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MULTIFRACTAL ANALYSIS OF RESOLUTION DEPENDENCE IN SATELLITE IMAGERY

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Abstract. Augmenting a satellite's resolution reveals increasingly detailed structures that are found to occupy a decreasing fraction of the image, while simultaneously brightening to compensate. By systematically degrading the resolution of visible and infra red satellite cloud and surface data as well as radar rain data we define resolution-independent co-dimension functions that describe the spatial distribution of image features as well as the resolution dependence of the intensities themselves. The scale invariant functions so obtained fit into theoretically predicted universality classes. These multifractal techniques have implications for our ability to meaningfully estimate cloud brightness fraction, total cloud amount, as well as other remotely sensed quantities. A preliminary account of this work can be found in Gabriel et al., (1988a). See also Gabriel (1988).

Introduction

The problem of resolution dependence of satellite (and other) remote geophysical measurements arises because the relevant emitting and reflecting radiance fields have structures typically down to millimetric scales or less which are considerably below the resolving power of current instruments. With the advent of high resolution civilian satellites such as the Satellite Probatoire d'Observation Terrestre- (SPOT) which has a resolution of 10 meters and with the attendant development of quantitative algorithms for exploiting data, the problem of resolution dependence becomes crucial. For example, the International Satellite Cloud Climatology Project calls for measuring monthly cloud cover from satellite radiances, with accuracies of a few percent. Other studies require the radiation budget of the atmosphere over various spatial scales. Cloud classification algorithms are also known to be strongly resolution dependent (e.g. Hughes (1983)). Shih et al. (1988) show that cloud fractions can frequently vary by factors of two with changes in resolution by a factor of ten. As we will see below, this implies a dimension of 2-log2/log10=1.7 which is not uncommon.

In spite of the growing recognition of the problem, the only systematic studies of resolution dependence have been in the perimeters of brightness regions rather that of the regions themselves (Lovejoy (1982), Rhys and Waldvogel (1986), Yano and Takeuchi (1988), Welch et al. (1988)). Furthermore, attention has been confined to the determination of geometric properties of the perimeters, particularly their characterisation by a single fractal dimension. Below, we extend these studies in two directions. First, using a technique called "functional box-counting" (Lovejoy et al. 1987), we directly investigate the resolution dependence of the bright regions themselves as a function of brightness thresholds, not just their perimeters. Second, we evaluate the codimension function characterising the generator of the multifractal radiance fields, showing that the latter fits into theoretically predicted, two parameter universality classes. Elsewhere (Lovejoy and Schertzer 1988) some of these early studies are re-examined in light of the above. The two main conclusions are that a) the multifractal nature of the brightness regions leads to systematic and large corrections to previous estimates of the fractal dimensions of the perimeters, and b) neglect of the multifractal nature of the fields can lead to spurious breaks in the scaling.

Outline of Multifractal Framework

Consider a satellite with resolution scale L and denote by η the smallest scale of the inhomogeneities of the radiation field with R being its average scale over L (a single image element), R_0 over the entire image and $T'=R/R_0$ is the relative radiance which may exceed unity. We may then define the brightness fraction exceeding a given threshold T by:

$$F_T = \frac{\text{Number of resolution elements } T'>T}{\text{Total image elements}} \qquad (1)$$

and, provided that $L \leq \eta$, F_T will be independent of L. However, clouds are turbulent fields which interact with the radiation field down to scales of the order of millimeters-hence, even for SPOT, $L >> \eta$. More precisely, the inhomogeneities of the various atmospheric fields introduce specific types of strong scale dependencies ("scaling") which are associated with scale invariant quantities (such as fractal dimensions, or spectral slopes...) which typically appear as exponents in power laws of scale ratios. Since the complex radiative transfer processes involve only the dimensionless optical thickness (i.e. these processes do not in themselves break the scaling by introducing a characteristic length) and, the underlying terrain also involves very small scale inhomogeneities, we may also expect the radiance field to have scaling, fractal structures over wide ranges. Alternatively, if considered from the perspective of the photon path length, we expect it to be a random quantity fluctuating down to similar scales. Some relevant power law spectral analyses of cloud liquid water and radiance measurements may be found in King et al. (1981) and Campbell et al. (1988).

We now seek to express F_T in resolution independent terms. Recall that fields with multiple fractal dimensions "multifractals" can be mathematically interpreted (Frisch and Parisi (1985)) as a superposition of singularities of different

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orders (γ) with dimensions D(γ). This is somewhat analogous to the decomposition of functions into sums of sinusoids by Fourier analysis, except that the superposition cannot be interpreted as a sum. By a singularity of order γ, we mean that in the neighbourhood of the singularity, the field diverges as $l^{-\gamma}$ where l is the distance from the singularity.

Indeed, in Schertzer and Lovejoy (1987) it is shown that in multiplicative cascade processes, singularities develop directly as the cascade proceeds to smaller scales. Physically, the cascade concentrates matter, energy and possibly radiation fluxes into smaller and smaller regions of space. Gabriel et al. (1986, 1988a,b) show that radiative transfer in fractal clouds gives rise to multifractal radiation fields. Schertzer and Lovejoy (1987) and Wilson et al. (1988) discuss numerical methods for modelling the passive scalar clouds associated with the different cascade universality classes.

When such a field is averaged by a sensor over scales L, it will therefore measure a radiance proportional to $L^{-\gamma}$ where γ is the dominant singularity in the footprint. Introducing the dimensionless ratio $\lambda = L/L_0$ (<1) where L_0 is the image scale, we obtain $T=\lambda^{-\gamma}$, or:

$$\gamma = -\frac{\log T}{\log \lambda} \qquad (2)$$

We then use (2) to express T, in terms of γ, obtaining F_γ whose scale dependence is in turn described by a simple power law:

$$F_\gamma(\lambda) = \frac{\lambda^{-D(\gamma)}}{\lambda^{-2}} = \lambda^{c(\gamma)} \qquad (3)$$

where we have introduced the completely (scale invariant) codimension function $c(\gamma)=2-D(\gamma)$, where D(γ) is the fractal dimension of the (sparse) regions where T exceeds $\lambda^{-\gamma}$. Note that as the resolution is increased ($\lambda \to 0$), $F_\gamma(\lambda) \to 0$ for all $c(\gamma)>0$, while, $T \to \infty$. This superposition of singularities of different orders is exactly the type of behaviour expected from a multiplicative cascade type process. Study of such processes shows Schertzer and Lovejoy (1987) that c(γ) falls into the the following universality classes:

$$c(\gamma) = c(0)\left(1+\frac{\gamma}{\gamma_0}\right)^{\alpha} \qquad (4)$$

where $\alpha \geq 2$, or $\alpha < 0$ with the value $\alpha=2$ corresponding to the case of gaussian cascade generator, and c(0) and γ_0 are parameters characterising respectively the intermittency and smoothness of the process. This result is the multiplicative analogue of the standard central limit theorem for the addition of random variables. In the latter case, the universality classes comprise both the familiar gaussian as well as the less familiar Stable-Levy distributions of parameter α' where $(\alpha)^{-1} + (\alpha')^{-1} = 1$. Since $0<\alpha' \leq 2$ for stable Levy distributions, we obtain the restriction $\alpha \geq 2$ or $\alpha < 0$ (see Schertzer and Lovejoy 1988). The case $\alpha<0$ involves generators with infinite means, and is not relevant here.

Results

To show empirically that eq. (3) holds, define for a given T (or equivalently γ), a series of lower resolution images (size L) by covering the high resolution image, by disjoint "boxes" (squares here), of size L. This is the "functional box counting" described in Lovejoy et al. (1987) which directly estimates $F_\gamma(\lambda)$ from the number of boxes $N_T(L)$ of size L

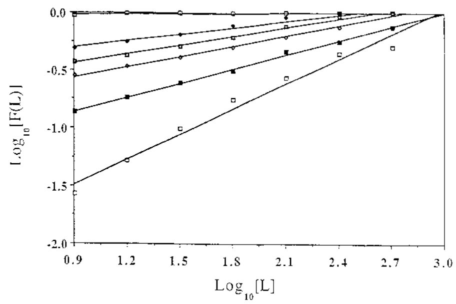

Fig. 1a: A plot of $F_T(L)$ v.s. L for 6 radiance thresholds with L (in km) varying from 8 to 512 km at visible wavelengths. The different symbols, top to bottom, indicate thresholds increasing by intervals of 6 satellite counts (these are proportional to the square root of themeasured radiance). The minimum goes channel count is 24 (ground) and the maximum 52 (bright cloud). Corresponding to these counts, the relative brightness ratio is 4.7. The straight lines indicate that over this range, the scaling is accurately followed. This image was from the Montreal region, summer, and was largely cloud covered.

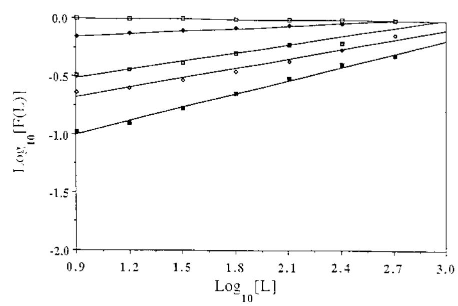

Fig. 1b: The same as 1a, except for the corresponding infra red image. The straight lines are for satellite channel counts of 80, 95, 110, 125 and 160, (top to bottom respectively) corresponding to a black body temperature of roughly 17, 9, 2, -5 and -23 degrees C. Here the lowest radiances (proportional to the fourth power of the temperature) comes from the sparsest (highest) cloud tops.

needed to cover the regions exceeding T ($F_\gamma(\lambda)=N_\gamma(L)/L^{-2}$). From (3), we obtain: $c(\gamma) = -\log F_\gamma(\lambda)/\log \lambda$.

Fig. 1a,b shows the results when the resolution of GOES (geostationary satellite) visible and infra red images are degraded from 8 to 512km on 1024X1024 km image section. The straightness of the lines shows that the scaling is very accurately followed over this range of scales. As indicated in figures 2a,b as T (or γ) increases, the absolute slopes c(γ) monotonically increase: the most intense regions are the most sparse, hence yield the lowest dimensions and highest codimensions. The only exceptions to this fairly accurate scaling were in cases of high thresholds with only small amounts of cloud present in the scene. This is not surprising since theoretically we only expect scaling to hold in the limit of either a very large range of scales or an ensemble of systems each with a limited range of scales.

Since many empirical algorithms designed to exploit remotely sensed data relate radiances above fixed thresholds with specific physical features (such as clouds), it is of some interest to establish whether any such regions can be defined

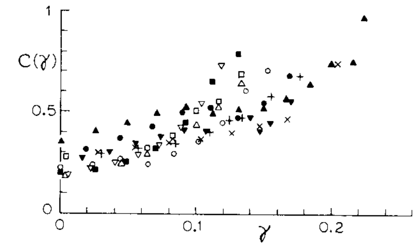

Fig. 2a: The functions c(γ) for the 10 visible cases analysed in the text.

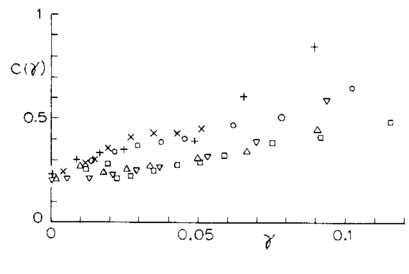

Fig. 2b: Same as 2a, but for the 4 infra red cases.

independently of the satellite resolution (i.e. are there values of γ for which $c(\gamma)\approx 0$ and hence $F_\gamma(\lambda)\approx$ constant?). If any of the radiances used by such an algorithm have $c(\gamma)>0$ then the algorithm will contain hidden resolution dependencies. Accordingly, χ^2 tests were used to test the hypothesis that $F_\gamma(\lambda)=A_\gamma$ where A_γ is the resolution-independent brightness fraction sought. In each of the cases we examined (10 in the visible, 4 in the infra red, spanning nearly cloud free to nearly completely cloud covered situations over Montreal), such a hypothesis could be rejected at very high levels of confidence for all except the very lowest T values (see figure 3 for examples). The results were similar at both wavelengths. <u>For virtually all features of the radiance fields, we could with high confidence, statistically reject the hypothesis that resolution independent brightness fractions exist.</u>

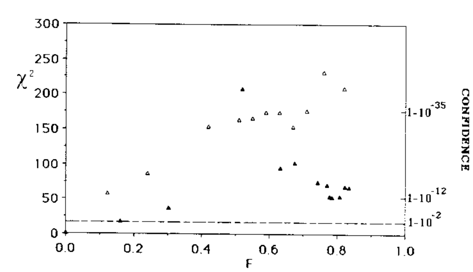

Fig. 3: Results of χ^2 tests calculated on the same satellite derived cloud scenes at visible (unfilled triangles) and infra red (filled triangles) wavelengths with 6 degrees of freedom. The χ^2 statistics provides a means of testing the hypothesis that A_γ is resolution independent. Since in general F_γ is proportional to $l^{c(\gamma)}$, this is equivalent to testing whether $c(\gamma)=0$. Note that the abscissa corresponds to $f_\gamma=1-F_\gamma$, or the fraction of those points which are below a given threshold. The dashed lines correspond to the 0.01 level of significance for 6 degrees of freedom.

We now show that the empirical c(γ) functions fit into the universality classes (4). This is important theoretically because it confirms the predictions of cascade theories (Schertzer and Lovejoy 1985a,b ,1987), and is important practically, because it permits determination of the fundamental parameters characterising c(γ). If there is no universality, then we are left in the unattractive situation in which an infinite number of geometrical parameters (the dimension function) is required to specify the statistical properties of the field. Empirically, the existence of universality classes allows us to determine this entire function by only using only two parameters. Theoretically, universality classes are directly related to the dynamical multiplicative cascade processes themselves.

For the satellite data used here, the empirically accessible range of γ 's is quite small (the maximum is ≈0.4). This makes it difficult to accurately estimate α since the latter measures the concavity of c(γ) which is only pronounced for large γ. The situation is only marginally better for the radar rain reflectivities we examined (using the same data discussed in Lovejoy et al. (1987)), in which a range of γ nearly four times this size was found to be largely compensated by correspondingly larger values of γ_0. To obtain well-defined parameter estimates, we therefore made the plausible assumption that generators were in the gaussian domain of attraction (i.e. α=2). Except for the most extreme fluctuations, the case α=2 is the most consistent with the widespread log-normal phenomenology of cloud statistics. Furthermore, for each satellite (and radar) image, we empirically estimated the parameter γ_0 via a least squares regression usingthe formula $c(\gamma)=c(\gamma)/c(0)$. The standard errors were: ±0.037, ±0.063, ±0.062 for visible, infra red, and radar data respectively. These are all comparable to the errors in estimating (γ) from functional box-counting which is typically of the order ±0.05.

c(0) is the empirically determined co-dimension of the field at image averaged brightness (since $R=R_0\Rightarrow T=1$, $\gamma=0$). We then plot the curves $\langle c(\gamma)_n\rangle$ v.s. $\langle(1+\gamma/\gamma_0)^2\rangle$ fig. 4a,b,c, where the angle brackets indicate ensemble averaging (here over all available cases). As predicted, the curves all closely follow the line x=y (shown for reference). This shows that the main difference between the primarily cloudy, and cloud free cases were in the values of the parameters, which for the visible images were on average 0.21±.07 and 0.41±.16, respectively for γ_0 whereas c(0) was nearly constant (0.21±.04 and 0.27±.04, respectively).

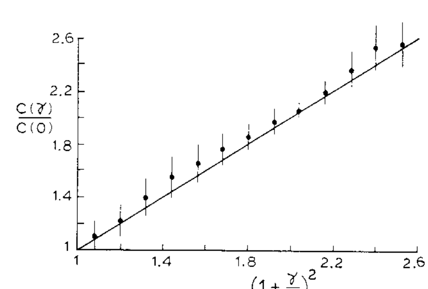

Fig. 4a: The mean normalised codimension, $\langle c(\gamma)_n\rangle$ for the 10 visible cases.

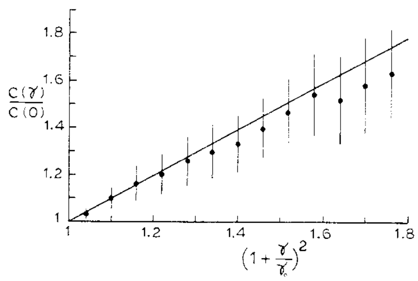

Fig. 4b: Same as 4a, but for the 4 infra red cases.

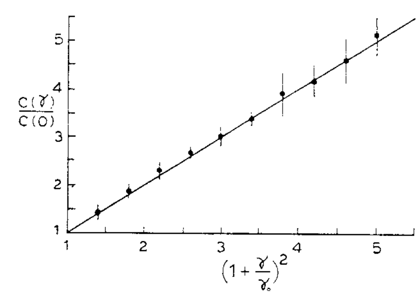

Fig. 4c: Same as 4a, but for 10 radar rain reflectivity fields discussed in detail in Lovejoy et al. (1987).

Conclusions

We have shown clear evidence for the scaling nature of the resolution dependence of cloud, rain and surface features over a wide range of scales, radiance intensities, and at several different wavelengths. These resolution dependencies were quantitatively characterised by the resolution independent co-dimension function $c(\gamma)$. Unlike the radiance fields, physically significant quantities (such as cloud fraction) are resolution independent. Our results therefore underline the necessity of developing resolution independent approaches and algorithms (based on $c(\gamma)$) for exploiting remotely sensed data. Such resolution independent methods will also be important in calibrating remote measurements by sparse in situ networks (Lovejoy et al. 1986a,b). Since we also show that the empirical $c(\gamma)$ functions fit into theoretically predicted universality classes, the number of significant parameters involved may be quite modest.

Due to the complex non-linear interactions between the remotely sensed radiances and the fields of interest, resolution independent approaches must involve the development of continuous multifractal models such as those described in Schertzer and Lovejoy (1987)) and Wilson et al. (1988). This will allow us to explicitely model the associated radiation fields over wide ranges of wavelengths and scales (e.g. as described in Gabriel el al. 1988a,b). Such models will also be important in developing optimal sampling, averaging and calibration procedures. We believe that the results of this paper give a new impetus to the development of multifractal methods for analysing and modelling remotely sensed fields.

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