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Multifractal analysis of the Greenland ice-core project climate data

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Abstract. Recent climatic records from the Greenland ice-core project (GRIP) ice core show that the climate proxy temperatures $\delta^{18}O$ ($^{18}O/^{16}O$ ratios) display sharp gradients and large fluctuations over all observed scales. We show that these variations are scale invariant over the range ≈400 yr to ≈40 kyr. The fluctuations corresponding to these scales are studied using multifractal analysis techniques. We estimate universal multifractal indices which characterize all the fluctuations for all the scales in this range and which are close to those obtained for turbulent temperatures at much shorter time scales. We speculate briefly on the origin of these common features intervening all over the observed range from seconds to kyr.

Introduction

Whilst the generally accepted approach to climate change emphasizes astronomical variations in solar forcing and climate response at a small number of frequencies (e.g. at periods of about 20, 40 and 100 kyr, see e.g. Berger et al. (1984)), the empirical variability in fact occurs over all observed scales. The recent GRIP Greenland ice core (Johnsen et al., 1992; GRIP members, 1993; Dansgaard et al., 1993) highlights this since it provides high resolution $\delta^{18}O$ data (Fig. 1) which is a proxy for the temperature variations (Dansgaard, 1964; Jouzel et al., 1987). The ≈3,000 m long ice core gives information up to ≈250 kyr, the dating being performed using stratigraphy back to 14.5 kyr BP (Johnsen et al., 1992) and beyond that with a calculated time scale using ice flow modeling (Dansgaard and Johnsen, 1969; Dansgaard et al., 1993). A 200 yr constant time step continuous time series is then constituted, about 1,200 data points long.

An alternative to the focus on specific frequencies and specific peaks was proposed by Lovejoy and Schertzer (1983, 1986): it described the fluctuations in a scale invariant (scaling) framework which emphasized the extreme nonlinear variability over wide ranges of scales (i.e. a scaling background spectrum). Since then, other researchers have broadened the scope of this scaling framework (e.g. Ladoy et al., 1986, 1991; Fluegeman and Snow, 1989; Bodri, 1993, 1994; Schulz et al., 1994; Ditlevsen and Svensmark, 1995; Shabalova, 1995). The primary conclusions of these early studies were a) the proxy temperatures are scale invariant over roughly the range 400 yr to ≈ 40 kyr, and b) the hemispheric temperatures (which are not local) are likely scale invariant from at least 5 yr to 40 kyr (Lovejoy and Schertzer, 1986). Note that recent analyses of the biannual resolution upper part of the GRIP core does indeed show evidence of such a break in the range 100 - 400 yrs (Ditlevsen and Svensmark, 1995). By implication the "spectral plateau" — the spectral flattening observed in local (not hemispheric) temperatures for periods shorter than ≈ 400 yr — is simply due to the lack of spatial averaging which smoothes out higher frequency variations in the hemispheric temperatures (Lovejoy and Schertzer, 1986). Due to strong intermittency, scaling studies require long series (and preferably many independent realizations); previous studies suffered from the much lower quality of data that was available in the early to mid 1970's. In this study we also take advantage of the important advances that have occurred in scaling, the multiple scaling approach (multifractals), and particularly the discovery of universality classes for multifractal processes.

Multifractality of the Fluctuations of the GRIP Data

In order to test the scaling of the GRIP ice core data, we estimate the power spectrum ($E(f)$, f is the frequency) in a log-

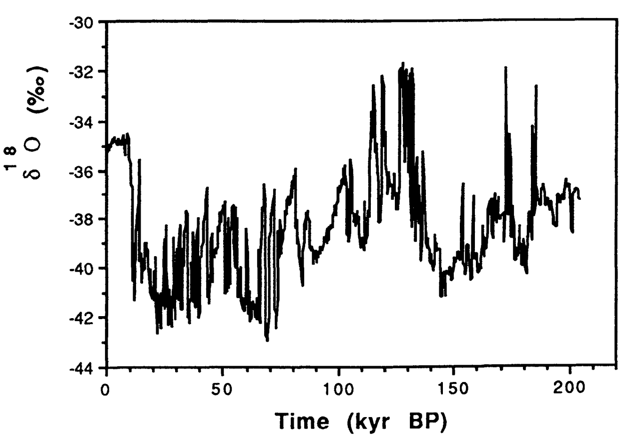

Figure 1. High resolution (200 yr average) $\delta^{18}O$ record coming from the recent GRIP Greenland ice core. Sharp fluctuations occuring on small time scales are clearly visible.

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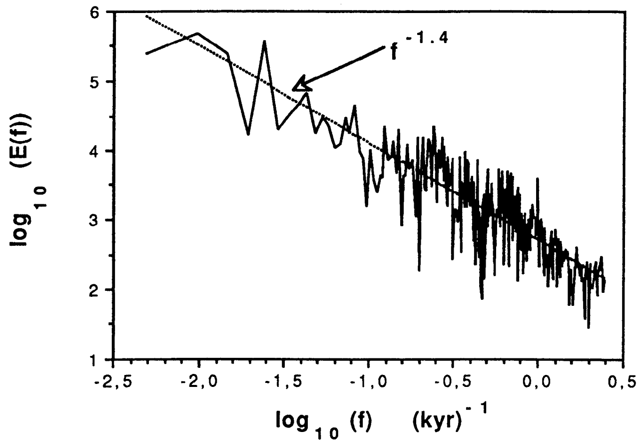

Figure 2. The power spectrum of the $\delta^{18}O$ data, shown in a log-log plot. The global straight line $f^{-1.4}$ is an indication of scaling. There is no evidence of periodic variations of frequencies of (20 kyr)$^{-1}$ or (40 kyr)$^{-1}$.

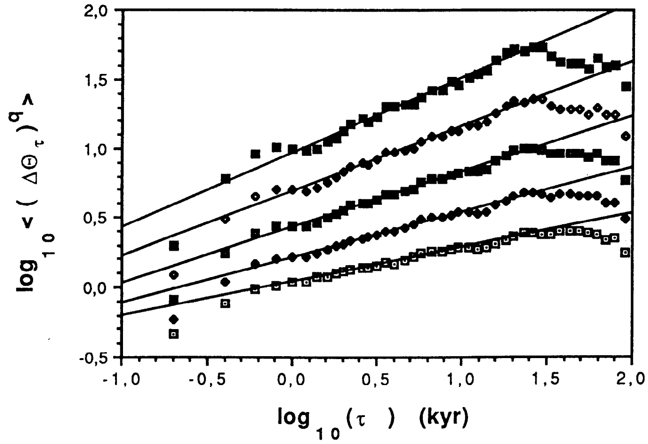

Figure 3. The structure functions $<(\Delta\theta_\tau)^q>$ vs. τ in a log-log plot for q=1, 1.5, 2, 2.5, and 3. Linear trends are clearly visible for all order of moments, for a range of scales going from $\approx$400 yr to $\approx$40 kyr. The straight lines indicate the best linear regression over this range of scales for each value of q. This gives in particular: $H=\zeta(1)=0.24\pm0.02$, and $\zeta(2)=0.40\pm0.03$.

log plot (Fig. 2). The observed power law trend ($E(f)\propto f^{-\beta}$ with $\beta\approx1.4$) shows the scaling of the spectral power density. There is no strong evidence of periodic variations at any particuliar frequency (including those at $\approx$(20 kyr)$^{-1}$ or $\approx$(40kyr)$^{-1}$, corresponding to the precession of equinoxes (Hays et al., 1976)); the "background" seems quite dominant. Another comment is that β is somewhat smaller than the value ≈1.8 obtained by Lovejoy and Schertzer (1986) with ice cores (Johnsen et al., 1972) and ocean cores (Shackleton and Opdyke, 1973). This difference might be due to the better quality and resolution of the present dataset, and the fact that the latter attempted to combine diverse data sets over a very wide range of scales using a single spectral exponent. This may also be a nonlinear paleotemperature calibration effect. Other problems include uncertainty in the dating of the deep layers, and even speculations as the influence of cosmic rays (Sonett et al. (1987)). But up to now, despite possible limitations, $\delta^{18}O$ is still generally considered as a valuable proxy for climate temperature variations.

The power spectrum is a second order moment (proportional to the square of the amplitude of a given frequency fluctuation). For multifractals — the general framework for scaling time series and functions — there is nothing special about 2$^{\text{nd}}$ order moments and we generalize this statistical moments approach with q^{th} order structure functions $<(\Delta\theta_\tau)^q>$ $=<|\theta(t+\tau)-\theta(t)|^q>$ where, for a duration τ between 200 yr and 250 kyr, the fluctuations of the temperature θ are averaged over all the available values ("<.>" indicates statistical or spatial averaging). Here we unfortunately estimate the ensemble averages using a single realization, hence there is imperfect scaling and statistical uncertainty in parameter estimates. For scaling processes, the scale invariant structure function exponent $\zeta(q)$ (e.g. Monin and Yaglom (1975)) is defined by:

$$<(\Delta\theta_\tau)^q>=<(\Delta\theta_T)^q>\left(\frac{\tau}{\mathrm{T}}\right)^{\zeta(q)} \qquad (1)$$

where T is the largest period (external scale) of the scaling regime. The first moment gives the scaling "Hurst" exponent $H=\zeta(1)$ corresponding to the scaling of the average absolute fluctuation: indeed, if $H\neq0$ the latter will depend on the duration τ, it therefore characterizes the degree of (scale by scale) non conservation of the process (this exponent is different from the "R/S" exponent, see below). The second moment is linked to the power spectrum scaling exponent by $\beta=1+\zeta(2)$. For simple scaling (monofractal) processes such as Brownian motion and its generalizations, the scaling exponent of the structure function $\zeta(q)$ is linear. For multifractal processes, this function is nonlinear and concave.

Figure 3 shows $<(\Delta\theta_\tau)^q>$ for values of q between 1 and 3. Over a range of scales going from $\approx$400 yr to $\approx$40 kyr a linear trend, indicating scaling is clearly visible for all orders of moments (we take $T\approx40$ kyr). The straight lines indicate the best linear regression over this range of scales for each value of q. The first result we report here is the value of $H=$

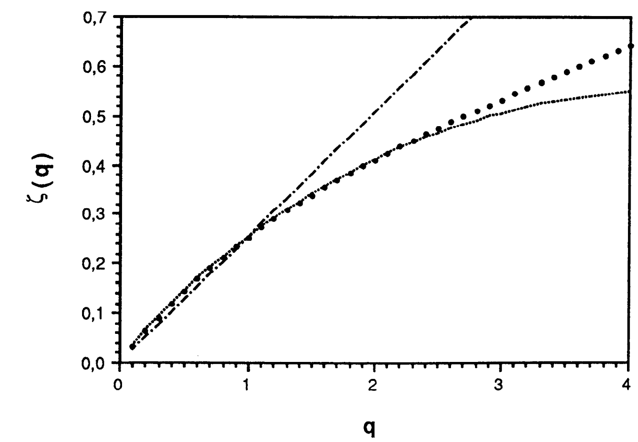

Figure 4. The scaling exponent structure function $\zeta(q)$ empirical curve (dots), compared to the monofractal curve $\zeta(q)=qH$ (dashed and dotted line), and to the universal multifractal function obtained with $\alpha=1.6$ and $C_1=0.05$ in equation (2) (dotted curve). The empirical curve is nonlinear, indicating multifractality. For moments of order q>1, the departure from monofractal behaviour is strong. The universal multifractal fit is excellent until moment order $q_{max}\approx2.5\pm0.3$, corresponding to a multifractal phase transition occuring because of sample limitations.

0.24 ± 0.02. Here as below, the error bars come from the different portions of the dataset analyzed separately — because the range of scales considered goes up to 40 kyr and we have a 250 kyr long dataset, we can estimate this value for 6 nonoverlapping intervals. This value is smaller than the less precise estimate $H \approx 0.4$ of Lovejoy and Schertzer (1986) with ice cores (Johnsen et al., 1972) and ocean cores (Shackleton and Opdyke, 1973), which was obtained by explicitly ignoring multifractal corrections: the approximation $H \approx \zeta(2)/2$ was used. Fluegeman and Snow (1989) analysed paleoclimatic data coming from Pacific core V28-239 (Shackleton and Opdyke, 1976), and using rescaled range (R/S) analysis estimated another scaling exponent (Hurst, 1951; Mandelbrot and Wallis, 1969). Unfortunately the significance of the R/S exponent for multifractal processes is not clear; recent studies show that the R/S exponent is generally not directly linked to H (Schmitt et al., 1995).

The scaling of the second order moment confirms the estimate from the power spectrum: $\zeta(2) = 0.40 \pm 0.03$. This $\zeta(q)$ estimate was performed for a range of q values from 0 to 4 with a 0.1 increment. The corresponding $\zeta(q)$ curve is shown in Fig. 4. The empirical curve is clearly nonlinear, especially for moments $q > 1$, showing that the climate temperature fluctuations are multifractal over the range ≈ 400 to $\approx 40{,}000$ yr. Multifractals are a superposition of singularities, representing structures of greatly varying intensities and scales. Our multifractal statistics are compatible with the abundance of structures of different strengths and scales clearly visible in the original time series (Fig. 1).

Universal Multifractal Parameters

Multifractal processes possess stable and attractive generators (Schertzer and Lovejoy, 1987, 1989, Schertzer et al., 1995). For these "universal" multifractals, the departure from linearity of $\zeta(q)$ is given by two parameters:

$$\zeta(q) = qH - \frac{C_1}{\alpha - 1}\left(q^{\alpha} - q\right) \qquad (2)$$

where $0 \le \alpha \le 2$ is a degree of multifractality ($\alpha = 2$ corresponds to the so called "log-normal" case, 0 to the monofractal "beta" model) and $0 \le C_1$ characterizes the intermittency of the mean. Equation (2) gives the non-linear (intermittency) corrections to the linear simple scaling trend. We now estimate the universal multifractal indices α and C_1 from equation (2): we see that a plot of $\log[q\zeta'(0) - \zeta(q)]$ vs. $\log(q)$ will have slope α and C_1 can be estimated by the intercept ($\zeta'(0)$ is defined only when $\alpha > 1$, but we see later that this is indeed the case). Figure 5 shows that $\alpha \approx 1.6 \pm 0.1$ and $C_1 \approx 0.05 \pm 0.01$, the error bars come by comparing these estimates with those obtained from the Double Trace Moment technique (Lavallée, 1991; Lavallée et al., 1992), which we also applied to the data. The empirical curve corresponding to these values in eq. (2) is shown in dotted line in Fig. 4: the fit is very good until moment order $q_{max} \approx 2.5 \pm 0.3$ after which the empirical curve is linear. This linear behaviour of the empirical scaling exponent structure function $\zeta(q)$ for sufficiently high order moments is well-known (Schertzer and Lovejoy, 1989) and is due to sampling limitations as is associated with multifractal phase transitions (Schertzer and Lovejoy, 1992, 1994). Indeed, Lovejoy and Schertzer (1986) found some evidence for divergence of statistical moments of order >4; if substantiated by large enough sample sizes, this would be associated with first order multifractal phase transitions and (nonclassical) self-organized critical behavior. (Note added in proof: using shorter ice-core paleotemperatures from glaciers, nearly the same universal multifractal parameters have been found by D. Lavallée (private communication)).

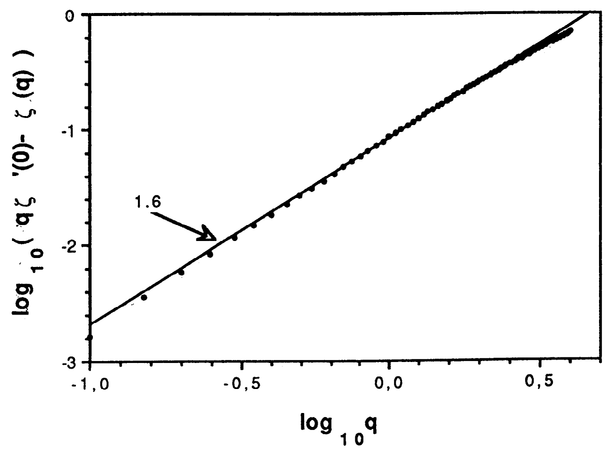

Figure 5. Empirical curve of $\log[q\zeta'(0) - \zeta(q)]$ vs. $\log(q)$ giving directly the universal multifractal indices α and C_1: α is the slope of the straight line and C_1 is estimated by the intercept. We obtain $\alpha \approx 1.6$ and $C_1 \approx 0.05$.

The universal multifractal parameters can be compared to those of turbulent temperature: H is slightly smaller than the theoretical value corresponding to temperature in fully developed turbulence $H = 1/3$ (Obukhov, 1949; Corrsin, 1951), and the other parameters are close to those corresponding to atmospheric turbulent temperatures ($\alpha \approx 1.2$ and $C_1 \approx 0.04$, see Schmitt et al. (1992)).

The fact that paleo and turbulent temperatures have such similar statistical properties is of course very intriguing. Although it is beyond the limits of this letter, we emphasize that in itself this empirical result casts doubt on the many attempts to separate meteorological and climatological types of fluctuation. Such attempts are simply not supported by the statistical evidence. However, we do not yet know the origin of these similarities whose study is both outside the scope of GCM's (with their paramatrisation of the "existing climate") as well as outside of the scope of low dimensional systems of ODE's (which parametrize the "weather"). In fact, to evaluate the effect of the statistics of the small scale space-time patterns on the longer time scale global statistics requires simultaneous simulations of both. Such simulations are however now feasible using new multifractals techniques (e.g. Schertzer and Lovejoy, 1995).

Conclusions

These results show that the fluctuations of the climate temperature recorded in the Summit ice core are scaling over wide range of scales (between about 400 yr and 40 kyr), and that these fluctuations are multifractal. Furthermore, the entire hierarchy of multifractal exponents characterizing all the climate fluctuation statistics over the range ≈ 400 yr to ≈ 40 kyr fits readily in the framework of universal multifractals. Over a wide range of scales, the many degrees of freedom of the complex climatic system can therefore be described by a small number of statistical exponents. This characterizes quantitatively the various climate temperature fluctuations which are likely to occur between 400 yr and 40 kyr, and shows that for this range of time scales, any division into seperate climate and noise processes is ad hoc. We briefly discussed the similarities between meteorological and climatological statistics.

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