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SCALE INVARIANCE OF BASALTIC LAVA FLOWS AND THEIR FRACTAL DIMENSIONS

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Abstract. The perimeter and area of basaltic lava flows and flow fields from Piton de la Fournaise (Indian Ocean), Mt Etna (Italy) and Puu Oo (Hawaii) obtained from geological maps have been demonstrated to be scaling (power law) functions of the resolution of geological maps. This fractal property shows that structures of all sizes are important. The fact that distinct fractal dimensions exist for different flows is consistent with the idea that, like the topography, the lava dynamics are multifractal.

Introduction

Lava flows are geological phenomena characterized by complex shapes and structures spanning wide ranges of scale. As with many other geophysical phenomena [Schertzer and Lovejoy, 1991] such as seismic events [Kagan and Jackson, 1991], topography [Mareschal, 1989; Lavallée et al., 1992], fractures [Nolte et al., 1989], turbulence [Schmitt et al., 1992, Sreenivasan, 1991; Schertzer and Lovejoy, 1987, 1985a, b] and ocean waves [Glazman, 1991] we may expect that lava flows have significant scale invariant regimes. Indeed, Kilburn [1990] has argued that from millimetres to tens of centimetres, aa lavas from Mt Etna showed scale invariant "cauliflower" morphologies. The complex morphology of lava flows depends on many factors [Chester et al., 1985; Kilburn, 1991] including rheology, gravity and underlying topography. The rheology is a nonlinear function of temperature and chemical composition as well as volatile, crystal and vesicle content [Chester et al., 1985]; for example, a small difference in the initial temperature may lead to large morphological differences [Archambault and Tanguy, 1976; Pinkerton and Sparks, 1978, Shaw et al., 1968]. Previous studies of lava flows have neglected this lava heterogeneity by using empirical averages (e.g., averaged effusion rates [Chester et al., 1985; Kilburn, 1991; Wadge, 1978; Lopes and Guest, 1982] and volumes [Lopes and Guest, 1982; Malin, 1980]) and homogeneous models [Dragoni et al., 1986].

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Resolution dependence of area and perimeter

Using geological maps, we analysed individual basalt lava flows from Piton de la Fournaise at 1:25 000 scale (Indian Ocean, Stieltjes [1985]) and basalt flow fields from Mt Etna (Italy, Romano et al. [1979]) and Puu Oo (Hawaii, Wolfe [1987]) at 1:50 000 scale. The maps were digitized on 512 by 512 point grids. Our objective was to calculate the areas (A) and perimeters (P) of the lavas at different resolutions. Area sets were defined by all elements on a flow (A), and perimeter sets by all elements on a flow with at least one nearest neighbour not on a flow (P). We then applied "box counting" to each set. Box counting is a standard way of degrading the resolution by covering a set with an increasing number of boxes. The number of area and perimeter boxes, $N_A(L)$ and $N_P(L)$ respectively, were determined over a range of resolution (L) corresponding to 512^2 boxes down to 1 box for each flow (Figure 1); this is equivalent to using maps at different scales. Fractal sets follow the general scaling law $N(L) \propto L^{-D}$ where D is the scale invariant fractal dimension. This is equivalent to the following scaling relations for the area A(L) and the perimeter P(L) of basaltic lava flows at different L:

$$A(L)= N_A(L)\, L^2 \propto L^{2-D_A} \qquad (1)$$

$$P(L)= N_P(L)\, L \propto L^{1-D_P} \qquad (2)$$

Plots of logA(L) vs log L and logP(L) vs logL will therefore be linear. Figure 2 (Etna, 1900-74) confirms that such a behaviour is respected for L varying from 43 m to 22 km with $D_A \approx 1.58$ and $D_P \approx 1.42$. The small amount of scatter about the fitted lines is to be expected since D_A and D_P are statistical exponents; exact scale invariance only holds over a group of statistically identical flows.

In this example, A is proportional to $L^{0.42}$ and P to $L^{-0.42}$. If this behaviour is continued to arbitrarily high resolution (smaller L), since $D_A<2$ and $D_P>1$, A would tend to zero and P to infinity, varying continuously with the resolution. This would be the case for a mathematical fractal. In real lava flows, however, as generally in geophysical fractals, equations 1 and 2 break down at sufficiently small (but yet unknown) L; the true A and P will be determined by D_A and D_P and this inner limit. This breakdown presumably corresponds to scales comparable to the width of the thinnest

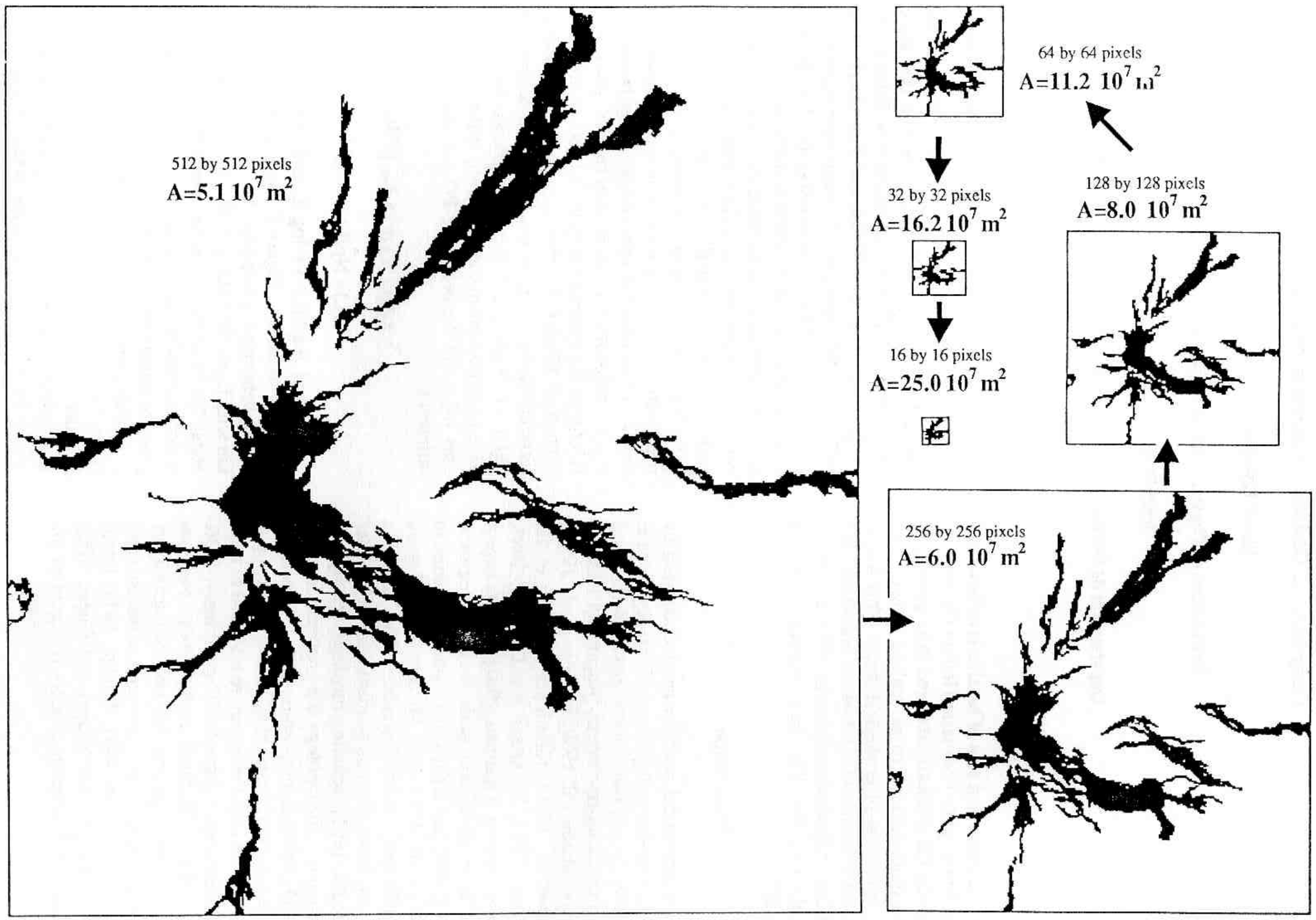

Fig. 1: Sketch of the lava flow field from Etna (1900-1974) showing resolution degradation in six steps from a grid of 512 by 512 boxes (resolution of 43 meters) to a grid of 16 by 16 boxes (resolution of 1376 meters). As resolution decreases, area (A) increases. A was calculated from eq. 1.

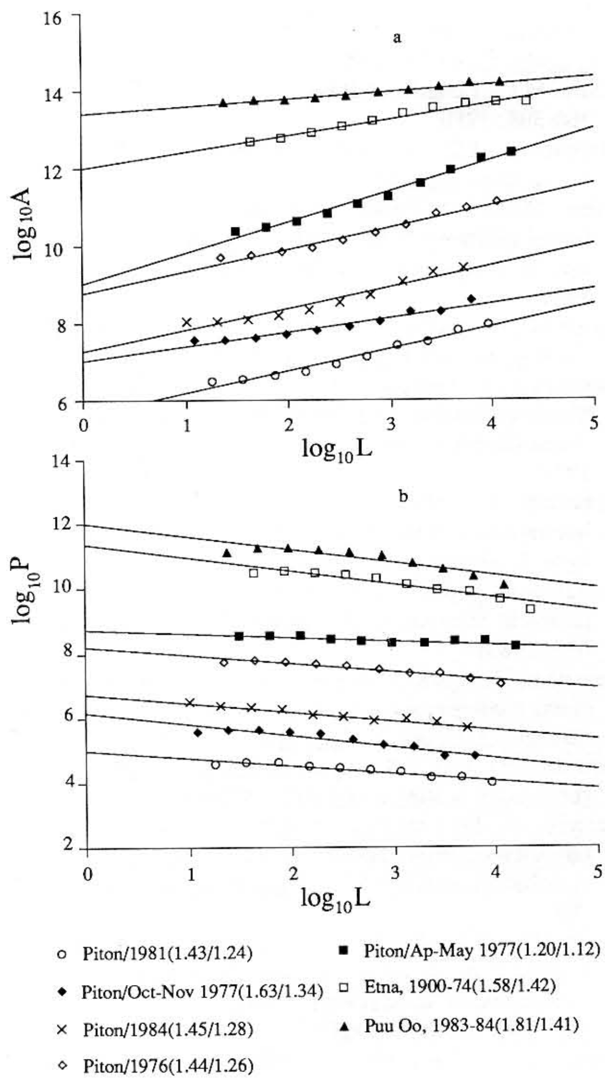

Fig. 2: (a). $\log_{10}$-$\log_{10}$ diagram of A(L) vs L for lava flows of Piton de la Fournaise, 1984,1981, October/November 1977, April/May 1977, 1976, and lava flow fields of Etna (1900-1974) and Puu Oo (1983-1984). For clarity the different flows were separated as logA, logA+1, logA+2, logA+3, logA+4, logA+5 and logA+6 for the successive examples. D_A is the first term in brackets; (b). $\log_{10}$-$\log_{10}$ diagram of P(L) vs L for the lava flows as in a. For clarity the different flows were separated as logP, logP+1, logP+2, logP+3, logP+4, logP+5 and logP+6 for the successive examples. D_p is the second term in brackets.

flow sections (the thinnest channels, levees, lobes). For example, lava flows mapped at L=43 m (512^2 pixels) and L=1.4 km (16^2pixels) yielded values of A differing by a factor of about 5 (Figure 1); if the inner limiting size of lava heterogeneity (the largest size of homogeneity) were 32 (=512/16) times smaller (≈1 m), than the 43 m map, the area would be differ by a further factor of 5. Judging by field evidence of lava channel widths from many flows, we expect this inner limit to be in the range of 1-5 meters or less. The scaling of the areas has obvious implications for estimates of lava volumes and eruption rates which are usually inferred from the areas by assuming constant average lava depths. Such estimates will have little meaning if either the resolution at which they were obtained or the smallest size of the heterogeneity are unknown.

Our results reflect the importance of all structures from large to small. This is to be expected since scaling laws and fractal dimensions are naturally associated with phenomena such as turbulence which are produced by strong nonlinear dynamics characterized by heterogeneous structures: in basaltic lava flows, these are bifurcations, breaches and overflows [Chester et al., 1985; Kilburn, 1991]. In contrast, the behaviour of a laminar (non-fractal) lava flow is fundamentally different since it leads to the usual dimensions $D_A = 2$ and $D_P=1$ and (according to eqs. 1, 2) the resolution independence of A and P.

Multifractal aspects

The general theoretical framework for scale invariant fields (and hence presumably the dynamics) is multifractals whereas for scale invariant sets, it is fractals. Although the lava flows themselves are fractal sets (a given point is either on a flow or not), the dynamics depend on various fields with values varying from point to point such as topography, temperature, crystal content, etc.; a priori they are therefore multifractal. Monofractal dynamics would be a very unusual special case. Indeed, Lovejoy and Schertzer (1990) and Lavallée et al. (1991) have shown that the fractal dimension of regions exceeding a fixed altitude decreases with the altitude threshold; the topography is therefore multifractal (the latter even indicates that it is a special "universal" multifractal). Finally, our fractal dimensions varying from 1.20 to 1.81 (D_A) and from 1.12 to 1.42 (D_P, see Figure 2) are quite consistent with multifractal dynamics. We found no obvious explanation for the precise way the different fractal dimensions varied with respect to either the composition of the flows (tholeiitic for Puu Oo, tholeiitic to alkaline for Piton de la Fournaise and alkaline for Etna) or with the average slopes of the underlying ground. For example, two flows from Piton de la Fournaise with virtually identical local slopes and erupted at the same elevation have different D_A (1.63, 1.44 for Oct-Nov 1977 and 1976). Perhaps the results are not too surprising since multifractal dynamics are inherently extremely variable (intermittent): the same process typically leads to results with large variations.

It is possible that lava flows (as in Figure 1) can be modelled as viscous fingers [Daccord et al., 1986] although on a multifractal support. Indeed, the similarity between viscous fingers and volcano morphology may have a dynamical basis. This is possible because viscous finger dynamics are modelled by the Laplace equation. Recently Bonafede and Boschi (1991) have also used Laplace equations (but with different boundary conditions) for a porous medium volcano morphology model.

Conclusions

By studying the complexity and heterogeneity of basaltic lava flows, we have found that areas and perimeters are power law functions of the resolution over a range of 12 m to 50 km. Although the scale invariance is well respected, the fractal dimensions vary from one lava flow to another. Knowledge of these dimensions and the inner limit where the scaling breaks down is necessary for estimating areas, perimeters and other characteristics. Finally, the observed spatial scale invariance should be accounted for in dynamic models of basaltic lava flows; such models must incorporate strong nonlinearities (not necessarily associated with advection) and will yield highly non-laminar (non-smooth) flows.

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