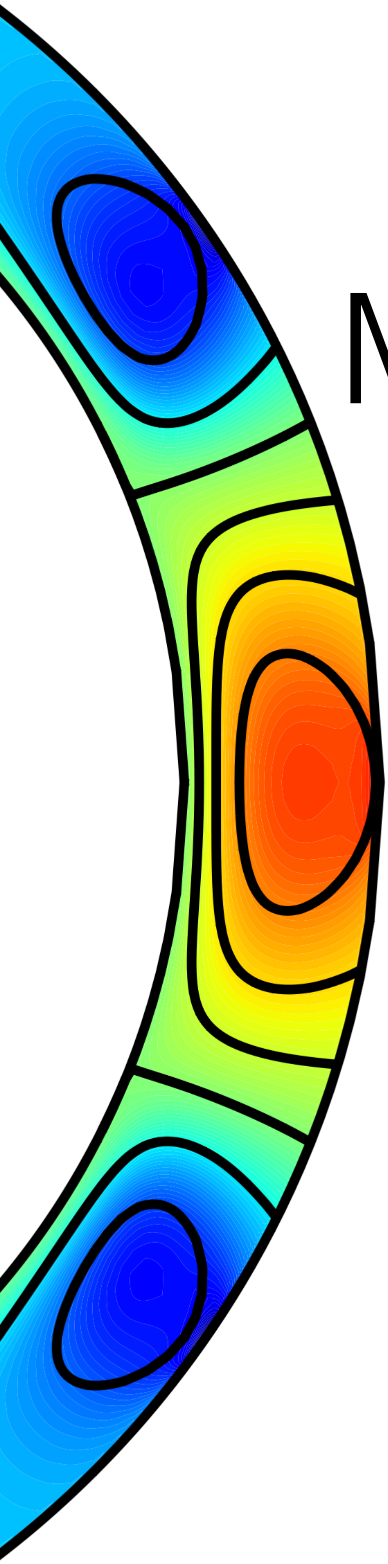


Magnetic Field Evolution in Neutron Star Crusts

Andrew Cumming (McGill University)

Kostas Gourgouliatos (CRAQ Fellow, McGill University)

A. Reisenegger, C. Armaza (PUC, Chile),
J. Valdivia (U. de Chile), M. Lyutikov (Purdue)

G&C 2014c, MNRAS, in press (arXiv:1406.3640)
G&C 2014b, PRL, 112, 171101
G&C 2014a, MNRAS, 438, 1618
Gourgouliatos et al. 2013, MNRAS, 434, 2480

Outline

1. Motivation for studying neutron star magnetic field evolution
2. The role of the Hall effect and a puzzle
3. A Hall attractor for neutron star crusts
4. Connection to observations: braking indices, long period cutoff

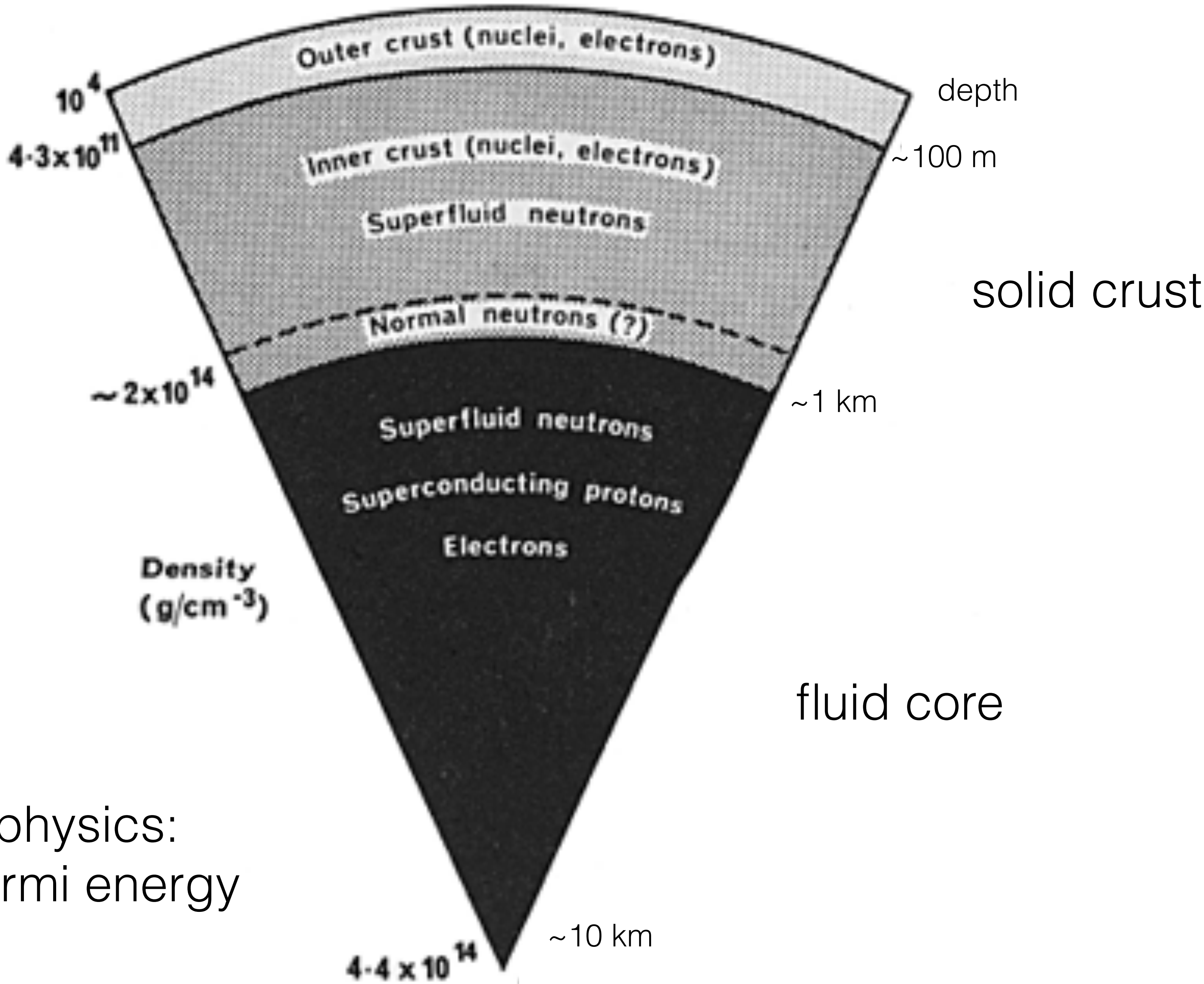
Studies of neutron stars address a range of different questions

They are **extreme** objects
- laboratories for testing the physics of dense matter
- extremely strong magnetic fields
- strong gravity

As **endpoints** of stellar evolution
- a much more diverse population than first thought

Interesting physics!
- nuclear physics, magnetohydrodynamics, plasma physics, condensed matter physics ...

Neutron star structure



Key piece of physics:
increasing Fermi energy
with depth

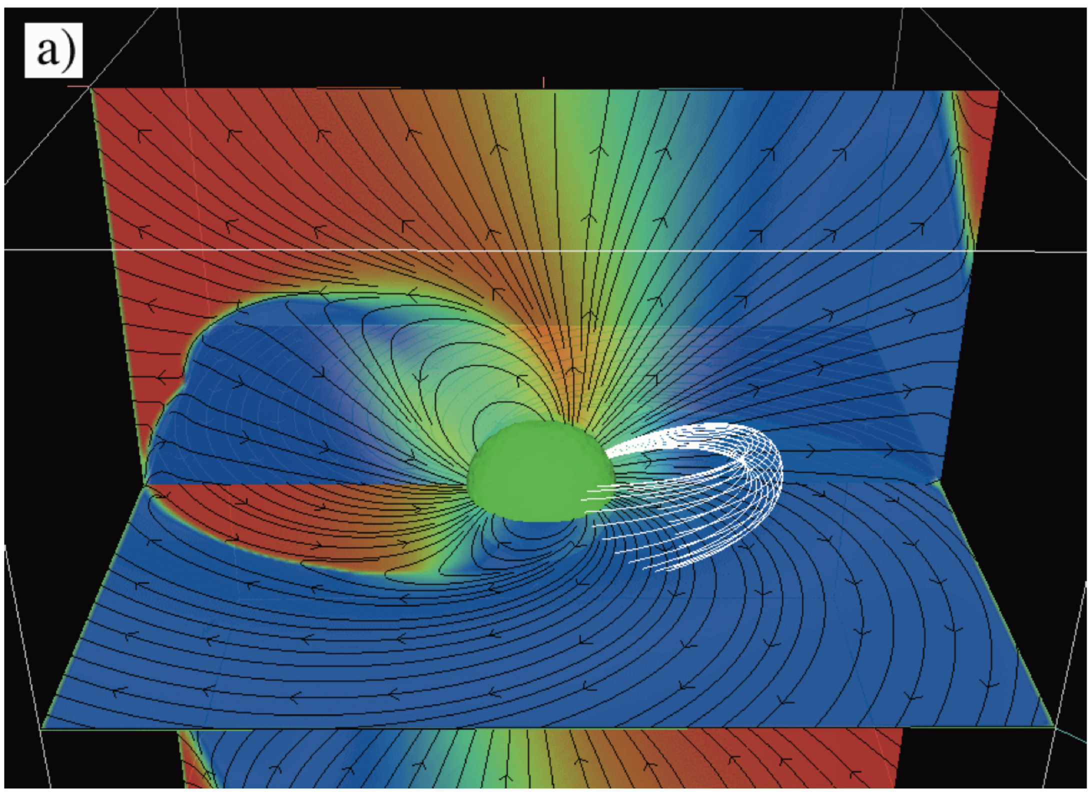
Pines (1980)

Our knowledge of neutron star magnetic fields mostly comes from their spin-down

Magnetic dipole spin down

$$B = 3.2 \times 10^{12}\ \mathrm{G}\ \left(\frac{P\dot{P}}{10^{-14}}\right)^{1/2}$$

MHD simulations of pulsar magnetospheres confirm this estimate to within a factor of 2 (Spitkovsky 2006)



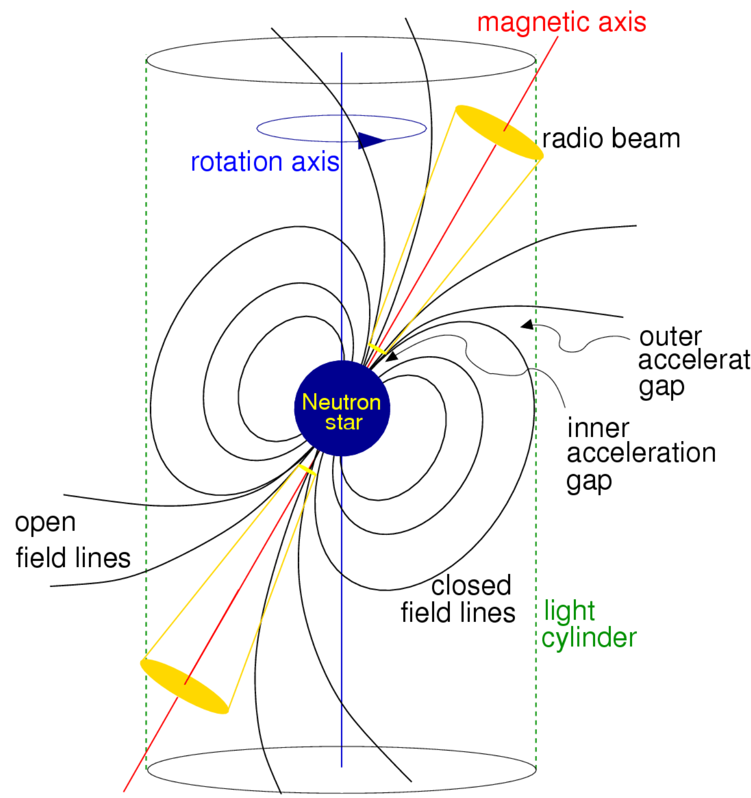
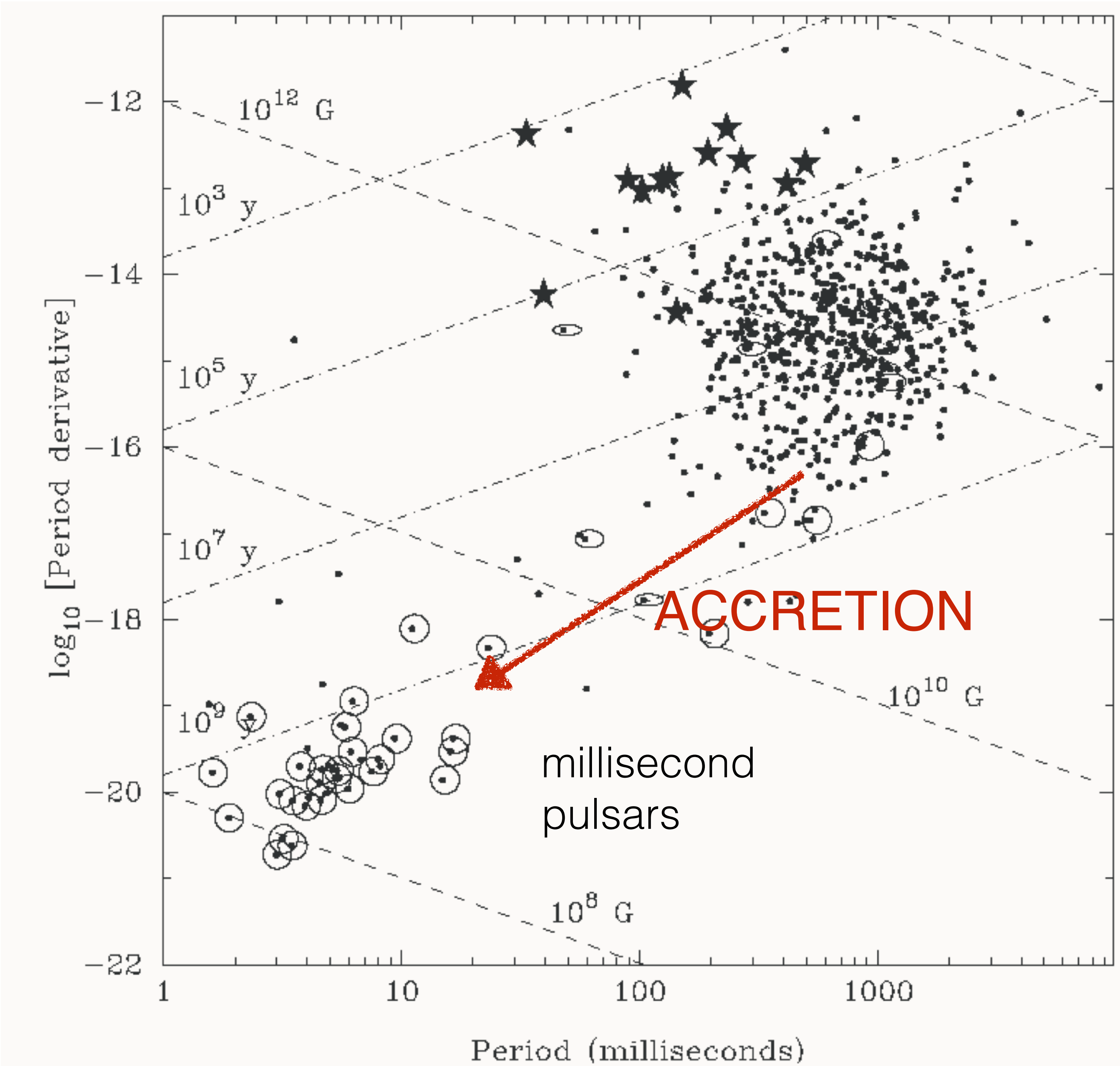


Figure: Lorimer & Kramer Handbook of Pulsar Astronomy



canonical radio pulsars

radio pulsar dipole spin down

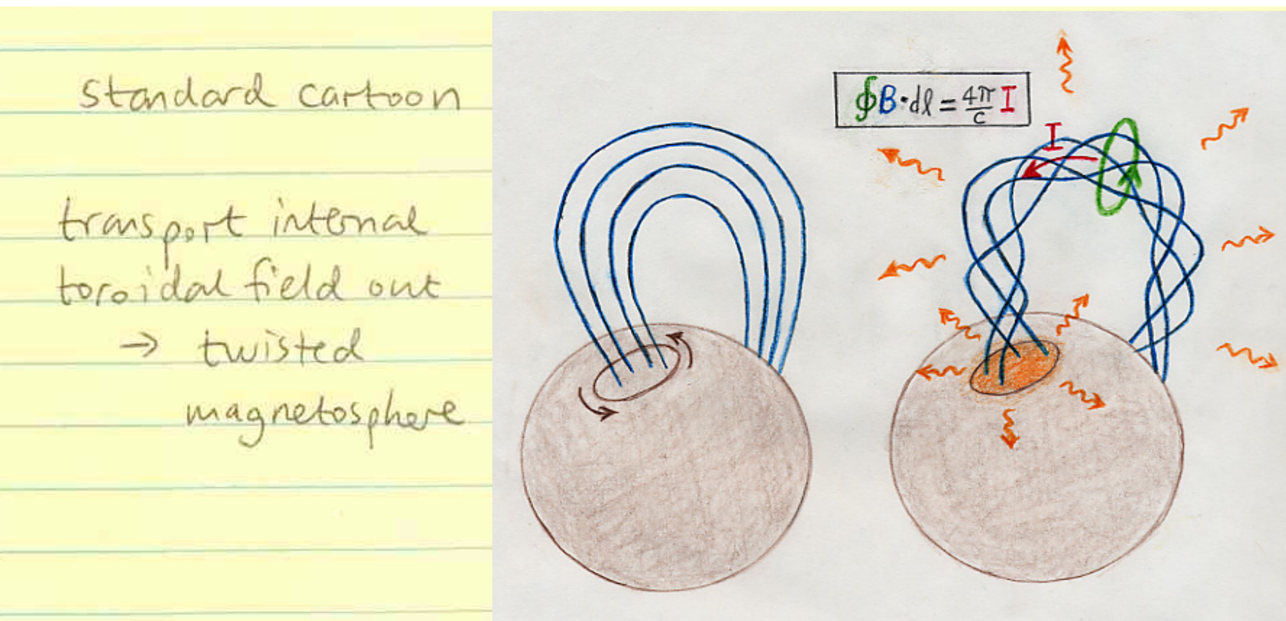
$$\dot{\Omega} \propto -B^2\Omega^3$$

Figure from Freire (2005)

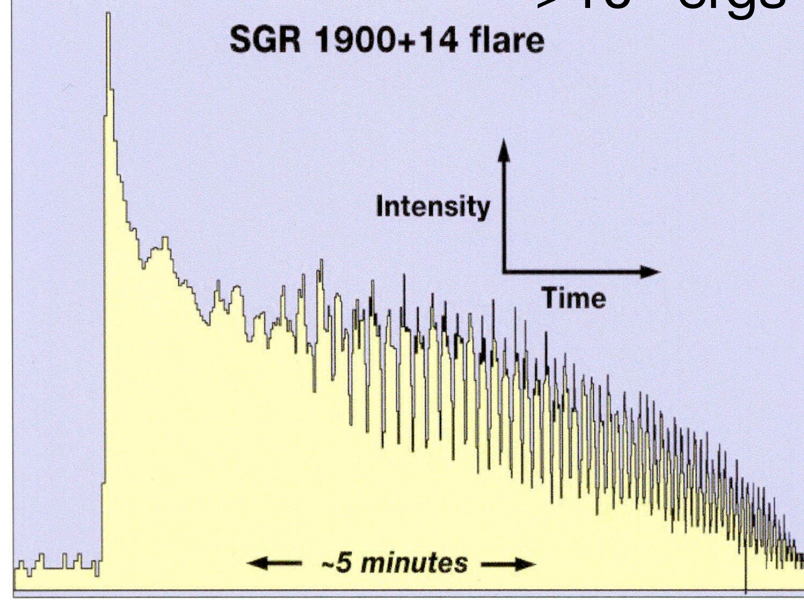
Magnetar basics

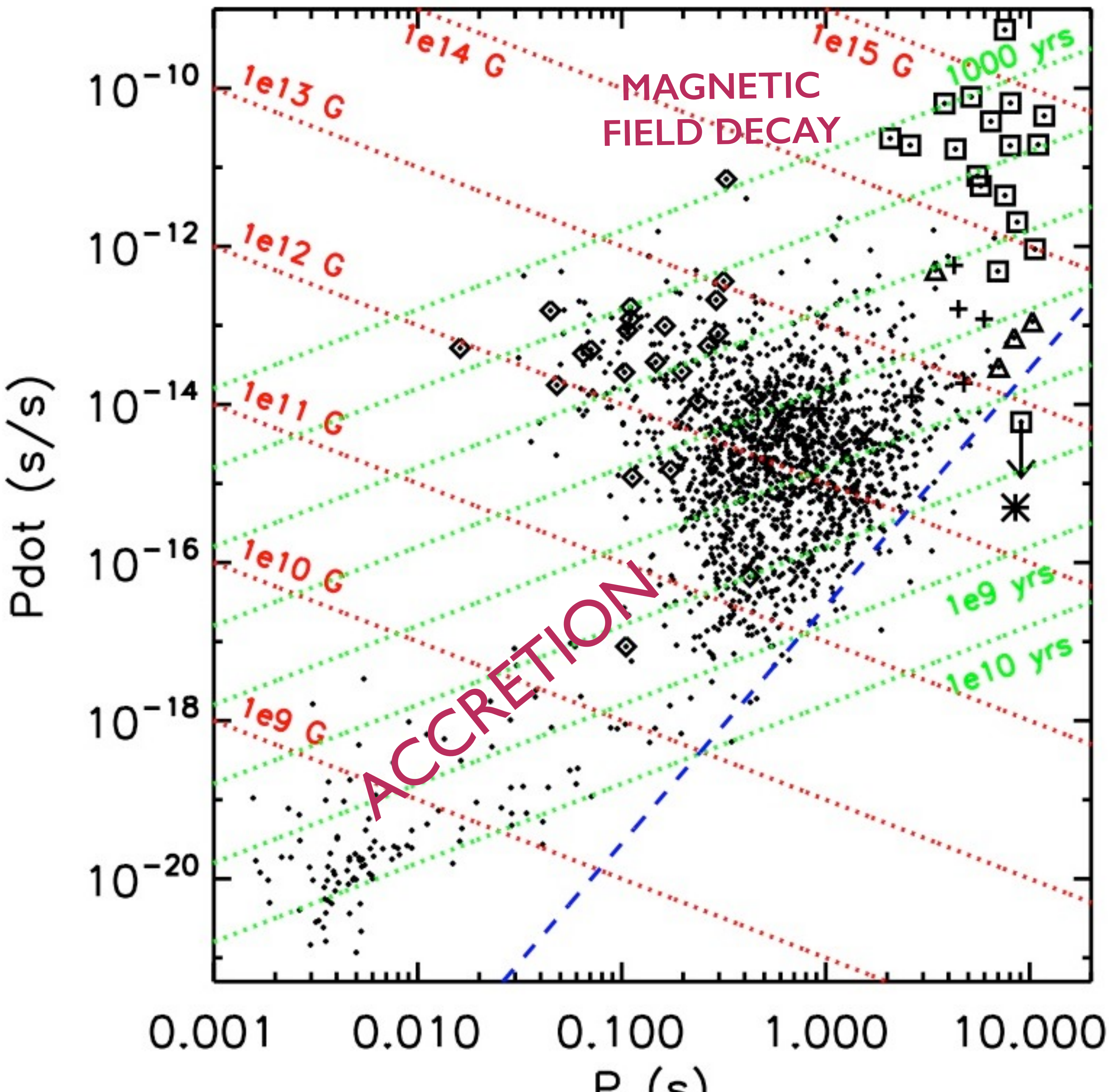
- Neutron stars powered by magnetic field decay (L >> spindown power)
- Two flavours: AXPs and SGRs
- Spin down rate indicates B > 10^{14}G
- Show a range of bursting and flaring behavior: perhaps most famous are the rare “giant flares”, but also long term (100 -1000 day) radiative outbursts

for a review, see Woods & Thompson 2006, Mereghetti 2008



$>10^{45}$ergs





1e14 G
1e15 G
1e13 G
1000 yrs
MAGNETIC
FIELD DECAY
1e12 G
1e11 G
1e10 G
1e9 G
ACCRETION
1e9 yrs
1e10 yrs
Pdot (s/s)
P (s)
0.001
0.010
0.100
1.000
10.000
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}

A diversity of neutron stars

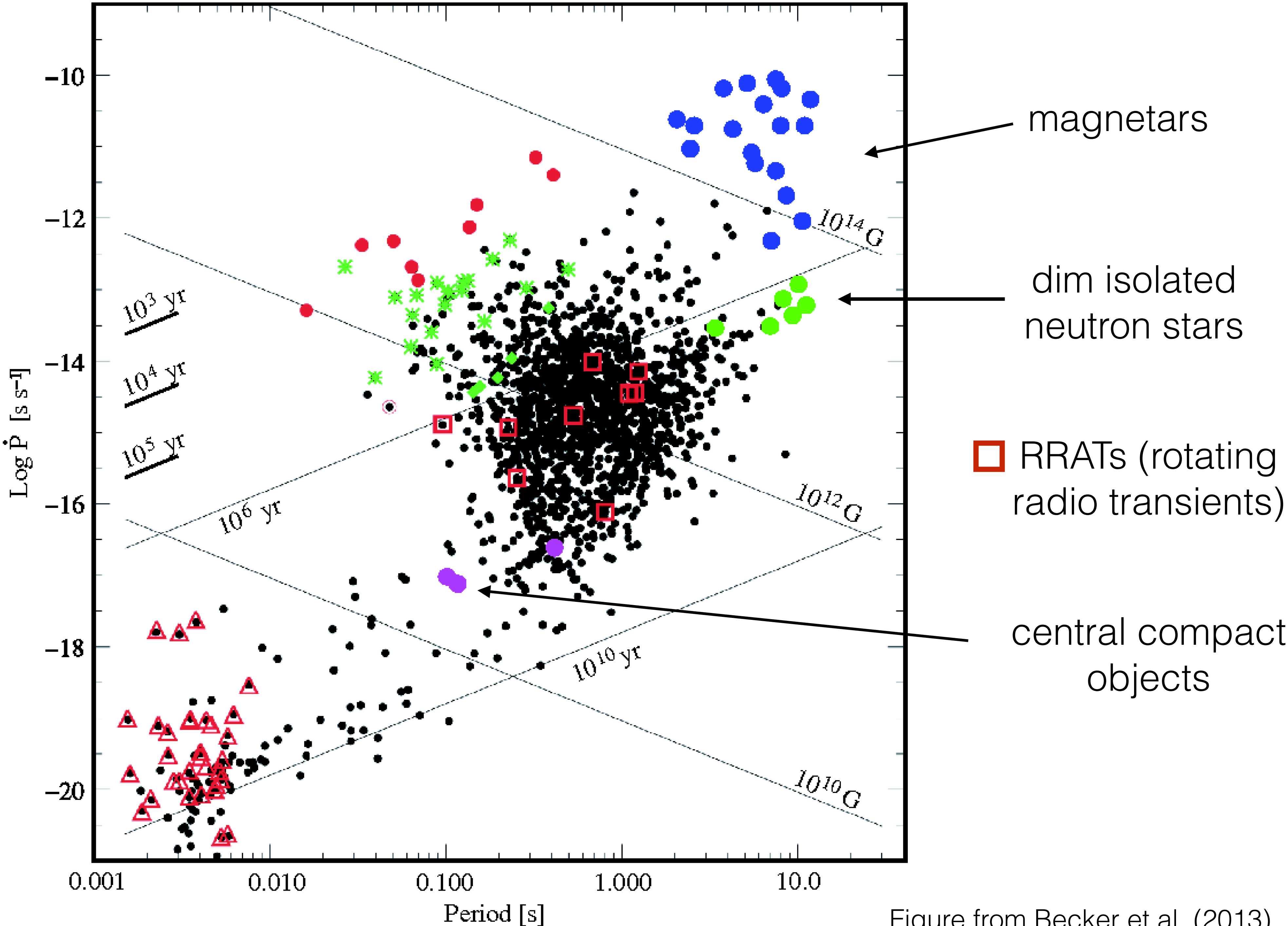
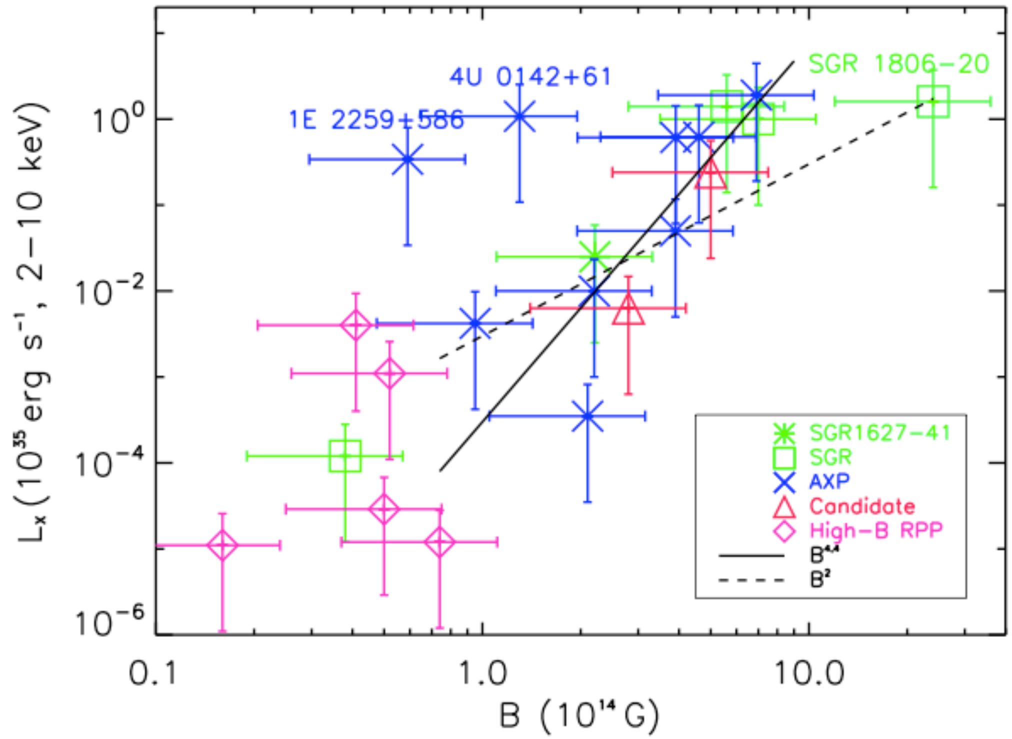


Figure from Becker et al. (2013)

Does the dipole magnetic field determine the properties of the neutron star?

The quiescent luminosity of magnetars and high B pulsars knows about the magnetic field strength

An et al. (2012)
see also Pons et al. (2007)



But there are transitional sources:

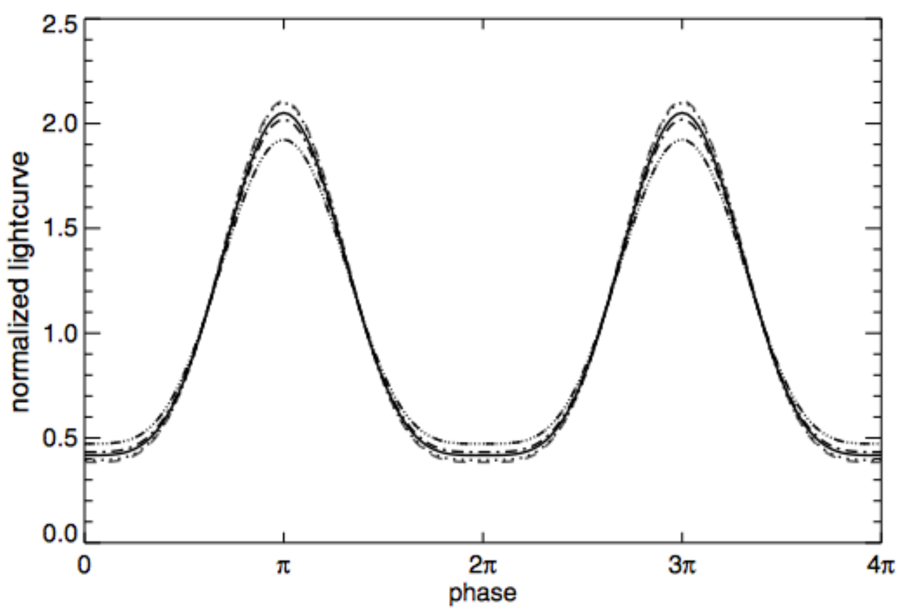
weak field magnetars: SGR 0418 has B=7.5x10^{12} G Rea et al. (2010)
Swift J1822 has B= 1.3x10^{13}G Scholz et al. (2014)

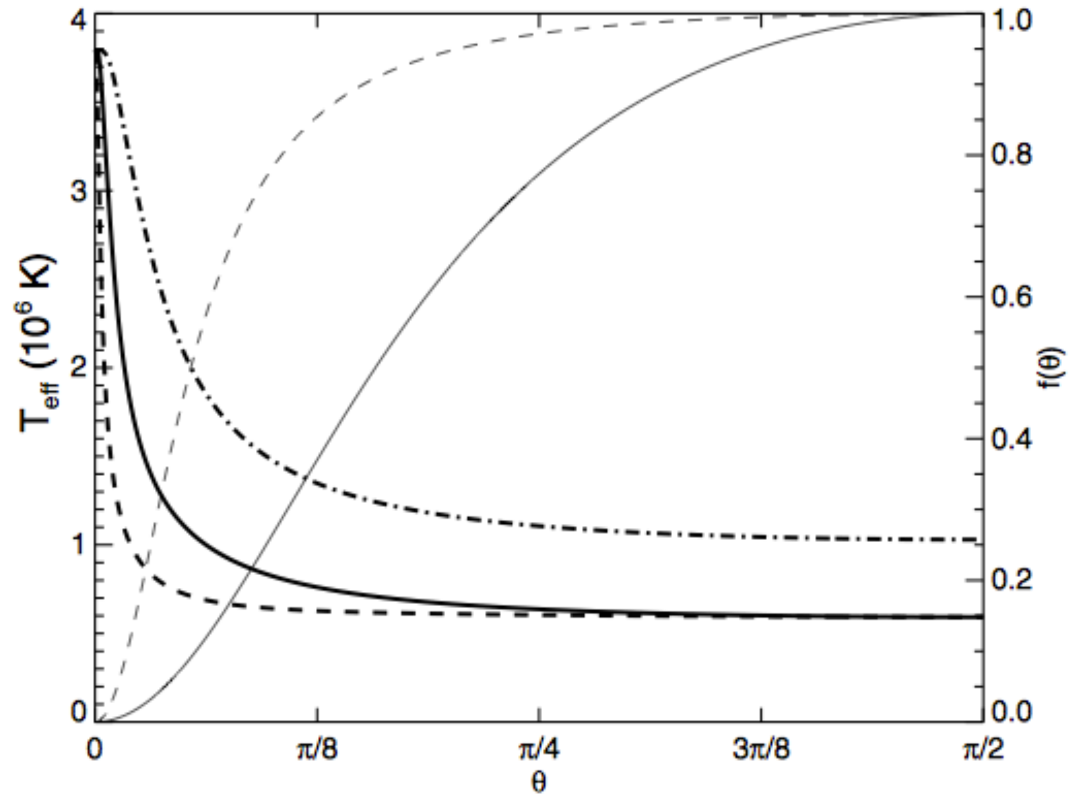
high B pulsars with magnetar-like outbursts Gavriil et al. (2008)

Modelling of pulse profiles suggests strong subsurface fields may be present

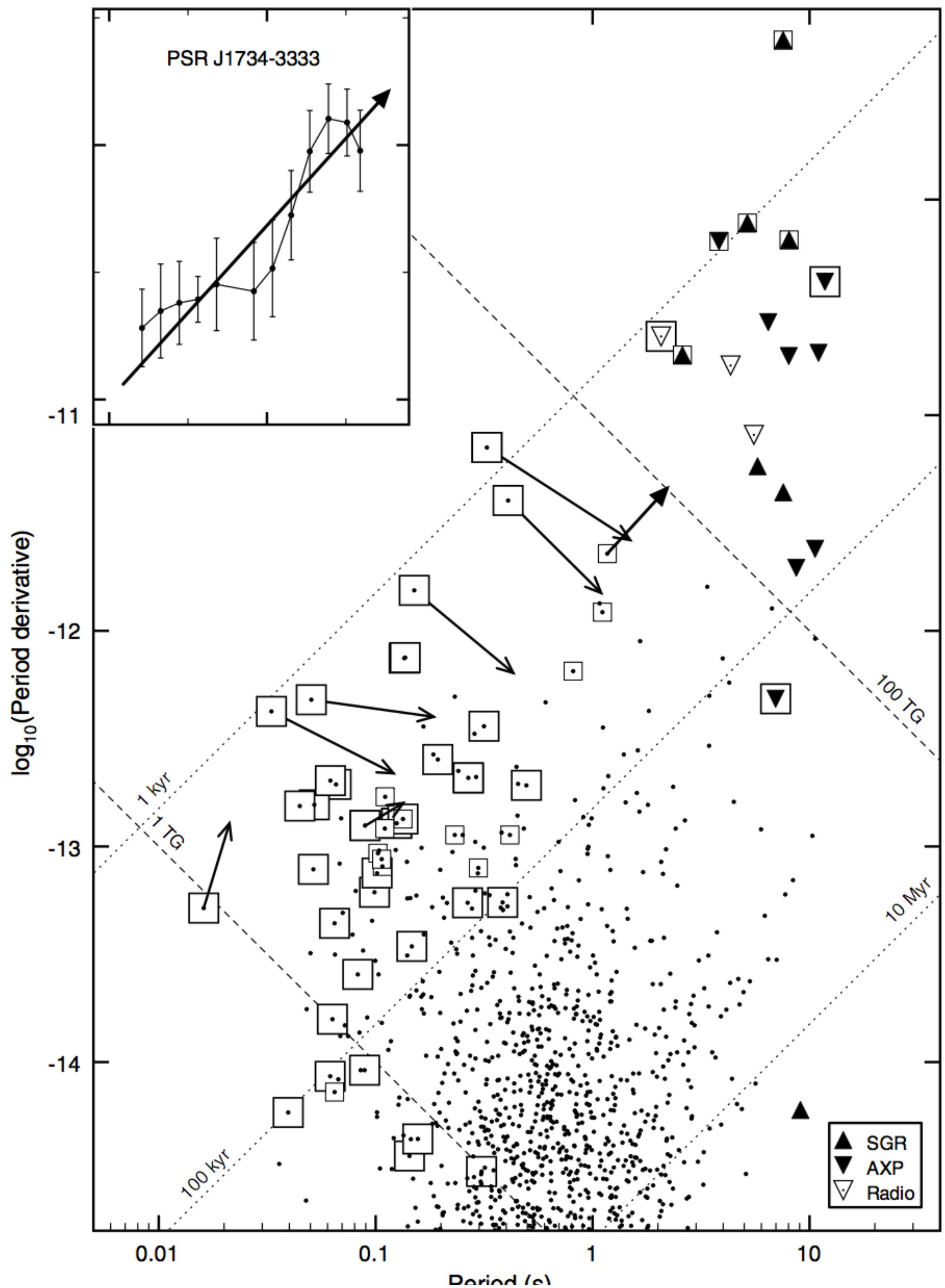
e.g. Shabaltas & Lai (2013)

high pulse fraction in Kes 79 CCO
=> peaked temperature distribution
=> subsurface 10^{14}G toroidal field to prevent heat flow except at the poles





e.g. Geppert et al. (2006) make a similar argument for dim isolated NSs



Braking indices n<3

$$\frac{d \ln \dot{P}}{d \ln P} = 1 - n$$

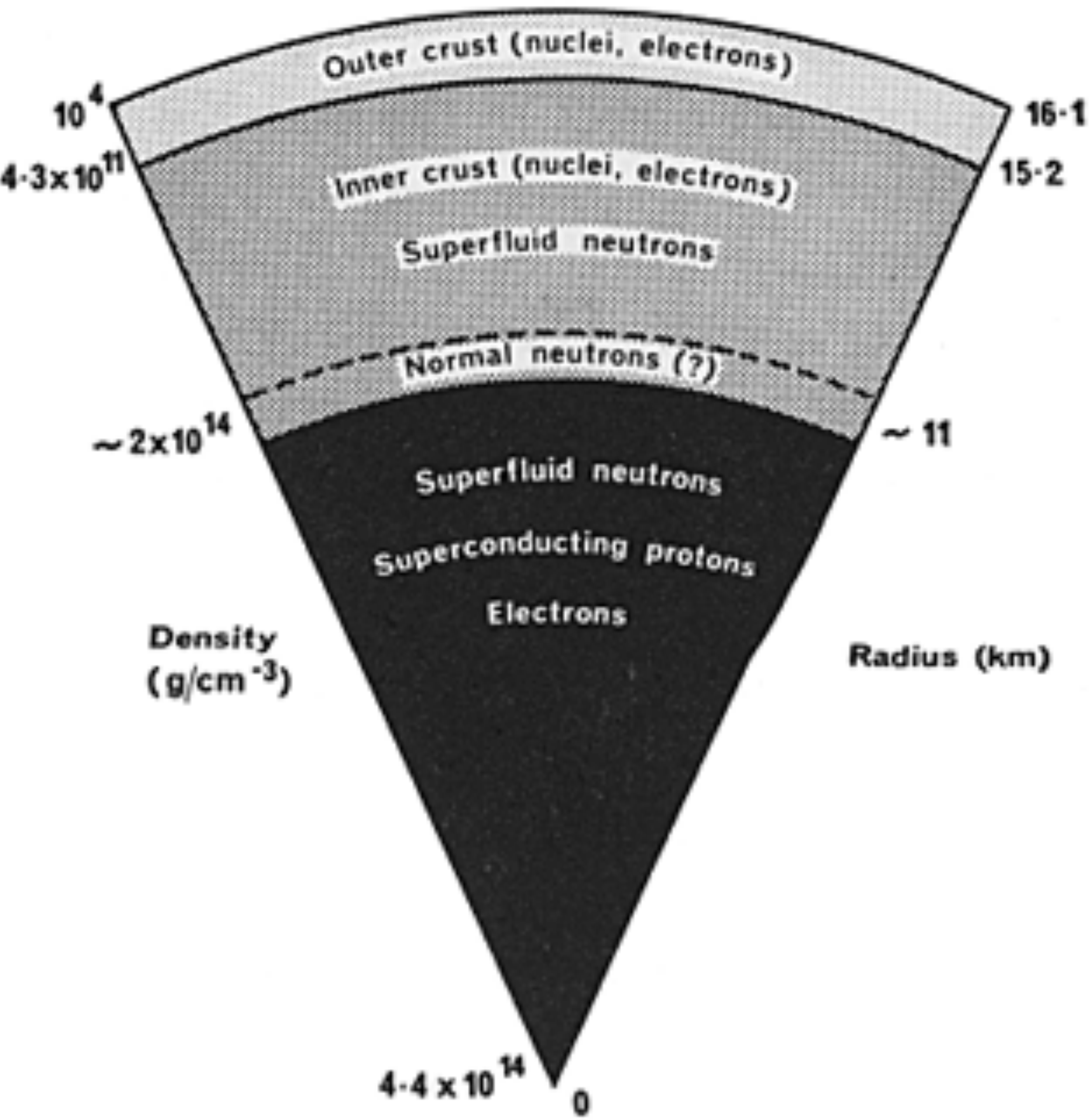
radio pulsar dipole spin down predicts n=3

$$\dot{\Omega} \propto -B^2\Omega^3$$

n<3 => increasing magnetic field over time?

Espinosa et al. (2011)

Mechanisms for magnetic field evolution in neutron stars



solid crust:

- Ohmic decay (electron collisions)
- Hall effect

$$t_{\text{Hall, inner}} = \frac{1.2 \times 10^7 \text{ yr}}{B_{12}} \rho_{14}^{7/3}$$

core:

- ambipolar diffusion
- weak equilibrium plays a key role

$$t^s_{\text{ambip}} \sim 3 \times 10^9 \frac{T_8^2 L_5^2}{B_{12}^2} \text{ yr}$$

Goldreich & Reisenegger (1992)

Open questions in magnetic field evolution

* **How does it happen?** Main physical processes identified: Ohmic decay (core+crust), Hall effect (crust), Ambipolar diffusion (core) (Goldreich & Reisenegger 1992)

but still many questions about how they operate.
Detailed simulations for the crust with coupled magnetic and thermal evolution (e.g. Vigano, Pons et al. 2013)

* **How do we connect theory to observations?** e.g. Hall effect is the prime culprit for driving field evolution in magnetars. How do we test this? Are there clean signatures in the observations?
    * e.g. Perna & Pons (2012) magnetic evolution => crust breaking => magnetar bursts
    * e.g. spin period cut off at ~10s due to field decay? (Vigano et al. 2013)
    * cooling of transient outbursts =>location of energy deposition (Pons & Rea 2012)

* **What are the initial conditions?** Crust vs. core fields, what are the allowed geometries at different ages?

  (get out what you put in)

Two complementary approaches

* Push ahead towards “whole star” simulations
* Try to derive general constraints, e.g. on the allowed initial fields, long term evolution, how strong a toroidal field is allowed at a given age, amount of energy available to power magnetic activity, etc..

Outline

1. Motivation for studying neutron star magnetic field evolution
2. **The role of the Hall effect and a puzzle**
3. A Hall attractor for neutron star crusts
4. Connection to observations: braking indices, long period cutoff

Evolution of the crust field due to the Hall effect

Ohm's law

$$E = \frac{J}{\sigma} + \frac{J \times B}{e n_e c}$$

Faraday's law

$$\frac{\partial B}{\partial t} = -c \nabla \times E$$

Hall term is

$$\frac{\partial B}{\partial t} = \nabla \times (v_e \times B)$$

=> flux freezing into the electron fluid

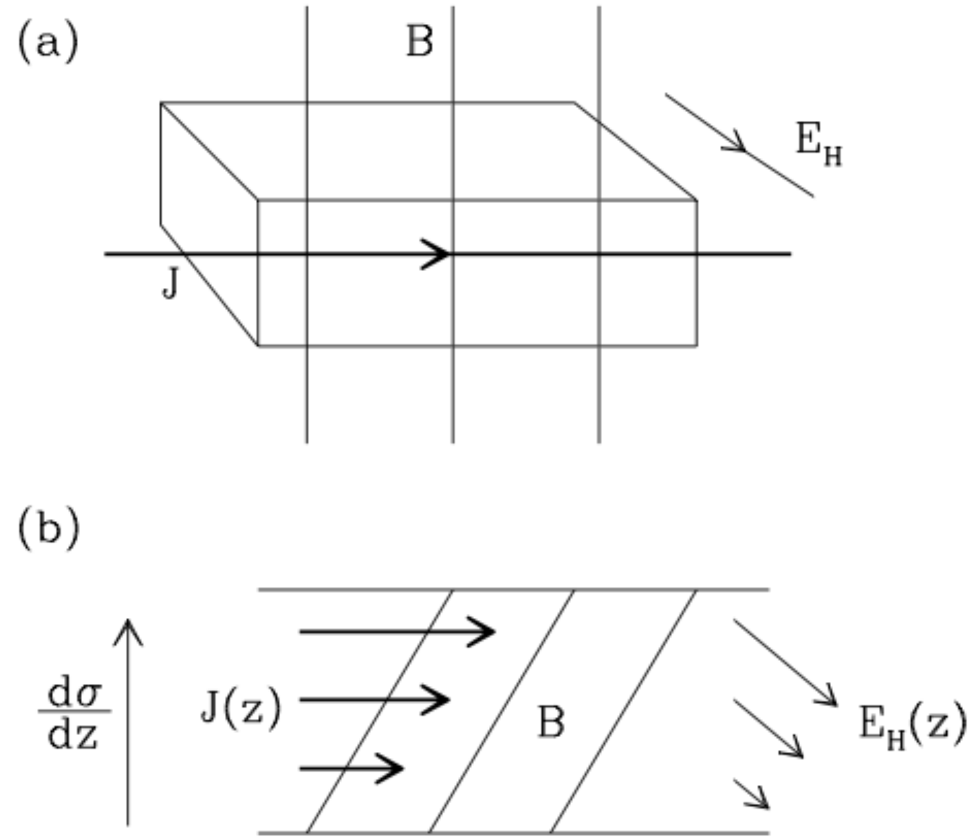
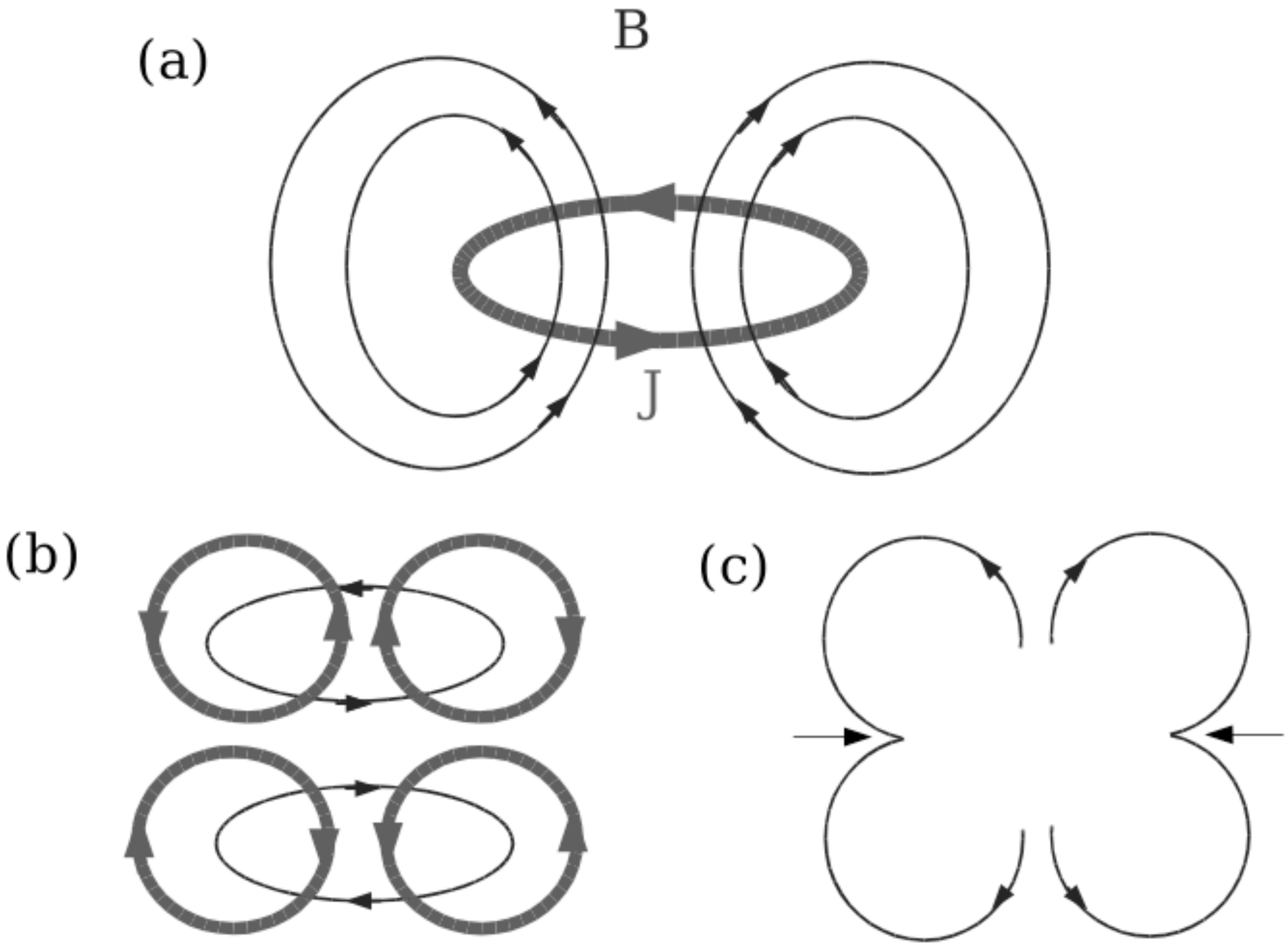


FIG. 1.—Simple example of the Hall effect. (*a*) The usual laboratory set up in which the Hall electric field balances the magnetic force on the conducting electrons. (*b*) If the conductivity varies with height, the electron velocity depends on height, shearing the magnetic field on a timescale $t_{\rm Hall} = L/v_e = n_e e L/J$.

$$J = -n_e e v_e$$

e.g. Evolution of a dipole field due to the Hall effect



Magnetic field moves with the electron fluid in the crust

Magnetic energy transferred to smaller scales

Cumming et al. (2004)

$$t_{\mathrm{Hall}} \approx 5 \times 10^8 \frac{L_5^2}{B_{12}} \left(\frac{\rho}{\rho_{\mathrm{nuc}}} \right) \mathrm{yr}$$

Goldreich & Reisenegger (1992)

A turbulent cascade in electron MHD?

Goldreich & Reisenegger (1992) proposed a Hall cascade

$$\frac{\partial \boldsymbol{b}}{\partial \tau} = -\boldsymbol{\nabla}_\xi \times [(\boldsymbol{\nabla}_\xi \times \boldsymbol{b}) \times \boldsymbol{b}] + \frac{1}{\mathscr{R}_B} \nabla_\xi^2 \boldsymbol{b} . \qquad (43)$$

$$\frac{\partial \boldsymbol{\omega}}{\partial \tau} = \boldsymbol{\nabla}_\xi \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \frac{1}{\mathscr{R}} \nabla_\xi^2 \boldsymbol{\omega} , \qquad (45)$$

=> complete dissipation of the field on the (outer) Hall timescale

than the linear diffusion term for $\mathscr{R} \gg 1$. We speculate that, where $\mathscr{R}_B \gg 1$ in the solid crust, the generic magnetic field evolves through a turbulent cascade. In other words, nonlinear couplings transfer magnetic energy from larger to smaller scales where it is ultimately dissipated by ohmic decay. The similarity between equations (43) and (45) leads us to speculate that the generic magnetic field is turbulent for $\mathscr{R}_B \gg 1$. The material in the remainder of this section is based on that speculation. It is so intriguing that we present it in advance of serious investigation.

Small scale structures undergo Ohmic decay and can propagate to low densities in the outer crust

$$\mathscr{R}_B \sim 4 \times 10^2 \, \frac{B_{12}}{T_8^2} \left(\frac{\rho}{\rho_{\rm nuc}}\right)^2$$

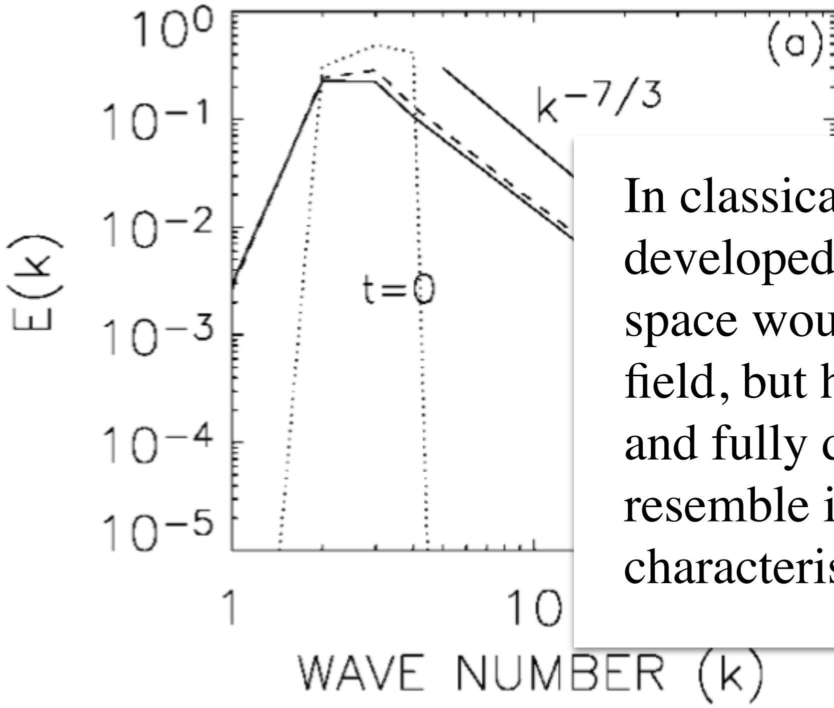
By considering weakly interacting Hall waves, they predicted a k^{-2} spectrum

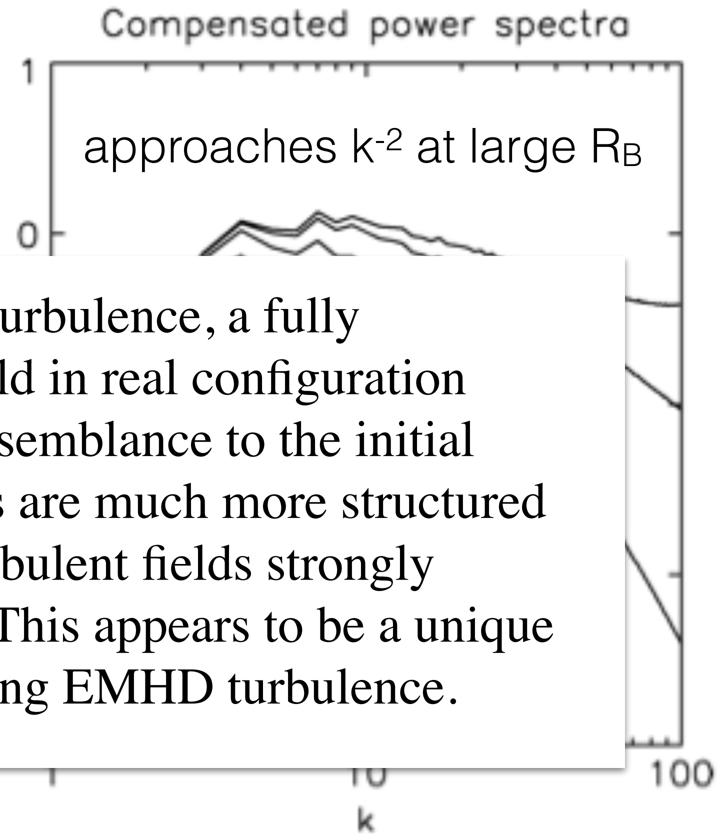
(earlier, Jones 1988 also discussed transport of magnetic energy by Hall waves)

Numerical simulations of the Hall cascade

A cascade is confirmed by simulations in Cartesian boxes, but spectral index is debated

Biskamp et al. (1996, 1999), Cho & Lazarian (2004,2009), Cho (2011)

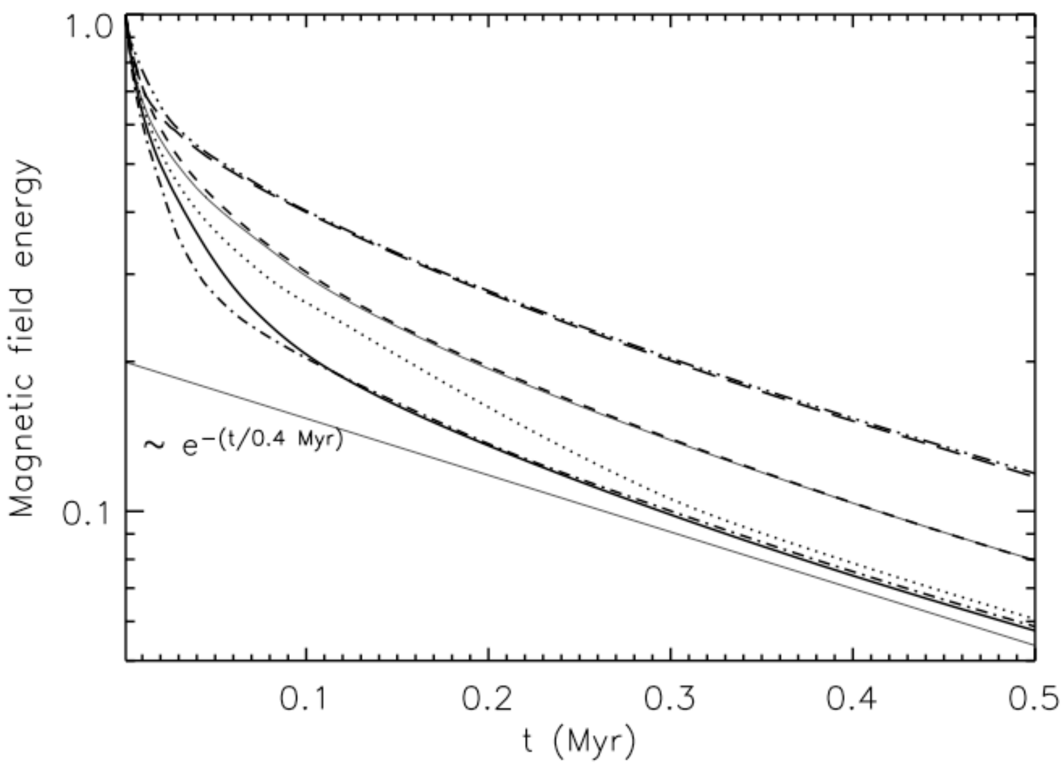




In classical and MHD turbulence, a fully developed turbulent field in real configuration space would bear no resemblance to the initial field, but here the fields are much more structured and fully developed turbulent fields strongly resemble initial fields. This appears to be a unique characteristic of decaying EMHD turbulence.

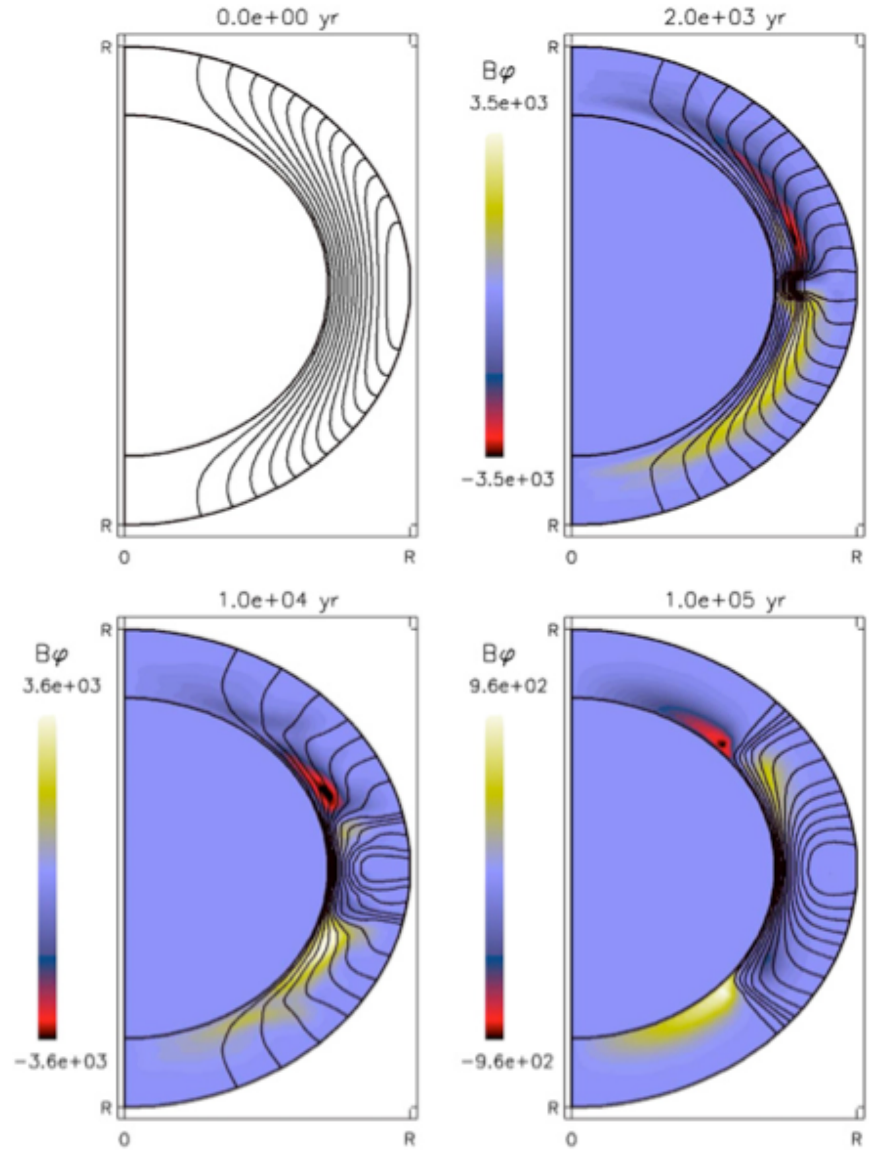
Wareing & Hollerbach (2009abc) argue that the dissipative cutoff is an artifact of assumed hyperdiffusivity and local coupling.

Simulations of magnetic field evolution in the crust



Field evolution is rapid initially,
but slows down
=> Hall effect "saturates"

Pons & Geppert (2007)



Vigano, Pons, Miralles 2012

see also Urpin & Shalybkov 1991, Naito & Kojima 1994, Shalybkov & Urpin 1997, Hollerbach & Rudiger 2002,2004, Geppert et al. 2013

Electron density gradient leads to steepening of toroidal fields => another way to get rapid dissipation

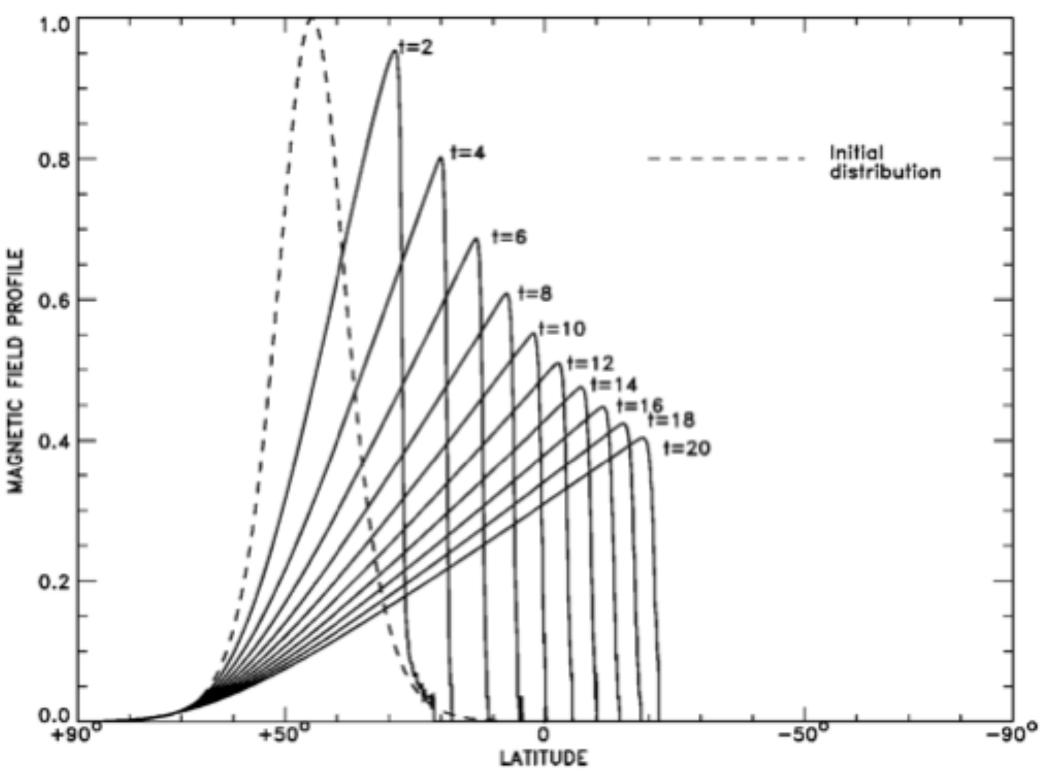
$$\boldsymbol{w} \equiv R^2 \mathcal{B} \nabla\chi \times \nabla\phi.$$

Toroidal fields evolve according to

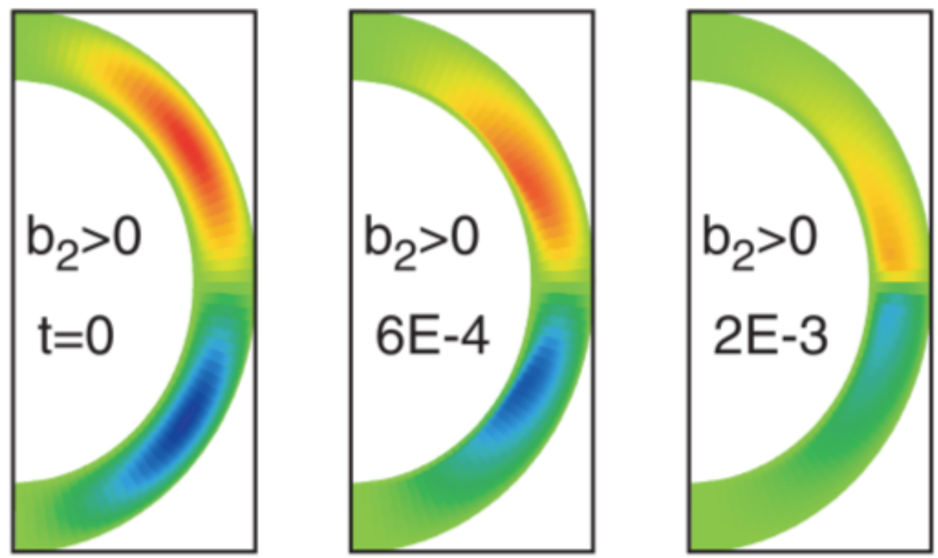
$$\frac{\partial \mathcal{B}}{\partial t} + \boldsymbol{w} \cdot \nabla \mathcal{B} = R^2 \nabla \cdot \left(\frac{\eta}{R^2} \nabla \mathcal{B} \right)$$

$$\chi = \frac{c}{4\pi e n_e r^2 \sin^2\theta}$$

Reisenegger et al. (2007)

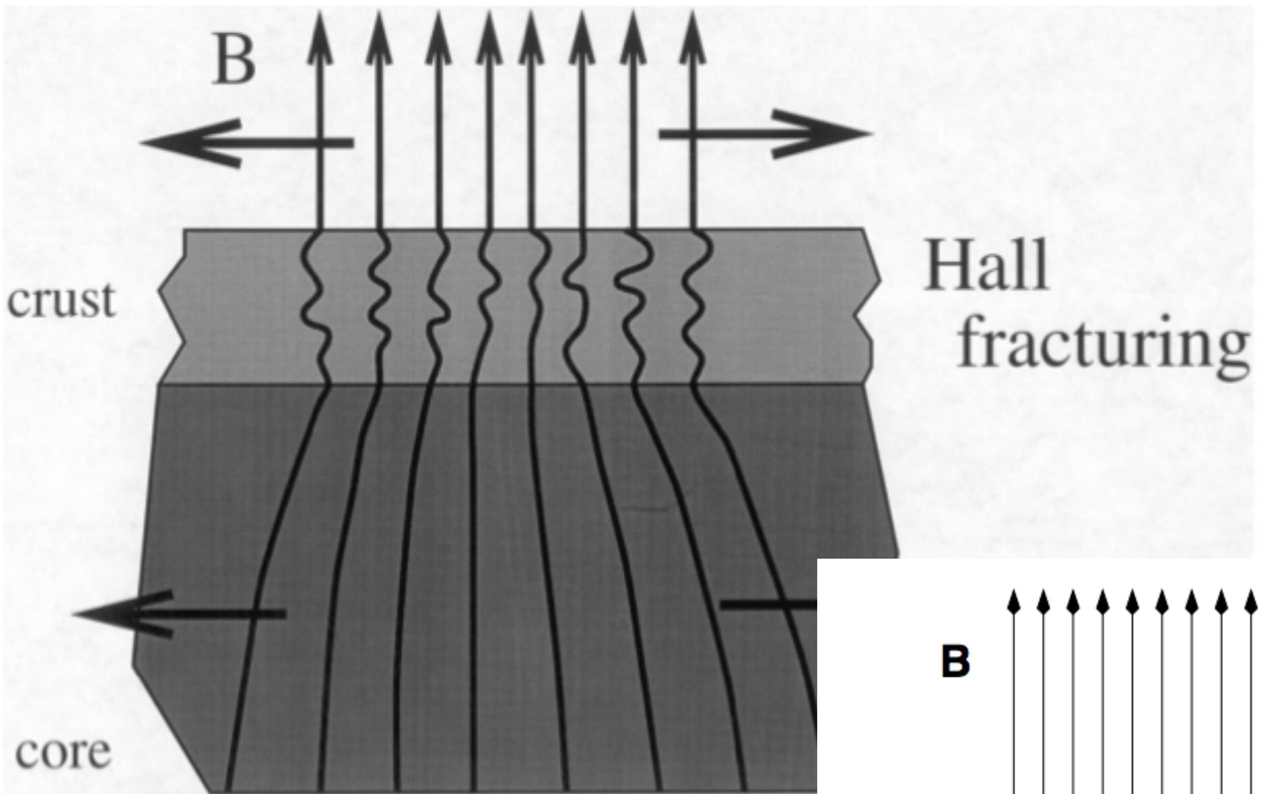


Vainshtein et al. (2000)



Kojima & Kisaka (2012)

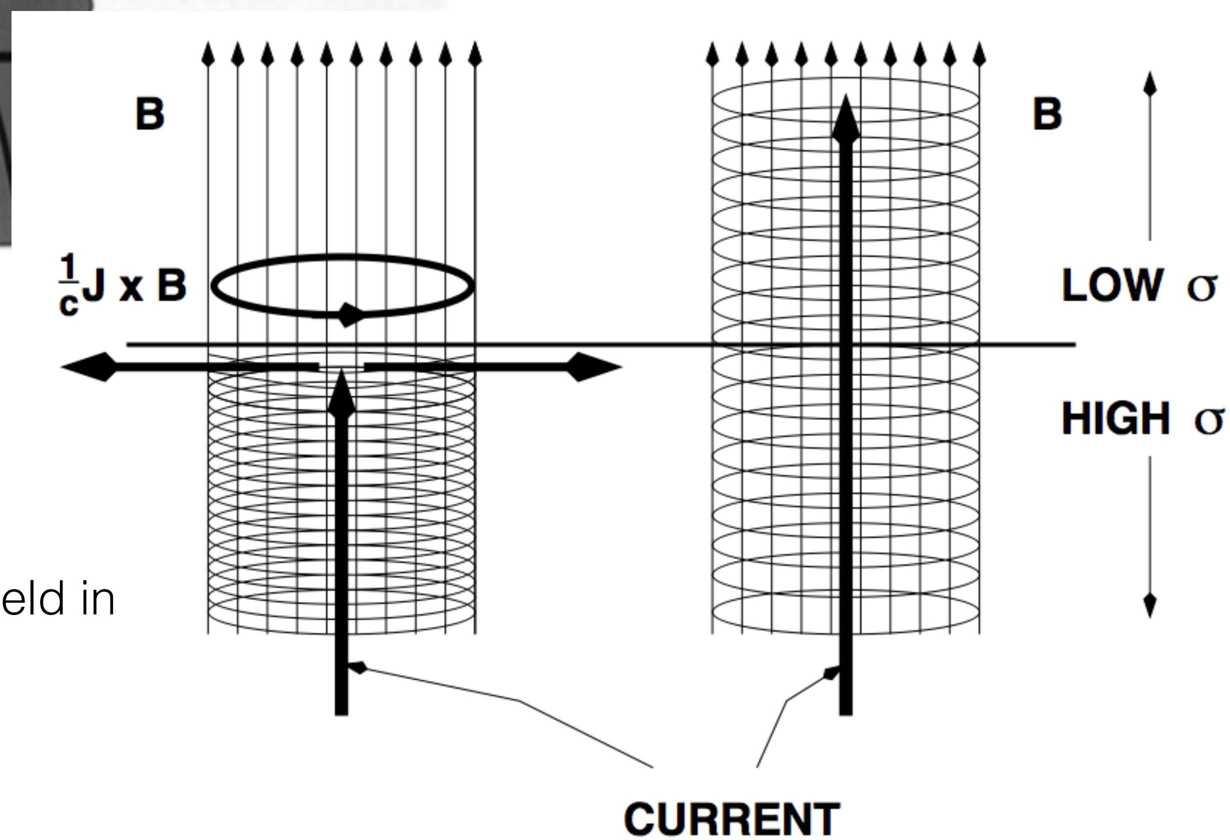
The role of the Hall effect: turbulent or not?



Thompson & Duncan (1996)

Does the Hall effect lead to enhanced dissipation?

Thompson, Lyutikov, & Kulkarni (2002)

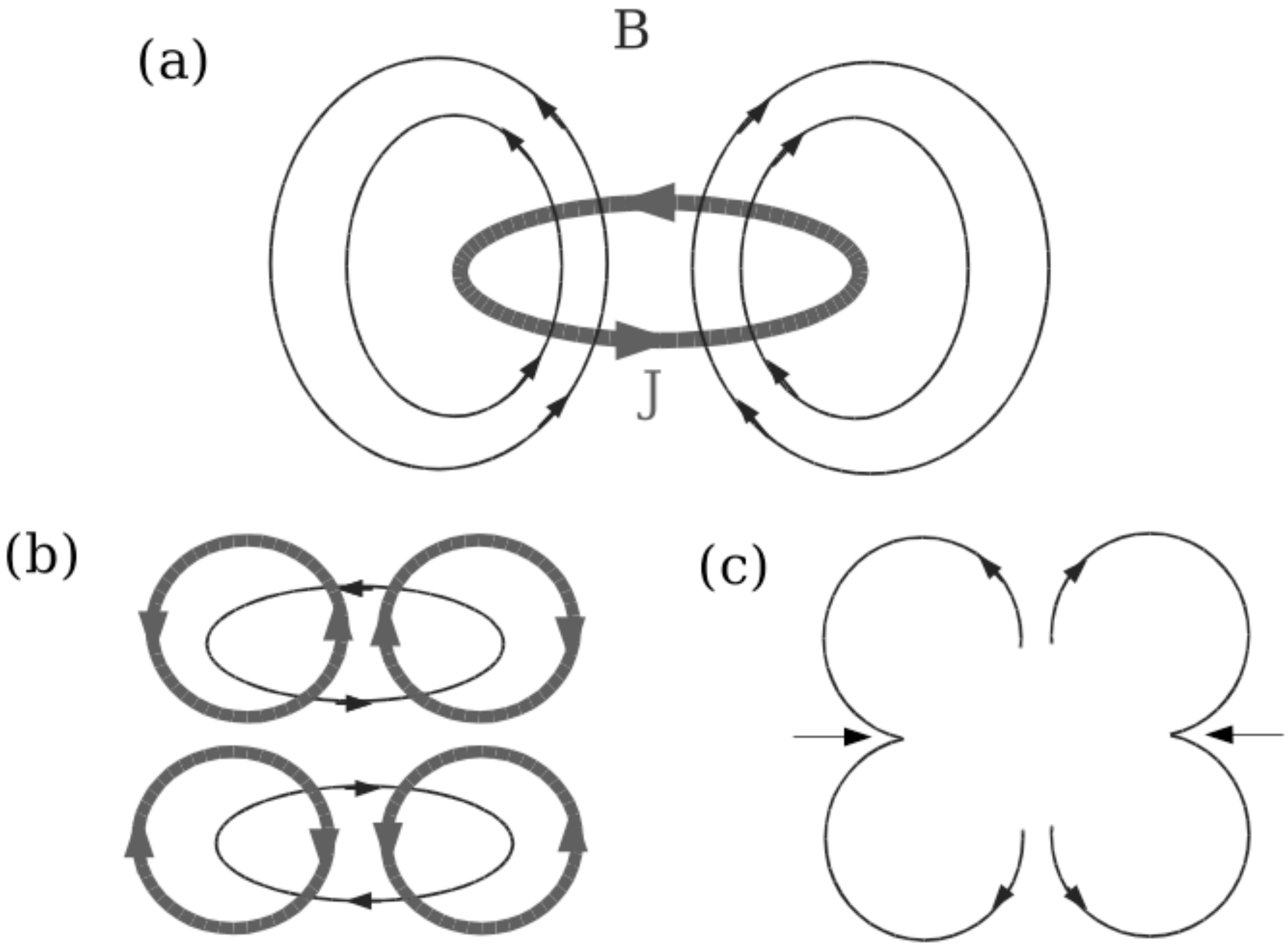


Large scale transport of field in the crust?

Outline

1. Motivation for studying neutron star magnetic field evolution
2. The role of the Hall effect and a puzzle
3. **A Hall attractor for neutron star crusts**
4. Connection to observations: braking indices, long period cutoff

e.g. Evolution of a dipole field due to the Hall effect



Magnetic field moves with the electron fluid in the crust

Magnetic energy transferred to smaller scales

Cumming, Arras, Zweibel (2004)

$$t_{\rm Hall} \approx 5 \times 10^8 \frac{L_5^2}{B_{12}} \left(\frac{\rho}{\rho_{\rm nuc}}\right) \,{\rm yr}$$

Goldreich & Reisenegger (1992)

What is the role of Hall equilibria?

Example: rigidly rotating electrons

* In axisymmetry, the Hall term can be written in terms of the angular velocity of the electrons as

$$\frac{\partial B_\phi}{\partial t} = r \sin\theta (\boldsymbol{B}_P \cdot \nabla)\Omega_e \qquad v_e = \Omega r \sin\theta$$

* A purely poloidal dipole field with rigidly rotating electrons is a Hall equilibrium

(A dipole has $J_\phi \propto v_e \propto \sin\theta$)

Cumming, Arras & Zweibel (2004)

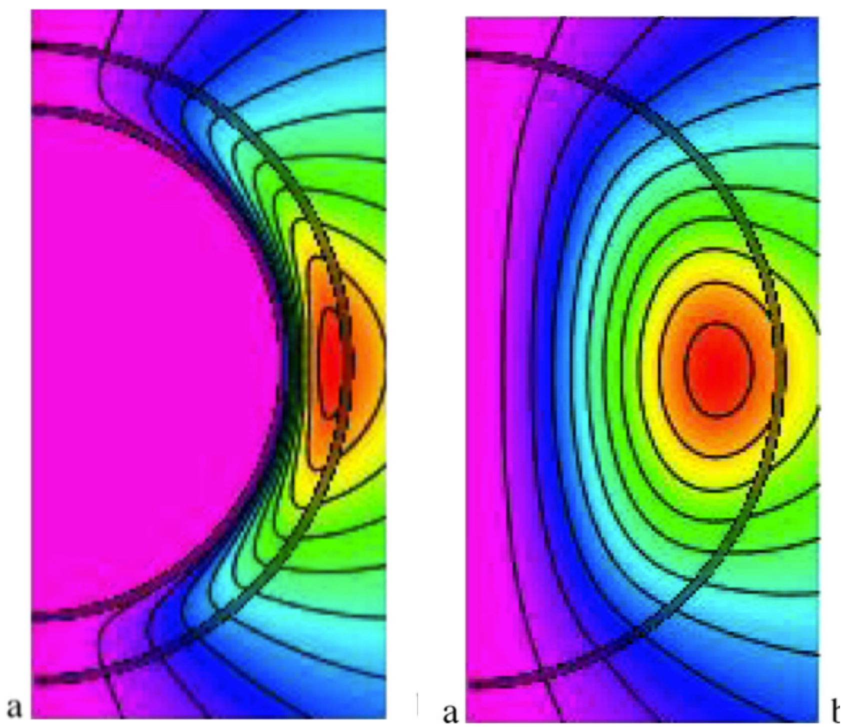
* Important because it suggests that the Hall effect doesn't necessarily lead to decay of all the magnetic energy

* **But** - are there really preferred states in electron MHD? Unlike MHD, "electron MHD does not have an energy principle" (Lyutikov 2013) — why would the equilibrium be a preferred state?

Hall equilibria

Gourgouliatos et al. (2013)

* External dipole fields require rigidly rotating electrons
* As in MHD, toroidal fields are located in closed loops of the poloidal field.
* The toroidal field can be locally stronger than the poloidal, but the total energy in the toroidal field is a small fraction (<few %) of total magnetic energy



* Analogous to MHD equilibria for barotropic stars (e.g. Lander & Jones 2009)

Evolution of axisymmetric fields in the crust

write B in terms of scalar functions Psi and I

$$\boldsymbol{B} = \nabla\Psi \times \nabla\phi + I\nabla\phi$$

$$j_T = \frac{c}{4\pi}\nabla \times \boldsymbol{B_P} = -\frac{c}{4\pi}\Delta^*\Psi\nabla\phi$$

$$\Omega = -\frac{j_T}{n_e e r \sin\theta} = \chi\Delta^*\Psi$$

$$\Delta^* = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$$

write the density gradient as

$$\chi = \frac{c}{4\pi e n_e r^2 \sin^2\theta}$$

Assume n_e and sigma do not depend on time, typically 100x100 grid in (r,mu)

Poloidal field:

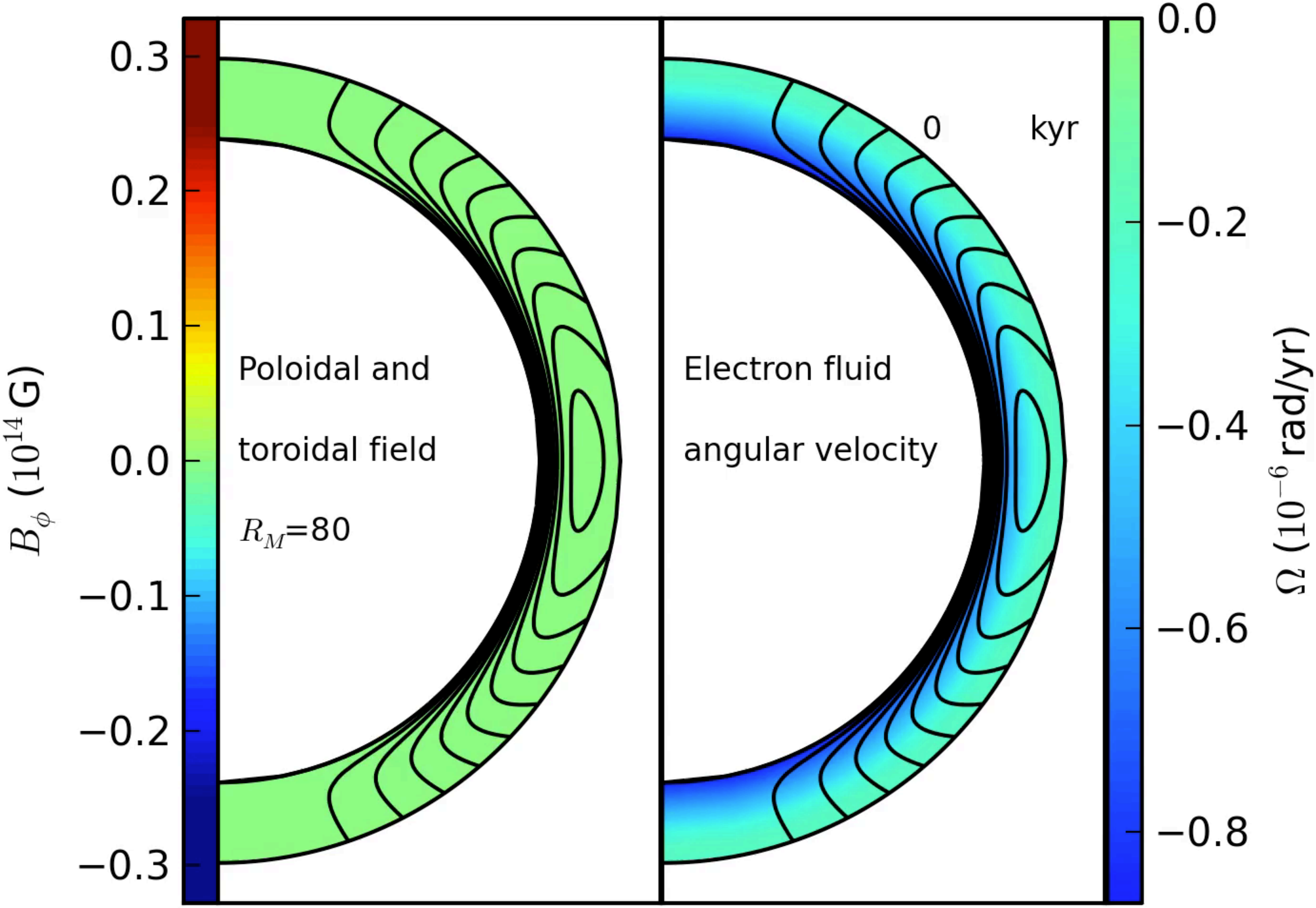
$$\frac{\partial\Psi}{\partial t} + r^2\sin^2\theta\,\chi(\nabla I \times \nabla\phi)\cdot\nabla\Psi = \frac{c^2}{4\pi\sigma}\Delta^*\Psi,$$

Toroidal field:

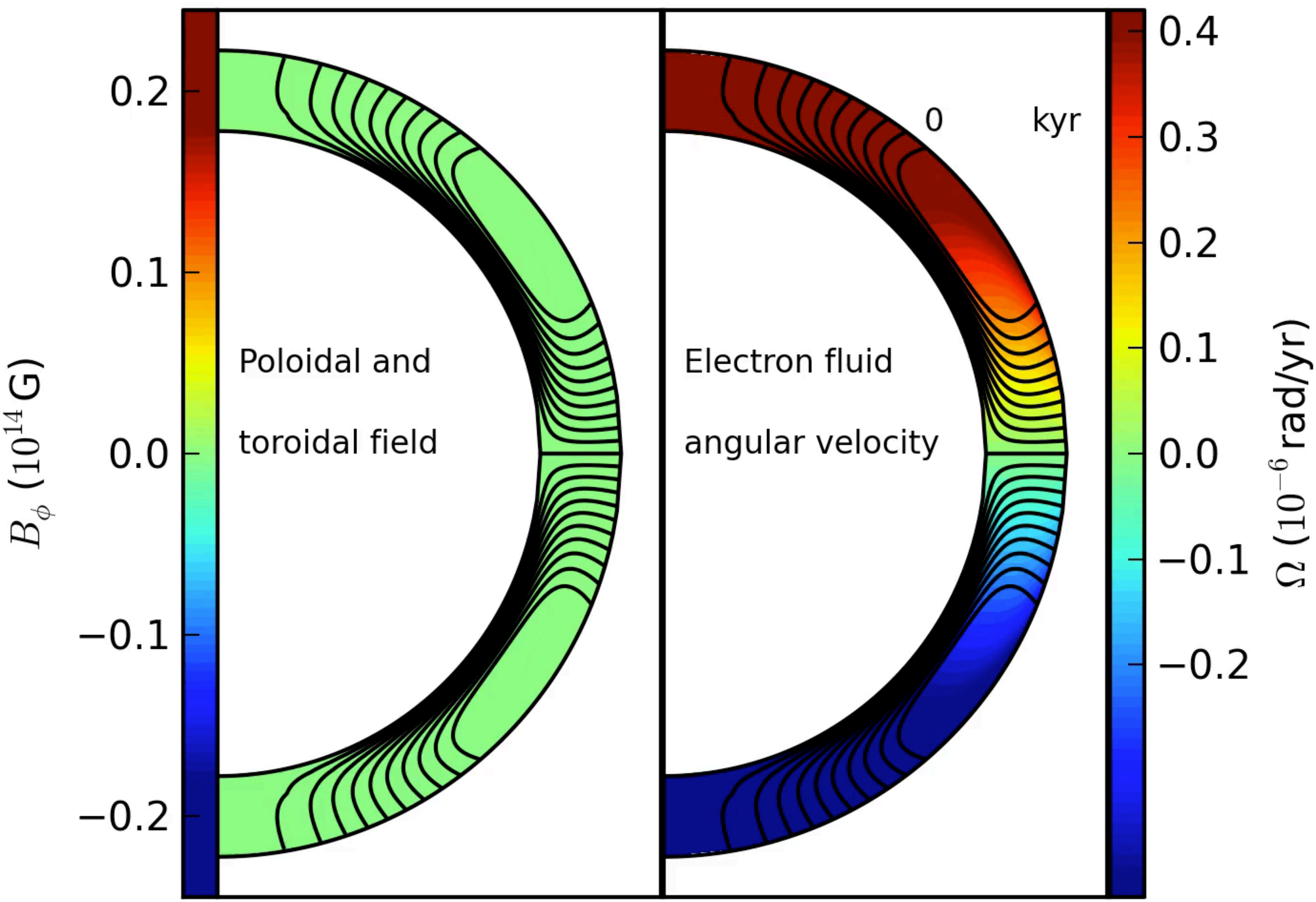
$$\frac{\partial I}{\partial t} + r^2\sin^2\theta((\nabla\Omega \times \nabla\phi)\cdot\nabla\Psi + I\,(\nabla\chi \times \nabla\phi)\cdot\nabla I)$$

$$= \frac{c^2}{4\pi\sigma}\left(\Delta^* I + \frac{1}{\sigma}\nabla I \times \nabla\sigma\right).$$

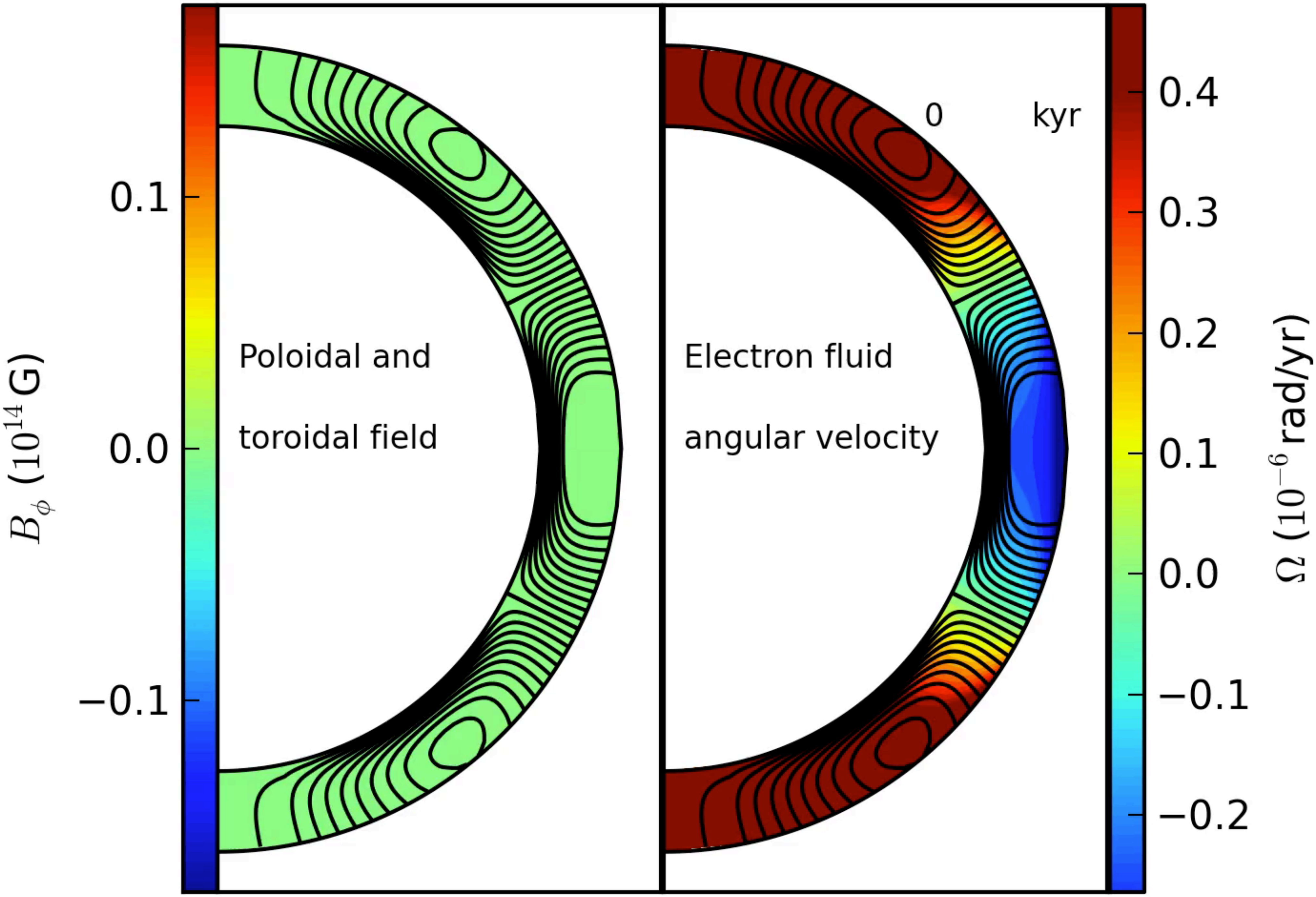
GC (2014)



0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
B_ϕ (10^{14}G)
Poloidal and
toroidal field
R_M=80
Electron fluid
angular velocity
0
kyr
0.0
−0.2
−0.4
−0.6
−0.8
Ω (10^{-6}rad/yr)

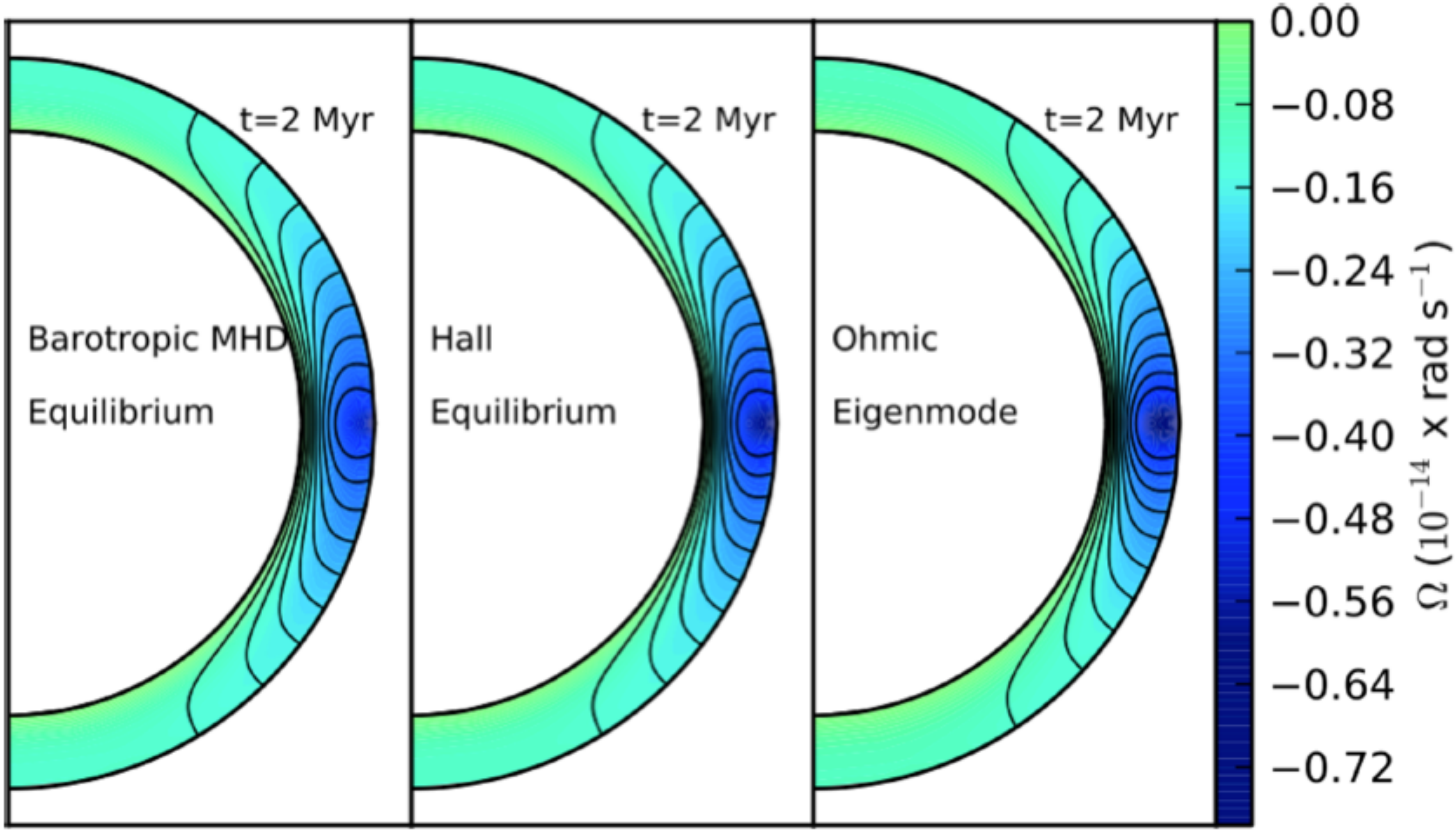


Poloidal and
toroidal field
Electron fluid
angular velocity
0
kyr
B_ϕ (10^{14} G)
0.2
0.1
0.0
−0.1
−0.2
Ω (10^{-6}rad/yr)
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2



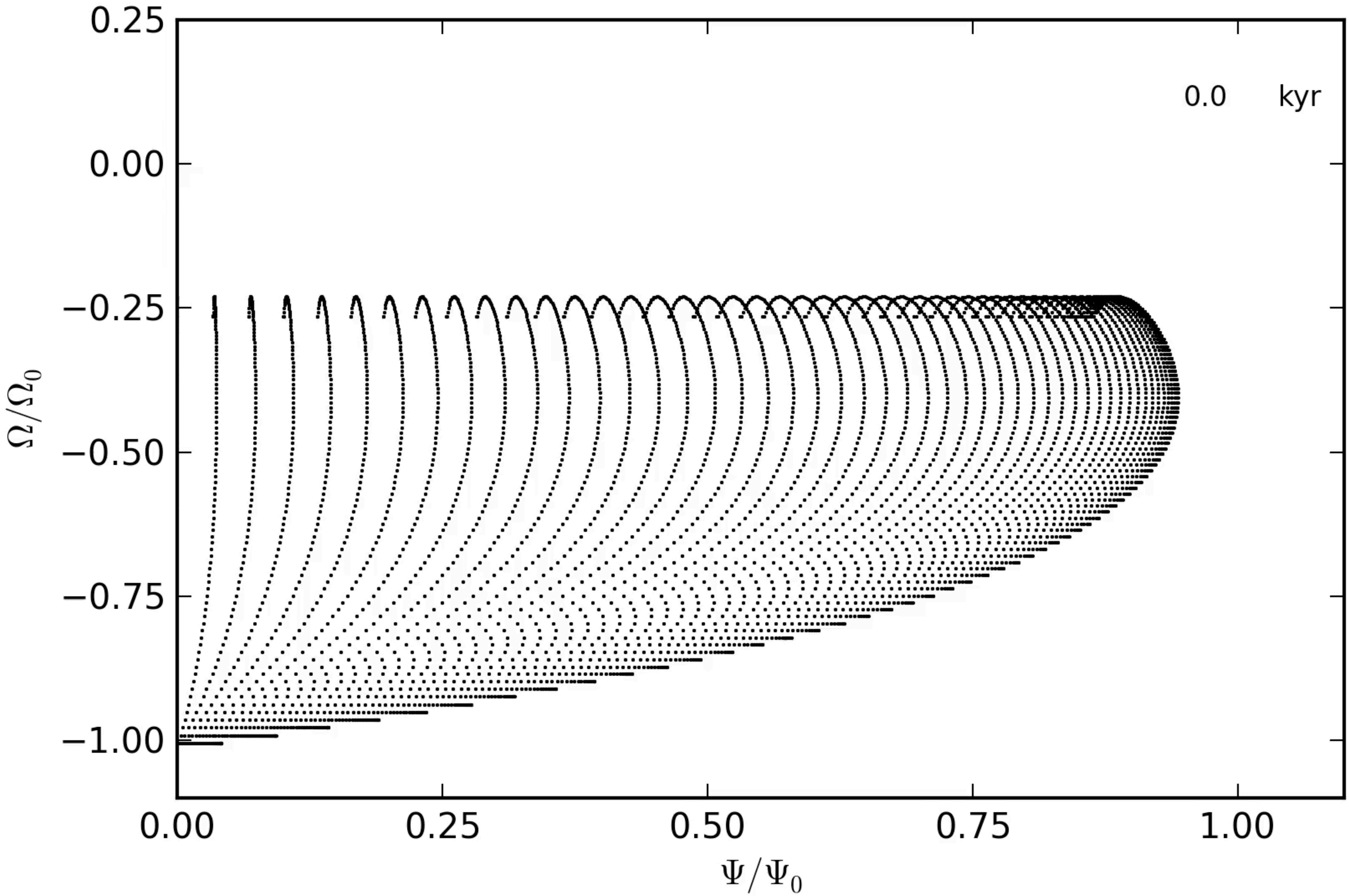
0.1
0.0
−0.1
B_ϕ (10^{14}G)
Poloidal and
toroidal field
Electron fluid
angular velocity
0
kyr
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
Ω (10^{-6}rad/yr)

The Hall effect quenches because the field evolves to a state of isorotation

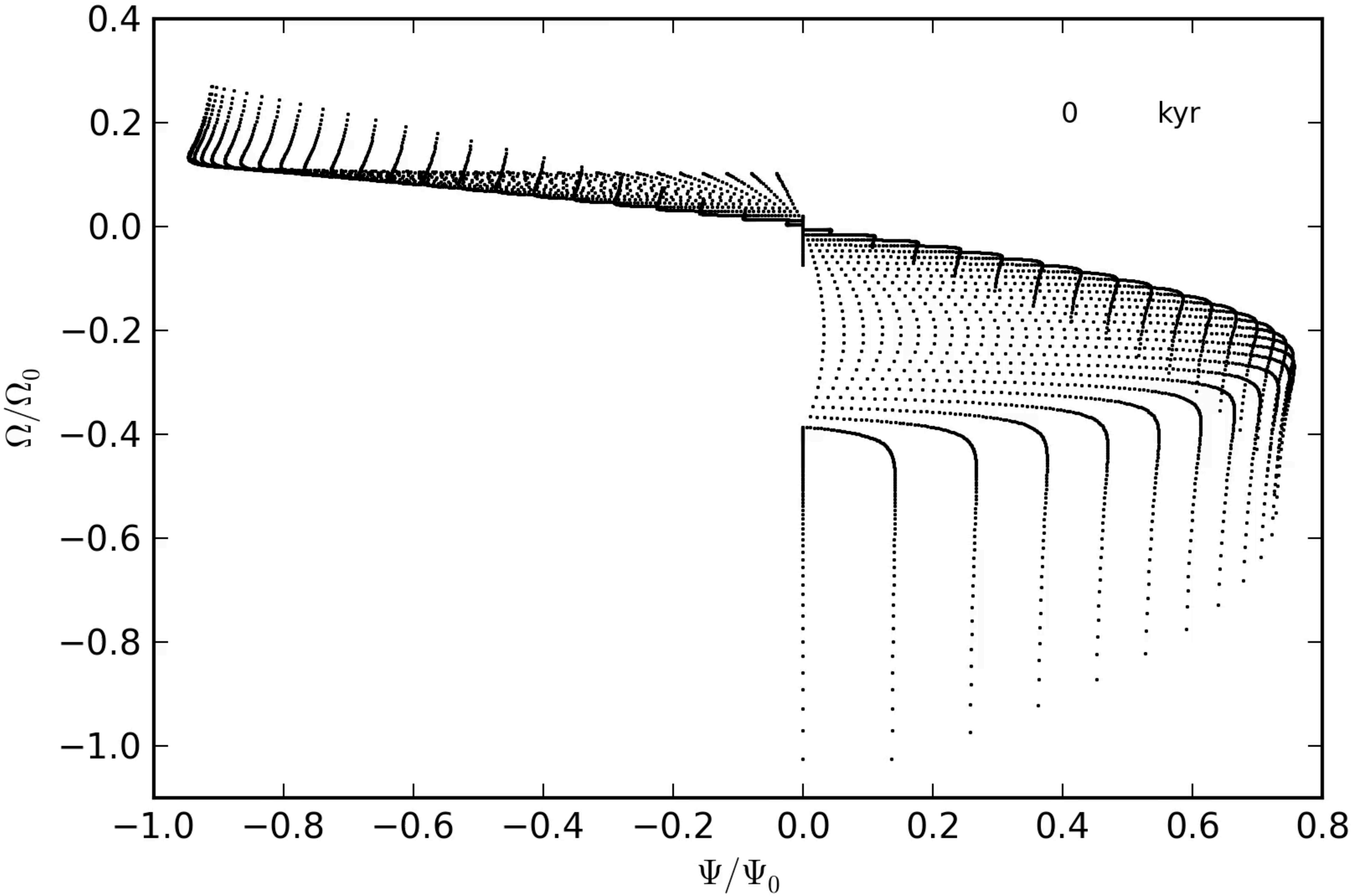


=> a Ferraro's law for EMHD $(B_P \cdot \nabla)\Omega_e = 0$

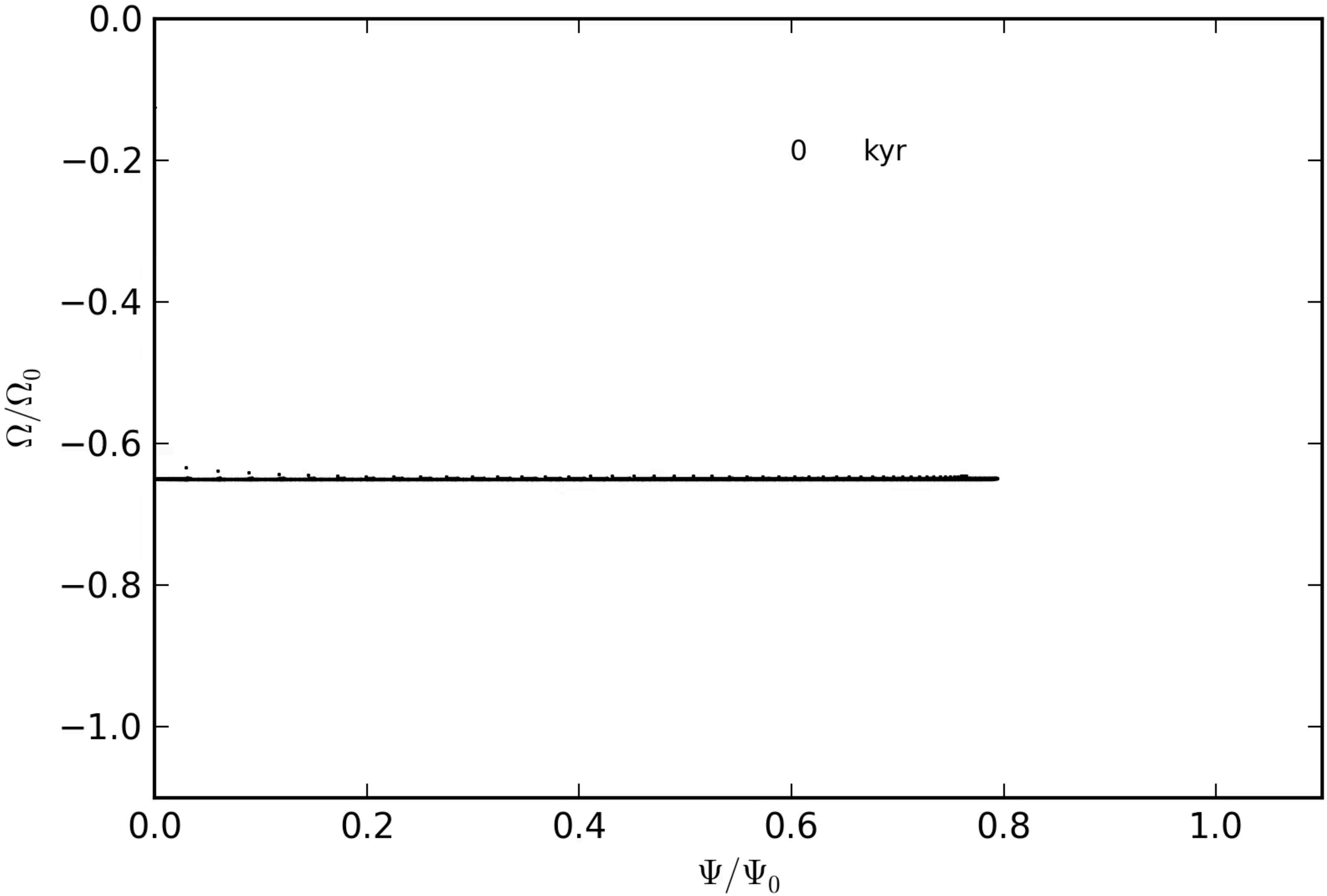
Dipole field, evolution to isorotation

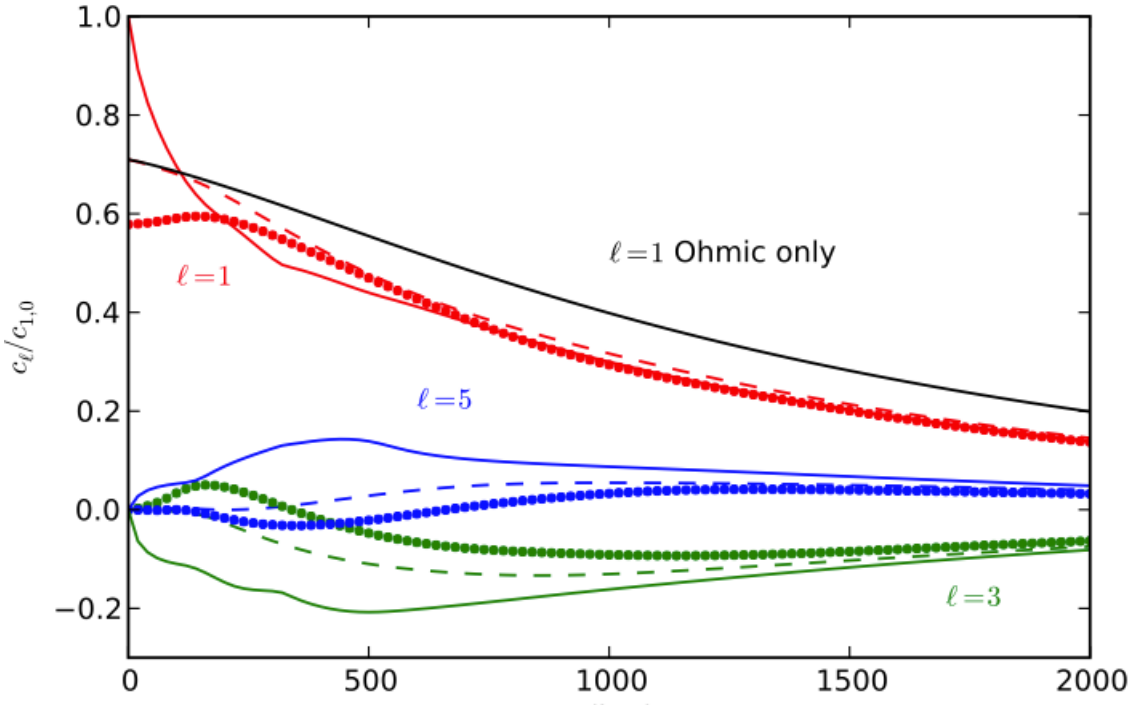


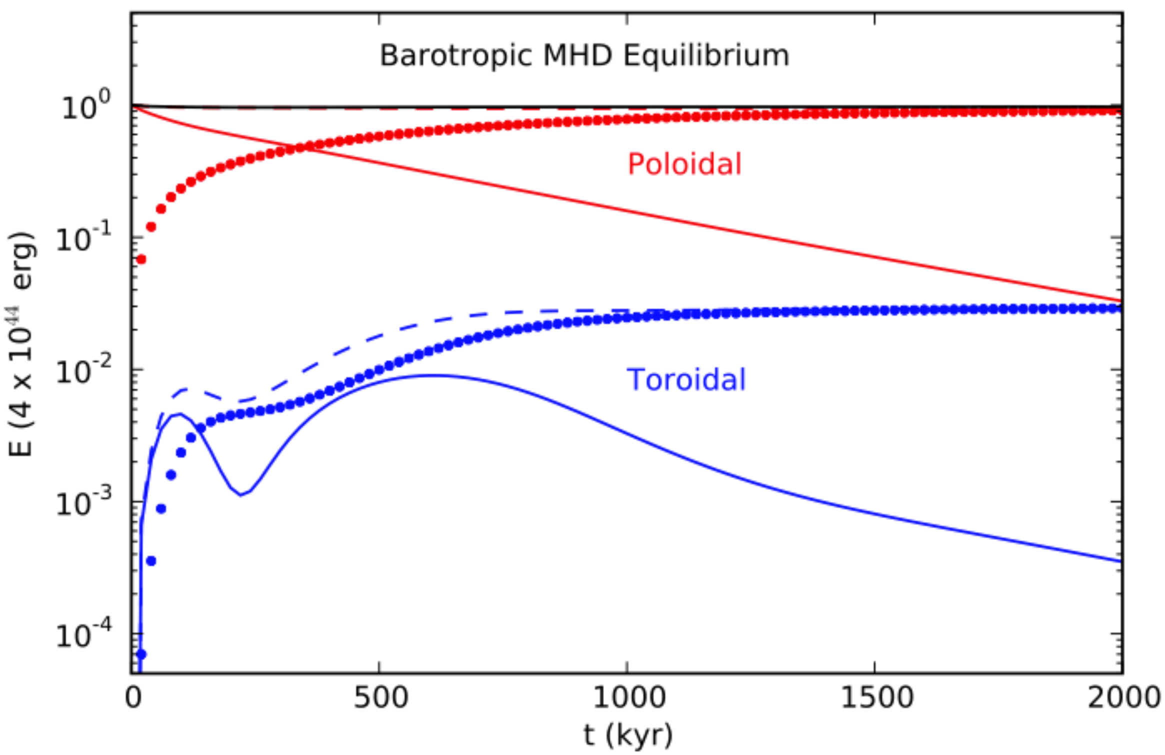
Octupole field



Rigid rotation is a Hall steady-state, but not an attractor







The evolution is not only to an isorotating state, but a particular isorotating state: the “attractor”

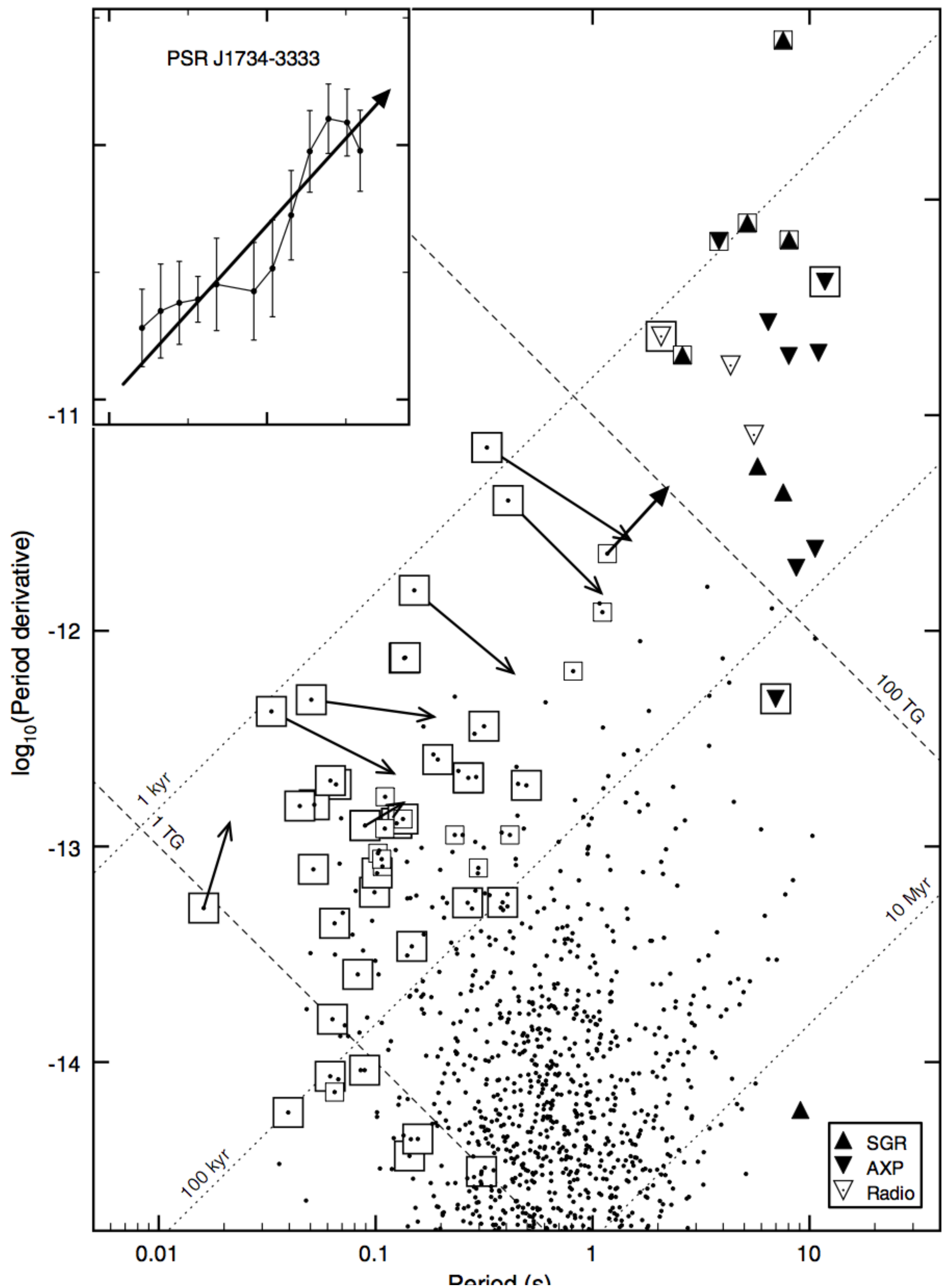
This state is characterized by ~equal and opposite mixtures of l=1 and l=3, with small amounts of higher multipoles

and a small ~1% toroidal field energy

Gourgouliatos & Cumming 2014a

Outline

1. Motivation for studying neutron star magnetic field evolution
2. The role of the Hall effect and a puzzle
3. A Hall attractor for neutron star crusts
4. **Connection to observations: braking indices, long period cutoff**



Braking indices n<3

$$\frac{d\ln\dot{P}}{d\ln P} = 1 - n$$

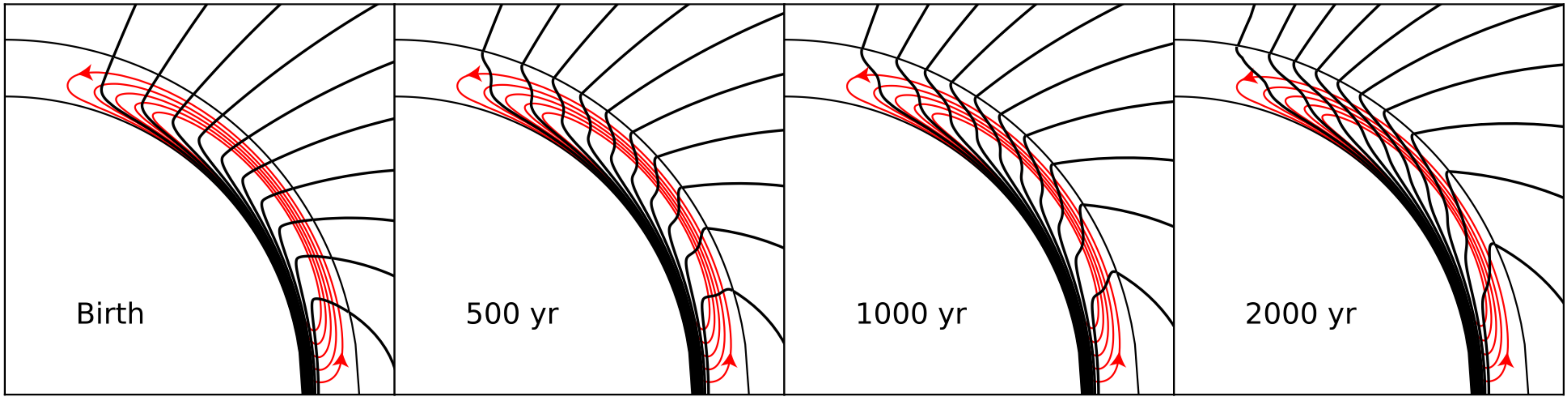
radio pulsar dipole spin down predicts n=3

$$\dot{\Omega} \propto -B^2\Omega^3$$

n<3 => increasing magnetic field over time?

Espinosa et al. (2011)

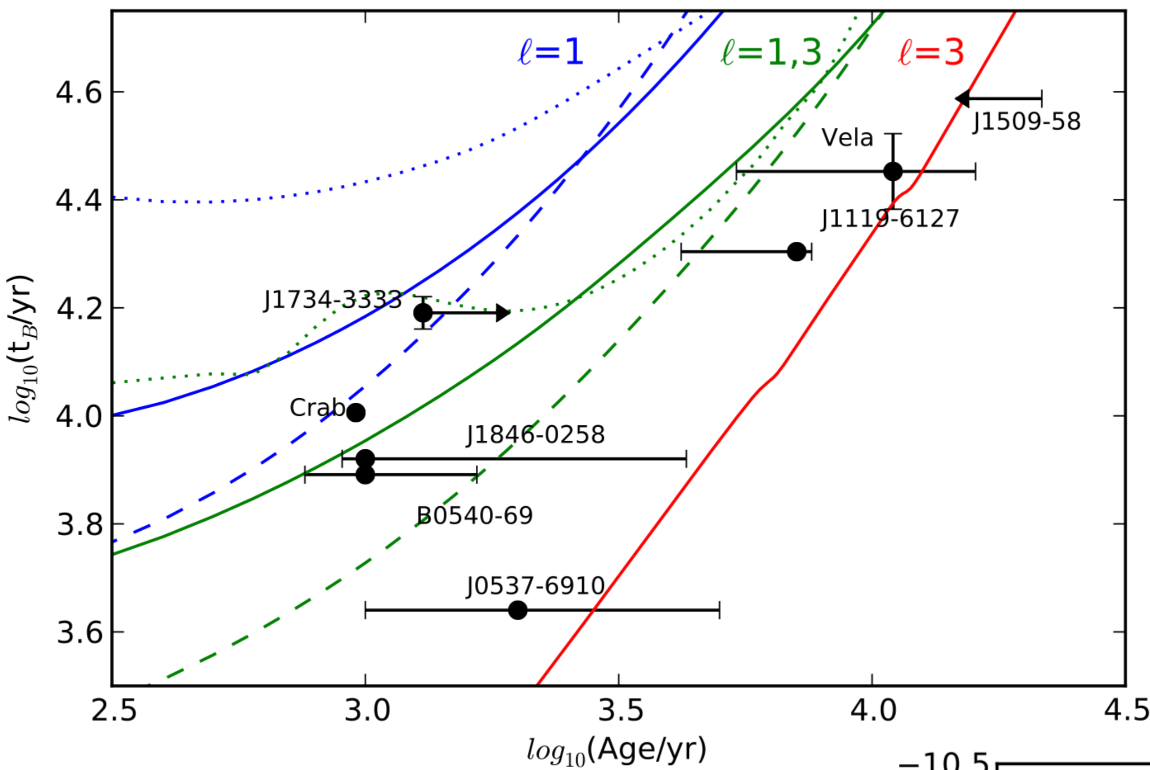
Hall drift and the braking indices of young pulsars

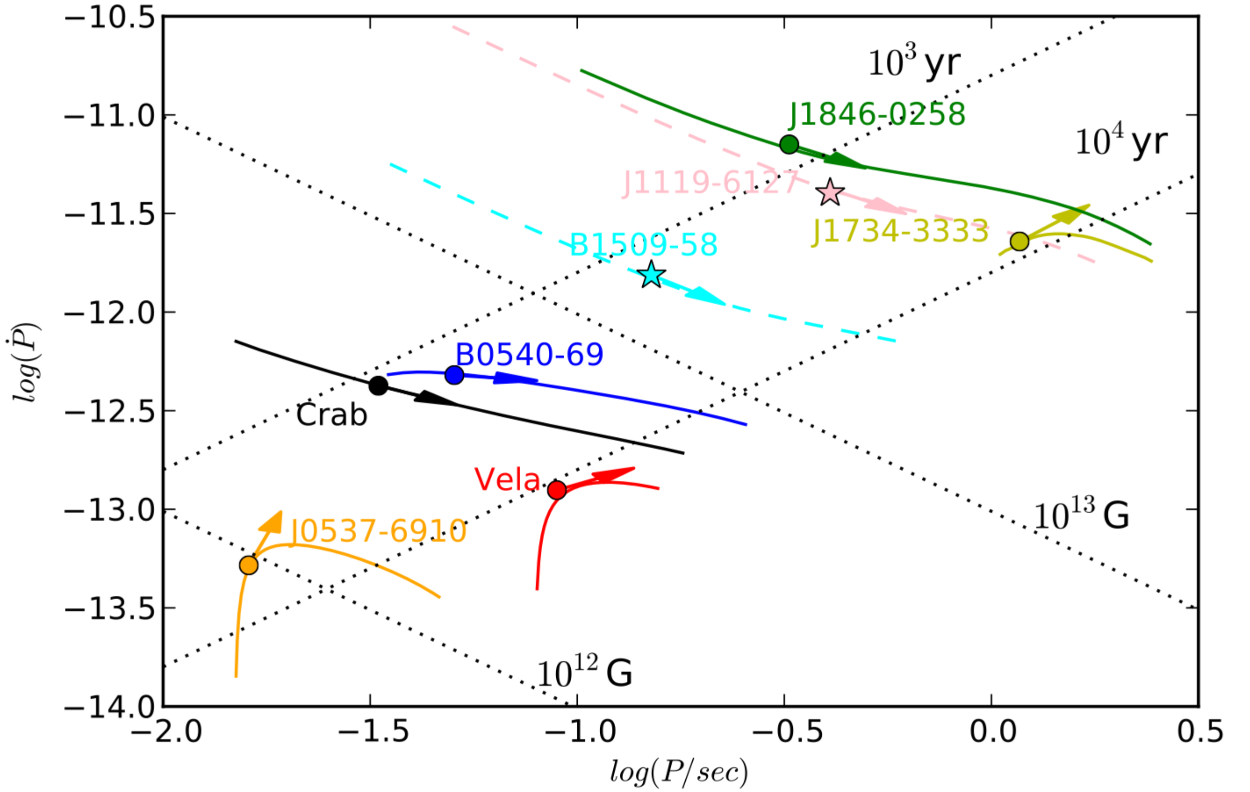


shortest timescale is set by the shear strength of the crust

$$t_{\rm Hall} = 2200 \text{ years} \left(\frac{B_\phi}{10^{13}\text{ G}}\right)^{-1} \left(\frac{B_P}{10^{13}\text{ G}}\right)^{2} \left(\frac{\epsilon}{0.1}\right)^{-1}$$

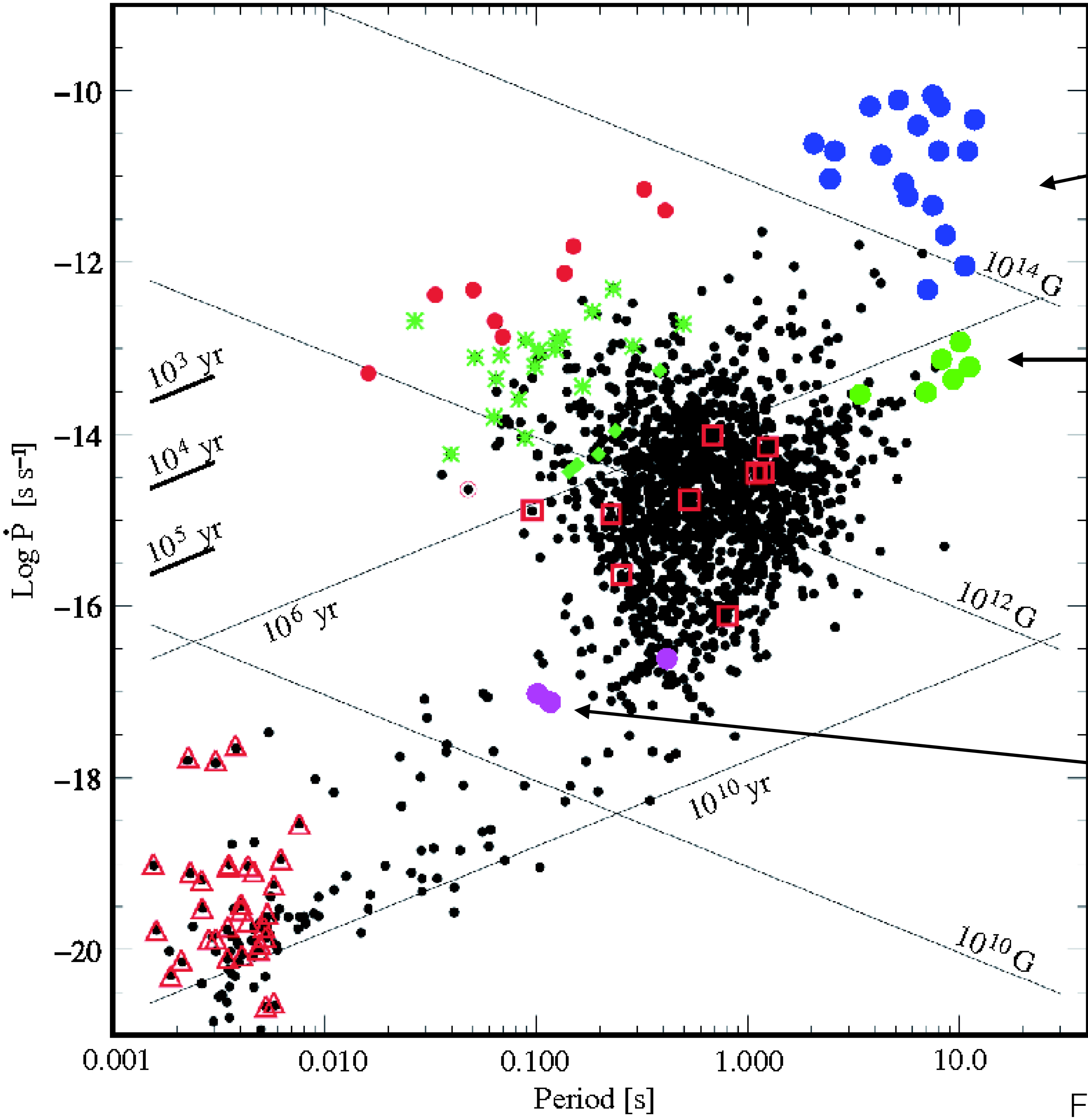
Gourgouliatos & Cumming 2014 arXiv::1406.3640





Gourgouliatos & Cumming 2014 arXiv::1406.

A diversity of neutron stars



magnetars

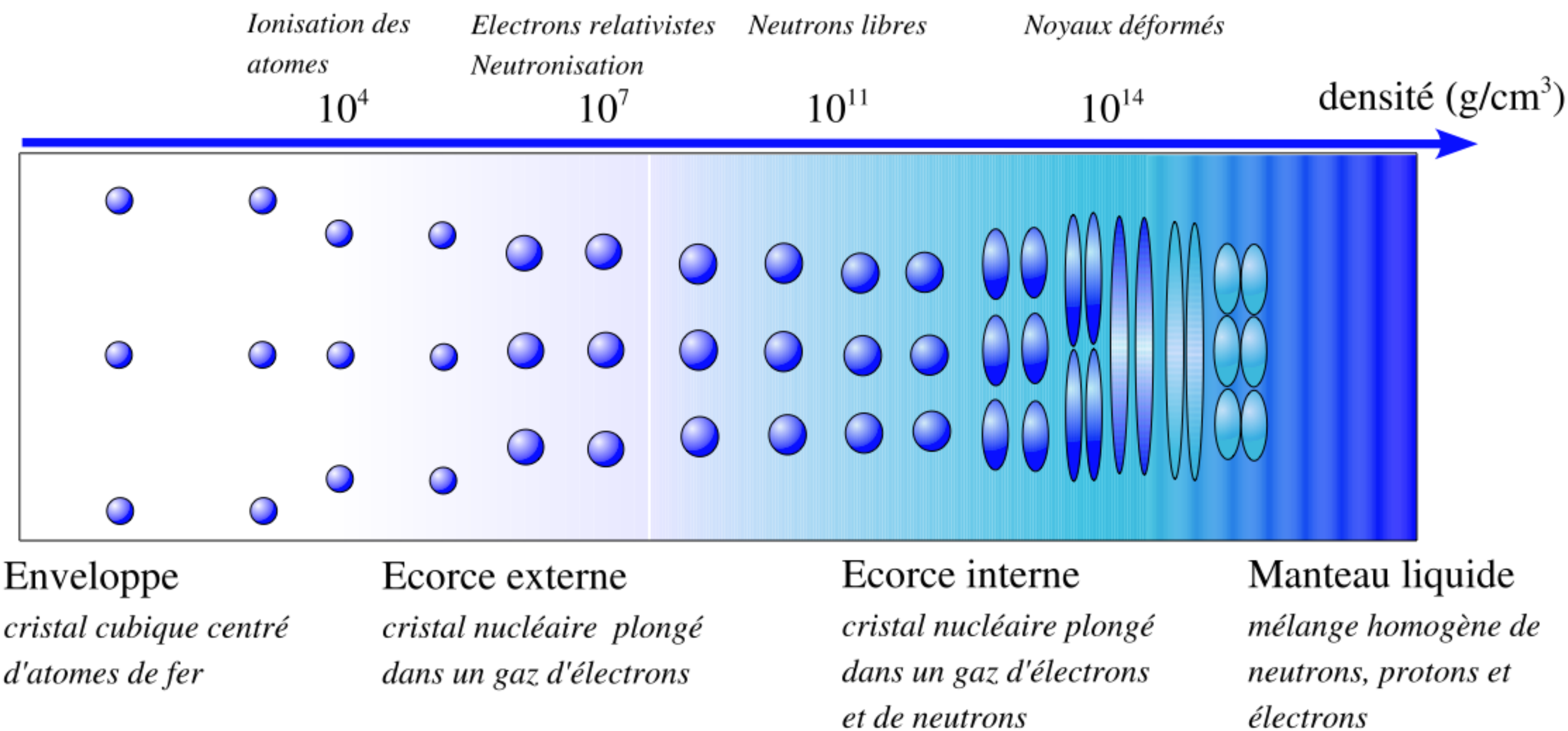
dim isolated neutron stars

RRATs (rotating radio transients)

central compact objects

Figure from Becker et al. (2013)

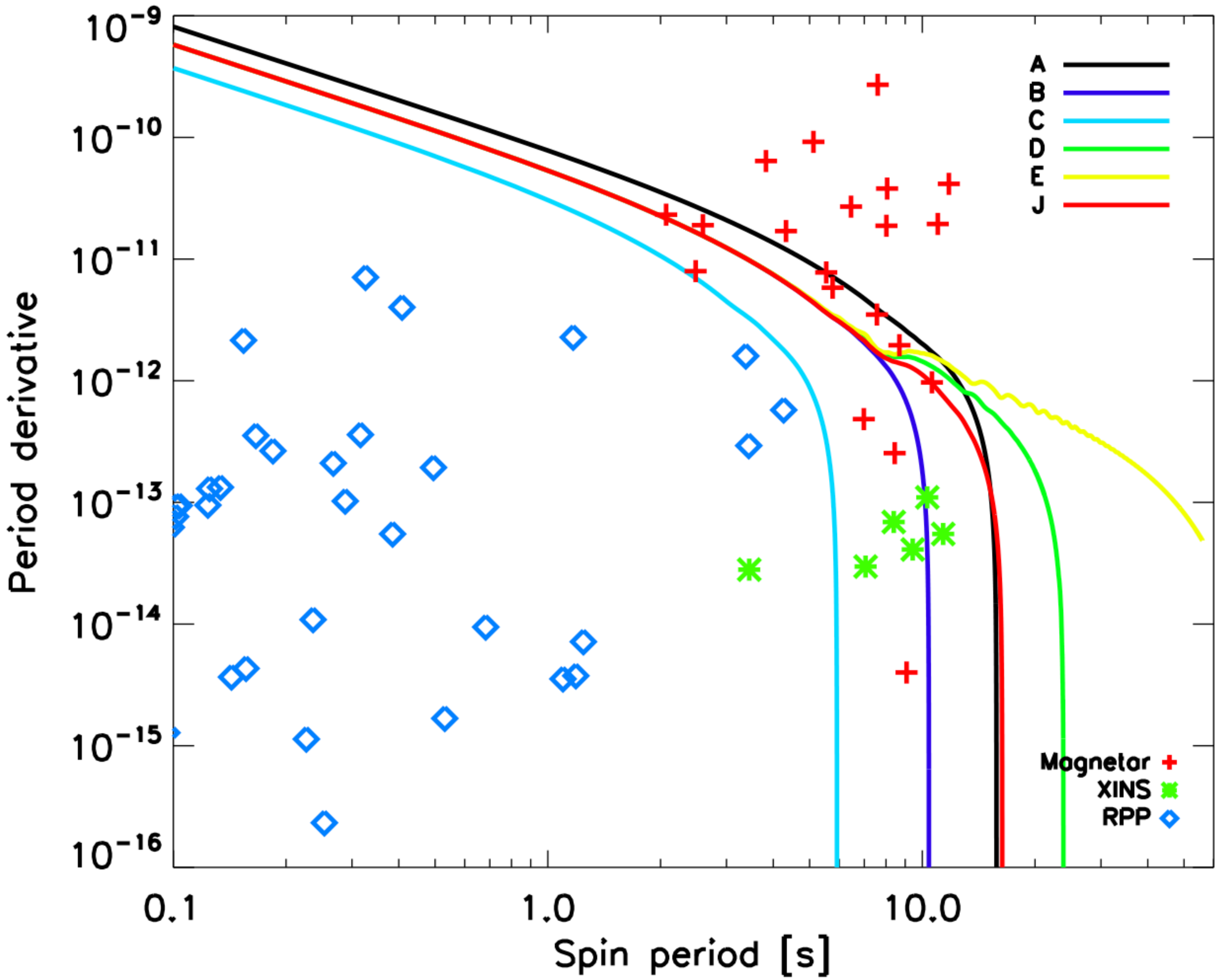
The neutron star crust connects the dense matter interior to the surface



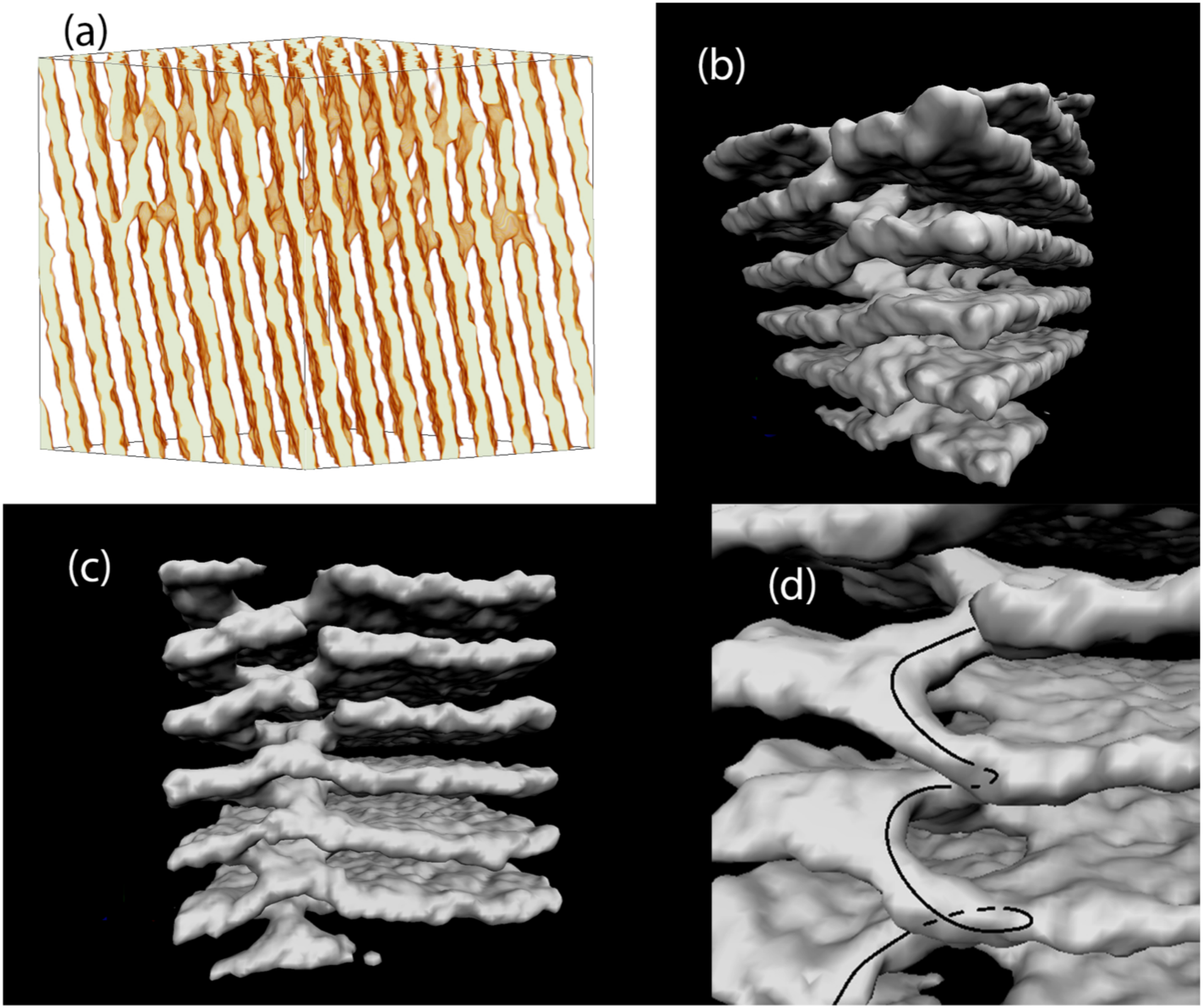
(N. Chamel)

Many unanswered questions about the matter in neutron star crusts: transport properties, physical structure, composition and shape of the nuclei

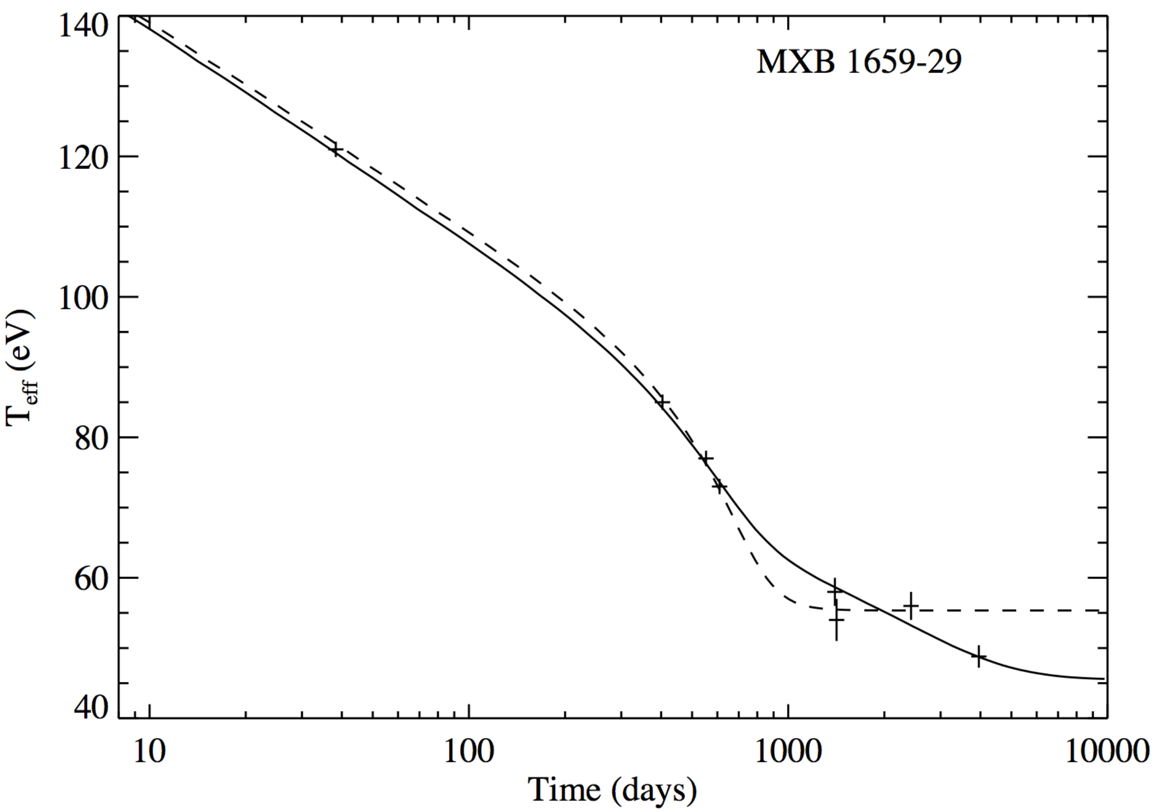
A highly resistive layer of pasta at the base of the crust?

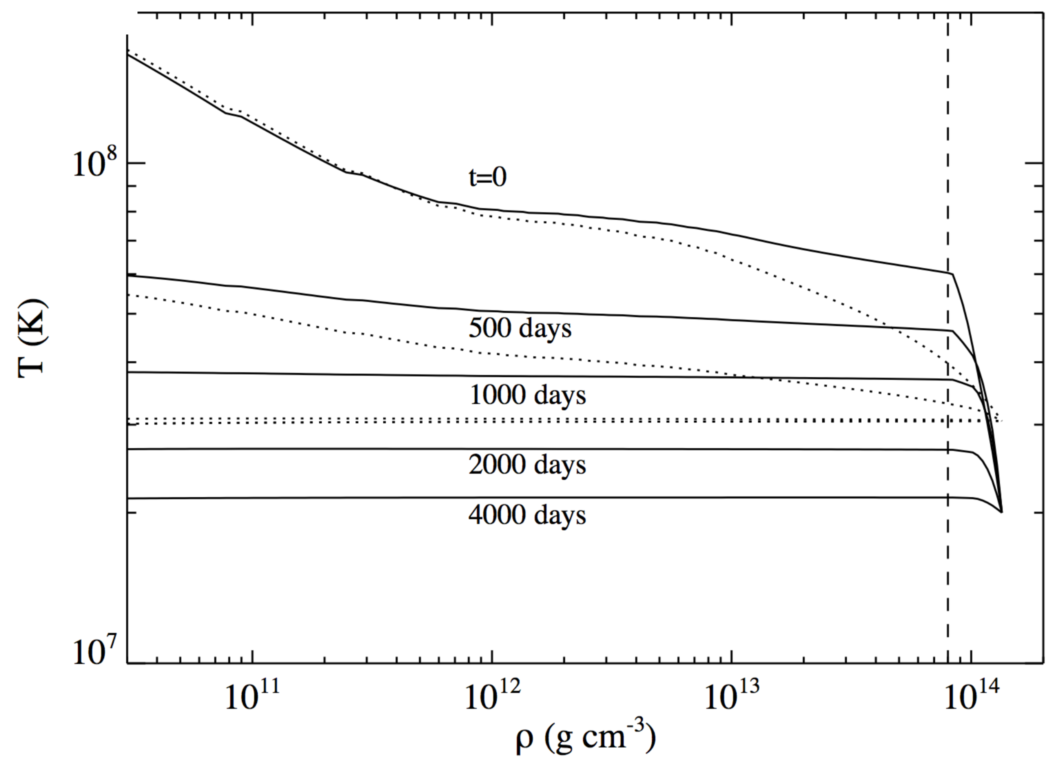


Pons et al. (2013)



Horowitz et al. (2014)





Horowitz et al. (2014)

Summary

- An interesting time to study magnetic field evolution in neutron stars!
- Magnetic field evolution plays a role in phenomenology of a much wider range of neutron stars than previously thought
- Interplay between neutron star interior physics and properties of the population

- Hall evolution in the crust leads to a “Hall attractor”
- Implications:
  - Unique field geometry for middle-aged pulsars? $(\ell,\ell+2)$
  - Long-lived toroidal field not possible in the crust; expect energies <~1%
  - Comparing initial state to Hall attractor gives a measure of “free energy” to power magnetar activity
- Questions: how is the attractor approached (Marchant et al. 2014), what happens in 3D? Interaction with crust yielding/breaking?

- What are observables of field evolution? (Long period cutoff; braking indices, thermal relaxation in magnetars)
- What happens in the “transition objects”? (e.g. weak dipole field magnetars)
- Can we relate to interior physics (e.g. pasta conductivity)