Multifractal analysis of phytoplankton biomass and temperature in the ocean

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Abstract. Many attempts have been made to relate phytoplankton biomass ditributions to their turbulent environments. These studies have not taken the intermittent nature of turbulent processes into account, and hence poorly approximate inhomogeneous patterns. Since these oceanic fields are scaling for a wide range of scales, and scaling processes are believed to generically yield universal multifractal (characterized by three basic exponents), it is natural to analyse temperature and phytoplankton biomass in such a framework. Over the range 0.5s to 11h30', the temperature followed a single scaling regime, whereas the phytoplankton had both a low and high frequency regime (the break occurring at about 100s). We estimated the universal multifractal parameters finding that fluorescence was nearly dynamically passive (i.e. similar to temperature) on smaller scales but biologically active at larger scales.

Introduction

Marine systems exhibit intimate relationships between physical and biological processes (Legendre and Demers, 1984; Mackas *et al.*, 1985), as shown by the coupling between the distribution of phytoplankton populations and the structure of their physical environment over a wide range of spatial and temporal scales (Haury *et al.*, 1978). On small scales in fully turbulent flows, the fluctuations of phytoplankton biomass, estimated by Fourier power spectral analysis (Platt, 1972; Platt and Denman, 1975), are considered similar to those of a passive scalar. However, power spectral analysis is only adequate to characterize turbulent processes for homogeneous models. Contrary to classical theoretical concepts regarding turbulent events as homogeneous processes (Obukhov, 1949; Corrsin, 1951), it has been shown that the rate of energy transfer and the variance fluxes of passive scalars from large to small scales exhibit — at all scales — sharp fluctuations called intermittency which generate inhomogeneity (Batchelor and Townsend, 1949; Kolmogorov, 1962; Obukhov, 1962).

Indeed, the well-known "-5/3" power spectrum (i.e. a second order moment) associated with the behavior of a passive scalar only characterizes variability in a very limited way (except in the case of "quasi-Gaussian" statistics). Indeed the determination of the probability distribution require the

determination of moments of all orders. With the assumption of scaling, this determination reduces to the estimation of an *a priori* infinite hierarchy of exponents which remain numerically indeterminable, especially for the highest orders corresponding to the most extreme variability. Multifractal fields then require an infinite hierarchy of exponents, rather than the single exponent of fractal patterns (Mandelbrot, 1983). However, in the framework of universal multifractals (Schertzer and Lovejoy, 1987, 1989), this hierarchy is characterized by only a few relevant exponents. In this letter we shall show how the intermittent distribution of temperature and phytoplankton biomass can be characterized with universal multifractals. We study a time series of temperature (a passive tracer of the fluid flow) and *in vivo* fluorescence (regarded as a phytoplankton biomass proxy) simultaneously recorded at an anchor station in a tidally mixed coastal water, the Southern Bight of the North Sea, using a CTD recorder (Sea-Bird 19) and a fluorometer (Sea Tech), respectively. The sampling frequency being 2 Hz, our analysis is based on a time series of 82,976 measurements.

Scaling and Multiscaling of Temperature and Phytoplankton fields

The estimation of the fluorescence power spectrum ($E(f)$, f is frequency) in a log-log plot shows two scaling regimes (Fig. 1). Over smaller scales (1s to 100s), the observed power law trend ($E(f) \propto f^{-\beta}$ with $\beta \approx 1.75$) shows that the temporal distribution (or spatial, *via* the classical Taylor's hypothesis) of phytoplankton and temperature are close to the expected Obukhov-Corrsin "-5/3 power law" for passive scalars. In
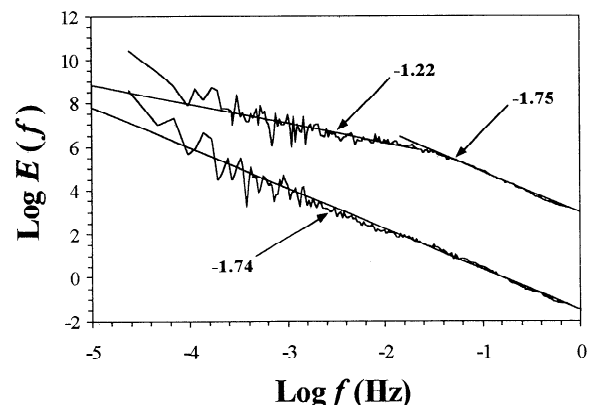

Figure 1. The power spectra $E(f)$ (f is frequency) of the fluorescence and the temperature data, shown in a log-log plot. The fluorescence data are scaling from 0.01 Hz to 1 Hz with a spectral slope $\beta \approx 1.75$ and for frequency smaller than 0.01 Hz with a spectral slope $\beta \approx 1.22$. The temperature data are scaling with $\beta \approx 1.74$.

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both cases, the spectra are slightly steeper than -5/3: this is expected in case of multifractal intermittency corrections (Frisch *et al.*, 1978; see below). On the other hand, over larger scales (> 100s), the scaling of the spectrum ($\beta \approx 1.22$) roughly agrees with theoretical and experimental studies predicting that the fluorescence spectrum will be flatter than the spectrum of a passive scalar, and supporting a predominant influence of biological factors on phytoplankton variability (Powell *et al.*, 1975; Denman and Platt, 1976; Denman *et al.*, 1977). Power spectral analysis corresponds to the second order structure function. It is generalized with the help of the qth order structure functions $< \left(\Delta S_\tau \right)^q > = < |S(t+\tau) - S(t)|^q >$ (Kolmogorov, 1962; Obukhov, 1962) where for a duration τ the fluctuations of the scalar S are averaged over all available values ("$\langle . \rangle$" indicates statistical or spatial averaging). For scaling processes, we consider the scale invariant structure functions exponents $\zeta(q)$ defined by $< \left(\Delta S_\tau \right)^q > = < \left(\Delta S_T \right)^q > \left(\tau/T \right)^{\zeta(q)}$, where T is the largest period (external scale) of the scaling regime (Monin and Yaglom, 1975). For simple (monofractal) processes such as Brownian motion, the scaling exponent of the structure function $\zeta(q) = qH$ is linear: $\zeta(q) = qH$ where $H = \zeta(1)$ is a parameter describing the scale dependence of the average fluctuations ($H = 1/3$ for a scalar passively advected by non-intermittent turbulence). For multifractal processes, $\zeta(q)$ is nonlinear and concave.

We performed this analysis for various moments of order from 0 to 5 (with an increment of 0.1) and showed the corresponding scaling in Fig. 2. These empirical curves also confirms the results of power spectral analysis and shows that there is a unique scaling regime for all scales for temperature, and two scaling regimes for fluorescence. We then plotted in Fig. 3 the functions $\zeta(q)$ — the slopes of the straight lines of Fig.2. The clear nonlinearity of this function is a first indication that variability in phytoplankton biomass and oceanic temperature can be characterized as multifractals. Moreover, the two curves corresponding to temperature and fluorescence on scales smaller than 100 seconds are very close to each other; within experimental error, they cannot be distinguished (we quantify this below with universal multifractal parameters). Thus, for all moments (and hence, all
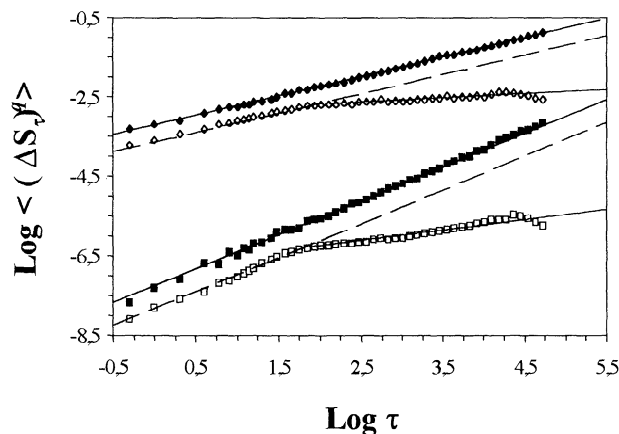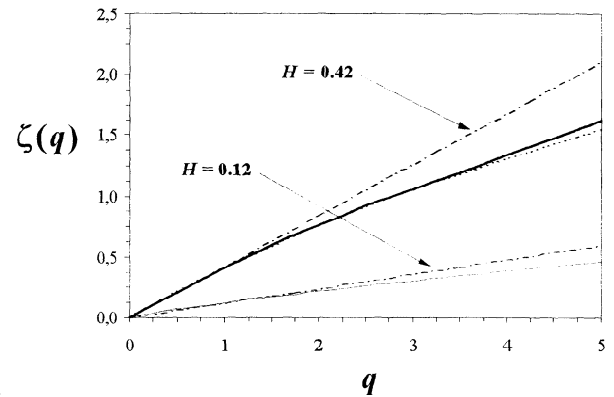
Figure 3. The empirical curves of scaling exponent structure functions $\zeta(q)$ for temperature (thick continuous line), small-scale (dashed line) and large-scale fluorescence (thin continuous line) compared to the theoretical monofractal linear curve $\zeta(q) = qH$ with $H = 0.42$ and $H = 0.12$ (discontinuous lines). The nonlinearity of the empirical curves indicates multifractality.

intensities), phytoplankton and temperature intermittencies have nearly the same probabilities. This confirms that small scale fluorescence is a passive scalar, as claimed by previous studies using only power spectra. On the other hand, the specific nonlinearity of the scaling exponent $\zeta(q)$ for *in vivo* fluorescence at scales greater than 100 seconds shows that phytoplankton variability can be regarded as a multifractal even for scales dominated by biological activities.

Universal Multifractal Parametrization

Universal multifractals are specific types of multifractals. They are likely to be ubiquitous, because they are stable and attractive limit classes obtained with continuous multiplicative scaling processes (Schertzer and Lovejoy, 1987, 1989). In this framework, $\zeta(q)$ depends only on three parameters in the following way: $\zeta(q) = Hq - K(q)$ where $K(q) = C_1(q^\alpha - q)/(\alpha - 1)$ ($\alpha \neq 1$) and the function $H = \zeta(1)$ defines the scaling of the mean field ($H = 0$ for a conservative field, i.e. its mean $< \phi_\tau >$ is strictly scale invariant) and $K(q)$ expresses intermittency. The first index $C_1 (0 \leq C_1)$ measures the mean homogeneity of the field: the larger C_1, the more the mean field is inhomogeneous or fractal (C_1 is the fractal codimension of the mean of the process), C_1 is then the measure of the sparseness of the field. The index $\alpha (0 \leq \alpha \leq 2)$ expresses the deviation from the mean of the field values; as α decreases, the high values of the field do not dominate as much as for larger values of α. More precisely, α measures the "degree" of multifractality, i.e. how fast increases the inhomogeneity with the order of the moments. Indeed, $\alpha = 0$ corresponds to the monofractal β-model (Novikov and Stewart 1964; Mandelbrot 1974; Frisch *et al.*, 1978) and reaches its maximum ($\alpha = 2$) for the (misnamed) log-normal model (Kolmogorov, 1962; Obukhov, 1962). α, which characterizes the generator of the process, is called the Lévy index (see *e.g.* Feller (1971) for stable variables). First, taking $H = \zeta(1)$ gives $H_T = 0.42 \pm 0.02$ for temperature and $H_{FSS} = 0.41 \pm 0.02$ for small-scale fluorescence. These values are very close to each other, significantly larger than the 1/3 value for homogeneous Obukohv-Corrsin passive scalar turbulence, and

Figure 2. The structure functions $< \left(\Delta S_\tau \right)^q >$ vs. τ in a log-log plot for $q = 1$ and 3 (from top to bottom) for temperature (black symbols) and for fluorescence (open symbols). Linear trends are clearly visible for temperature for all orders of moments, whereas fluorescence exhibits two scaling tendencies.

slightly larger than the value reported in Schmitt et al. (1996) for atmospheric temperature ($H = 0.38 \pm 0.02$). For large-scale fluorescence, we obtain $H_{FLS} = 0.12 \pm 0.01$. This is a small value but nevertheless shows that the mean of the fluctuations are scaling ($H = 0$ for scale-independent average fluctuations). We then estimate the values of α and C_1 by considering the field ϕ_τ obtained from the scalar field by performing a fractional differenciation of order H ($\phi_\tau = \Delta S_\tau / \tau^H$; see Schertzer and Lovejoy, 1987). The values of α and C_1 can be directly obtained from this field, by applying the Double Trace Moment analysis technique (Lavallée et al., 1992): this consists of raising the field at the smallest scale to the ηth power, and then considering the qth order of moments of the result at all scales. This gives a new scaling exponent function dependent on these two parameters: $K(q,\eta) = K(q\eta) - qK(\eta)$, ($K(q) \equiv K(q,1)$) and leads to $K(q,\eta) = \eta^\alpha K(q)$. Thus, $K(q,\eta)$ vs. η appear as a linear log-log plot with a slope of α. This is shown in Fig. 4, for $q = 2, 2.5$ and 3. Estimates of α are given by the slopes of the straight lines and C_1 is given by the value of log $K(q)$, which is the intercept of the straight line. Using this method we find $\alpha_T = 1.7 \pm 0.05$ and $C_{1(T)} = 0.04 \pm 0.01$ for temperature data and $\alpha_{FSS} = 1.8 \pm 0.05$ and $C_{1(FSS)} = 0.04 \pm 0.01$ for fluorescence data at small scales. These values are very close to each other and confirm that the statistics of fluorescence at small scales are similar to those of a passive scalar. The values of α we obtain here for oceanic temperature are larger than previous reports of α for atmospheric turbulence (Schmitt et al., 1993, 1996; Tessier et al., 1993) within the range 1.3-1.5. It is nevertheless smaller than $\alpha = 2$ corresponding to log-normal statistics (Baker and Gibson, 1987; Gibson, 1991).

At small scales, the similarity of the universal multifractal parameters of temperature and fluorescence reflects profound couplings between the space-time structure of phytoplankton populations and the structure of their physical environment. In this multifractal framework, phytoplankton biomass then appears to be passive and inhomogeneously (i.e. non-randomly) distributed, contrary to the basic perception of small scale interactions in plankton ecology which is based on a passive but homogeneous distribution of phytoplankton. This inhomogeneity then may have important effects on concentration-dependent processes such as phytoplankton coagulation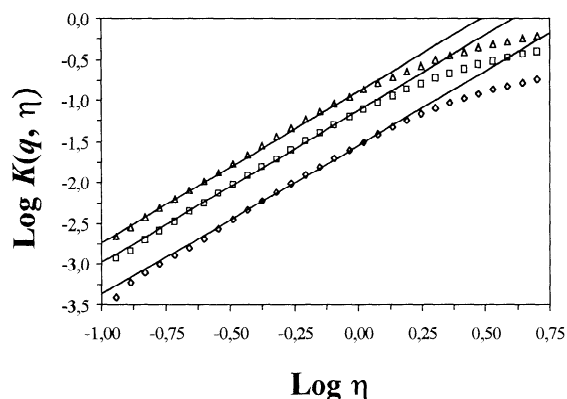 (e.g. Jackson and Lochmann, 1993). These findings also provide a complementary approach to previous theoretical studies of the effects of turbulence (regarded as a random process generating homogeneity) on contact rates between predators and preys (Rothschild and Osborn, 1988).

For large-scale fluorescence, we first averaged the data up to the scale of 100 s, to place it in what we identified as the second regime, and then applied the Double Trace Moment analysis technique. We obtained $\alpha_{FLS} = 0.8 \pm 0.02$ and $C_{1(FLS)} = 0.02 \pm 0.01$. These values show that the statistics of small-scale fluorescence differ significantly from those of large-scale: not only does the average of the fluctuations vary differently with scale (the value of H), but also with the parameters characterizing the type of intermittency. They indicate "lower" multifractality ($\alpha_{FLS} < \alpha_T$) and larger mean homogeneity ($C_{1(FLS)} < C_{1(T)}$) than temperature at the same scales. At these scales, the combination of biological processes — such as growth, sinking, or community interactions — provides an aggregative effect which dominates the dispersive effect of turbulence (Denman and Platt, 1975; Lekan and Wilson, 1978).

Conclusions

We have shown that the intermittency of phytoplankton and temperature fields in the Southern Bight of the North Sea are multifractal over the whole range of measurements scales. Furthermore, our analysis of the physical as well as the biological processes gives credence to the prediction that these stochastic processes are universal multifractals; we estimated the three fundamental parameters, which in this framework characterize all the statistics of the fields. Our analysis of temperature and phytoplankton biomass differs from the previous multifractal analysis of zooplankton data by Pascual et al. (1995) in several ways. First, the use of universal multifractals makes the analysis much more robust; only three basic parameters are needed to characterize the whole variability of the fields. Next, we directly analysed the data using structure functions as is usually done in turbulence studies (e.g. Monin and Yaglom, 1975), and estimated the slopes of their Fourier power spectra which are very suitable to make comparisons with other fields. Both approaches provide nevertheless a stochastic basis for multifractal simulations: this allows efficient simulation of intermittent fields on a wide range of space-time scales, contrary to the usual deterministic models which are restricted to specific scales. Furthermore, the universal multifractal framework is useful for simulations (Wilson et al., 1989; Pecknold et al., 1993), because it provides very precise control of the variability by the way of continuous multiplicative processes (with the help of the basic parameters). This approach then allows to include the multiscale detailed variability of natural processes and therefore opens a new perspective for modeling oceanic ecosystems.

These results show not only that biological as well as physical processes can be characterized by three universal exponents, but also that these exponents help to discriminate between a regime where physics is dominant (smaller scales, higher variability) and a regime where on the contrary biology is dominant, damping out variability of turbulent physical processes (larger scales, lower variability). The cross-over between passive and biologically active regimes for fluorescence (i.e. phytoplankton biomass) seems to intervene at a period of about 100 seconds. We have no clear

Figure 4. The curves $K(q,\eta)$ vs. η in a log-log plot for $q = 2, 2.5$ and 3 (from bottom to top) for temperature field. The slope and intercept of the lines give respectively estimates of α and C_1.

interpretation of this characteristic time scale to propose; it is indeed significantly smaller than the generation time of the phytoplankton populations (≈ 1 day).

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