EXTREME VARIABILITY OF CLIMATOLOGICAL DATA: SCALING AND INTERMITTENCY

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ABSTRACT. The atmosphere displays linked spatial/temporal variability over wide ranges in scale. In this paper we study the scaling behaviour of the fluctuations. These properties are characterized by spectral exponents, and the extreme fluctuations by exponents of the corresponding probability distributions. Using both temperature and rain rate data from climatologically representative locations, we find very similar behaviour including relatively small dispersions in the estimated exponents. We also show that the effect of spatial averaging is primarily to reduce the amplitudes of both the fluctuations and their spectra by constant factors, a behaviour which is consistent with global space/time scaling. Finally we argue that these non-standard statistical procedures may be indispensable in taming extremely variable climatic and meteorological data.

1. INTRODUCTION

We are primarily interested in studying the variability of climate over "regional" spatial scales (e.g. up to the scale of France) and temporal scales up to the order of several decades of years. However, atmospheric fields are extremely variable over a wider range[1]. The phenomenology of atmospheric phenomena includes existence of sharp gradients, sudden transitions, erratic fluctuations, many with structures spanning a wide range of scales. This extreme variability cannot be tamed with standard statistical techniques involving assumptions about the existence of characteristic decorrelation times and distances (hence implicitly of exponential decays of correlations) as well as the existence of characteristic amplitudes of fluctuations (hence of exponential decays of the probability distributions). This extreme variability (intermittency) of climatological data (temperatures and precipitations) discussed here has two basic features:

1- The fluctuations (ΔX) of the fields X span a wide range of scales. The energy spectra displays regimes over which the energy varies in a simple power law manner: over this range, there is no characteristic time scale; the field is scale invariant (scaling).

2- At a given scale the fluctuations ΔX may span a wide range of intensities. The probability distributions are fat-tailed (intermittent) and large fluctuations will occur; the sample spectrum (e.g. in one dimension the modulus squared of the corresponding fourier component) will have large random peaks.

Scaling and intermittency have until recently been associated isotropic (self-similar) processes defined by a single fractal dimension. Such models have been useful in modelling various geophysical fields such as landscape(topography) or clouds. Recent developments have shown that such mono-scaling behaviour is the exception rather the rule. Instead, multifractal fields with a hierarchy of fractal dimensions generically arise as a result of cascade processes of broadly the same type as those that concentrate energy, moisture and other conserved fluxes into smaller and smaller regions of the atmosphere. These multifractal cascades have many interesting properties including very strong intermittency.

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[1] 10 orders of magnitude in spatial scale (1mm to 10,000km), coupled with 7 orders of magnitude in temporal scale (1 second to 1 month if we consider climatological variations, the range would be much larger)

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The object of this paper is to characterize the scale invariant regimes of the climate with a few exponents. As long as we are interested in the behaviour of single moments (such as the exponent of the second order moments that characterize the scaling of the spectra), a single exponent is sufficient. More refined (multifractal) analyses will be performed in future. The data used here are three hourly or daily average temperatures and daily accumulations of precipitation from recording stations in France, Saint-Pierre and Miquelon, New-Caledonia, and Antilles. The work reported here is an elaboration of results previously reported in Ladoy (1986) and in Ladoy et al. (1986).

2. METHODS FOR STUDYING SCALE DEPENDANCE

2.1. Simple scaling

A specific type of scaling called "simple scaling" or "scaling of the increments" may be defined as follows. For a function of a single variable $X(t)$ ($T(t)$ for temperature, $R(t)$ for the rain rate), a particular type of scale invariance arises when fluctuations (ΔX) at small scales ($\Delta t/\lambda$, $\lambda > 1$) are related to those at large scale (Δt) via the following relation:

$$\Delta X(\Delta t/\lambda) \overset{d}{=} \Delta X(\Delta t) / \lambda^{H} \quad (2.1)$$

where:

$$\Delta X(\Delta t) = X(t_1) - X(t_0) \qquad \Delta t = t_1 - t_0$$
$$\Delta X(\Delta t/\lambda) = X(t_2) - X(t_0) \qquad t_2 = t_0 + (t_1 - t_0)/\lambda \quad (2.2)$$

and the sign "$\overset{d}{=}$" indicates equality in probability distributions. The random variables are equal in this sense when $Pr(u>q) = Pr(v>q)$ for any threshold q. "Pr" means probability. The parameter H is a constant[2] called the - unique - scaling parameter, and is $0 \le H \le 1$.

2.2. Energy spectra

The energy spectrum $E(\omega)$ of fluctuations ΔX where ω is a frequence is useful. The spectrum is scaling when it varies in a power law manner, i.e. it is of the form:

$$E(\omega) \propto \omega^{-\beta} \quad (2.3)$$

When $E(\omega)$ is of this form over a given frequency range, there is no characteristic time and hence within the range, the process is scale invariant ("scaling"). In simple scaling (and when the variance $<\Delta X^2>$ is finite), the exponents H and β are related by the following formula[3]:

$$\beta = 2H+1 \quad (2.4)$$

For example, the familiar Kolmogorov spectrum $E(\omega) \approx \omega^{-5/3}$ for the spectrum of turbulent wind fluctuations implies $H=1/3$.

2.3. Probability distributions

The direct analysis of probability distributions is best accomplished by using log-log plots such as those shown in fig. 1.

2 Either in the framework of monofractality or for the extreme fluctuations.

3 The formula is a simple consequence of 2.1 and the fact that the energy spectrum is the Fourier transform of the autocorrelation funcyion .

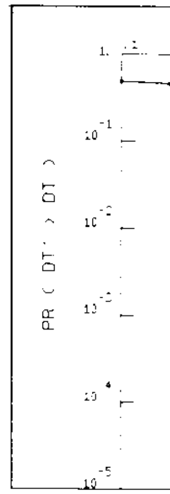

Fig. 1: The probability distributions of
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data). Positive and negative differences ar
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This method enables[4] us to estimate
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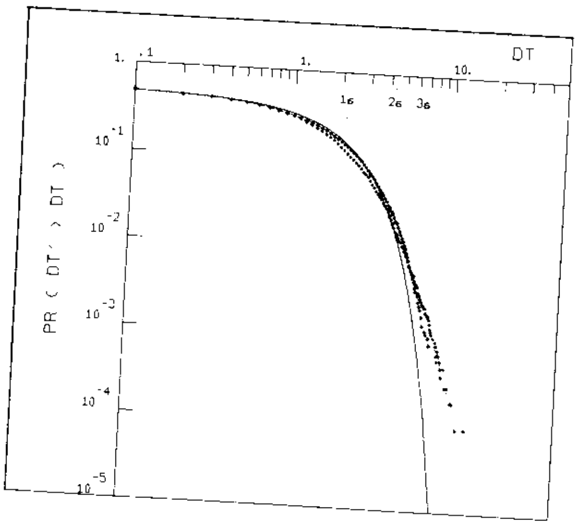

Fig. 1: The probability distributions of daily temperature differences in daily mean temperatures from Macon France for the period 1949-1979 (10,957 days). Daily means are computed by averaging consecutive three hourly data). Positive and negative differences are shown as separate curves. A best fit gaussian is shown for reference indicating that the extreme fluctuations correspond more than 7 standard deviations; which for a gaussian, would have a probability level of $\approx 10^{-20}$. The distribution is far from gaussian, since it rather has hyperbolic tails as displayed by the extremes which are nearly straight on this log-log plot.

This method enables[4] us to estimate α, the hyperbolic intermittency parameter (the negative slope for large ΔX in fig. 1). Hyperbolic intermittency means that large fluctuations in ΔX are distributed as follows:

$$\Pr(\Delta X' > \Delta X) \propto \Delta X^{-\alpha} \tag{2.5}$$

$\Pr(\Delta X' > \Delta X)$ is the probability of a random fluctuation $\Delta X'$ exceeding a fixed ΔX. When the fluctuations are of this type, the phenomena is so intermittent that high order moments $<\Delta X^h>$ diverge (tend to ∞ as the sample size is increased, see Feller (1971)) for all $\alpha \geq h$ (this gives rise to the "pseudo-scaling" (Schertzer and Lovejoy 1987, Lavallée et al., this volume). Fluctuations of this type with scaling spectra and hyperbolic intermittency are expected to occur due to the action of cascade processes concentrating energy fluxes, temperature variance fluxes (more generally of the dynamically relevant conserved flux), from large to small scales. Ever since Richardson, the idea of cascade processes transfering energy from large to small scales has been central to theories of fully developed turbulence (leading notably to Kolmogorov's 1941 $\omega^{-5/3}$ energy spectrum). Extreme variability results because as the number of cascade steps tends to infinity, the energy becomes distributed over a (mathematically) singular measure which may be characterized by a hierarchy of fractal dimensions. In these multiplicative (and multifractal) processes, the hierarchy of exponents specifies the variation with scale of the statistical moments (and hence probability distributions). In such processes, equation 2.1 (involving a single parameter H related to a single fractal dimension) applies only to the extreme tails of the probability distributions. In spite these

[4] it also allows us to evaluate H, as well as the limits to scaling.

approximations, this paper will use the simple scaling of eq. 2.1 to study climatological data over a range of time and space scales.

3. CLIMATOLOGICAL TEMPERATURES

3.1. Scale invariance of the local temperature

Figure 2 shows the energy spectrum of temperature fluctuations from the climatological recording station in Macon France over the same period as for the distributions shown in fig. 1. At the high frequency end (6 hours)$^{-1}$ to about (2 weeks)$^{-1}$, the spectrum follows a straight line on a log-log plot (fig. 2) corresponding to $E(\omega) \propto \omega^{-1.7}$. Over the range (2 days)$^{-1}$ to (6 hours)$^{-1}$, the spectrum is dominated by the diurnal peak and various sub harmonics, especially (12 hours)$^{-1}$, and (8 hours)$^{-1}$. This "red noise" is very close to the $\omega^{-5/3}$ spectrum predicted for the temperature fluctuations in a turbulent fluid when the temperature acts as a "passive scalar".

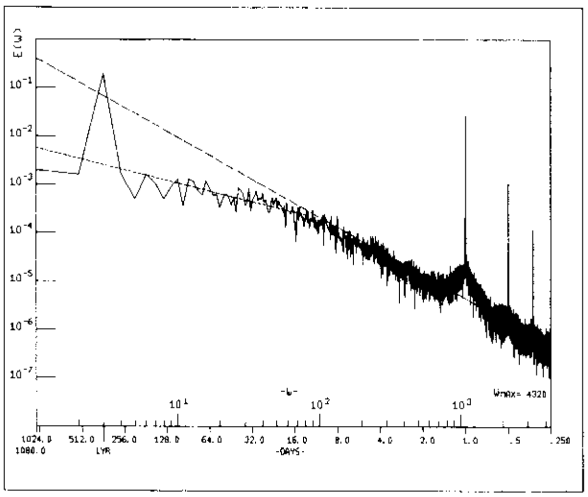

Fig. 2: The average of ten consecutive 3 years spectra for the three hourly temperature data used in fig. 1. Note the annual, diurnal peaks as well as the sub-harmonics of the latter. The two straight lines are fit to the data corresponding to periods greater and less than 14 days, and correspond to $\omega^{-0.72}$ and $\omega^{-1.78}$ respectively

The high frequency scaling regime clearly breaks down for $\omega \leq$(2 - 3 weeks)$^{-1}$ and the spectrum becomes nearly flat. This "spectral plateau" - quasi "white noise" spectral region (Lovejoy and Schertzer 1986b) -is very roughly constant spectrum at these time scales. The smooth transition at $\omega \approx$(3 weeks)$^{-1}$ is the "synoptic maximum" - see Kolesnikova and Monin (1965) - these authors estimated the period of the maximum to be between 1 and 3 weeks. It is generally agreed that this is the minimum time scale for the planetary sized fluctuations.

Figures 3, 4 and 5 display spectra of temperature variance for the three very climatologically different recording stations those of Noumea in New Caledonia, St. Pierre and Miquelon and the historical Paris-Montsouris station. We can see the same general shape of the spectra, β varying from ≈1.45 to≈1.8.

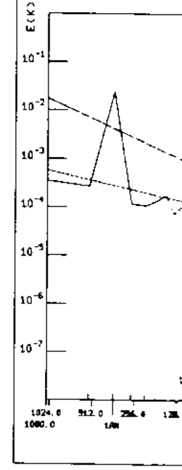

Fig.3: The average of ten consecutiv
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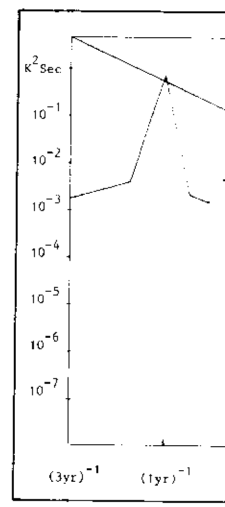

Fig.4: The average of ten consecutive
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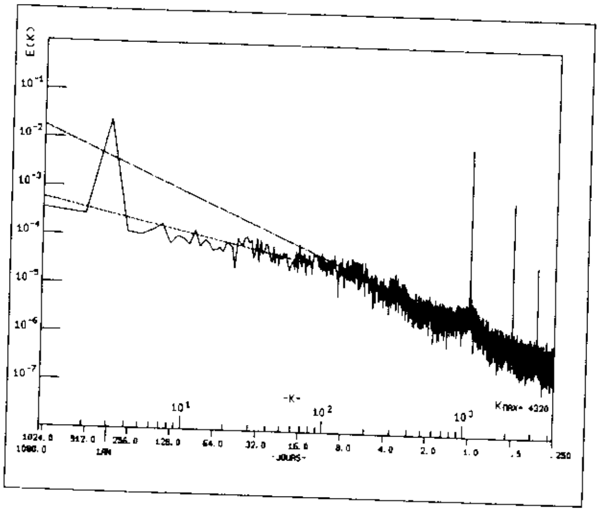

Fig.3: The average of ten consecutive 3 years spectra for the three hourly temperature at Noumea, New Caledonia. Note the annual, diurnal peaks as well as the sub-harmonics of the latter. The two straight lines are fit to the data corresponding to periods greater and less than 14 days, and correspond to $\omega^{-0.66}$ and $\omega^{-1.45}$ respectively.

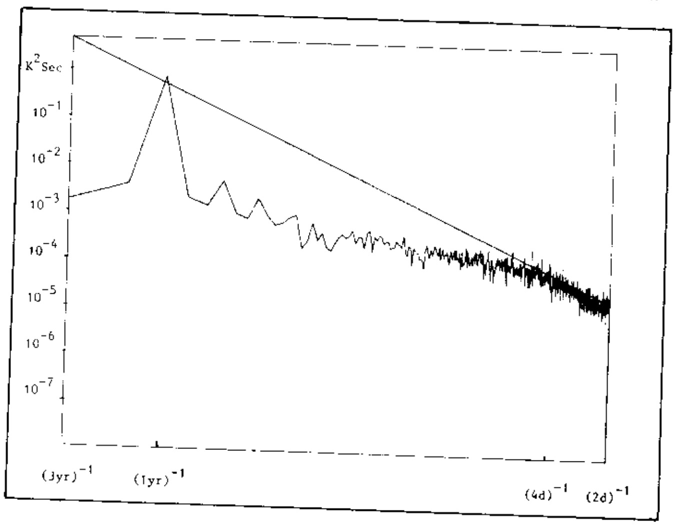

Fig.4: The average of ten consecutive 3years spectra for the daily temperature at Saint Pierre and Miquelon. Note the annual, diurnal peaks as well as the sub-harmonics of the latter. The straight line is fit to the data corresponding to periods less than 14 days and corresponds to $\omega^{-1.80}$.

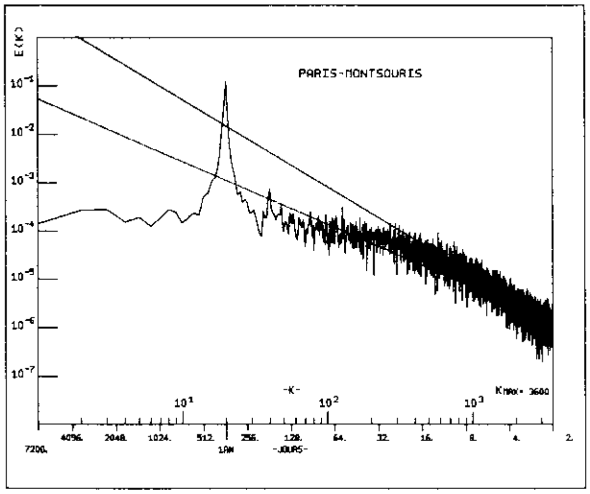

Fig 5: The average of 5 consecutive of 20 years (≈7,200 days) spectra for the daily temperature at Paris-Montsouris, (for the last century:1873-1972). Note the annual peak (and lack of 11-year peak). The straight line is fit to the data corresponding to smaller periods less than 20 days and corresponds to $\omega^{-1.8}$.

3.2 Regional temperatures

Temperature is of necessity a spatially averaged quantity. As temperatures are averaged over larger regions (eventually covering the entire globe, see Jones et al. (1982)), the amplitude of ΔT for a fixed Δt will decrease because averaging smooths out large local variations. To study the effect of such spatial smoothing, we performed averages of 53 daily recording stations over France (fig. 6), a scale refered to as regional. The general level of the energy spectrum is less than that of the local series. Note that the synoptic maximum has advanced somewhat to higher frequencies (to about $(10\text{days})^{-1}$).

This hyperbolic fall-off is quite different than that of a gaussian distribution. The fairly low value of the empirical (absolute) slope ($\alpha \approx 4.5$) shows that the intermittency is very strong. In figure 7 we plotted the probability distribution of the regional ΔT. We note that because of scaling, the only difference with the local distribution is a constant factor a linear shift on a log-log plot. The spatial averaging smooths out the fluctuation hence the amplitude is decreased.

To illustrate the difference between gaussian and hyperbolic tails, we computed the return probability of a extreme fluctuation.Over 1951-1980, the extreme (daily) fluctuation in (daily) mean temperatures was found to be 14°C. With the hyperbolic law, the extreme fluctuation predicted in a century long record is 17.8°C, whereas using the conventional Gumbel law for return probabilities, the return period for such a large fluctuation is 237 years!

The critical meteorological situation on January the 12th in 1987 in France is also very illustrative. Indeed, a seventeen days long cold wave had been severe for 6 days with dayly average temperature less than -10°C. The Loire river began to froze and threaten to prevent the incoming of fresh water indispensible to cool down the nuclear core of the power plant Saint Laurent des Eaux. It turns out that Electricité de France, the state owned agency who runs the power plant, didn't forecast such a drastic event in underestimating the occurences of extrem fluctuationes.

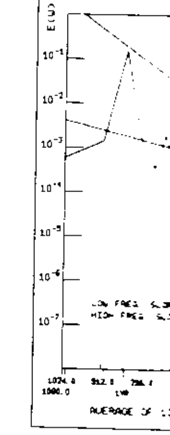

Fig.6: The average of ten consecutive
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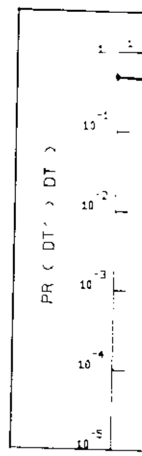

Fig. 7: The probability distributions
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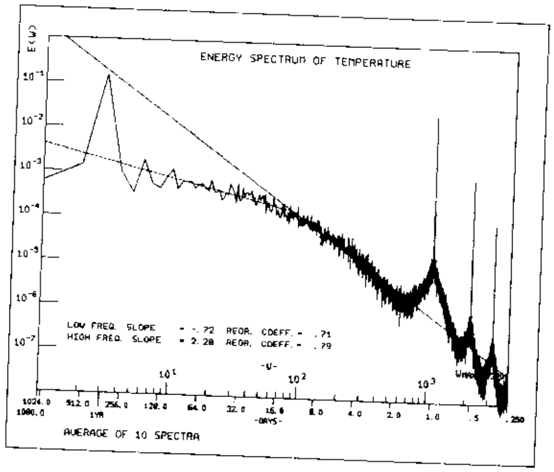

Fig.6: The average of ten consecutive 3 years spectra for the three hourly regional temperature obtained by averaging over 53 meteorological measuring stations. Note that the spectrum maintains its shape, but is displaced towards lower energies. In particular, the slope is unaffected, the straight line fit to the data corresponding to periods less than 22 days which corresponds to $\omega^{-1.8}$. As expected,because time and space are statiscally linked larger frequencies ($\leq 4(\text{day})^{-1}$) are damped by the spatial averaging

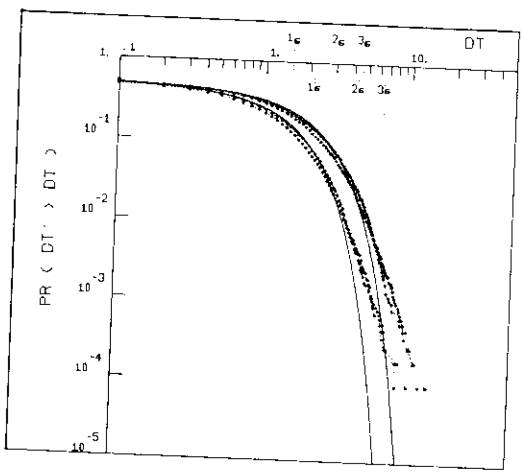

Fig. 7: The probability distributions of daily temperature differences (in daily mean temperatures) from Macon, France (rightmost curves), and regionally averaged in space over 53 stations (leftmost curves) for the period 1949-1979 (10,957 days). Daily means are computed by averaging consecutive three hourly data. Positive and negative differences are shown as separate curves. A best fit gaussian is shown for reference indicating that the extreme fluctuations correspond to more than 7 standard deviations; which for a gaussian, would have a probability level of $\approx 10^{-20}$. Both distributions are far from gaussian, as shown by their nearly straight extremes on this log-log plot indicating that they are hyperbolically distributed. Notice that the two sets of curves are virtually identical except for a left/right shift corresponding to a constant factor. This is exactly what we would expect if the temperature is scaling in timt and space.

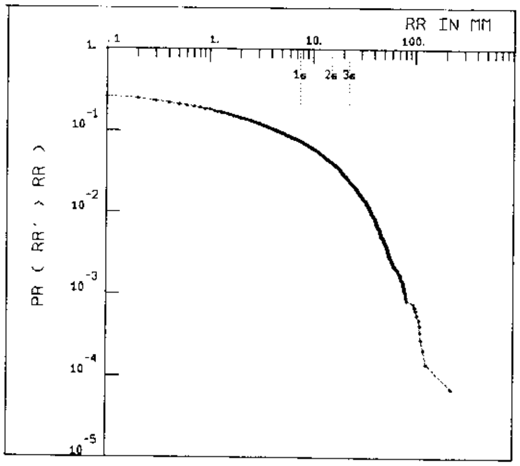

Fig. 8: The probability distributions of the fluctuations of the daily rain accumulation at Nimes-Courbessac for 40 years (1949-1988 hence 14,245 days) in a log-log plot. This distribution displays an asymptotic hyperbolic behaviour, with exponent $\alpha \approx 2.6$.

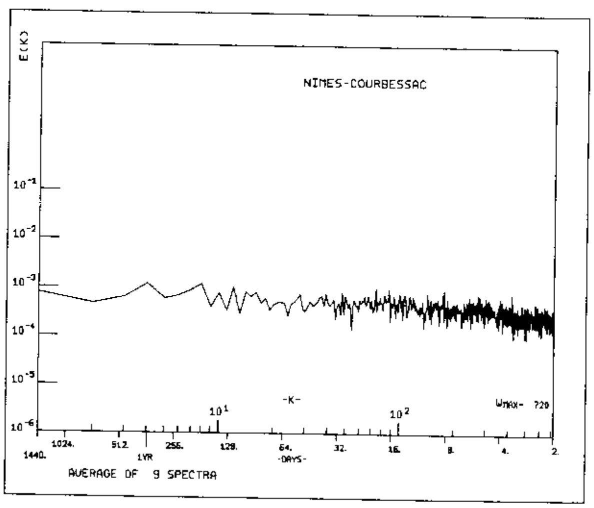

Fig.9: The average of six consecutive 4 years spectra of the daily rain accumulation at Nimes-Courbessac. Note there is no obvious annual peak, neither synoptic maximum. The scaling regime seems to span the whole range, but with a small slope $\beta \approx .3$.

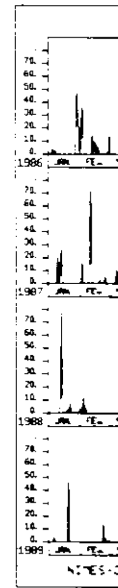

Fig.10a: Total amount of daily precipita

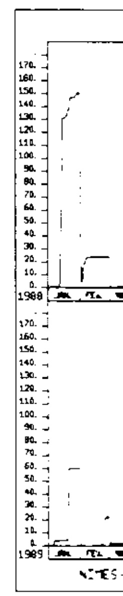

Fig.10b: Monthly accumulations of preci

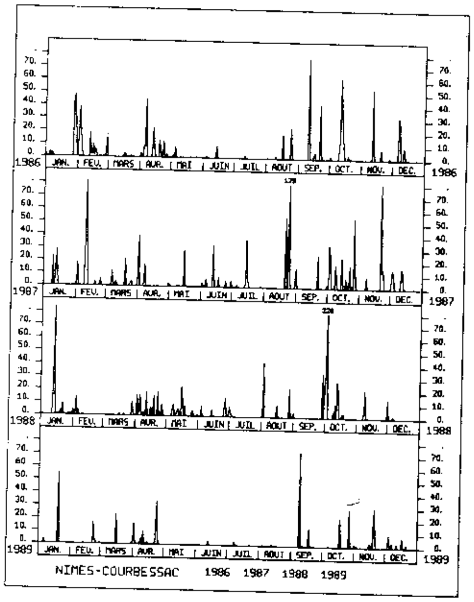

Fig.10a: Total amount of daily precipitation (in mm) over a period of four years.

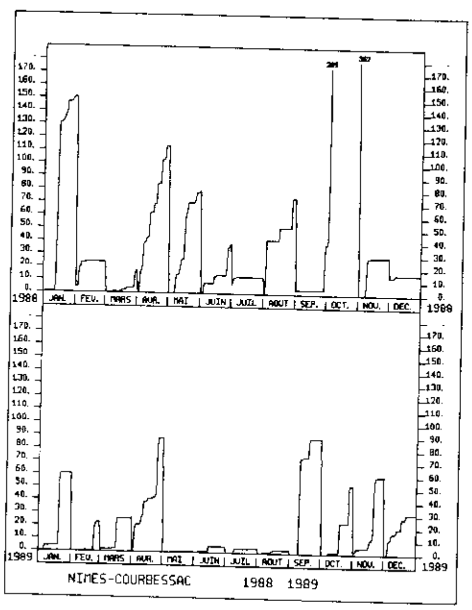

Fig.10b: Monthly accumulations of precititation (in mm) for two years

4. PRECIPITATION

In order to study the climatological behaviour of the rain field, we study on the one hand the probability distribution $\Pr(\Delta R' > \Delta R)$ for day to day fluctuations of daily rain accumulations, which shows a hyperbolic fall-off with $\alpha \approx 3.5$ (see fig. 8). On the other hand we analyse the spectrum of this fluctuations (see fig. 9) which is rather flat ($\beta \approx .3$). The extreme fluctuations of the rain field are often underestimated. However, the rather recent sudden flood which struck the town Nimes, in southern France, is one of the many vivid Natur manifestations: 420 mm of rain within a day, including 228 mm within 6 hours ... and consequently several casualties and 1.5 billion FF in damages. Some people speculated on return period for such a large fluctuation. According to Hémain (1989), this period is of the order of a century, in opposition to optimistic estimations of the order of the 10 centuries! The hyperbolic law estimated from our data set confirms the pessimistic point view. Figs. 10a,b illustrate the strong intermittancy of precipitation on both the daily and monthly basis.

5. CONCLUSIONS

In this paper we characterized the scaling behaviour of the climatological fluctuations of temperature and rain rates with the help two fundamental exponents (H,α). The former indicates how the amplitude of the fluctuations depend on scale, the latter characterizes the behaviour of the probability distribution, hence the extreme fluctuations. We find that quantitatively similar behaviour from different climatologically representative locations. It is important to realize that these findings could not be obtained by means of standard statistical procedures.

6. ACKNOWLEDGMENTS

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7. REFERENCES

Feller, W., 1971: An introduction to probability theory and its applications. vol.2, Wiley, New-York, 567pp.

Kolesnikova, V. N. ,and A. S. Monin, 1965: The spectra of micrometeorological synoptic and climatic oscillations of meteorological fields. Meteorological Research, 16, 30-56. Soviet. Geophys. Committee, Acad. Sci. USSR, Canadian Meteorological Translation 17, 1971.

Jones P. D., T. M. L. Wigley, and P. M. Kelly, 1982: Variations in surface air temperatures: Part I, Northern hemisphere, 1881-1980. Monthly Weather Rev., 110, 59-70.

Hémain, J. C., 1989: L'événement du 3 octobre 1988. La Houille Blanche, 6, 423-428

Ladoy, Ph., D. Schertzer, S. Lovejoy, 1986a: Une étude d'invariance locale-régionale des températures, La Météorologie, 7, 23-34.

Ladoy, Ph., 1986: Approche non-standard des séries climatologiques, invariance d'échelle et intermittence. Thesis, Paris IV, 194pp.

Lovejoy, S., D. Schertzer, 1986a: Scale invariance, symmetries fractals and stochastic simulation of atmospheric phenomena. Bull AMS, 67, 21-32.

Lovejoy, S., D. Schertzer, 1986b: Scale invariance in climatological temperatures and the local spectral plateau. Annalea Geophysicae, B-4 , 401-410.

Schertzer, D., S. Lovejoy, 1985: The dimension and intermittency of atmospheric dynamics. Turbulent Shear Flow, 4, 7-33, B. Launder ed., Springer, NY.

Schertzer, D., S. Lovejoy, 1987: Singularités anisotropes, et divergence de moments en cascades multiplicatifs. Annales Math. du Qué., 11, 139-181.

ON THE EXISTENCE OF LOW DIM

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University of Western O
London, Ont., Canada N

ABSTRACT. Recent work has highli
theory to study climate. We review t
dimension of the reconstructed attracto

1. INTRODUCTION

Interest has recently focused on wheth
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system, then it should be describable b
might be represented by a low dime
submanifold of the total available phas
Analyses of hydrodynamic experi
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Couette flow also indicate values betw
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D. Schertzer and S. Lovejoy (eds.), Non-Lin