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ELLIPTICAL TURBULENCE IN THE ATMOSPHERE

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ABSTRACT

We present a new scheme of atmospheric turbulence: small scales structures are continuously flattened at larger and larger scales. This continuous deformation may be characterised by defining an " elliptical dimension " D_{el} . We show both theoretically and empirically that $D_{el} = 23/9 \simeq 2.56$. Atmospheric motions are therefore never " flat " ($D_{el}=2$), nor isotropic ($D_{el}= 3$).

Intermittency, in the context of this " elliptical turbulence " is discussed: dimension of the support of turbulence and divergence of higher statistical moments of the different fields.

INTRODUCTION

The classical scheme of atmospheric motions (e.g. Monin (1972)), considers the large scale as two-dimensional, and the small scale as three-dimensional, a transition named, for obvious reasons, a " dimensional transition" by Schertzer and Lovejoy (1983), is expected to occur in the meso-scale, possibly in association with a "meso-scale gap" (Van der Hoven (1957)).

This scheme favours the simplistic idea that at planetary scales the atmosphere look like a thin envelope, whereas at human scales, it looks more like an isotropic volume.

A dimensional transition, if it were to occur, would be likely to have fairly drastic consequences because of the significant qualitative difference of turbulence in two and three dimensions (Fjortoft (1953), Kraichnan (1967), Batchelor (1969)): the all important streching and folding of vortex tubes, in three dimensions cannot occurs in two dimension.

Since the 50's, there has been a wide debate over the effective dimension of atmospheric turbulence, due in particular to the extension of two-dimensional results to the case of quasi-geostrophy (Charney (1971), Herring (1980)).

Although, a dimensional transition should be readily observable, experiments over the last 15 years have failed to detect it (Pinus (1968), Vinnechenko (1969), Morel and Larchevêque (1974), Macpherson and Issac(1977), Gage (1979), Gilet et Al. (1980), Lovejoy (1982), Larsen et Al. (1982), Lilly and Peterson (1983), Nostrum and Gage (1983)). There is a large body of evidence consistent with a uniform scaling on the horizontal up to, at least 1500 km.

Schertzer and Lovejoy (1983) have examined the different theoretical ideas underlying these experimentals results. They pointed out that the prevalence of this uniform scaling should be connected with the fact that many non-linear equations do not introduce a characteristic length, and thus admit scaling solutions.

In principle, this scaling could be broken either by non-scaling boundary conditions or a non-scaling forcing. However, Mandelbrot (1982) has found evidence that the topography is scaling up to planetary scales, and different analyses (Gautier (1982-personnal communication), Lovejoy(1982), Schertzer and Simonin (1982), and Simonin (1982)) of the sources and sinks of diabatic heating indicate also a scaling behaviour.

Because of the non-linear coupling between the different meteorological fields, the exisence of a characteristic length scale in one, is likely to manifest itself in the others. It is therefore likely that over a given range all fields are scaling.

We may add that our current understanding of intermittency (e.g. Batchelor (1969) or Curry et Al.(1982)) as the frequent transitions between quiescence and chaos leads us to doubt the existence of a well-defined transition (such as the " meso-scale gap")).

THE VERTICAL STRUCTURE

Perhaps the most serious objection to the hypothesis of a scaling behaviour of atmospheric motions arises from the special role of the vertical axis. Indeed, there has been a deluge of papers based on non-scaling techniques which reject implicitely a priori any possibility of vertical scaling (e.g. " one point closures "). In what follows, it will be apparent that this rejection has had unfortunate consequences.

The vertical direction plays a key role for the following reasons:

i) The gravity field defines a direction at every point.
ii) The atmosphere is globally stratified.
iii) It has a well defined thickness (exponential decrease of the mean pressure).
iv) The fundamental sources of disturbances are the vertical shear and the buoyancy force (e.g. the Kelvin-Helmotz instability).

In the following, we examine the possibility that the atmosphere is in fact scaling in the vertical as well as in the horizontal direction. To do so, we examine the wind and temperature fields, attempting to capture two basic and conceptually distinct properties of these fields:

Scaling: The scaling relation relates the fluctuations ΔX of a field X for large scales $\lambda\Delta z$ ($\lambda>>1$) to the small scales Δz by:

$$X(\lambda\Delta z) \overset{d}{=} \lambda^H \Delta X(\Delta z)$$

where H is the scaling parameter and "$\overset{d}{=}$" means equality in probability distributions. Note that the exponent of the corresponding power spectrum ($-\beta$) is related to H by $\beta = 2H + 1$, in the case of finite variance.

Intermittency: This is directly connected with the probability law. One is particularly interested in the tail of this law, since it controls the relative frequency of the extreme (intermittent) behaviour. For instance, if the distribution has an algebraic fall-off at large fluctuations, then the degree of intermittency can be characterised by the exponent α (the hyperbolicity):

$$Pr(\Delta X' > \Delta X) \sim \Delta X^{-\alpha}; \quad \Delta X \gg 1$$

where "Pr" denotes "probability".

This kind of distributions have been invoked in other fields of physics (e.g. " Holtsmark distribution", Feller (1971), see Mandelbrot (1982) for other examples), and are usualy called " hyperbolic distributions". Behaviour of this sort was predicted for the non-linear flux of energy in turbulence according to a phenomenological model of intermittency (Mandelbrot(1974)).

Note that, in the case of hyperbolic distributions, all moments of order α or higher diverge, a fact that that has important consequences. Levy (1937) and Feller (1971) are standard texts, in the case $\alpha < 2$, (cf. the Levy-stable laws which form a convolution semi-group).

We shall primarly be interested in the vertical fluctuations of the horizontal velocity field (dv) and in the buoyancy force per unit mass acting across a layer of thickness dz : $df = gd\,Ln\,\theta$, where θ is the potential temperature, and g the accelaration of gravity. These quantities are related to two fundamental frequencies: that of the vertical shear (s) and the Brunt-Vaisala frequency (n):

$$s = dv/dz \text{ and } n^2 = gdLn\theta/dz = df/dz$$

The ratio of the squares of these freqüencies defines the dimensionless Richardson number :

$$Ri = n^2/s^2$$

The shear frequency characterises the dynamical processes, and the Brunt-Vaisala frequency, the stability (and gravity waves). The dominant process has the highest frequency. To determine their scaling regime, Fourier analysis could be used. Here, we analyse directly the scaling of the probability law by measuring quantities across atmospheric layers of thickness Δz. This has the advantage that it enables the scaling parameters (H's) and the hyperbolicities (α's) to be obtained simultaneously. We therefore define:

$$s^2(\Delta z) = v^2(\Delta z)/\Delta z^2; \quad n^2(\Delta z) = g\,\Delta Ln\theta(\Delta z)/\Delta z$$
$$Ri(\Delta z) = n^2(\Delta z)/s^2(\Delta z)$$

DATA ANALYSIS

S, n, Ri were evaluated from the high resolution radiosonde data obtained in the 1975 experiment in Landes, France. θ, $\underline{v}$ and the humidity were obtained every second ($\sim$3m in the vertical) and processed to yield low noise data every 5s (15-20m, see Tardieu (1979) for more details). All estimates of n, s, Ri were made over layers at least 50m. thick, and from the ground up to the arbitrary height of 6km. The data examined are from 80 soundings taken at 3 hour intervals at Landes.

From the Log-Log plots shown in Fig. 1, 2, 3 it can easily be verified that the probability distributions of Δv, Δf and Ri exhibit both scaling and hyperbolic behaviour. The easiest way to see this, is to recall that for hyperbolic distributions :

$$Pr(\Delta X' > \Delta X) \sim (\Delta X/\Delta X^*)^{-\alpha_X}$$

ΔX^* is the "width" of the distribution, or the amplitude of the fluctuations. Scaling implies then that the width grows with the separation as :

$$\Delta X^*(\Delta z) \sim \Delta z^H$$

this is observed by the constant shift H Log2 for each doubling of the separation Δz. The value $(-\alpha)$ is the slope of the straight line asymptote.

We obtain:

$$H_v \simeq 3/5 \qquad \alpha_v \simeq 5$$
$$H_{Ln\,\theta} \simeq 9/10 \qquad \alpha_{Ln\,\theta} \simeq 10/3$$
$$H_{Ri} \simeq 1 \qquad \alpha_{Ri} \simeq 1$$

The H's and α's are given rational expressions, since, as explained below, they can often be deduced by dimensional considerations.

Another quantity of interest is the flux of non-linear transfer of energy (ε). This is the fundamental dynamical quantity in a turbulent cascade of energy from large to small scales. We obtained a probability distribution by reploting Merceret's aircraft data. The result, see Fig. 4, leads to:

$$Pr(\bar{\varepsilon}' > \bar{\varepsilon}) \sim \bar{\varepsilon}^{-\alpha_{\bar{\varepsilon}}} \quad \alpha_{\bar{\varepsilon}} \sim 5/3$$

Merceret (1976) obtained his value of $\bar{\varepsilon}$ by calculating averaged spectrum of horizontal wind fluctuations every second ($\sim$ 100m).

We assume this average to have been taken over an horizontal straight line (the bar notation indicates a one dimensional spatial average).

THE TURBULENT RICHARDSON NUMBER

The fluctuations fo the Richardson number are very large, since $\alpha_{Ri} \sim 1$, even its mean may not converge (if $\alpha_{Ri} \leq 1$). This fact seems to have been recognised since a long time if one considers the series of "modified" Richardson numbers (based on the ratio of two statistics and not the ratio ot the random variables s and n), for instance the Richardson of flux R_f.

The erratic nature of Ri is directly related to the phenomenon of intermittency, since it controls the onset of turbulence. Its law can be easily understood since s and n are weakly correlated ($\rho = .048 \pm .018$) and have a non-zero probability density at the origin (e.g. Student's distribution) and thus lead to a Cauchy-type law.

ELLIPTICAL TURBULENCE

Considering the velocity-field, we find an exponent of the vertical scaling: $H_v = 3/5$. This exponent is confirmed by Adelfang (197?) up to 14 km, and Endlich et Al.(1969) found $\beta = 5/2$ (thus , $H = 3/4$) up to 16 km. The slight discrepancy of the latter result seems due to considerable interpolation of the data. In any case, no evidence of characteristic vertical length scales is found.

A similar result was predicted almost 25 years ago for the directionaly averaged spectrum in the so-called "buoyancy subrange", by Obukhov (1959), and independenly by Bogliano (1959). The vertical scaling can be thus deduced by the same derivation or by considering directly the physically meaningful quantity ϕ the flux of buoyancy force variance . The latter derivation has the advantage that it does not depend on the Boussinesq approximation:

$$\bar{\phi}(\Delta z) = \bar{\tau}^{-1}(\Delta z)\, \Delta f^2(\Delta z)$$

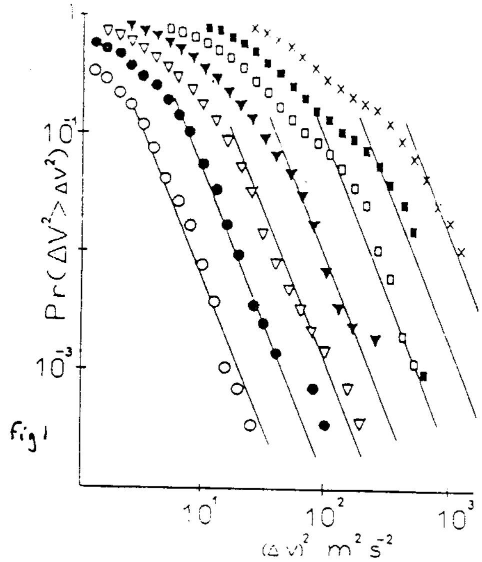
where $\bar{\tau}(\Delta z)$ is a characteristic time for the transfer process. Dimensional analysis yields :

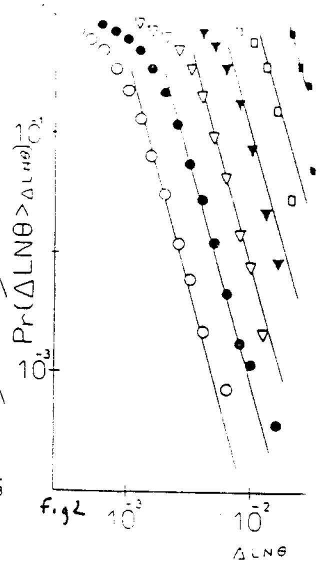
$$\Delta v(\Delta z) \overset{d}{=} \bar{\phi}(\Delta z)^{1/5} \Delta z^{3/5} \qquad (1)$$

While the quite different Kolmogorov scaling is supposed to hold in the horizontal:

$$\Delta v(\Delta x) \overset{d}{=} \bar{\varepsilon}(\Delta x)^{1/3} \Delta x^{1/3} \qquad (2)$$

Objects which scale in the same way in all directions are called self-similar fractals because the large scale can be simply viewed as a magnification of the small scale. In the atmosphere, we have argued that scaling, although present in all directions, and over a wide range of lengths, is quite different in the vertical and horizontal. Large scale structures can no longer be





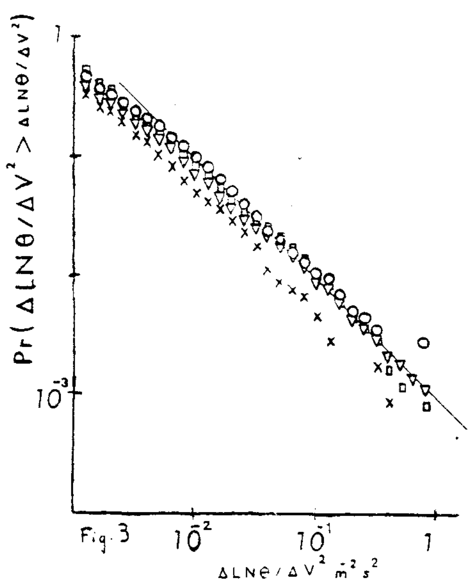


Fig.1 : The probability distribution of fluctuations in the quantity $\Delta v^2(\Delta z)$ where v is the horizontal velocity, for different vertical layers as follows : O : $\Delta z =$ 50m, ●: $\Delta z =$ 100m, ▽ : $\Delta z =$ 200m, ▼: $\Delta z =$ 400m, □: $\Delta z =$ 800m, ■: $\Delta z =$ 1600m, X : $\Delta z =$ 3200m.

Fig.2: The probability distribution of fluctuations of th buoyancy force . Same symbols as in Fig.1.

Fig.3: The probability distribution of fluctuations in the quatity $\Delta \mathrm{Ln}\ \theta(\Delta z) / \Delta v^2 = \mathrm{Ri}\,\Delta z^{-1}$ Ri being the Richardson number. Same symbols as in the previous figures. The curves for 100, 400, 1600m have been suppressed for clarity of presentation.

simply regarded as large-scale copies of smaller ones. In addition to magnification, we must also stretch. This can be expressed as follows :

$$\Delta \underline{v}(\underline{\underline{G}}_\lambda . \Delta \underline{x}) \overset{d}{=} \lambda^H \Delta \underline{v}(\Delta \underline{x}) \tag{3}$$

($\underline{x} = (x, y, z)$), the group ($\underline{\underline{G}}$, λ^H) is a kind of renormalization group, and behaves as :

$$\underline{\underline{G}}_\lambda \sim \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{H_z} \end{bmatrix} \quad \text{with } H_z = H/H_v$$

It introduces an elliptical geometry, futher exploited in Schertzer et Al.(1983) with more general expression of $\underline{\underline{G}}_\lambda$. Here, $H = 1/3/3/5 = 5/9 \Rightarrow D_{el} = 23/9$.

Fig. 5 shows how a small vertically oriented cross-section of an eddy is transformed at larger scales. The magnification and stretching process transforms the vertically oriented "convective" type eddy into a large horizontally oriented "Hadley" type eddy. Note that $\underline{\underline{G}}$ transforms the statistical properties of the eddies.

This thransformation increases the volume of an eddy by the factor :

$$\lambda . \lambda . \lambda^{H_z} = \lambda^{D_{el}} \; ; \; D_{el} = 2 + H_z$$

In an isotropic three dimensional turbulence, $H_z = 1$ $D_{el} = 3$, and in the isotropic two-dimensional case, $H_z = 0$ and $D_{el} = 2$. Writting the above relationship in the form :

$$\mathrm{Det}\ \underline{\underline{G}} = \lambda^{D_{el}}$$

we are lead to the more general definition of D_{el} :

$$D_{el} = \mathrm{Tr}(d\underline{\underline{G}}_\lambda / d\lambda)_{\lambda=1}$$

The number $N(\ell)$ of eddies of horizontal scale ℓ may now be written :

$$N(\ell) \sim \ell^{-D_{el}}$$

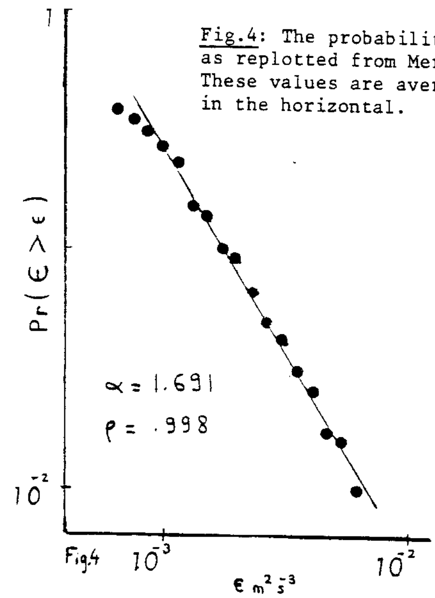


Fig.4: The probability distribution of ϵ as replotted from Merceret (1976) Fig.14. These values are averages over ~ 100m in the horizontal.

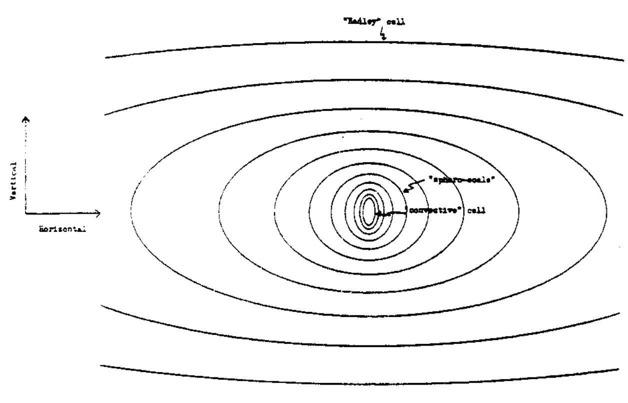


Fig.5: Isocontours of $\underline{\underline{G}}$, which may be interpreted as the shapes of the vertical cross-sections of averaged eddies at different scales. "Hadley cell and "convective cell" are suggestive labels for very large, and respectively very small structures.

THE SPHERO-SCALE

The distinction between isotropy and anisotropy is profound. An horizontal cross-section will have an area proportional to ℓ^2 whereas a vertical cross-section will have an area proportional to ℓ^{1+H_z}. Their ratio gives a quantitative measure of the verticalness of the vortex.

There therefore exists a scale at which this ratio is 1, the turbulence appears the as isotropic three-dimensional : the horizontal and vertical fluctuations have the same amplitude. This scale, that we call "sphero-scale" for obvious reasons, will depend on the relative fluctuations of $\bar{\varepsilon}$ and $\bar{\phi}$ (due to equations 1 and 2) both of which show large fluctuations. The size of this scale may vary in an extremely erratic manner, unless $\bar{\varepsilon}$ and ϕ are totaly dependent.

STOCHASTIC STRATIFICATION

By the action of $\underline{G}_\lambda$, stratification may be seen rather as the result of a stochastic process, hence a " stochastic stratification ". Fig. 6a shows an isotropic gaussian field with H = 1/2 and D_{el} = 2, Fig.6b shows the corresponding field for D_{el} = 3/2 (the same white noise field was used in both cases to generate via Fourier transforms, the random fields).

Investigations of this process for the logarithm of the pressure and the temperature will be pursued elsewhere. Note that the hydrostatic relation is recovered by assuming that the vertical scaling of Log p has the parameter: $H_{v(\mathrm{Log}\ p)} = 1$.

INTERMITTENCY

Intermittency expresses the fact, that , roughly speaking, the turbulence doesn't fill all the volume of the space available to it , the "spottiness" of turbulence (Batchelor and Townsend(1949), Batchelor (1953)). This is related to Leray's (1933) conjecture on the existence of a set of singularities of the Euler equations.

Kolmogorov (1962) and Yaglom (1966) presented a corrected spectrum to take into account intermittency, by assuming a log-normal distribution of $\bar{\varepsilon}$. Orszag (1970), Mandelbrot (1974) have pointed out several theoretical difficulties with this hypothesis.

In particular, Mandelbrot, building up on an earlier, explicit model for " spottiness " (Novikov and Steward (1964)) showed that log-normality may only be expected under rather special conditions, whereas hyperbolic behaviour was likely. This latter possibility was unfortunately dropped in Kraichnan((1974) and Frisch et Al (1978), and subsequent works which retained only the notion of the fractal dimension of the support of turbulence, D_s.

Mandelbrot's model is in fact quite general and can be divided in two cases. The first is "curdling", it generates eddies strictly into either completely "dead" or uniformly "active"regions at each stage of the cascade. It is often refered as the "β-model". This is the only case where no divergence of moments occurs whatever the dimension of the spatial average. The second case is "weighted curdling" where active regions no longer have uniform intensity. Schertzer and Lovejoy(1983), who extended this model to the case of "elliptical turbulence", stressed the fact that in this case divergence occurs for any spatial average of dimension D_A such as: $D_{el} - D_s < D_A < D_{el} - D_\infty$

where D_∞ characterises the dimension of the "very active regions". They proposed that this latter case could be called "α - model", because of the hyperbolic exponent it introduces.

Note that the various dimensions intervening in the case of the "α- model" may be interpreted in terms of a "multi-fractal" (Parisi, private communication, (1983)) composed of different fractals , on each of them the velocity field has a certain scaling (i.e. a certain - singularity type, described by its scaling parameter).

The experimental results obtained indicate:

$$23/9 = D_{el} > D_s > D_{el} - 1 \;;\; D_s > D_\infty$$

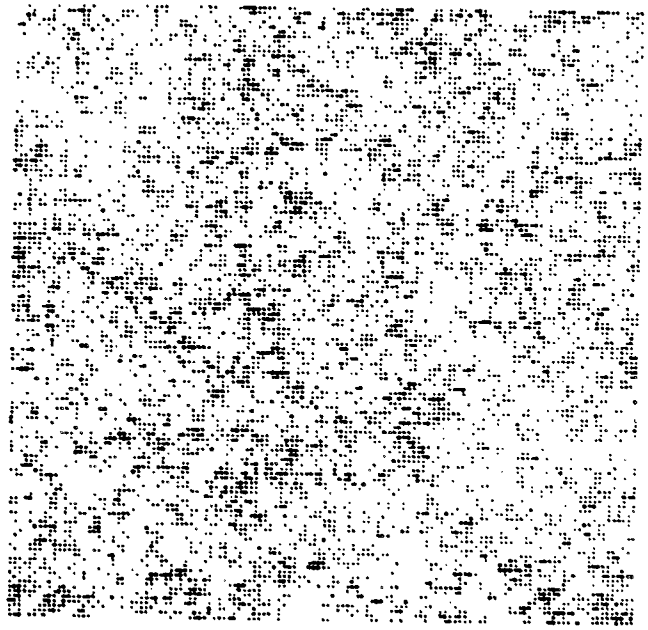


Fig.6a : A random isotropic field with H = 1/2, the intensity of the field is proportional to the shade of the grey.

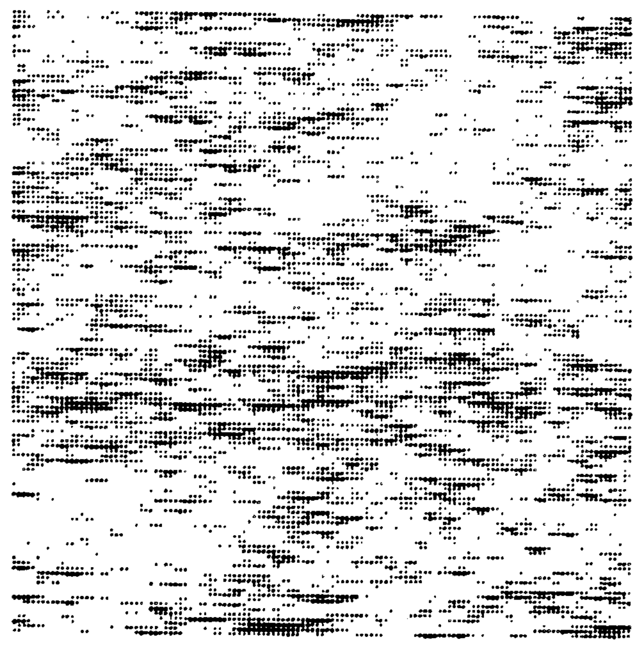


Fig.6b : An example of stochastic stratification. This figure is the same as Fig.6a, except that D_{el} = 1.5 , instead of 2 (both figures are obtained by taking the same gaussian white noise) . In one direction, H = 1 and in the direction perpendicular , H = 1/2 , the "sphero-scale" has a value of ~1% of the length of one side of the figure.

DIVERGENCE OF THE FIFTH MOMENT OF THE VELOCITY

Since we may assume, either on physical arguments or only dimensional arguments (see equation 2):

$$\bar{\varepsilon}(\Delta x) \overset{d}{=} \Delta v^3(\Delta x)/\Delta x$$

hence: $\alpha_{\bar{\varepsilon}} = \alpha_v/3$

thus our present results based on radiosonde data and Mercert's ones confirm : $\alpha_v \simeq 5$; $\alpha_{\bar{\varepsilon}} \simeq 5/3$; $\alpha_{\bar{\phi}} = 1$

This shows also that various fields may be not only related by simple algebraic equations satisfied by their scaling exponents, the same should be true for their hyperbolic exponents.

For, instance relation between dynamics and rain-field is urgently needed, because it is a case where numerical modeling is notoriously difficult. Indeed, Lovejoy (1981) shows thatchanges of the rate of rain from isolated storms have an hyperbolic law, (α_R = 5/3, but this quantity is a 2-D lagrangian statistic, and ε is a 1-D eulerian one in our study).

An interesting feature of hyperbolic distributions is that they are presumably related to the classical phenomenology of meteorological fields : Mandelbrot and Wallis (1969) pointed out that they have the effect of causing that the largest fluctuations have an overwhelming effect, which they called "Noah effect". This is investigated in detail for the horizontal rain areas by Lovejoy (1981), Lovejoy and Mandelbrot (1983).

Fig.7 shows an "hyperbolic fractal animal" obtained by stochastic simulation in an anisotopic space : one main "animal" dominates the smaller ones. It could correspond to a vertical section of a rain-field.

HYPERBOLIC RENORMALIZATION AND ELLIPTICAL TURBULENCE

The results obtained here throw into doubt the renormalization procedures used in turbulence. Usually called "spectral closures" (e.g. Herring et Al (1982)), they have been developed in a quasi-gaussian framework which is no longer tenable if the hyperbolic behaviour of the different fields is confirmed.

Conversely, placing renormalization procedures in an hyperbolic context should be particularly rewarding, since renormalization has encountered, in the quasi-gaussian context, three closely linked fundamental difficulties : random gallilean invariance, renormalization of the vertex, intermittency. Until now, the two former have been overcome only by more or less ad-hoc procedures. By stating the problem of intermittency as the problem of renormalizing in a hyperbolic context, it suggests to develop what one might call "hyperbolic renormalization".

On the other hand, the "fractally anisotropic" frame-work of the "elliptical turbulence" (i.e. a scaling anisotropy) may also be essential to overcome difficulties encountered in anisotropic cases where so far only formal manipulation of renormalization schemes have succeded (e.g. Kraichnan (1964)).

CONCLUSION AND COMMENTS

We have investigated the scaling and the hyperbolic behaviour of the vertical shear, the buoyancy force and the Richardson number.

We think that the hypothesis of a dimensional transition (2D/3D) between large scales and small scales is no longer tenable either theoretically or empirically.

The observed structure of the atmosphere can be explained by a simpler hypothesis; it is anisotropic and scaling throughout, a fact that can be characterised by the "elliptical dimension": $D_{el} = 23/9 = 2.56$, (i.e. "two plus the scaling of Kolmogorov over the one of Bogliano and Obeukhov).

On the other hand, intermittency plays a key role at the different scales, due to the low hyperbolic exponents observed. In particular as $\alpha_v \sim 5$, the fifth statistical moment of thevelocity field may diverge, and as $\alpha_R \sim 1$, even the statistical mean of the Richardson number may diverge. This points out that use of theory based on limited expansions in Ri (e.g. Lilly (1983)) are to be understood in a widely intermittent context : there is no uniform separation between waves and turbulent regimes.

This new scheme of atmospheric turbulence introduces some new notions which have been briefly discussed (e.g. the sphero-scale, which is not a characteristic scale), the "stochastic stratification").

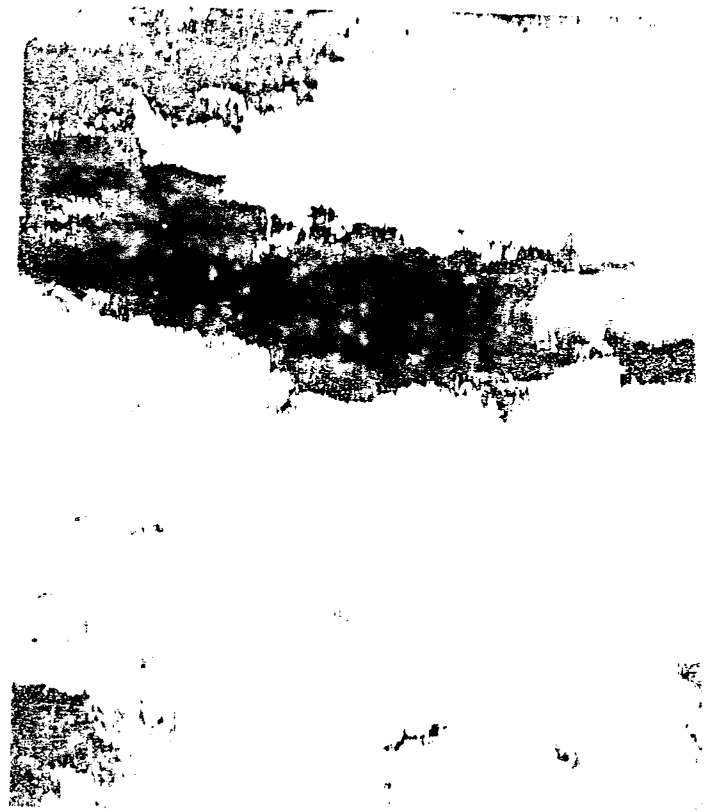


Fig. 7: An example of a hyperbolic fractal animal on an anisotropic space, with dimension D_{el} = 1.80. The log intensities are indicated by the intensities of grey. This model is on an 800X800 point grid, and the sphero-scale has the value of 30 pixels. The fine structure is therefore oriented perpendicularly to the overall shape.

On the other hand, intermittency plays a key role at the different scales, due to the low hyperbolic exponents observed. In particular as $\alpha_v \sim 5$, the fifth statistical moment of the velocity field may diverge, and as $\alpha \sim 1$, even the statistical mean of the Richardson number may diverge. This points out that use of theory based on limited expansions in Ri (e.g. Lilly (1983)) are to be understood in a widely intermittent context: there is no uniform separation between waves and turbulent regimes.

This new scheme of atmospheric turbulence introduces some new notions which have been briefly discussed (the "sphero-scale", "stochastic stratification"...) We questioned the relevance of certain of the usual assumptions of existing renormalisation methods in the case of strongly intermittent and anisotropic flows. It is hoped that the phenomenonology of the different animals crowding the meteorological zoo (e.g. fronts, bands, dust devils, blocks etc.) may be understood as the result of scaling in an anisotropic hyperbolically intermittent context (in particular, the "Noah effect").

Although far more work is needed to provide definate answers to these different questions, we may safely conclude that atmospheric turbulence is fractally homogeneous (highly intermittent), and fractally anisotropic (anisotropic scaling).

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