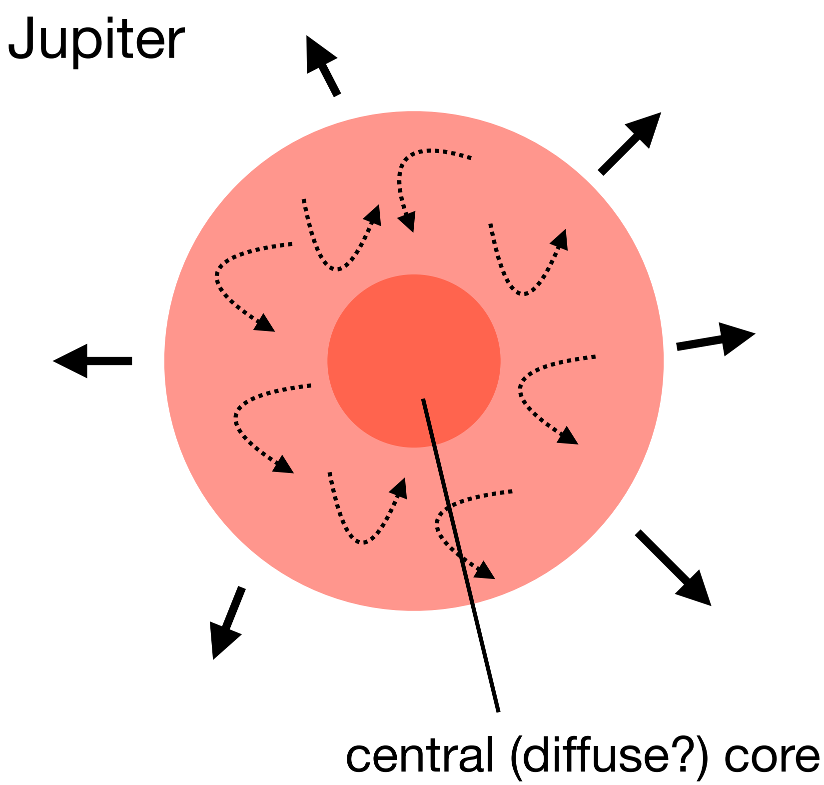
Composition gradients and convection in dense interiors: giant planets and white dwarfs

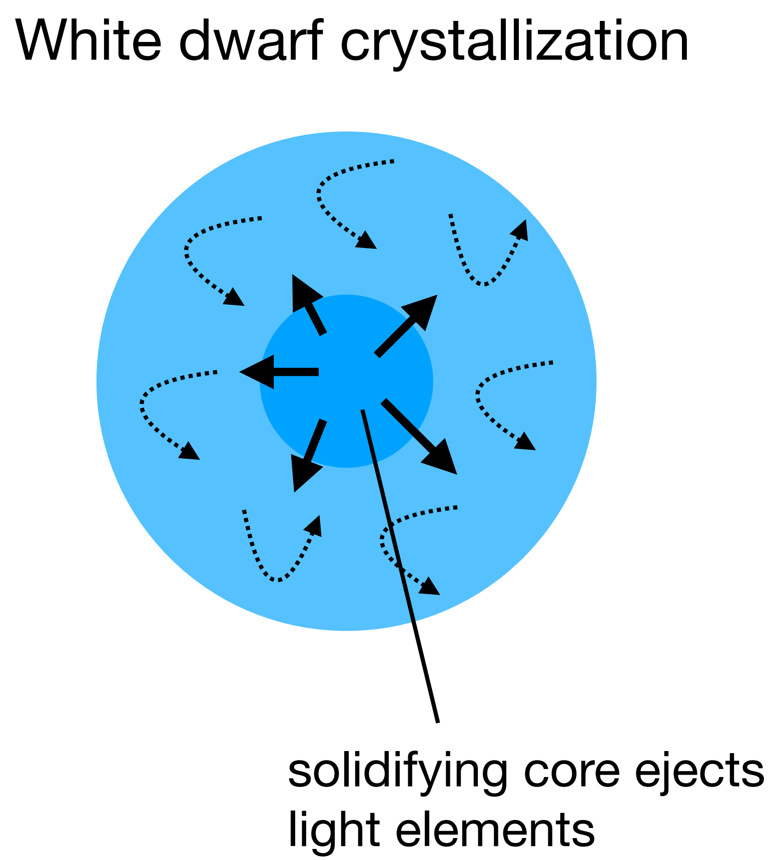
Andrew Cumming (McGill University)

Rafael Fuentes (UC Boulder), Matias Castro Tapia (McGill),
Evan Anders (KITP/UCSB), Bradley Hindman (JILA/UC Boulder)

This talk

- two problems in which composition gradients and convection interact in interesting ways
- driven by boundary fluxes: secular evolution of gradients and boundary locations
- try to use idealized (Boussinesq for now) simulations to inform how we treat these kinds of convection in evolutionary models





There is now convincing evidence that gas giant interiors are non-adiabatic

- Measurements of the gravity field of Jupiter by Juno suggest that the heavy element core maybe significantly extended — “diffuse” or “fuzzy” core

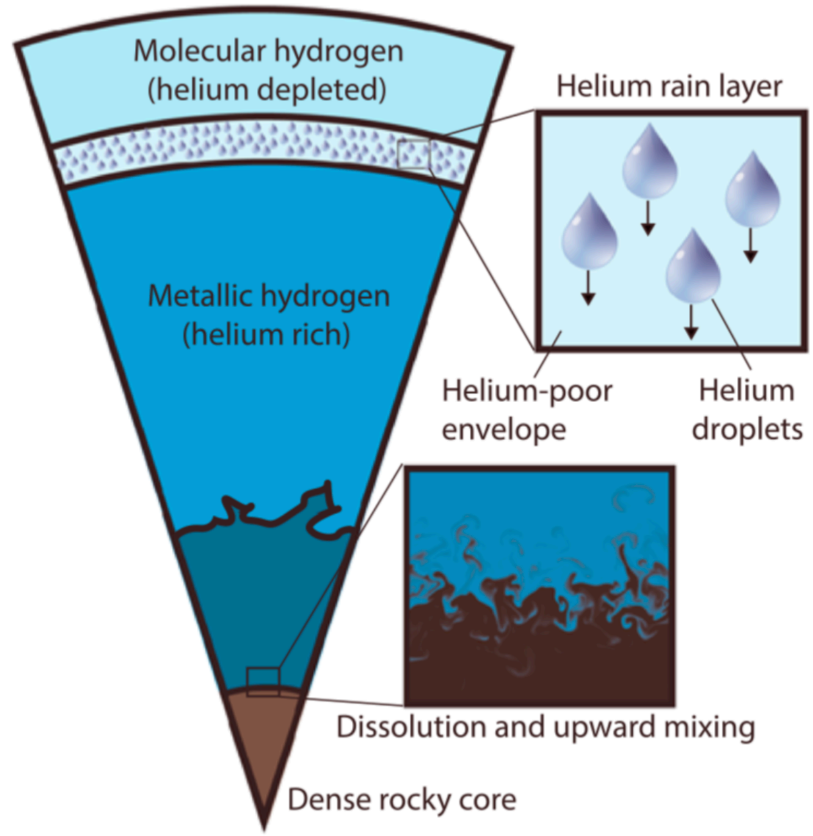
  Wahl et al. 2017; Debras & Chabrier 2019; Nettelmann et al. 2021

- Similar conclusion from Cassini data for Saturn

  e.g. Nettelmann et al. 2021

- Detection of oscillation modes of Saturn from density waves in Saturn’s rings also requires deep stratified layer Fuller 2014

- Exciting because it provides a possible clue about how the gas giants formed !



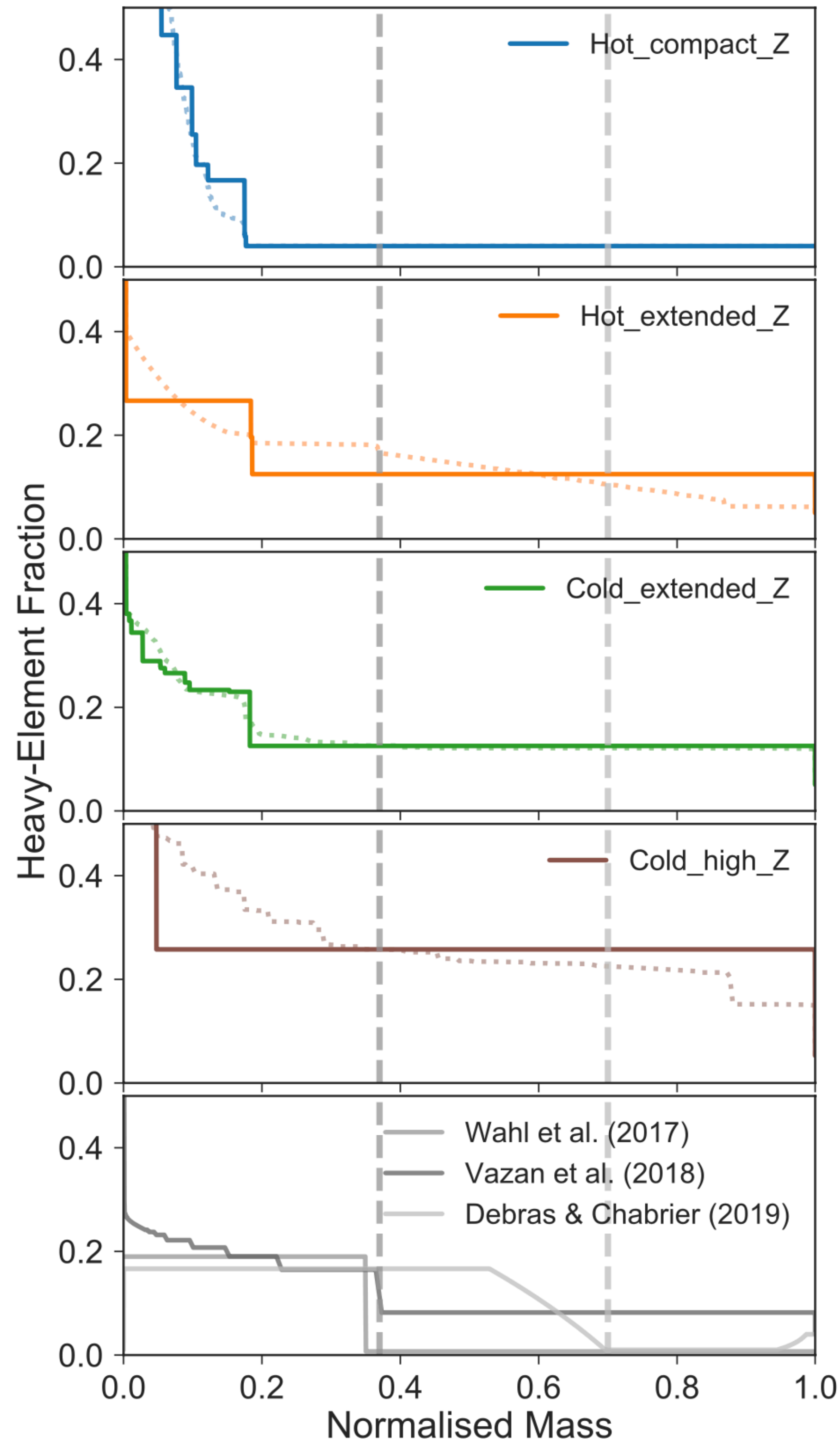
Modelling Jupiter's diffuse (?) core

- 1D evolution models have a hard time explaining a diffuse core

  Müller, Helled, Cumming 2020; see also Vazan et al. 2018; Stevenson et al. (2022)

- Start with a newly-formed Jupiter with a composition profile from a formation model. Then evolve for 5 Gyr using a stellar evolution code (MLT convection, Ledoux criterion).

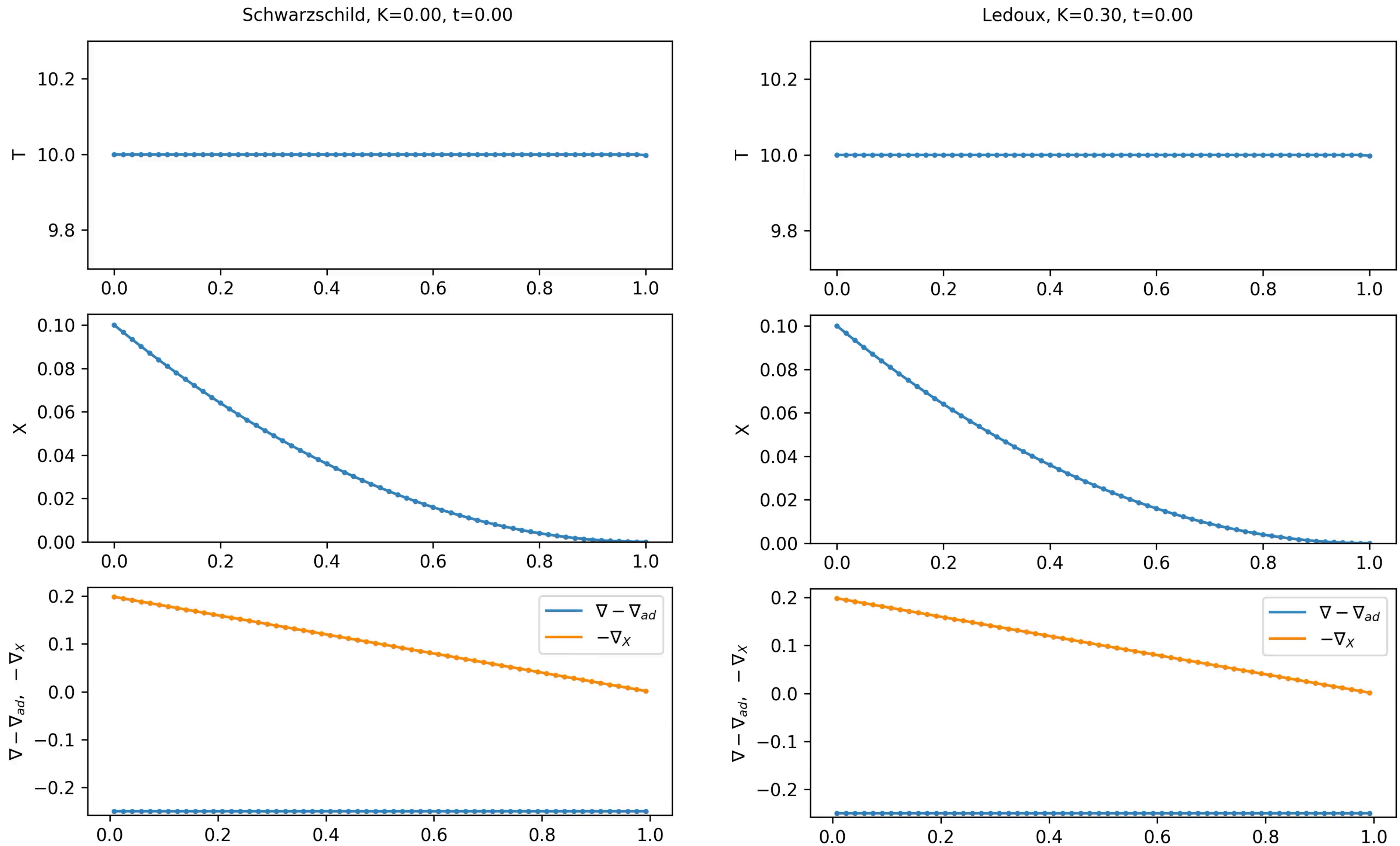
- Two outcomes: either **fully-mixed** or the initial gradient breaks up into a series of **layers** with little erosion



Layer formation is a competition between convection and thermal conduction

(see Vazan et al. 2018)

- Simplest model: heat transport on a grid by thermal diffusion with enhanced diffusivity in convection zones; compositional transport only in convection zone



Radko (2005) solved a similar set of equations to study the growth of staircases on a linear background

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} F_T - \mu \frac{\partial^4}{\partial z^4} T$$

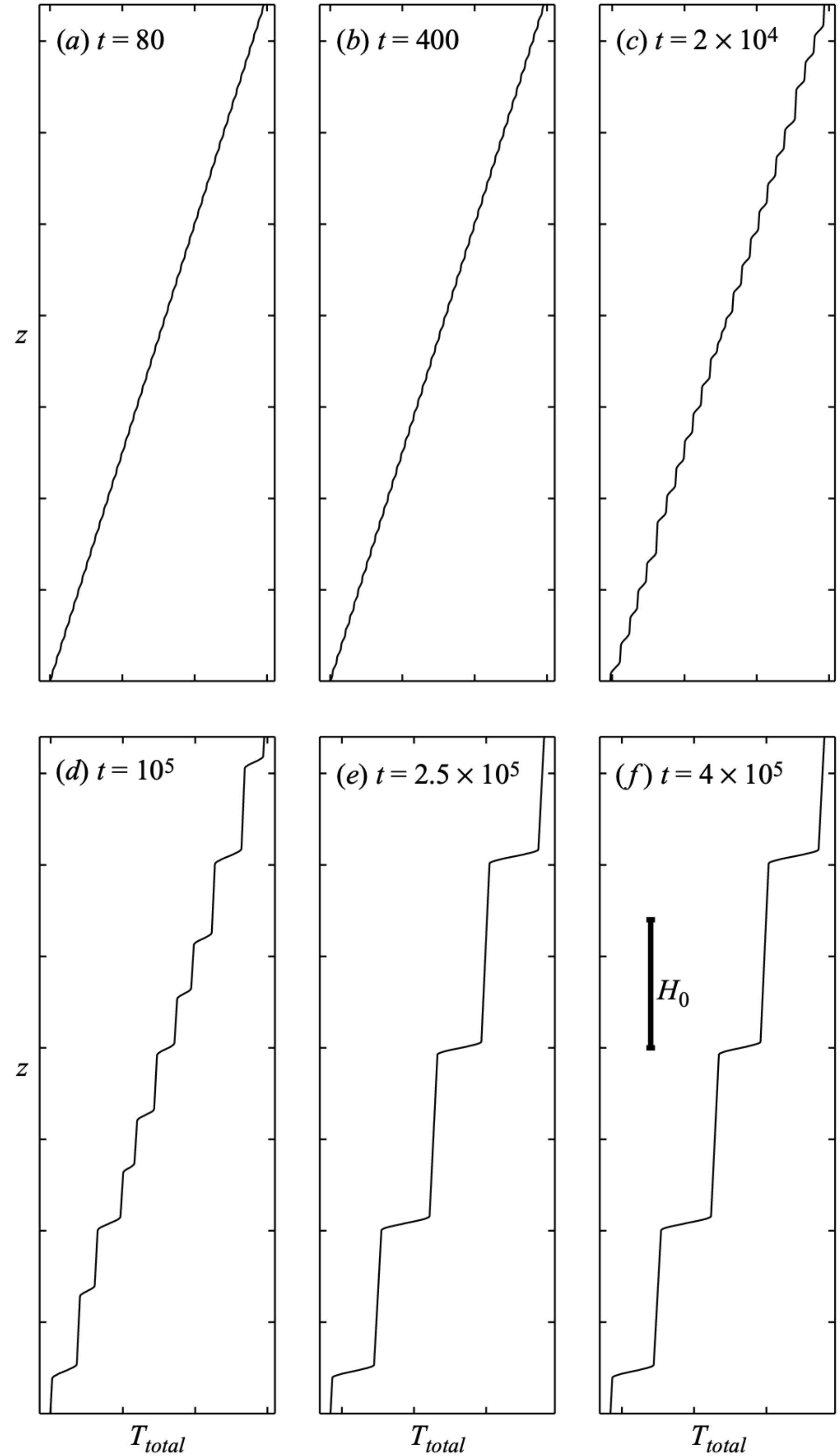
$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial z} F_S - \mu \frac{\partial^4}{\partial z^4} S.$$

Higher order terms stabilize short wavelength unstable modes and allows the steps to be resolved

Diffusive layer thickness controlled by the parameter μ

Without these terms, the number of steps increases with resolution

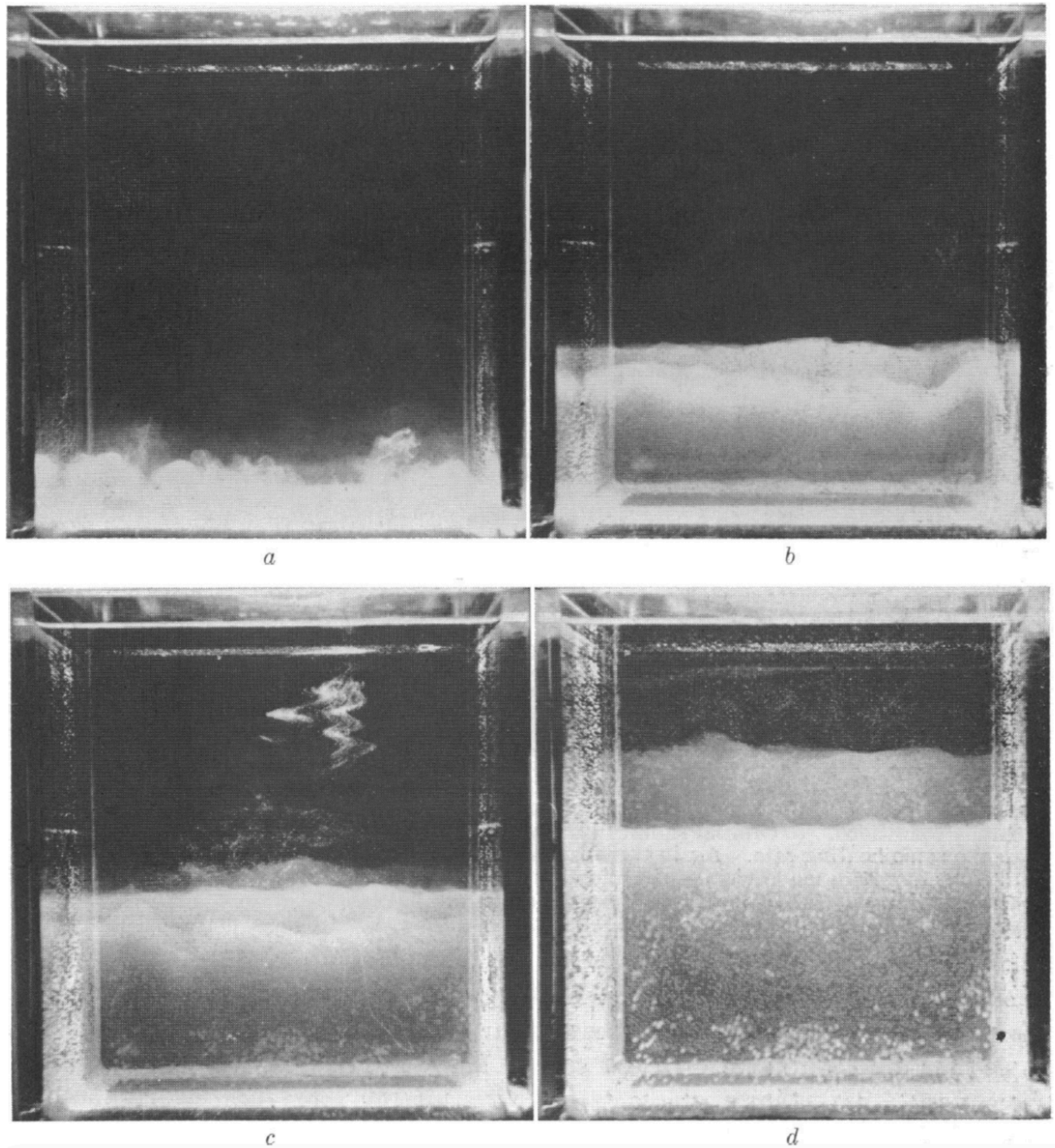
Non-local transport determines boundary width



Penetrative convection

- **Turner & Stommel (1964)** experiments on salt water heated from below: convection zone grows into the stable salinity gradient

- Observed formation of secondary layers ahead of the primary convection zone

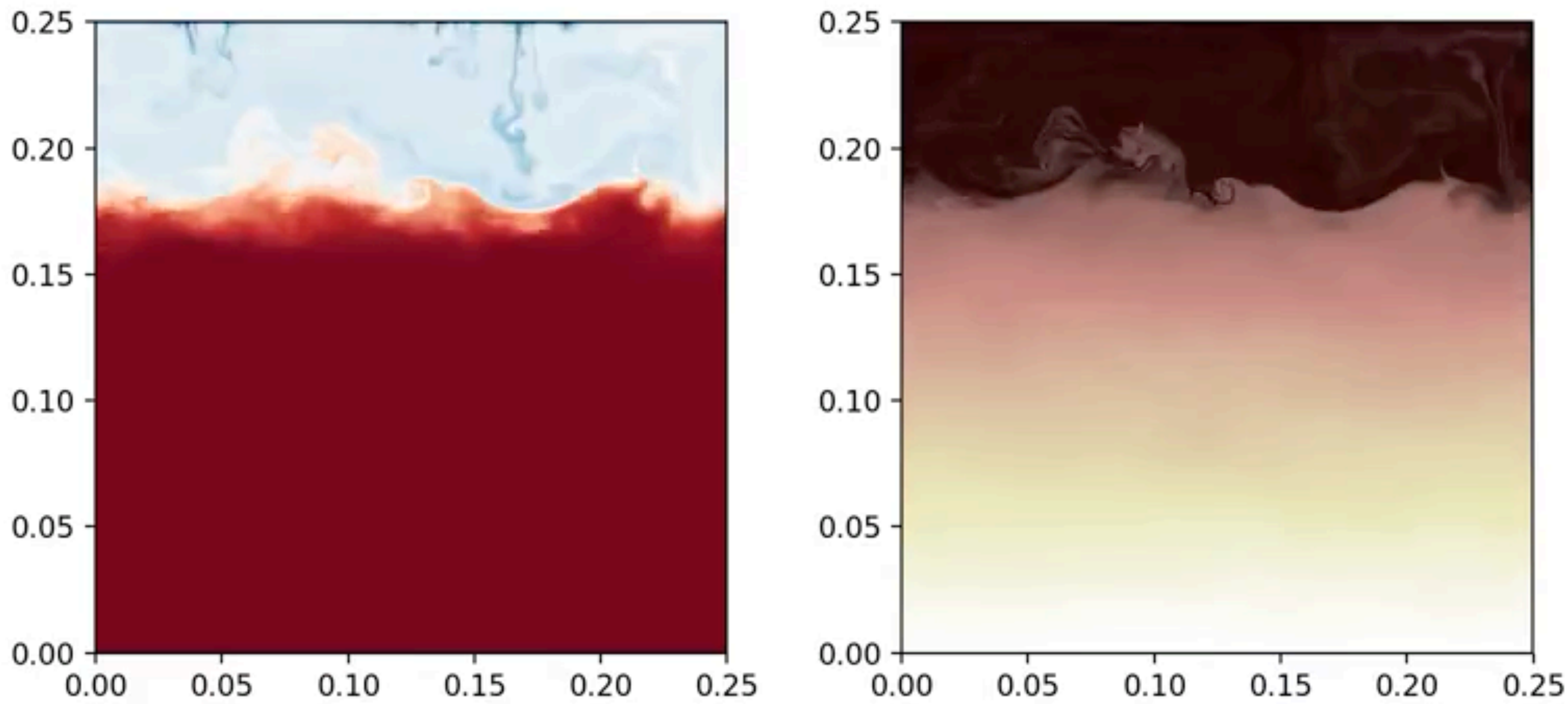


Boussinesq simulations of penetrative convection at low Pr

Fuentes & Cumming (2020,2021), Fuentes, Cumming & Anders (2023)

- Convection driven by constant cooling flux at the top; convection zone penetrates into stable salt gradient
- 2D simulations with the Dedalus code (https://dedalus-project.readthedocs.io)
- $\mathrm{Pr} = 0.1, 1, 7$; $\tau = \kappa_S / \kappa_T = 0.1$, aspect ratio L/H=2

- Study the penetration of the outer convection zone and how it depends on Pr
- Formation of layers underneath the outer convection zone
- Also studied the onset of shear as the aspect ratio evolves



How quickly does the outer convection zone move inwards?

Turner (1968) argument: $$\rho_0 c_P \Delta T h = F_0 t$$ Energy conservation

$$\Delta S = \frac{1}{2}\left|\frac{dS}{dz}\right| h$$

Salt is mixed in the convective region

When is the temperature difference enough to mix the underlying layer upwards?

$$R_\rho = \frac{\beta \Delta S}{\alpha \Delta T}$$

Two limits: $R_\rho = 1$ The temperature difference eventually overcomes the composition difference (Ledoux)

$R_\rho = 3$ Potential energy released by cooling goes into lifting heavy elements — i.e. entrainment; Fernando (1987)

=>

$$h(t) = (2R_\rho)^{1/2}\left(\frac{\alpha F_0}{\rho_0 c_P}\right)^{1/2}\left(\beta\left|\frac{dS}{dz}\right|\right)^{-1/2} t^{1/2}$$

How quickly does the outer convection zone move inwards?

- Extend analytic models of Turner/Fernando:

$$h\frac{d\Delta\overline{T}}{dt} = -\Delta\overline{T}\frac{dh}{dt} + \frac{F_0}{\rho_0 c_P}(1-\varepsilon)$$

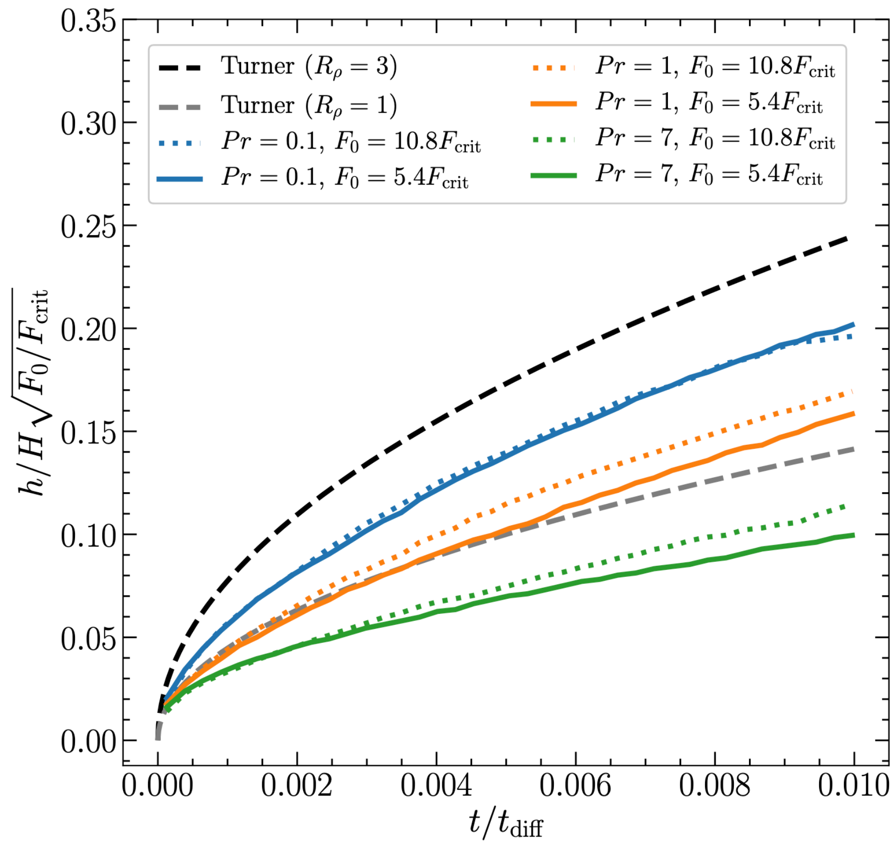
heat flux across the interface

$$-\Delta\overline{B}\,\frac{dh}{dt} = \gamma\left(\frac{g\alpha F_0}{\rho_0 c_P}\right) = \gamma(\rho v_{\rm conv}^3)$$

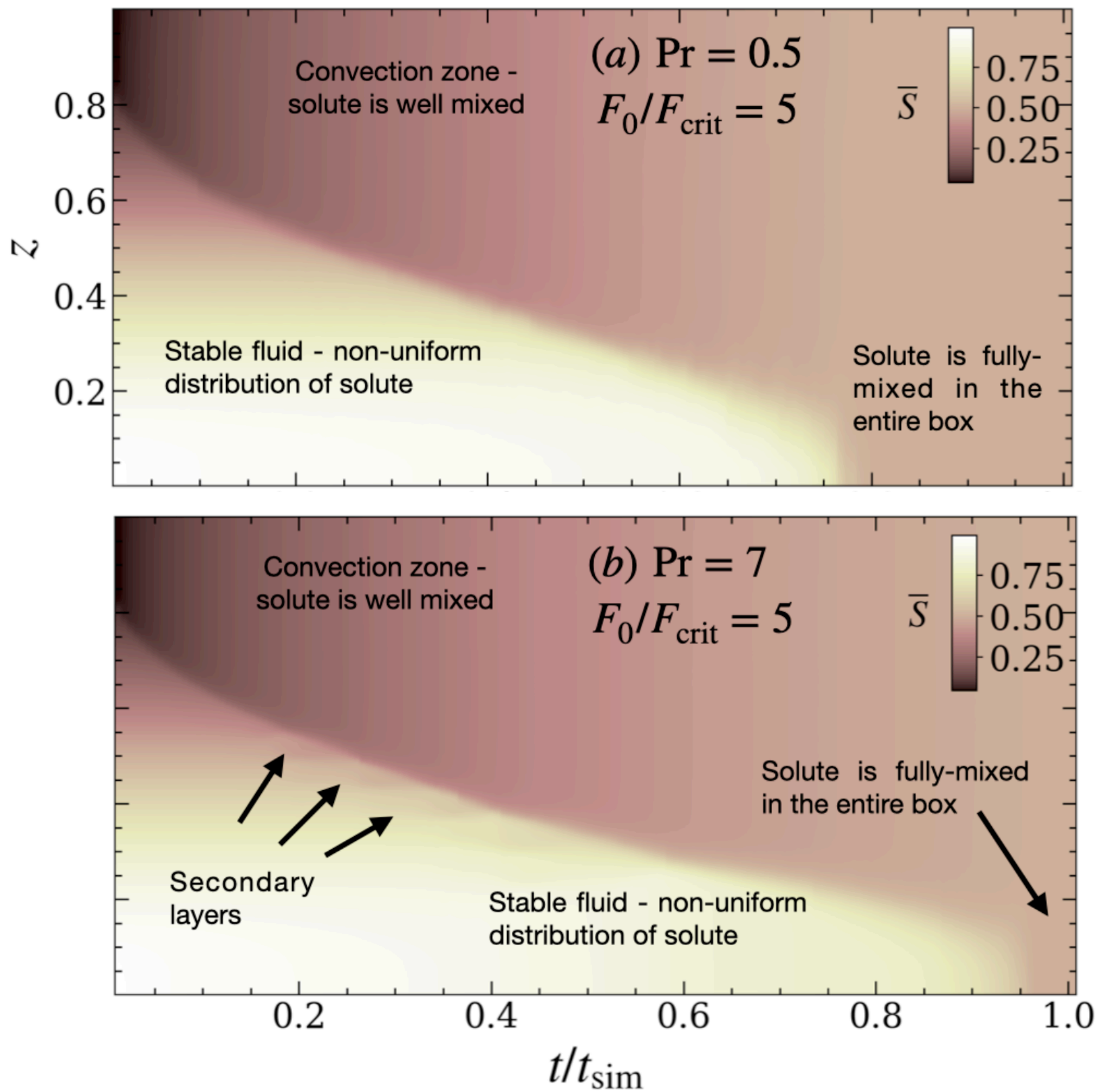
entrainment parameter: how much of the kinetic energy flux is used to lift material across the boundary

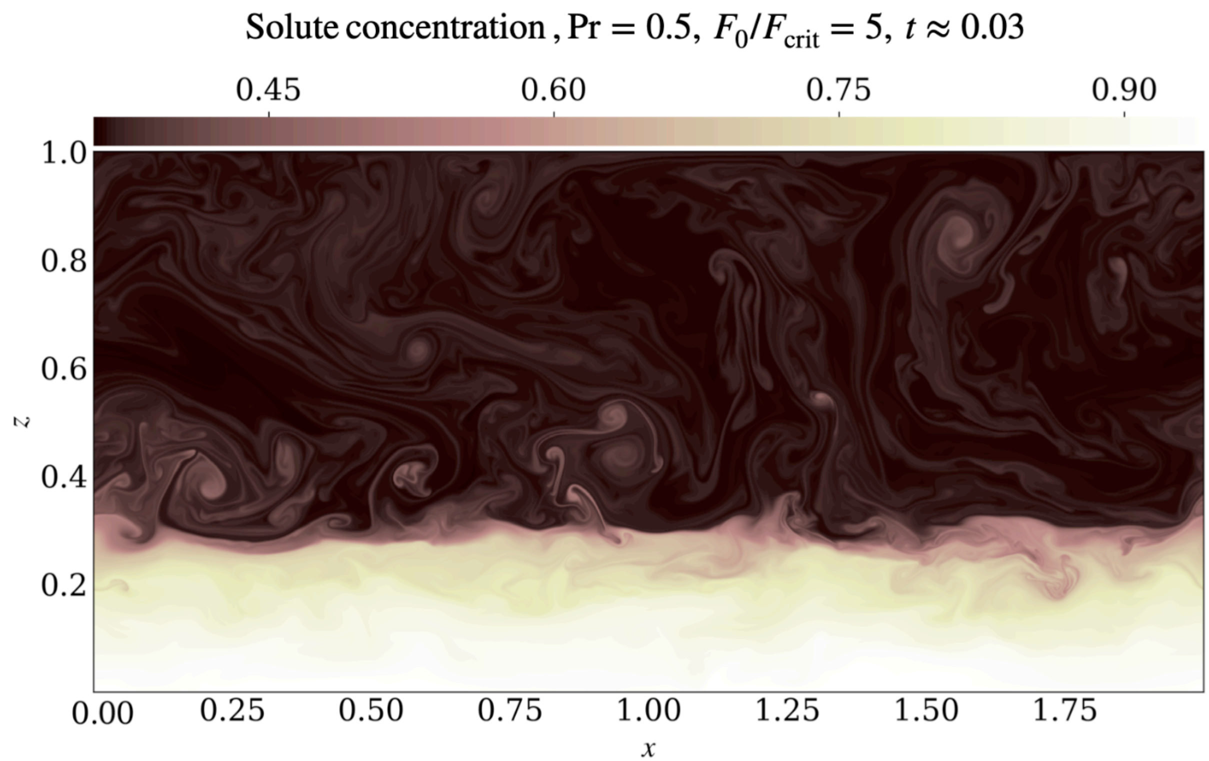
- Has solution with $h \propto t^{1/2}$ and

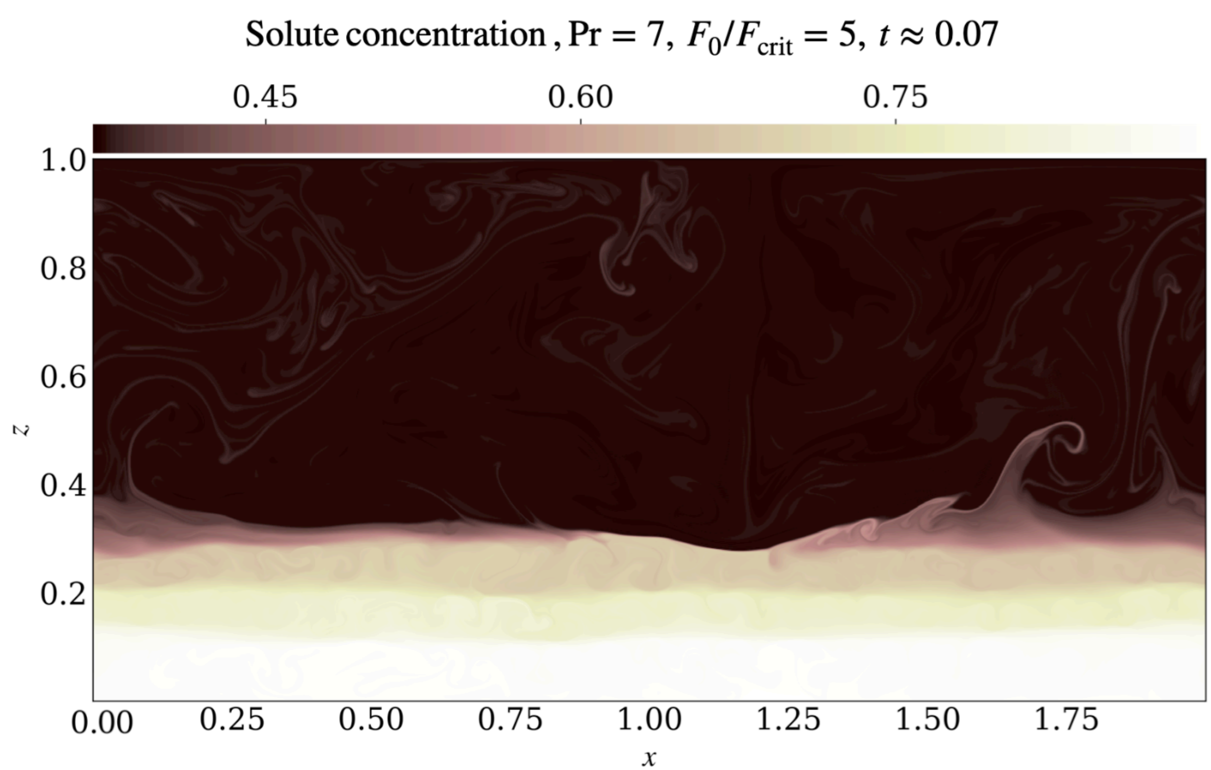
$$R_\rho = \frac{\beta\Delta\overline{S}}{\alpha\Delta\overline{T}} = \frac{1-\varepsilon+2\gamma}{1-\varepsilon}$$

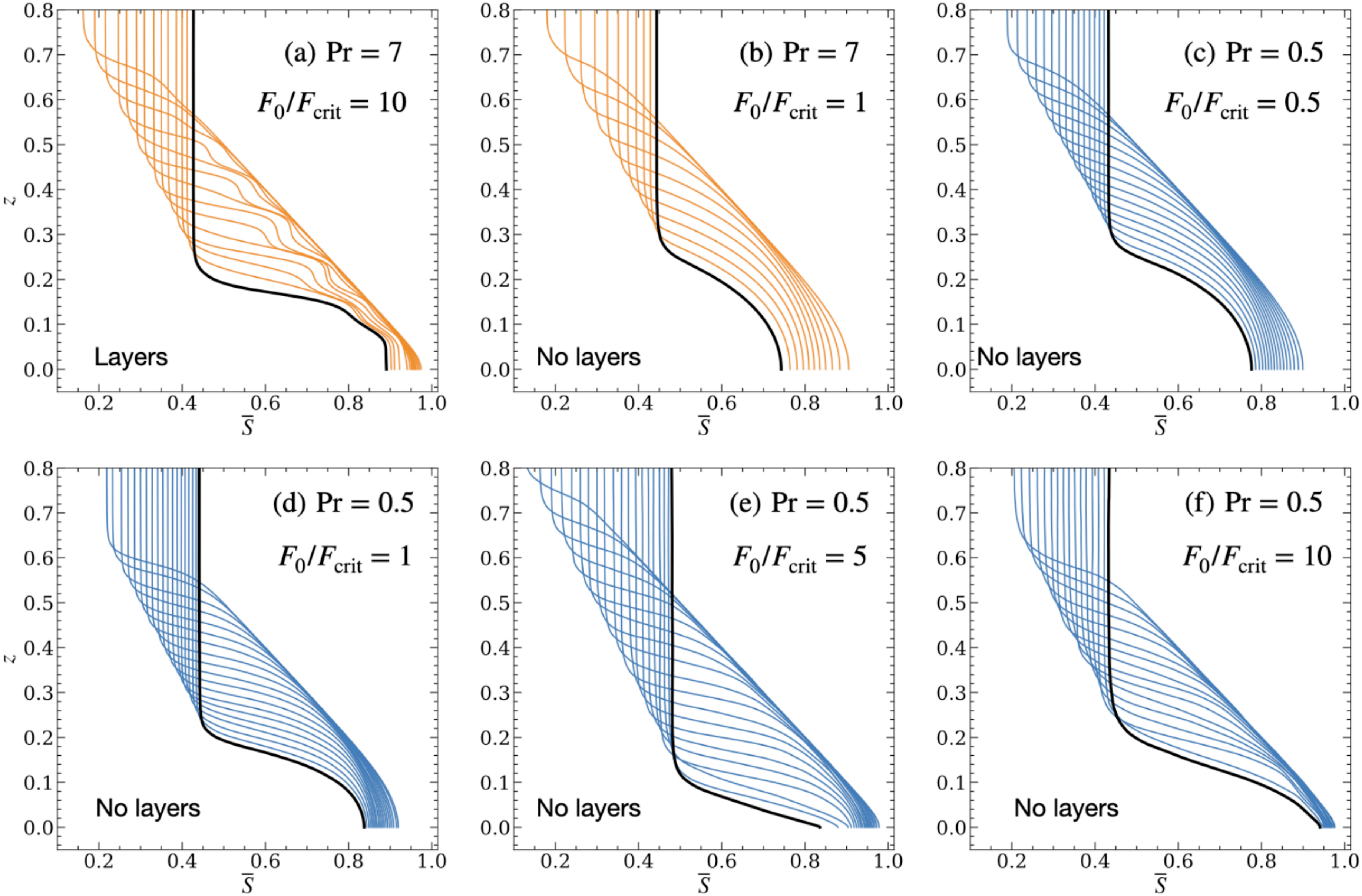


Pr	0.1	7
γ	0.85	0.1
ϵ	0.3	0.45
R_ρ	3.4	1.4


(a) Pr = 0.5
$F_0/F_{\text{crit}} = 5$
Convection zone - solute is well mixed
Stable fluid - non-uniform distribution of solute
Solute is fully-mixed in the entire box
$\bar{S}$
0.75
0.50
0.25
(b) Pr = 7
$F_0/F_{\text{crit}} = 5$
Convection zone - solute is well mixed
Secondary layers
Stable fluid - non-uniform distribution of solute
Solute is fully-mixed in the entire box
$\bar{S}$
0.75
0.50
0.25
z
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
t/t_{sim}


Solute concentration , Pr = 0.5, $F_0/F_{\rm crit} = 5$, $t \approx 0.03$
0.45
0.60
0.75
0.90
1.0
0.8
0.6
0.4
0.2
z
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
x


Solute concentration , Pr = 7, $F_0/F_{\rm crit} = 5$, $t \approx 0.07$
0.45
0.60
0.75
1.0
0.8
0.6
0.4
0.2
z
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
x


(a) Pr = 7
$F_0/F_{\rm crit} = 10$
Layers
(b) Pr = 7
$F_0/F_{\rm crit} = 1$
No layers
(c) Pr = 0.5
$F_0/F_{\rm crit} = 0.5$
No layers
(d) Pr = 0.5
$F_0/F_{\rm crit} = 1$
No layers
(e) Pr = 0.5
$F_0/F_{\rm crit} = 5$
No layers
(f) Pr = 0.5
$F_0/F_{\rm crit} = 10$
No layers
z
$\overline{S}$

Layer formation

For Pr = 7, layers form by Ledoux instability of the thermal diffusive boundary layer ahead of the convection zone

Temperature ahead of the front obeys

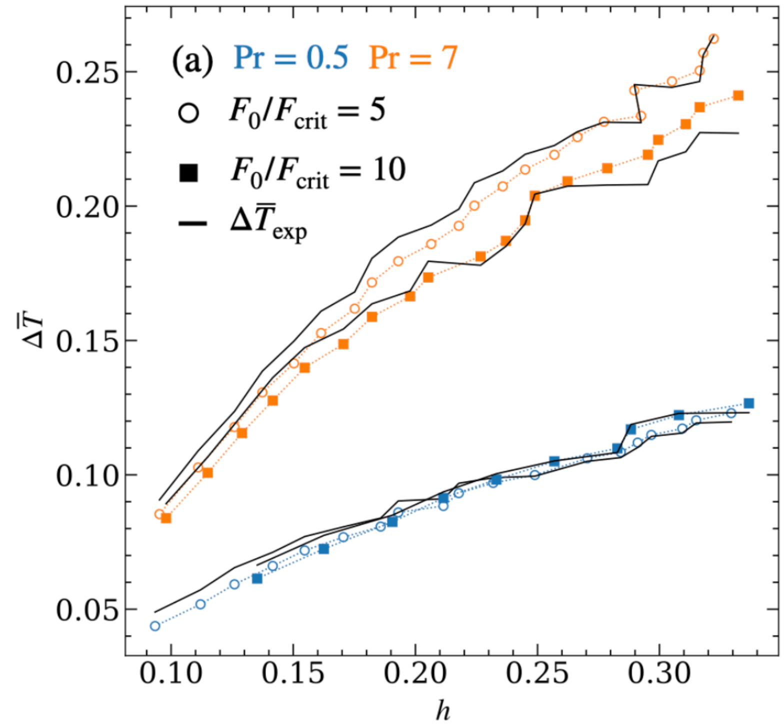
$$T(z^*)_{\rm exp} = \Delta\overline{T}\exp\left(-z^*/\dot{h}\right)$$

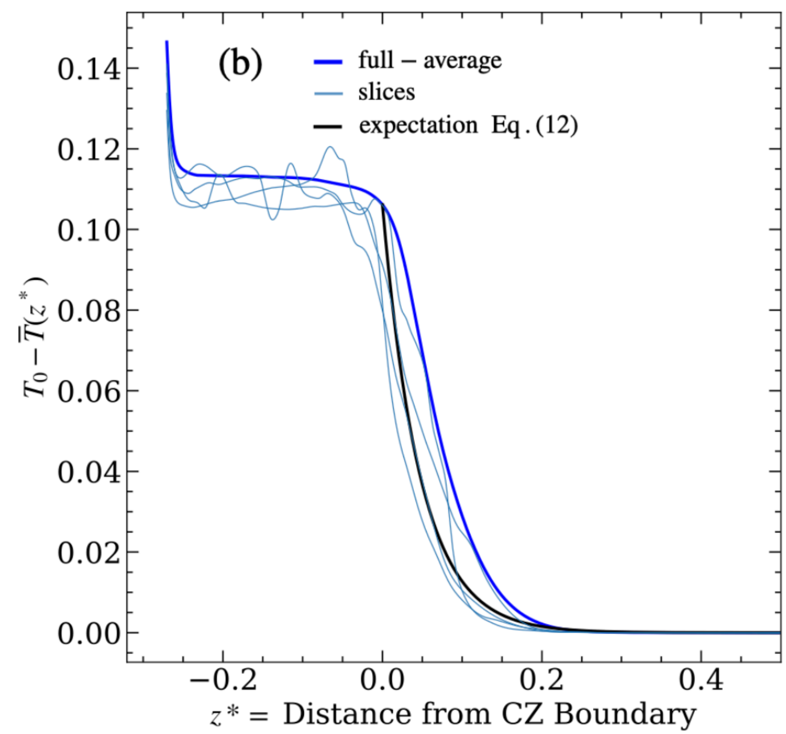
(Turner 1968)

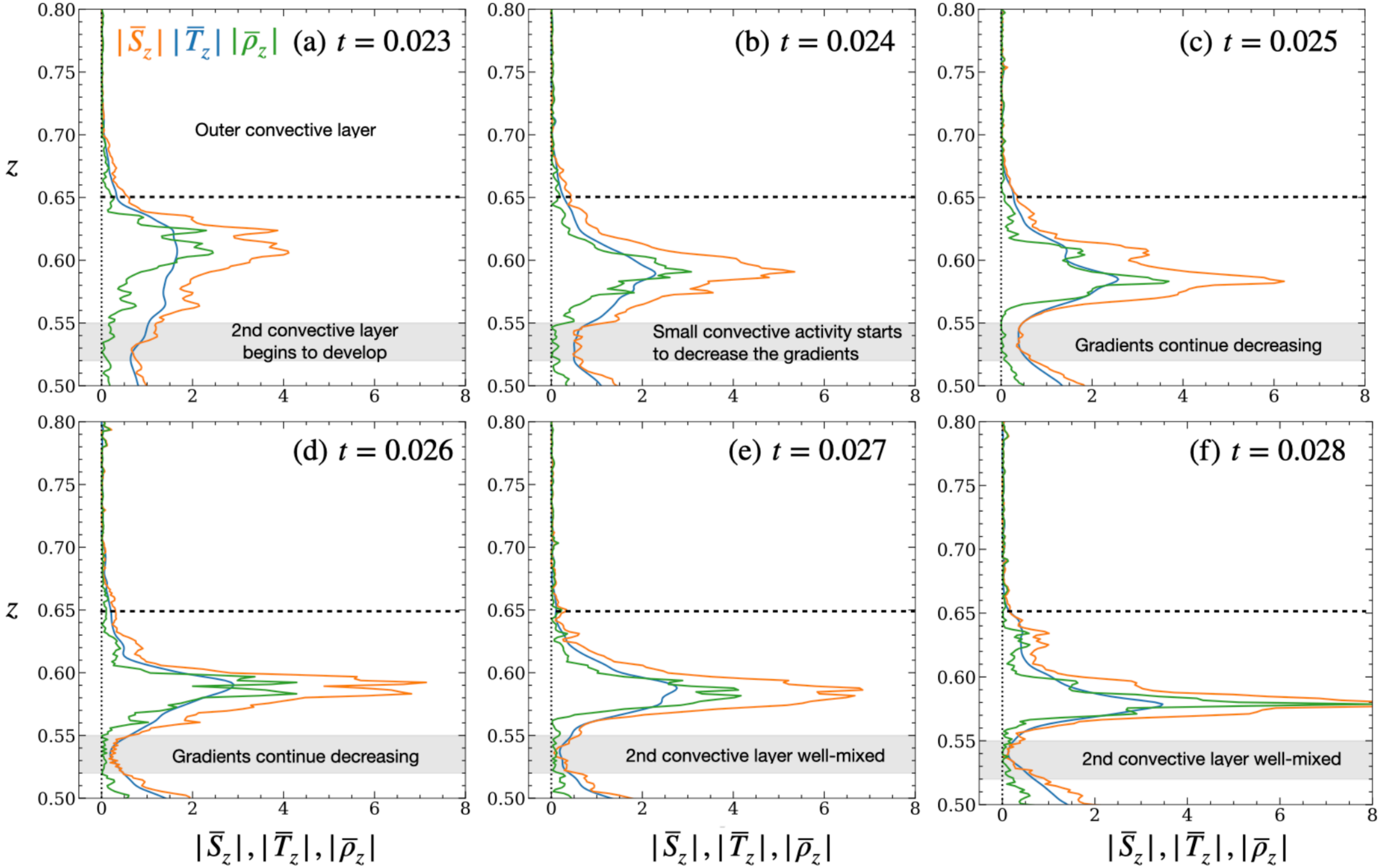
Two effects determine the temperature gradient:

(1) faster penetration => thinner boundary layer
(2) faster penetration => less cooling so smaller ΔT

At low Pr, (2) wins and layers never form







$|\bar{S}_z|$ $|\bar{T}_z|$ $|\bar{\rho}_z|$
(a) $t = 0.023$
Outer convective layer
2nd convective layer begins to develop
(b) $t = 0.024$
Small convective activity starts to decrease the gradients
(c) $t = 0.025$
Gradients continue decreasing
(d) $t = 0.026$
Gradients continue decreasing
(e) $t = 0.027$
2nd convective layer well-mixed
(f) $t = 0.028$
2nd convective layer well-mixed
z
$|\bar{S}_z|, |\bar{T}_z|, |\bar{\rho}_z|$

Effect of rotation

- Arguments based on CIA balance:

$$\frac{2\Omega U_R}{\ell} \sim \frac{U_R^2}{\ell_\perp^2} \sim \frac{\alpha g \delta T_R}{\ell_\perp} \quad \text{and} \quad F_H \sim \rho c_P \delta T_R U_R$$

=> $$U_R \sim \left(\frac{g\alpha F_H}{\rho c_P}\right)^{2/5}\left(\frac{\ell}{2\Omega}\right)^{1/5} \sim U_{NR}\left(\frac{U_{NR}}{2\Omega\ell}\right)^{1/5}$$

where $$U_{NR} \sim \left(\frac{\alpha g F_H \ell}{\rho c_P}\right)^{1/3}$$

convective turnover time ~ 1 year

rotation period ~ 10 hours

=> velocity changes by ~ 6

- Kinetic energy flux:

$$\frac{F_{K,R}}{F_H} \sim \left(\frac{\alpha g \ell}{c_P}\right)\left(\frac{U_R}{2\Omega\ell}\right)^{1/2} \sim \mathrm{Ro}^{1/2}\frac{F_{K,NR}}{F_H}$$

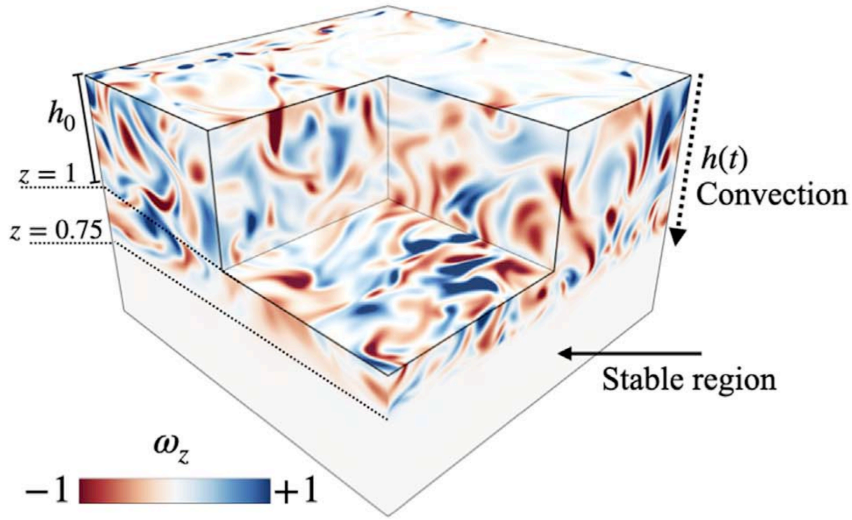
available KE flux for entrainment changes by 6^3 ~ 200

where for Jupiter Ro~1e-5

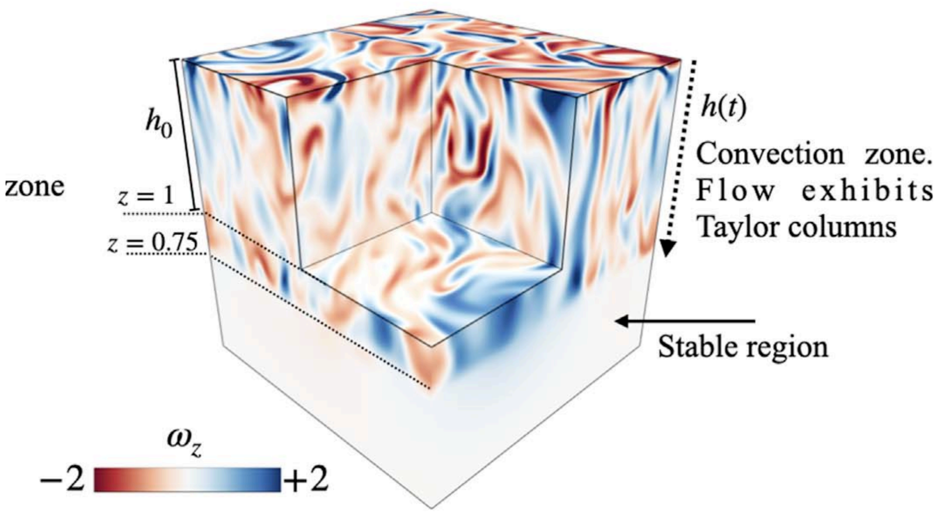
Fuentes et al. 2023 ApJL

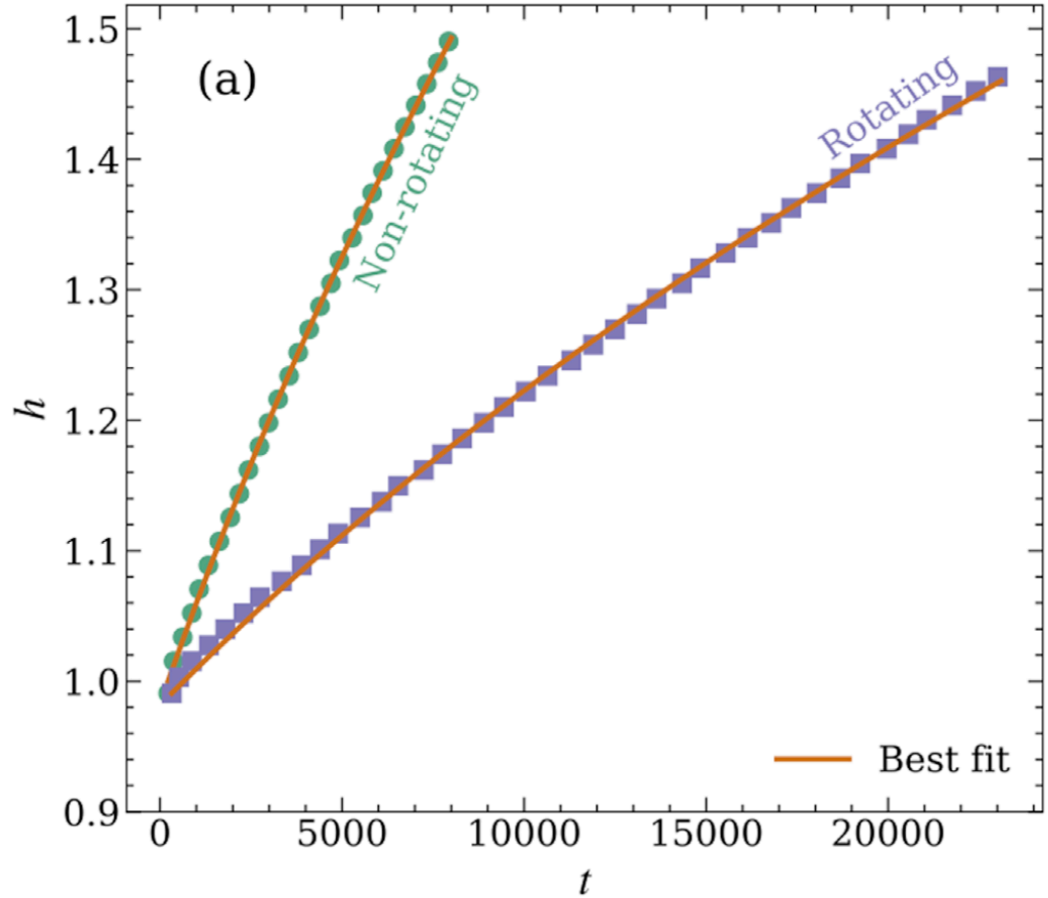
- New set of simulations with rotation (aligned with gravity)
- Ro ~ 0.07

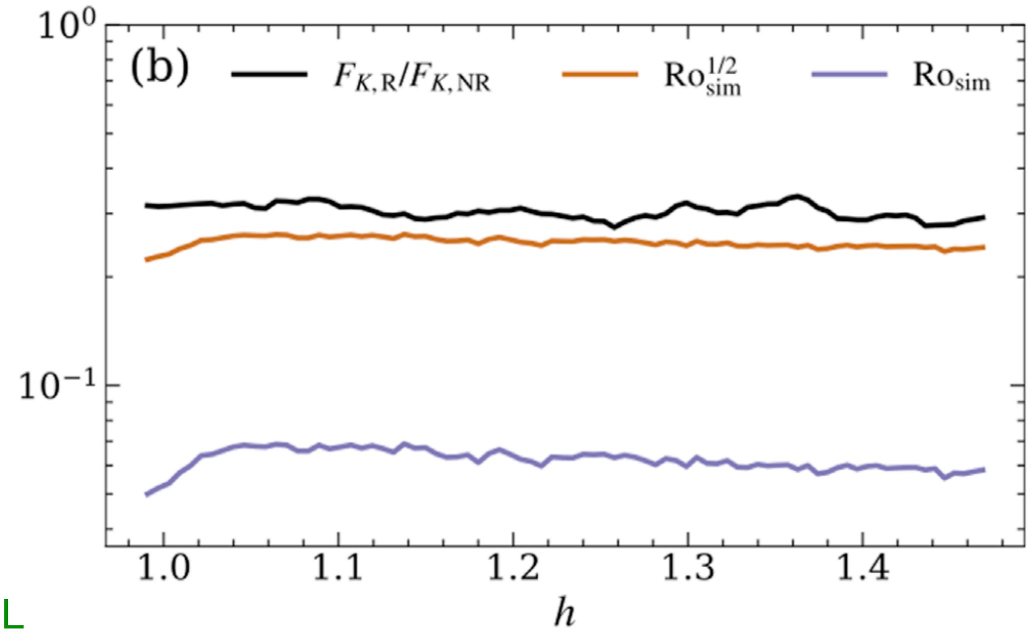
Non-rotating case



Rotating case







Fuentes et al. 2023 ApJL

Summary so far

- Penetrative convection happens more quickly at low Pr, entrainment efficiency ~10 times larger at Pr=0.1 than at Pr=7
- Layers form at Pr=7 due to Ledoux instability of the thermal boundary layer, does not happen at Pr=0.1
- Similar mechanism is seen in 1D evolution models, but need prescription for transport at the boundaries
- Reduction of convective velocity by rotation reduces penetration rate

Still many open questions:

- Compressibility

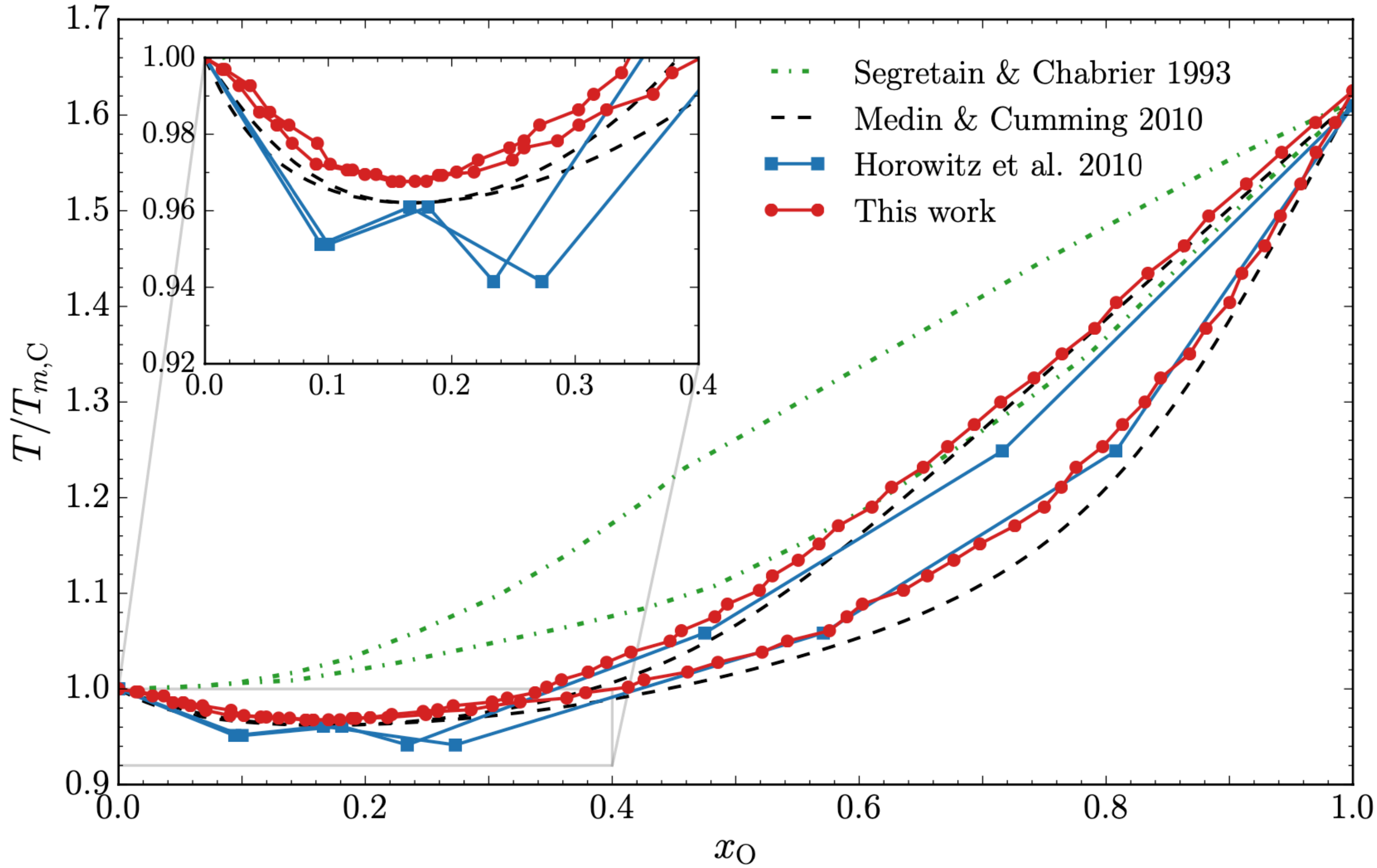
  "Dissipation number" $$\frac{F_{\rm KE}}{F} \sim \frac{\rho_0 v_{\rm conv}^3}{\rho_0 v_{\rm conv} c_P \delta T} \sim \frac{v_{\rm conv}^2}{c_P \delta T} \sim \frac{\alpha g H}{c_P} \sim 10^{-7}$$

- How to implement this in 1D models: similar entrainment law has been used in stellar codes, e.g. Scott et al. (2021) for core convection in massive stars $\dot{h} = v_{\rm conv} {\rm Ri}^{-1}$

  (a rewrite of our entrainment law)

- What is the scale of layers? Relation to layering from double-diffusive instabilities: Chabrier & Baraffe (2007), Leconte & Chabrier (2012,2013). Layer thickness $\delta_T \approx (\kappa_T l/v)^{1/2}$ => large number of layers

When CO white dwarfs freeze, the liquid is enriched in carbon, the solid in oxygen — buoyancy source for convection in interior



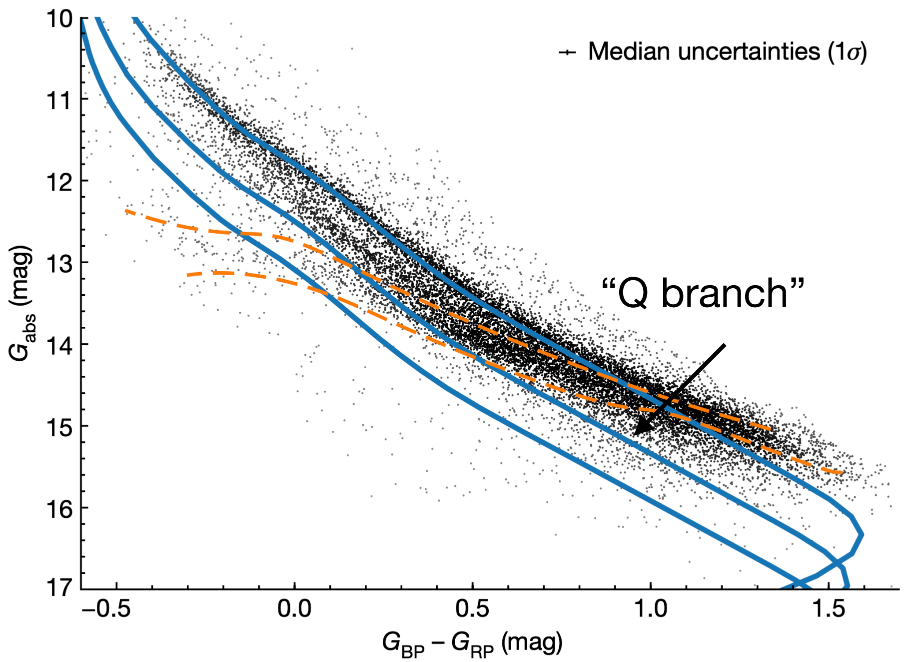
Blouin et al. (2020)

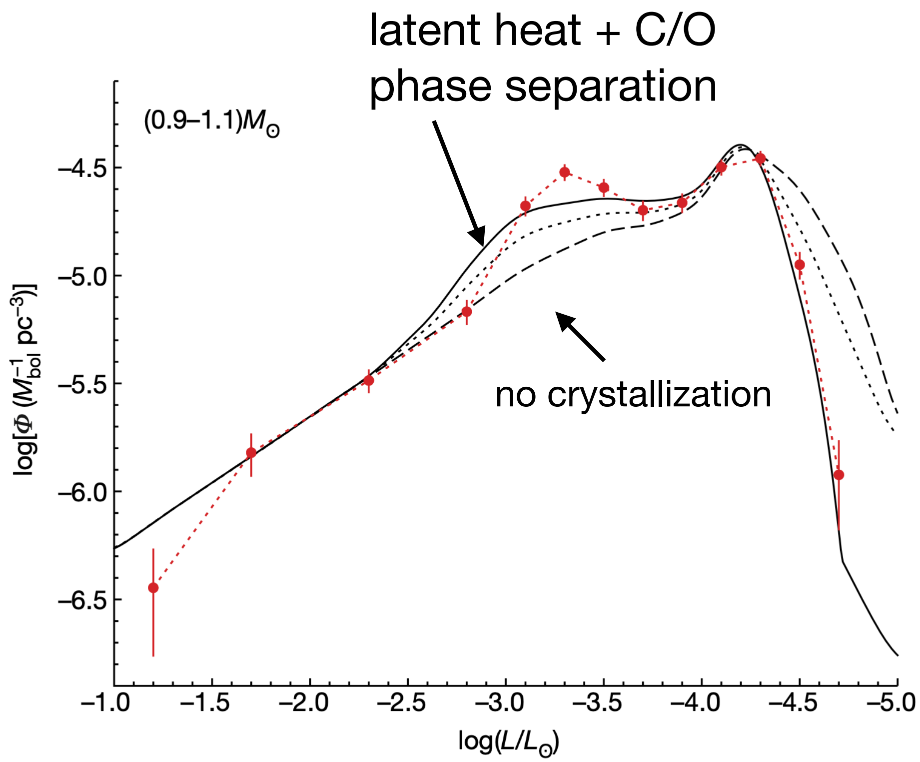
Good agreement between different methods

- semi-analytic using fit to one-component Monte Carlo
- molecular dynamics
- Gibbs-Duhem approach + Monte Carlo

GAIA found definitive evidence for crystallization in white dwarfs

- Cooling delay from latent heat and chemical separation of C and O on freezing





100pc sample of ~15,000 field WDs from GAIA Tremblay et al. (2019)

- A particular puzzle is the massive end of the Q branch where ~6% of the WDs need about 8 Gyr of additional cooling delay Cheng et al. (2019)

One possibility is "distillation" of light crystals that rise into the fluid layer (but requires enhanced metallicity, mergers?) Blouin et al. (2021), Shen, Blouin, Breivik (2023)

A crystallization-driven dynamo in white dwarfs?

- Most strongly magnetized WDs are crystallizing (B>10MG)

- Saturated dynamo

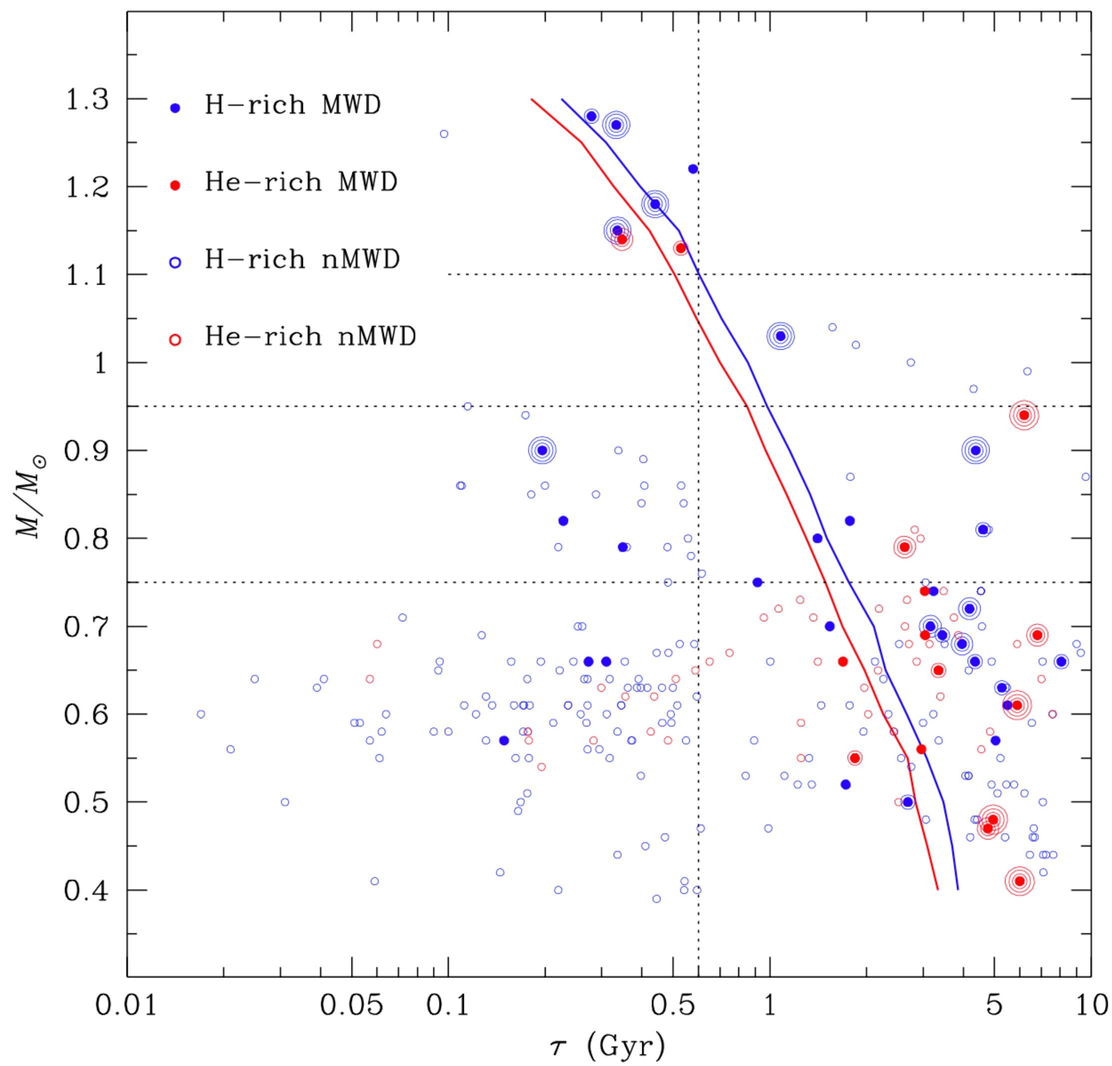
$$\frac{B^2}{4\pi} \sim \rho v_c^2 \sim \rho \left(\frac{F}{\rho}\right)^{2/3}$$

gives magnetic fields of a few MG, in the right range

Isern et al. 2017; Ginzburg et al. 2022

- Could also explain some incidence of magnetic white dwarfs in different kinds of binaries

Belloni, Schreiber et al. (2021,2022)



Bagnulo & Landstreet (2022)

(See also Amorim et al. 2023, Caron et al. 2023)

Crystallization-driven convection in WDs involves

- up-gradient heat transport

In MLT, the heat flux is $F_H \approx \rho v_c c_P T(\nabla - \nabla_{\rm ad})$

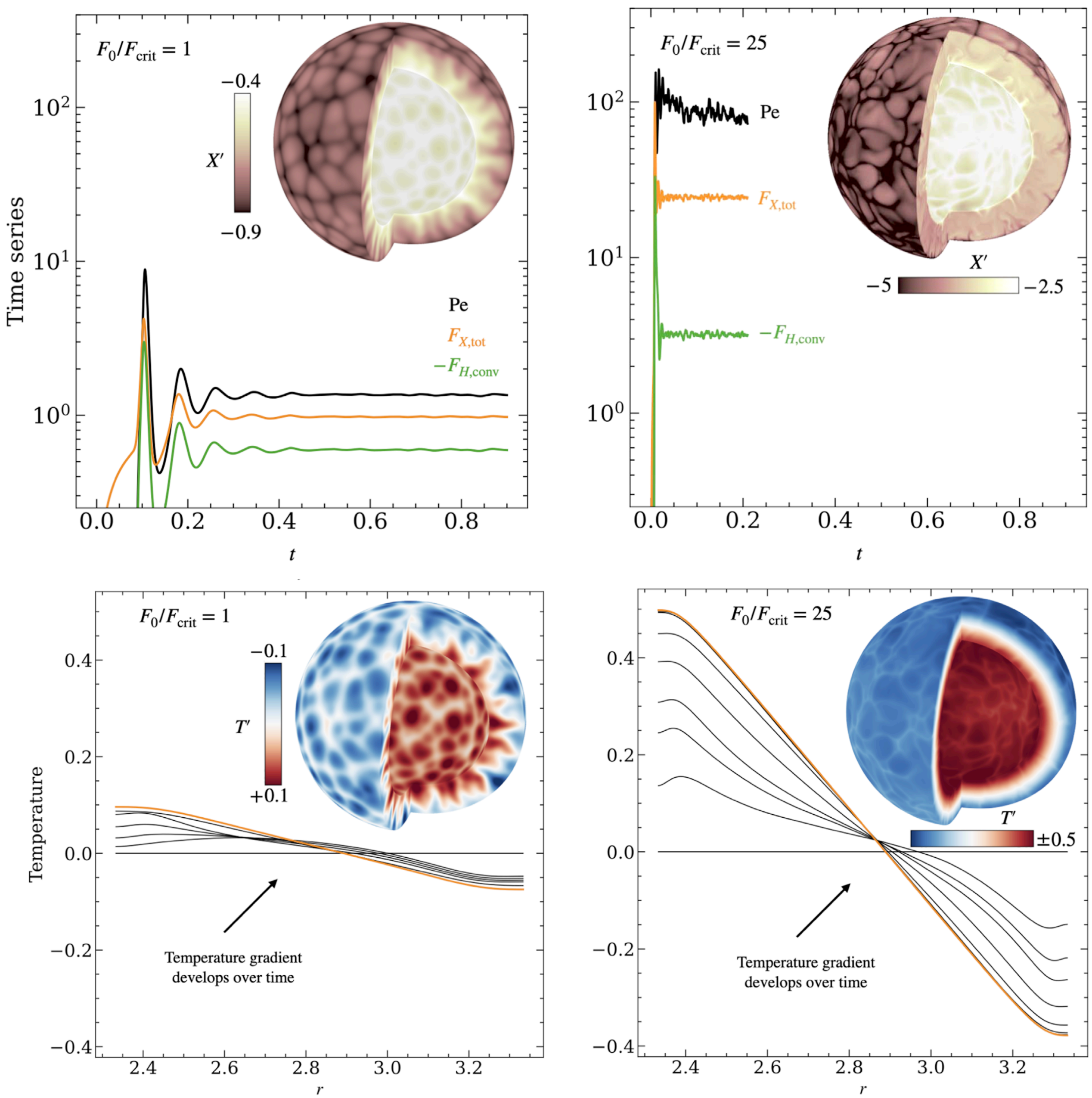
In a thermally-stable background $\nabla < \nabla_{\rm ad} \Rightarrow F < 0$

[same thing happens in overshoot layer in standard convection]

- high thermal conductivity => Pe small typically => fingering convection

$$\mathrm{Pe} = \frac{v_c \ell}{\kappa_T} \sim \frac{\text{thermal diffusion time}}{\text{convective turnover time}}$$

Boussinesq simulations of compositionally-driven convection



- compositionally-driven convection in a 3D shell with Dedalus, Boussinesq, inner radius = 0.7 x outer radius

$$\mathrm{Ra_T} = \frac{g\alpha\Delta r^4|\partial_r T_{\rm ad}|}{\kappa_T^2} = 1.5\times10^5$$

$$\mathrm{Sc} = \frac{\nu}{\kappa_X} = 1.6 \qquad \mathrm{Le} = \frac{\kappa_T}{\kappa_X} = 3.3$$

$$\mathrm{Pr} = 0.5$$

- composition flux applied into the lower boundary and out of the top boundary, then evolve to steady state

- initially isothermal, with insulating boundaries => temperature gradient develops to balance the inwards heat flux

Fuentes et al. (2023)

General behaviour matches mixing length predictions

Can write down a mixing length theory that smoothly transitions from overturning to fingering convection

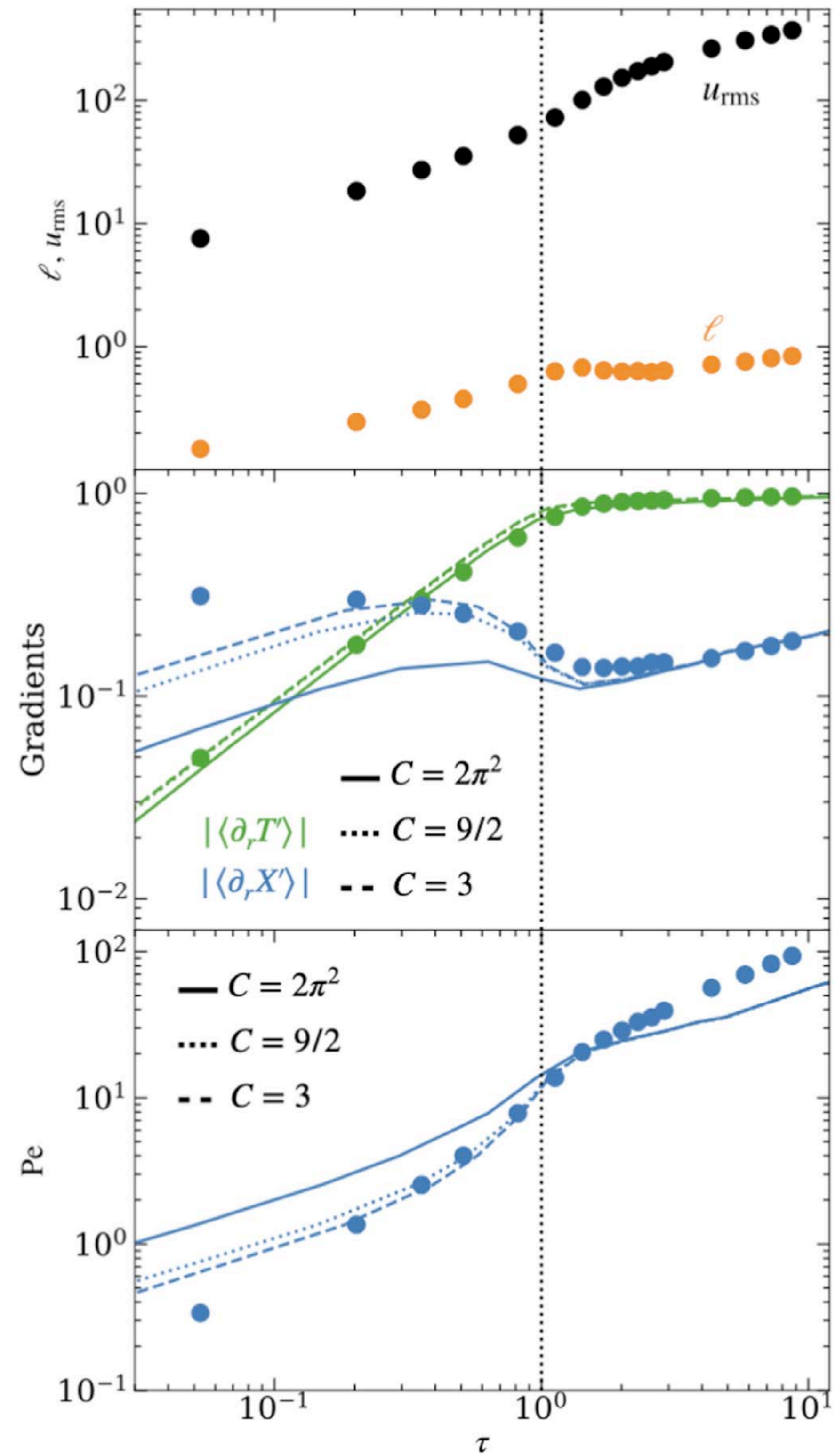
Fuentes et al. (2023)
(see also Mochovitch 1983)

Usual MLT in stellar codes cannot handle this kind of convection. e.g. heat flux is hard-wired to be outwards!

WD cooling codes do instantaneous mixing rather than following convection

Usual 3rd order equation in stellar MLT becomes 5th order

Castro-Tapia et al. (2023)



Effect of rotation: **increased** velocities ?

CIA balance

$$\frac{v_c^2}{L^2} \sim \frac{2\Omega v_c}{H_P} \sim \frac{g}{H_P}\frac{\chi_X}{\chi_\rho}(\nabla_X - \nabla_{X,\mathrm{crit}})$$

Convection is anisotropic with perpendicular scale

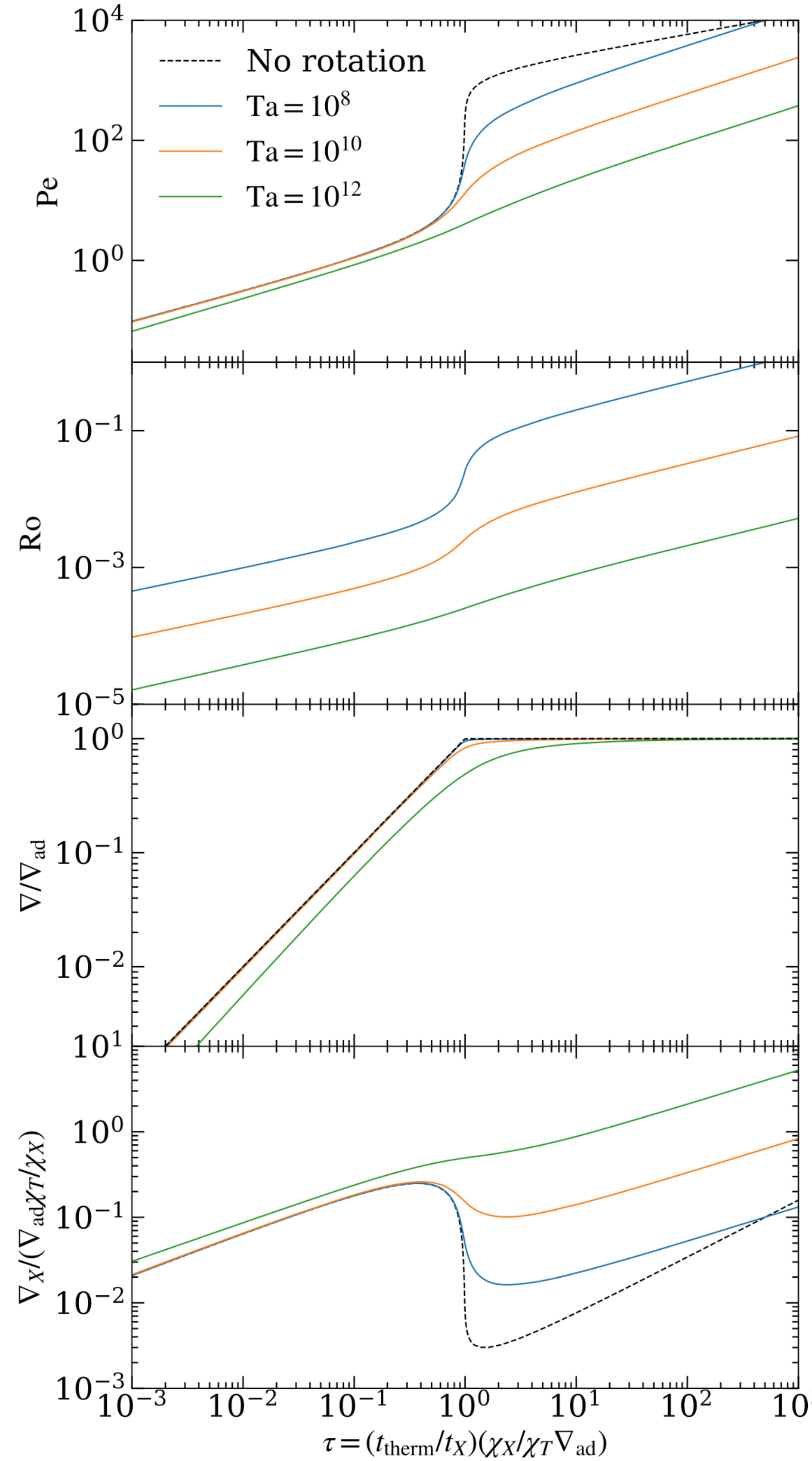
$$\frac{L}{H_P} \approx \left(\frac{v_c}{2\Omega H_P}\right)^{1/2} \approx \mathrm{Ro}^{1/2}$$

For a fixed composition flux:

$$F_X = \rho v_{\mathrm{conv}} L \frac{dX}{dr}$$

If L goes down, v must go up!

critical gradient, just enough to get convection going



Convective velocity estimates — dynamo or no dynamo?

Isern et al. (2017) and Ginzburg et al. (2022) write down

$$\frac{B^2}{4\pi} \sim \rho v_c^2 \sim \rho \left(\frac{F}{\rho}\right)^{2/3}$$

where the flux is the flux of gravitational energy from chemical separation (~ same as heat flux)

But this is not the correct estimate of the velocity;

$$F_{KE} \ll F_H$$

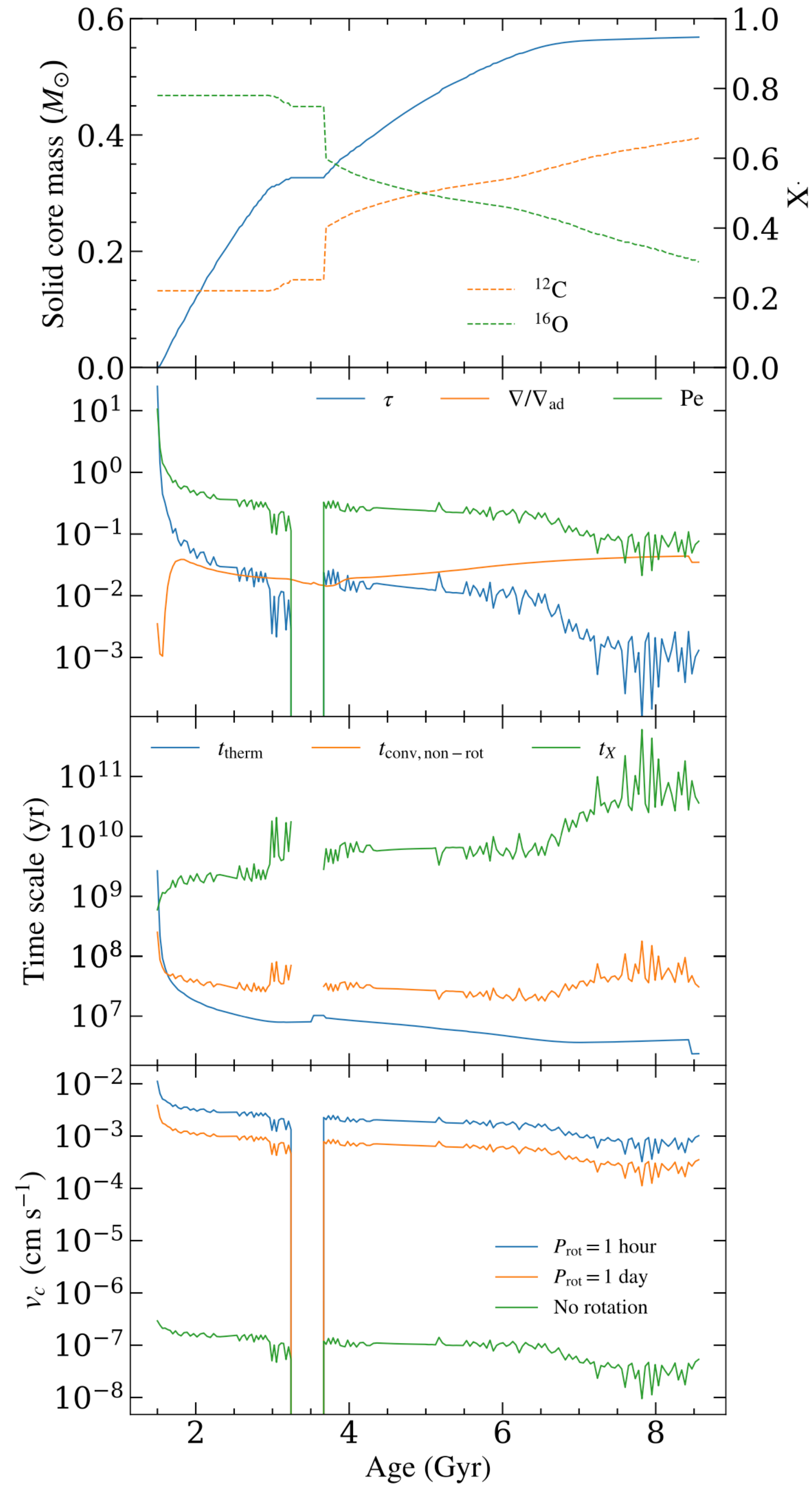
$$B^2/4\pi \sim \rho v^2$$

gives $B \sim 3\,\mathrm{G} \times \rho_6^{1/2} \sim 10^{-3}\,\mathrm{cm\,s^{-1}}$

(Note that RM is still >>1 even with these low velocities)

There is enough energy coming out to explain the B fields we see, but only a tiny fraction is in kinetic energy -> no dynamo ?

What is the correct force balance to write down?



Summary

- Penetrative convection is faster and less likely to form layers at low Pr
- When they do form, layers form dynamically rather than through double-diffusive instabilities

- The penetration rate is sensitive to the convective velocity, eg rotation reduces penetration rate significantly

- Need a better model for boundary layer mixing for planet evolution models

- Compositionally-driven convection in white dwarfs transports heat inwards

- Standard white dwarf cooling codes need modification to be able to follow the convection

- Predict that rotation increases the convective velocity rather than decreases it

- Can fingering convection support a dynamo and if so does the magnetic field track the overall energy flux or the kinetic energy flux?

"Beyond Boussinesq" is needed in both cases!