)m_e^2 - zm_h^2$

$= (x+z+y)k^2 + 2k\cdot(xq - zp) + zm_e^2 - (x+y)m_e^2 - zm_h^2 + xq^2$

$\Rightarrow k^2 + 2k\cdot(xq - zp) + (z-x-y)m_e^2 - zm_h^2 + xq^2$

define
$\ell = k + xq - zP$
$\Rightarrow k = \ell - xq + zP$

$= (\ell - xq + zp)^2 + 2(\ell - xq + zp)\cdot(xq - zp) + (z-x-y)m_e^2 - zm_h^2 + xq^2$

$= \ell^2 \cancel{- 2xq\cdot\ell} + 2zp\cdot\ell \cancel{- 2xzq\cdot p} + x^2q^2 + z^2p^2$
$\quad + 2x\ell\cdot q - 2z\ell\cdot p - 2x^2q^2 \cancel{+ 2xzq\cdot p} + 2zxp\cdot q - 2z^2p^2$
$\quad + (z-x-y)m_e^2 - zm_h^2 + xq^2$

$= \ell^2 + q^2(x - x^2) + 2zx\,pq + (-z^2 + z - x - y)m_e^2 - zm_h^2.$

$= \ell^2 + q^2(x - x^2 + zx) + (-z^2 + z - (1-z))m_e^2 - zm_h^2$

$= \ell^2 + q^2x(1+z-x) + (-z^2 + z + z - 1)m_e^2 - zm_h^2$

$$D = \ell^2 + q^2 x(1+z-(1-z-y)) + (-z^2+2z-1)m_e^2 - z m_h^2$$

$$= \ell^2 + q^2 xy - (1-z)^2 m_e^2 - z m_h^2$$

$$\boxed{= \ell^2 - \Delta}$$

where $\boxed{\Delta = -xy\, q^2 + (1-z)^2 m_e^2 + z m_h^2}$ ✓

$$\Rightarrow i\mathcal{M} = \int \frac{d^4\ell}{(2\pi)^4}\, \lambda^2 e\, \bar{u}(p') \frac{N}{(\ell^2-\Delta)^3} u(p)\ \delta(x+y+z-1)\, dx\,dy\,dz$$

where $N = (\not{k}' + m_e)\gamma^\mu(\not{k} + m_e)\big|_{k=\ell-xq+zp}$

$$= (\not{\ell} - x\not{q} + z\not{p} + \not{q} + m_e)\gamma^\mu(\not{\ell} - x\not{q} + z\not{p} + m_e)$$

Note: 1) $g-2 = 2F_2(0) \Rightarrow$ ignore terms linear in γ^μ

2) ignore terms linear in ℓ.

3) we will use the identities:

- $\not{a}\gamma^\mu = 2a^\mu - \gamma^\mu\not{a}$
  - $\hookrightarrow$ for $a=p$ we have $\not{p}\gamma^\mu = 2p^\mu - \gamma^\mu\not{p}$
  $\Rightarrow 2p^\mu - \gamma^\mu m.$
- $\gamma^\mu\not{a} = 2a^\mu - \not{a}\gamma^\mu$
  - $\hookrightarrow$ for $a=p'$ we have $\gamma^\mu\not{p}' \Rightarrow 2p'^\mu - m\gamma^\mu.$
- $\not{a}\not{a} = a\cdot a.$
- $\bar{u}(p')\not{p}' = m_e\bar{u}(p')$
- $\not{p}\,u(p) = m\,u(p).$
- $\bar{u}(p')\not{q}\,u(p) = 0.$

$$N = (\not{\ell}\gamma^\mu - x\not{q}\gamma^\mu + z\not{p}\gamma^\mu + \not{q}\gamma^\mu + m_e\gamma^\mu)(\not{\ell} - x\not{q} + z\not{p} + m_e)$$

$$= \not{\ell}\gamma^\mu\not{\ell} - x\cancel{\not{\ell}\gamma^\mu\not{q}} + \cancel{z\not{\ell}\gamma^\mu\not{p}} + \cancel{m_e\not{\ell}\gamma^\mu}$$

$$- \cancel{x\not{q}\gamma^\mu\not{\ell}} + x^2\not{q}\gamma^\mu\not{q} - xz\not{q}\gamma^\mu\not{p} - xm_e\not{q}\gamma^\mu$$

$$+ \cancel{z\not{p}\gamma^\mu\not{\ell}} - xz\not{p}\gamma^\mu\not{q} + z^2\not{p}\gamma^\mu\not{p} + z\not{p}\gamma^\mu m_e$$

$$+ \cancel{\not{q}\gamma^\mu\not{\ell}} - x\not{q}\gamma^\mu\not{q} + z\not{q}\gamma^\mu\not{p} + \not{q}\gamma^\mu m_e$$

$$+ \cancel{m_e\gamma^\mu\not{\ell}} - xm_e\gamma^\mu\not{q} + \cancel{m_e z\gamma^\mu\not{p}} + \cancel{m_e^2\gamma^\mu}$$

$$= \not{\ell}\gamma^\mu\not{\ell} + x^2\not{q}\gamma^\mu\not{q} - xz\not{q}\gamma^\mu m_e - xm_e\not{q}\gamma^\mu$$

$$- xz\not{p}\gamma^\mu\not{q} + z^2\not{p}\gamma^\mu m_e + zm_e\not{p}\gamma^\mu$$

$$- x\not{q}\gamma^\mu\not{q} + z\not{q}\gamma^\mu m_e + \not{q}\gamma^\mu m_e$$

$$- xm_e\gamma^\mu\not{q}$$

$$= \overset{(1)}{\not{\ell}\gamma^\mu\not{\ell}} + \overset{(2)}{\not{q}\gamma^\mu\not{q}(x^2 - x)}$$

$$+ \overset{(3)}{\not{q}\gamma^\mu m_e(-xz - x + z + 1)} \qquad 1 + z - xz - x$$

$$+ \overset{(4)}{\not{p}\gamma^\mu m_e(z^2 + z)} \qquad (1-x) + z(1-x)$$

$$\overset{(5)}{-xz\not{p}\gamma^\mu\not{q}}$$

$$\overset{(6)}{-xm_e\gamma^\mu\not{q}}$$

$$* \quad \boxed{q^\mu = (p' - p)^\mu}$$

① .

$\not l \gamma^\mu \not l$

$= 2 l^\mu \not l - \gamma^\mu \not l \not l$

$= 2 l^\mu l^\nu g_{\nu\eta} \gamma^\eta - l^2 \gamma^\mu$

$\rightarrow = \frac{2l^2}{4} g^{\mu\nu} g_{\nu\eta} \gamma^\eta - l^2 \gamma^\mu$

$= l^2 \left(\frac{\gamma^\mu}{2} - \gamma^\mu \right.$

$= -\frac{1}{2} l^2 \gamma^\mu$

ignore.

② . $\not q \gamma^\mu \not q$

$= (2 q^\mu - \gamma^\mu \not q) \not q$

$= 2 q^\mu \not q - \gamma^\mu q^2$

$\bar{u}(p') \not p u(p) = 0$

$\rightarrow -q^2 \gamma^\mu$

ignore

③ . $\not q \gamma^\mu$

$= (\not p' - \not p) \gamma^\mu$

$\rightarrow (m_e - \not p) \gamma^\mu$

$\rightarrow -\not p \gamma^\mu$

$= -2 p^\mu + \gamma^\mu \not p$

$\rightarrow -2p^\mu$

④ $\not p \gamma^\mu$

$= 2p^\mu - \gamma^\mu \not p$

$\rightarrow 2p^\mu - \gamma^\mu m_e$

$\rightarrow 2p^\mu$

⑤ $\not p \gamma^\mu \not q$

$= 2p^\mu \not q - \gamma^\mu \not p \not q$

$\rightarrow -\gamma^\mu \not p \not q$

$\rightarrow -\gamma^\mu (2p q - \not q \not p)$

$\rightarrow +\gamma^\mu \not q \not p$

$\rightarrow \gamma^\mu \not q m_e$

$\rightarrow 2 p'^\mu m_e.$

⑥ $\gamma^\mu \not q$

$= \gamma^\mu (\not p' - \not p)$

$= \gamma^\mu \not p' - m_e \gamma^\mu$

$\rightarrow \gamma^\mu \not p'$

$\rightarrow 2p'^\mu - \not p' \gamma^\mu$

$\rightarrow 2p'^\mu - m_e \gamma^\mu$

$\rightarrow 2p'^\mu$

$$\Rightarrow N \longrightarrow m_e(-2p^\mu)(-\underline{xz}-\underline{x}+z+1)$$
$$+\,m_e(2p^\mu)(z^2+z)$$
$$-\,\underline{xz}\,(2p'^\mu m_e)$$
$$-\,\underline{x}\,m_e(2p'^\mu)$$

$$= 2m_e p^\mu(-z^2 \cancel{+z} + xz + x \cancel{-z} - 1)$$
$$+\,2m_e p'^\mu(-xz - x)$$

$$= 2m_e\{p^\mu(z^2 - 1) + (xz+x)(p^\mu - p'^\mu)\}$$

$$= 2m_e\{p^\mu(z^2-1) - x(z+1)(p'^\mu - p^\mu)$$

$$= 2m_e\{p^\mu(z^2-1) + \frac{p'^\mu(z^2-1)}{2} - \frac{p'^\mu(z^2-1)}{2} - x(z+1)(p'^\mu - p^\mu)$$

$$= 2m_e\left\{\frac{(p^\mu+p'^\mu)(z^2-1)}{2} + \left(-x(1+z) + \frac{1-z^2}{2}\right)q^\mu\right)$$

$$= m_e\{(p^\mu+p'^\mu)(z^2-1) + (-2x-2xz+1-z^2)\,q^\mu\}.$$

$$= m_e\{(p^\mu+p'^\mu)(z^2-1) + \underbrace{(y-x)(1+z)\,q^\mu}\}$$

must vanish in accord w/ Ward identity
odd under $x \leftrightarrow y \Rightarrow$ the $dxdy$ integration is zero. ✓

$$1 - 2x - 2xz - z^2$$
$$1 - 2x - 2xz - z(1-x-y)$$
$$1 - 2x - \cancel{2}xz - z + \cancel{zx} + zy$$
$$1 - 2x - z - xz + zy$$
$$\cancel{x} + y + \cancel{z} - \cancel{2}x - \cancel{z} - xz + zy$$
$$y - x - xz + zy$$
$$y(1+z) - x(1+z)$$
$$(y-x)(1+z)$$

$$\Rightarrow N \to m_e\,(p^\mu + p'^\mu)(z^2-1).$$

Gordon Identity $\to m_e(z^2-1)\,(2m_e)\left(\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2m_e}\right)$

$$\to 2m_e^2(z^2-1)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m_e}$$

Finally, $iM = \int \frac{d^4 l}{(2\pi)^4}\,\lambda^2 e\,\frac{\bar{u}(p')\,N\,u(p)}{(l^2-\Delta)^3}\,\delta(x+y+z-1)\,dx\,dy\,dz$

$$\to \lambda^2 e \int \frac{d^4 l}{(2\pi)^4} \int dx\,dy\,dz\,\frac{\delta(x+y+z-1)}{(l^2-\Delta)^3}\,\bar{u}(p')\left[2m_e^2(z^2-1)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m_e}\right]u(p)$$

$$= \bar{u}(p')\,F_2(q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m_e}\,u(p)$$

where

$$F_2(q^2) = \int dx\,dy\,dz\,\delta(x+y+z-1)\,(z^2-1)\,2m_e^2\lambda^2 e\,\underbrace{\int \frac{d^4 l}{(2\pi)^4}\frac{1}{(l^2-\Delta)^3}}_{\frac{(-1)^3 i}{(4\pi)^{d/2}}\frac{\Gamma(3-\frac{d}{2})}{\Gamma(3)}\left(\frac{1}{\Delta}\right)^{3-\frac{d}{2}}}.$$

$$= \frac{-i}{16\pi^2}\,\frac{1}{2}\left(\frac{1}{\Delta}\right)$$

$$= \frac{-i}{32\pi^2\Delta}$$

$$= \int dx\,dy\,dz\,\delta(x+y+z-1)\left(-\frac{2m_e^2\lambda^2 e i}{32\pi^2}\right)\frac{z^2-1}{-xyq^2+(1-z)^2 m_e^2 + z m_h^2}$$

$$= \frac{m_e^2\lambda^2 e i}{16\pi^2}\int_0^1 dx\,dz\,\frac{1-z^2}{x(1-x-z)q^2 + (1-z)^2 m_e^2 + z m_h^2}$$

we want the $q \to 0$ limit

$$F_2(q\to 0) = \frac{m_e^2\lambda^2 e i}{16\pi^2}\int_0^1 dx\,dz\,\frac{1-z^2}{(1-z)^2 m_e^2 + z m_h^2}$$

$$F_2(0) = \frac{m_e^2 \lambda^2 e}{16\pi^2 m_h^2} \int_0^1 dx\,dy \frac{1-z^2}{(1-z)^2\frac{m_e^2}{m_h^2}+z}$$

$$= \frac{\lambda^2 e}{16\pi^2} \frac{m_e^2}{m_h^2} \int_0^1 dz \frac{1-z^2}{(1-z)^2\frac{m_e^2}{m_h^2}+z} \int_0^{1-z} dx$$

$$= \frac{\lambda^2 e}{16\pi^2}\left(\frac{m_e}{m_h}\right)^2 \int_0^1 dz \frac{(1-z)(1-z^2)}{(1-z)^2\left(\frac{m_e}{m_h}\right)^2+z}$$

$\to = 1 - z^2 - z + z^3$

$= 1 - z - z^2 + z^3$

$$= \frac{\lambda^2 e}{16\pi^2}\left(\frac{m_e}{m_h}\right)^2\left[\int_0^1 dz \frac{1}{(1-z)^2\left(\frac{m_e}{m_h}\right)^2+z} + \int dz \frac{(-z)(1+z-z^2)}{(1-z)^2\left(\frac{m_e}{m_h}\right)^2+z}\right]$$

$m_e \ll m_h$

$$\simeq \frac{\lambda^2 e}{16\pi^2}\left(\frac{m_e}{m_h}\right)^2\left[\int_0^1 dz \frac{1}{(1-z)^2\left(\frac{m_e}{m_h}\right)^2+z} - \underbrace{\int_0^1 (1+z-z^2)}\right]$$

$= 1 + \frac{1}{2} - \frac{1}{3}$

$= \frac{6+3-2}{6}$

$= \frac{7}{6}$

Let $u = z + (1-z)^2\left(\frac{m_e}{m_h}\right)^2$

$du = dz - 2(1-z)\left(\frac{m_e}{m_h}\right)^2 dz$

$= \left(1 - 2(1-z)\left(\frac{m_e}{m_h}\right)^2\right)dz$

$$\int_{\left(\frac{m_e}{m_h}\right)^2}^1 \frac{du/u}{1-2\left(\frac{m_e}{m_h}\right)^2(1-z)}$$

$$\simeq \int_{\left(\frac{m_e}{m_h}\right)^2}^1 \frac{du}{u}$$

$$= -2\ln\left(\frac{m_e}{m_h}\right)$$

$$= \frac{\lambda^2 e}{16\pi^2}\left(\frac{m_e}{m_h}\right)^2\left(-2\ln\left(\frac{m_e}{m_h}\right) - 7/6\right)$$

$$= \frac{\lambda^2 e}{8\pi^2}\left(\frac{m_e}{m_h}\right)^2\left(\ln\left(\frac{m_h}{m_e}\right) - 7/12\right)$$

The higgs contribution to the e^-'s anomolous magnetic moment is

$$2F_2(0) = \frac{\lambda^2 e}{4\pi^2}\left(\frac{m_e}{m_h}\right)^2\left(\ln\left(\frac{m_h}{m_e}\right) - 7/12\right)$$

b) QED accounts extremely well for the e^-'s anomalous mag moments

if $a = \frac{g-2}{2}$ and $|a_{exp} - a_{QED}| < 10^{-10}$

- what limits does this place on λ and M_h?
- in the simplest version of electroweak theory

$$\lambda = 3 \times 10^{-6}$$

$$M_h > 60\ GeV$$

Show that these values are not excluded.

The coupling of the Higgs to the muon is larger by a factor of M_μ / M_e

$$\lambda = 6 \times 10^{-4}$$

Thus, although exp knowledge of the muon anomalous mag moment is not as precise

$$|a_{exp} - a_{QED}| < 1 \times 10^{-10}$$

one can still obtain a stronger limit on M_h. Is it strong enough?

$$a_{higgs} = \frac{g-2}{2}$$

$$= F_2(0)$$

$$= \frac{\lambda^2 e}{8\pi^2} \left(\frac{M_e}{M_h}\right)^2 \left[\ln\left(\frac{M_h}{M_e}\right) - 7/3\right]$$

$$a_{higgs}(\lambda = 3 \times 10^{-6}, M_h = 60\ GeV) = 7.72229 \times 10^{-23}.$$

now, $|a_{exp} - a_{QED}| < 1 \times 10^{-10}$

$$\Rightarrow a_{higgs} < 1 \times 10^{-10}.$$

So the values of $\lambda = 3 \times 10^{-6}$ and $M_h = 60\ GeV$ are consistent.

being a bit more general we have:

$$a_{higgs} = \left(\frac{\lambda m_e}{m_h}\right)^2 \frac{e}{8\pi^2}\left[\ln\left(\frac{m_h}{m_e}\right) - 7/3\right] < 1\times10^{-10}.$$

we have 2 parameters λ and $\xi \equiv \frac{m_h}{m_e}$.

$$a_{higgs} = \frac{e}{8\pi^2}\left(\frac{\lambda}{\xi}\right)^2\left(\ln\xi - 7/3\right)$$

try to find a limit on λ by setting $m_h = 60\,GeV$.

$$\Rightarrow \xi = \frac{0.511\times10^6}{60\times10^9} = 5\times10^{-17}.$$

$$\Rightarrow a_{higgs} = (8.58\times10^{-12})\,\lambda^2 < 1\times10^{-10}.$$

$$\Rightarrow \lambda < \sqrt{\frac{1\times10^{-10}}{8.58\times10^{-12}}} = 3.41388$$

<u>Muon</u>: $\lambda \Rightarrow 6\times10^{-4}$ and $\xi \rightarrow m_\mu / m_h$

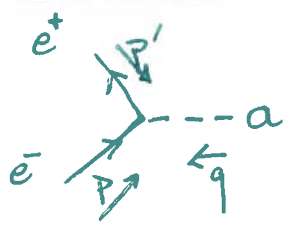
c) more complex versions contain pseudo-scalar called the axion:

$$H_{int} = \int d^3x \; \frac{i\lambda}{\sqrt{2}} \, a \, \bar{\psi} \gamma^5 \psi$$

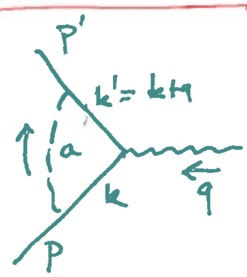
axion may be a light as e^- or lighter and may couple more strongly to the Higgs

Compute contribution to $g-2$ from axion

work out excluded values of λ and m_a.



$$= -i\left(\frac{i\lambda}{\sqrt{2}}\gamma^5\right) = \frac{\lambda}{\sqrt{2}}\gamma^5$$

$i\mathcal{M} =$ 

$$= \int \frac{d^4k}{(2\pi)^4} \bar{u}(p')\left[\left(\frac{\lambda\gamma^5}{\sqrt{2}}\right) \frac{i}{\not{k}' - m_e^2} (-ie\gamma^\mu) \frac{i}{\not{k} - m_e^2} \left(\frac{\lambda\gamma^5}{\sqrt{2}}\right) \frac{i}{(p-k)^2 - m_a^2}\right] u(p)$$

$$= \frac{\lambda^2 e}{2} \int \frac{d^4k}{(2\pi)^4} \bar{u}(p')\left[\frac{\gamma^5(\not{k}' + m_e)\gamma^\mu(\not{k} + m_e)\gamma^5}{(k'^2 - m_e^2)(k^2 - m_e^2)((p-k)^2 - m_a^2)}\right] u(p)$$

$$= \frac{\lambda^2 e}{\cancel{2}} \int \frac{d^4k}{(2\pi)^4} \int dx\,dy\,dz\, \delta(x+y+z-1)\, \cancel{2} \, \frac{\bar{u}(p')\, N\, u(p)}{D^3}$$

where $D = \ell^2 - \Delta$; $\Delta = -xyq^2 + (1-z)^2 m_e^2 + z m_a^2$

$$\boxed{\ell = k + xq - zp.}$$

$k = \ell - xq + zp$

$k' = k + q$

$= \ell + (1-x)q + zp$

$$N = \gamma^5 (\not{k}' + m_e) \gamma^\mu (\not{k} + m_e) \gamma^5$$

$$= \gamma^5 (\not{k}' + m_e) \gamma^\mu \gamma^5 (\not{k} + m_e)$$

$$= \gamma^5 (\not{k}' + m_e) \gamma^5 \gamma^\mu (\not{k} - m_e)$$

$$= (\gamma^5)^2 (-\not{k}' + m_e) \gamma^\mu (\not{k} - m_e)$$

$$= -(\not{k}' - m_e) \gamma^\mu (\not{k} - m_e)$$

$$= -(\not{l} + (1-x)\not{q} + z\not{p} - m_e) \gamma^\mu (\not{l} - x\not{q} + z\not{p} - m_e)$$

- ignore all terms proportional to $\not{l}$ as $d^4k = d^4l$
- ignore terms $\propto \gamma^\mu$ as we want the coeff of $\sigma^{\mu\nu} q_\nu$
- use ~~$\not{p}$~~ $\not{p}\, u(p) = m_e u(p)$

$$\Rightarrow N = -\{ \not{l}\gamma^\mu\not{l} - x(1-x)\not{q}\gamma^\mu\not{q} + z(1-x)\underline{\not{q}\gamma^\mu\not{p}} - m_e(1-x)\underline{\not{q}\gamma^\mu}$$
$$- xz\not{p}\gamma^\mu\not{q} + z^2\underline{\not{p}\gamma^\mu\not{p}} - m_e z\underline{\not{p}\gamma^\mu}$$
$$+ x m_e \gamma^\mu \not{q} - \underbrace{m_e z \gamma^\mu \not{p}}_{\text{ignore} \to \gamma^\mu\not{p} \to m_e\gamma^\mu} \}$$

$$\Rightarrow -\{ \cancel{\not{l}\gamma^\mu\not{l}}^{\,1} - x(1-x)\cancel{\not{q}\gamma^\mu\not{q}}^{\,2} + m_e(z(1-x) - (1-x))\not{q}\gamma^\mu{}^{\,3}$$
$$+ m_e(z^2 - z)\not{p}\gamma^\mu{}^{\,4} - xz\not{p}\gamma^\mu\not{q}^{\,5} + m_e x\gamma^\mu\not{q}^{\,6} \}$$

$$\Rightarrow -\{ m_e(z(1-x) - (1-x))(-2p^\mu) + m_e(z^2 - z)(2p^\mu)$$
$$- xz(2p'^\mu m_e) + m_e x(2p'^\mu) \}$$

$$= -2m_e \{ -(z(1-x) - (1-x))p^\mu + (z^2 - z)p^\mu - xzp'^\mu + xp'^\mu \}$$

$$= -2m_e \{ p^\mu (z^2 - z - z + zx + 1 - x) + p'^\mu (x - xz) \}$$

$$= -2m_e \{ p^\mu (z^2 - 2z + 1) + p^\mu (xz - x) - (xz - x) p'^\mu \}$$

$$= -2m_e \{ (z-1)^2 p^\mu + (p^\mu - p'^\mu)\, x(z-1) \}.$$

$$N = -2m_e\left\{ \frac{(z-1)^2}{2}(p^\mu + p'^\mu) + \left(\frac{(z-1)^2}{2} + xz - x\right)(p^\mu - p'^\mu)\right\}$$

$$= -m_e\left\{(z-1)^2(p^\mu+p'^\mu) + \underbrace{\left((z-1)^2 + 2zx - 2x\right)(p^\mu - p'^\mu)}_{\text{must vanish by Ward identity}}\right\}$$

$$\Rightarrow -m_e(z-1)^2(p^\mu+p'^\mu)$$

$$\Rightarrow -2m_e^2(z-1)^2\left(\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_e}q_\nu\right)$$

$$\Rightarrow -2m_e(z-1)^2\,\frac{i\sigma^{\mu\nu}}{2m_e}q_\nu$$

$$\Rightarrow i\mathcal{M} \to \bar{u}(p')\left[\int\frac{d^4l}{(2\pi)^4}\int dx\,dy\,dz\,\delta(x+y+z-1)\left(\frac{-2m_e^2(z-1)^2}{(l^2-\Delta)^3}\right)(\lambda^2 e)\right]u(p).$$

$$\Rightarrow F_2(q^2) = -2m_e^2\lambda^2 e\int dx\,dy\,dz\,\delta(x+y+z-1)(z-1)^2\underbrace{\int\frac{d^4l}{(2\pi)^4}\frac{1}{(l^2-\Delta)^3}}_{\frac{-i}{32\pi^2\Delta}}$$

$$= \frac{2i\,e\,m_e^2\lambda^2}{32\pi^2}\int dx\,dy\,dz\,\delta(x+y+z-1)\frac{(z-1)^2}{\left(-xyq^2+(1-z)^2m_e^2+zm_a^2\right)}$$

$$= \frac{e\,m_e^2\lambda^2}{16\pi^2}\int dx\,dz\,\delta(x+y+z-1)\frac{(1-z)^2}{-x(1-x-z)q^2+(1-z)^2m_e^2+zm_a^2}$$

$$q\to 0 \;\to\; = \frac{e\,m_e^2\lambda^2}{16\pi^2}\int dz\,\frac{(1-z)^3}{(1-z)^2m_e+zm_a^2}$$

$$= \frac{e\lambda^2}{16\pi^2}\int dz\,\frac{(1-z)^3}{(1-z)^2+z\,\frac{m_a}{m_e}}\qquad \frac{m_a}{m_e}\ll 1$$

$$\simeq \frac{e\lambda^2}{16\pi^2}\int dz\,(1-z)$$

$$= \frac{e\lambda^2}{32\pi^2}$$