Scaling turbulent atmospheric stratification: a turbulence/wave wind model

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Abstract

Twenty years ago, we argued that atmospheric dynamics are scaling and anisotropic over a wide range of scales characterized by an elliptical dimension D_s=23/9, shortly thereafter we proposed the continuous cascade "Fractionally Integrated Flux" (FIF) model and somewhat later, causal space-time extensions with D_{st}=29/9. Although the FIF model is more physically satisfying and has been strikingly empirically confirmed by recent lidar measurements (finding D_s=2.55±0.02, D_{st}=3.21±0.05) in classical form, its structures are too localized, it displays no wave-like phenomenology.

We show how to extend the FIF model to account for more realistic wave-like structures. The key point is that the FIF requires two propagators (space-time Green's functions). The first determines the space-time structure of the cascade of fluxes, this must be localized in space-time in order to satisfy the usual turbulence phenomenology. In contrast, the second propagator relates the turbulent fluxes to the observables (wind, passive scalar density), this propagator can be unlocalized in space-time (although the spatial part is the same as before, it is still localized in space, now in wave packets). We display numerical simulations which demonstrate the requisite (anisotropic, multifractal) statistical properties as well as wave-like phenomenologies.

1. Introduction

According to a growing body of analyses (e.g. [1,2,3,4,5]) 1D the horizontal sections, 1D time series of the atmosphere follow Kolmogorov laws: $\Delta v = \varepsilon^{1/3}\Delta x^{1/3}$ and (assuming no overall advection/wind): $\Delta v = \varepsilon^{1/2}\Delta t^{1/2}$ where ε is the energy flux. However, 1D vertical sections follow the Bolgiano-Obukov law: $\Delta v = \phi^{1/5}\Delta z^{3/5}$, ϕ the buoyancy variance flux. The generalization to arbitrary space-time displacements $\underline{\Delta R} = (\underline{\Delta r}, \Delta t)$, $\Delta \underline{r} = (\Delta x, \Delta y, \Delta z)$, and multifractal statistics is the 23/D spatial model, 29/9D space-time model. The model velocity (v) and the corresponding anisotropic extension of the Corrsin-Obukov law for passive scalar density (ρ) satisfies:

$$\Delta v(\underline{\Delta R}) = \varepsilon^{1/3}_{[\![\underline{\Delta R}]\!]}[\![\underline{\Delta R}]\!]^{1/3}; \quad \Delta \rho(\underline{\Delta R}) = \chi^{1/2}_{[\![\underline{\Delta R}]\!]}\varepsilon^{-1/6}_{[\![\underline{\Delta R}]\!]}[\![\underline{\Delta R}]\!]^{1/3} \tag{1}$$

ε and χ are the (multifractal) energy and passive scalar variance fluxes; the subscripts indicate the scale at which they are averaged. The key idea of the theory is that the physical scale function $[\![\underline{\Delta R}]\!]$ replaces the usual Euclidean distance in the classical isotropic turbulence laws. $[\![\underline{\Delta R}]\!]$ need only satisfy the general functional ("scale") equation:

$$[\![T_\lambda \underline{\Delta R}]\!] = \lambda^{-1}[\![\underline{\Delta R}]\!]; \quad T_\lambda = \lambda^{-G} \tag{2}$$

T_λ is the scale changing operator which transforms vectors into vectors reduced by factors of λ in scale; T_λ defines a Lie group with generators *G*. The trace of G is called the "elliptical dimension" D_{el}. The basic 29/9D model has G with eigenvalues 1, 1, H_z, H_t with H_z=5/9, H_t=2/3 so that Trace G=29/9. An overall advection (Gallilean transformation) is taken into account by a similarity transformation on G. This "Generalized Scale Invariance" (GSI) is the basic framework for defining scale in anistropic scaling systems[6].

The Fractionally Integrated Flux[7] (FIF) is a multifractal model obeying eq. 2 based on a subgenerator $\gamma_\alpha(\underline{r},t)$ which is a noise composed of independent identically distributed Levy random variables (index 0<α≤2; "universal" multifractals[7]). One next produces the generator $\Gamma(\underline{r},t)$:

$$\Gamma(\underline{r},t) = \gamma_\alpha(\underline{r},t) * g_\varepsilon(\underline{r},t); \quad \tilde{\Gamma}(\underline{k},\omega) = \widetilde{\gamma_\alpha}(\underline{k},\omega)\tilde{g}_\varepsilon(\underline{k},\omega) \tag{3}$$

by convolving ("*") $\gamma_\alpha(\underline{r},t)$ with the propagator (space-time Green's function) $g_t(\underline{r},t)$ where we have indicated fourier transforms by tildas. The conserved flux ε is then obtained by exponentiation:

$$\varepsilon(\underline{r},t) = e^{\Gamma(\underline{r},t)} \tag{4}$$

The horizontal velocity field is obtained by a final convolution with the (generally different) propagator:

$$v(\underline{r},t) = \varepsilon^{1/3}(\underline{r},t) * g_v(\underline{r},t); \quad \tilde{v}(\underline{k},\omega) = \widetilde{\varepsilon^{1/3}}(\underline{k},\omega)\tilde{g}_v(\underline{k},\omega) \tag{5}$$

In order to satisfy the scaling symmetries, it suffices for the propagators to be (causal) powers of scale functions:

$$g\left(\underline{\Delta R}\right)=h\left(t\right)\left[\!\left[\underline{\Delta R}\right]\!\right]^{-(D-H)};\quad \tilde{g}\left(\underline{\Delta R}\right)=\tilde{h}\left(t\right)*\left[\!\left[\underline{k},\omega\right]\!\right]^{-H} \tag{6}$$

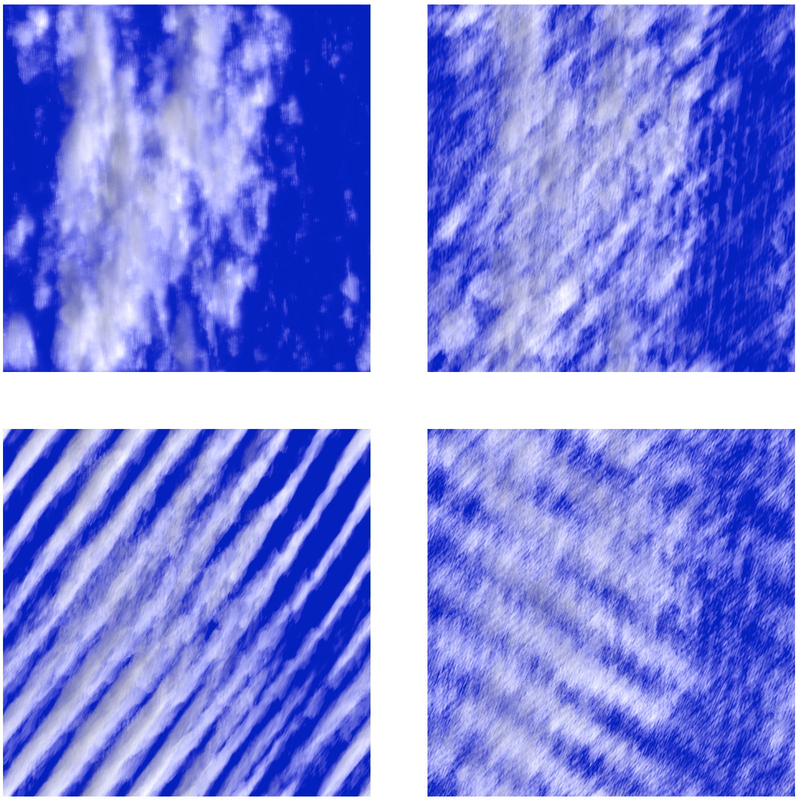

Fig. 1. This figure shows a horizontal cross section of FIF models with varying localization using $g_v\left(\underline{\Delta R}\right)=g_{v,wav}^{(H_{wav})}\left(\underline{\Delta R}\right)*g_{v,tur}^{(H_{tur})}\left(\underline{\Delta R}\right)$ and $H_{wav}+H_{tur}=H=1/3$, $H_t=2/3$; $C_1=0.1$, $\alpha=1.8$, with a small amount of differential anisotropy. Clockwise from the upper left we have H_{wav} = 0, 0.33, 0.52, 0.38. Rendering is with single scattering radiative transfer. ε is the same in all cases so that one can see how structures become progressively more and more wave-like. See the multifractal explorer: http://www.physics.mcgill.ca/~gang/multifrac/index.htm.

The real space and fourier scale function are different; the latter is symmetric with respect to the transpose of G. H must be chosen = $D(1-1/\alpha)$ for g_ε, and H=1/3 for g_v. h(t) is the Heaviside function necessary to account for causality.

2. An extreme unlocalized (wave) extension

Although the FIF is quite general, its classical implementation ([7, 8], fig.1 upper left) is obtained by using the same localized space-time scale function for both g_ε and g_v. We now allow g_v to be nonlocal in space-time (wave packets). The key is to break $[\![\underline{\Delta R}]\!]$ into a separate spatial and temporal parts; in a frame with no advection, the classical FIF uses the strongly localized:

$$g_{v,tur}^{(H)}\left(\underline{\Delta r},\Delta t\right)=h\left(\Delta t\right)[\![\underline{\Delta R}]\!]^{-(D-H)};\quad [\![\underline{\Delta R}]\!]=\left\|\underline{\Delta r}\right\|+\Delta t^{1/H_t} \tag{7}$$

For extreme nonlocalized (wave) models, we now choose:

$$[\![\underline{k},\omega]\!]=\left(i\left(\omega-\left\|\underline{k}\right\|^{H_t}\right)\right)^{1/H_t} \tag{8}$$

Taking the inverse Fourier transform the –H power with respect to ω and ignoring constant factors (see fig. 1 for numerical simulation):

$$\tilde{g}_{v,wav}^{(H)}\left(\underline{k},t\right)=h\left(t\right)t^{-1+H/H_t}e^{i\left\|\underline{k}\right\|^{H_t}t} \tag{9}$$

This propagator is a causal temporal integration of order H/H_t of waves.

We have shown that the FIF framework is wide enough to include wave effects, elsewhere[1], we show with dispersion relations sufficiently close to the standard gravity wave dispersion relations that the model can quite plausibly explain the empirical results in much of the atmospheric gravity wave literature.

3. References:

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