Fractals, Vol. 10, No. 3 (2002) 1–12

THE MULTIFRACTAL SCALING OF CLOUD RADIANCES FROM 1M TO 1KM

D. SACHS and S. LOVEJOY
Department of Physics, McGill University, Montreal

D. SCHERTZER
Laboratorie de Modelisation en Mechanique (L.M.M.),
Centre Nationale de Recherche Scientifique (CNRS), Paris

Abstract

The cloud radiances and atmospheric dynamics are strongly nonlinearly coupled, the observed scaling of the former from 1 km to planetary scales is *prima facae* evidence for scale invariant dynamics. In contrast, the scaling properties of radiances at scales $<$ 1 km have not been well studied (contradictory claims have been made) and if a characteristic vertical cloud thickness existed, it could break the scaling of the horizontal radiances. In order to settle this issue, we use ground-based photography to study the cloud radiance field through the range scales where breaks in scaling have been reported (30 m to 500 m). Over the entire range 1 m to 1 km the two-dimensional (2D) energy spectrum $(E(k))$ of 38 clouds was found to accurately follow the scaling form $E(k) \approx k^{-\beta}$ where k is a wave number and β is the spectral exponent. This indirectly shows that there is no characteristic vertical cloud thickness, and that "radiative smoothing" of cloud structures occurs at all scales. We also quantitatively characterize the type of (multifractal) scaling showing that the main difference between transmitted and reflected radiance fields is the (scale-by-scale) non-conservation parameter H. These findings lend support to the unified scaling model of the atmosphere which postulates a single anisotropic scaling regime from planetary down to dissipation scales.

Keywords: Clouds; Fractals; Multifractal; Scaling; Turbulence.

1. INTRODUCTION

1.1 The Role of Cloud Radiances in Investigating Characteristic Atmospheric Length Scales

There is no doubt about the basic inner and outer atmospheric length scales: the former — determined by turbulent dissipation — is of the order of a millimeter, whereas the latter is the size of the planet. Since in the intermediate range the corresponding dynamic equations have no characteristic scale, it is possible that the dynamics are scaling (and, following Richardson[1]) that they are ruled by cascade processes concentrating energy fluxes into smaller and smaller scales. Since such cascades are the generic multifractal process,[2,3] we may therefore expect wildly variable (strongly intermittent) multifractal statistics; in addition, due to the existence of stable, attractive cascade/multifractal processes, such multifractals are expected to fall into three-parameter universality classes. Indeed, numerous atmospheric studies (over various ranges of scale) have found precisely such behavior (atmospheric temperatures,[4,5] wind,[6–9] rain,[10] cloud liquid water content,[11] radiation fields,[12,13] and pollution concentration.[14–16]

In spite of the success of the universal multifractal model[a] and the absence of other length scales in the governing equations, there is still debate about the true range of scaling in the atmosphere. An obvious potential source of scale breaking is through boundary conditions such as the topography. However globally, the latter is also apparently scaling,[17,18] down to 100 m or less. Hence, unless we artificially break the scaling by special conditioning of the statistics (e.g. by specifying that we look a fixed distance on the lee side of a mountain chain or by subjectively selecting special cloud types, or using single realizations of special rare events, etc.) we will not expect to see systematic, genuine (i.e. non-random) scale breaks. Indeed, the classical argument for a break at intermediate scales is quite indirect relying instead on the strong anisotropy induced by gravity. The argument, going back to Fjortoft[19] and Kraichnan[20] starts with the recognition that isotropic two-dimensional (2D) and isotropic three-dimensional (3D) turbulence are theoretically quite different (the former allows no vortex stretching, it has two quadratic invariants rather than only one, hence a k^{-3} regime as well as a $k^{-5/3}$ regime, etc.). It is then argued that since the atmosphere looks "thin" at large enough scales and "voluminuous" at small scales, that the latter regimes are indeed roughly isotropic 2D and isotropic 3D turbulence. A "meso-scale gap" is therefore predicted separating the two regimes; presumably at scales comparable to the pressure scale height of ≈ 10 km.

Although there were indeed early empirical claims of meso-scale gaps (especially van der Hoven[21]), systematic studies starting with Goldschmitt (1968),[22,23] were not able to reproduce them. One of the reasons that the issue is still not completely settled is the difficulty in interpreting the rather sparse velocity data. For example,the GASP experiment,[24–26] while probably the most data intensive velocity spectra published to date, finds no break anywhere near the meso-scale. Although it does show a slight spectral steepening at low frequencies corresponding to scales of several hundred to about a thousand kilometers, Lovejoy et al[27] have argued that this could simply be the result of a systematic bias introduced because the data came from commercial aircraft which tend to deviate to avoid the centers of intense storms, thus somewhat reducing the energy at high wave numbers. Indeed, recently[28,29] special research data sets (unaffected by this problem) have been collected spanning the (even wider) range 100 m to 2000 km, and no gap has been found. More interestingly, the aircraft trajectories themselves were found to be fractal so that the spectral components had a very different interpretation.

In addition to the evidence directly from the velocity data, three other arguments make it unlikely that a meso-scale gap exists. The most fundamental is a theoretical argument which is the outcome of progress in scaling notions: the framework of Generalized Scale Invariance (GSI).[30,31] It is now known that scaling need not be isotropic; indeed, gravity acts at all scales implying that atmospheric scaling is *a priori* quite different in the vertical and horizontal directions. This is indeed in accord with a growing body of evidence.[8,30,32–34] There is therefore no theoretical or empirical justification for either the isotropic 2D or isotropic 3D turbulence models; the alternative "unified scaling" model[27,30]

[a]The main competing multifractal model is the log-Poisson model[35] but this is not stable/attractive and does not fit the empirical turbulence data as closely as the universal multifractal model (see Schertzer et al.[36] for detailed intercomparison).

is both anisotropic and scaling throughout; the atmosphere simply becomes more and more stratified at larger and larger scales.

Although the massive quantities of velocity data needed to completely and unambiguously resolve the issue empirically do not yet exist, there is strong indirect evidence that the corresponding (horizontal) velocity field is indeed scaling. This is because — as pointed out in Schertzer et al.[3] the classical size (L)–lifetime (τ) relations for various atmospheric structures (from dust devils through cyclones to planetary waves) are roughly power law in form: $\tau \approx L^{1-H}$ implying a scaling velocity $v \approx L^H$. In addition, the exponent H is found to have a value near 1/3 which is the value predicted by an anisotropic extension of Kolmogorov theory (i.e. assuming the energy flux is the fundamental dimensional quantity in the horizontal but not vertical directions.[30]).

The final argument against the existence of a meso-scale gap comes from the systematic use of cloud data. In order to overcome the difficulties in obtaining large enough velocity data sets, Lovejoy et al.[27] (building on the wide range cloud scaling found in Lovejoy[37] proposed that cloud fields (as infered by satellite cloud radiances) could be used instead. The theoretical argument is simple: since scale invariance is a symmetry principle and the cloud and velocity fields are strongly nonlinearly coupled, any break in one should be reflected in the other. In addition, if the 2D/3D transition theory is correct, then the cloud field is a particularly pertinent field to study since, unlike the vector velocity field, a passive scalar cloud[b] has a single quadratic invariant in both two and three dimensions. Hence, rather than a (possibly difficult to detect) transition from k^{-3} (large scale, 2D) to $k^{-5/3}$ (small scale, 3D spectrum), the transition would be quite drastic (occurring near the scale of injection of passive scalar variance). The observation of excellent scaling in the spectra of 15 cloud scenes as observed by AVHRR sensors over the Atlantic Ocean (at five different wavelengths from visible to thermal infra-red (IR), scale range 1.1–550 km), increased the total amount of data analyzed in this (critical mesoscale) range by an order of magnitude (a single 512×512 pixel image contains the same amount of data as the entire GASP experiment). Furthermore, the corresponding (fourier space) angle integration greatly calms the enormous intermittency which makes individual 1D spectra very hard to interpret.[c] In Lovejoy et al.,[38–40] the quantity of data has been increased by a further factor of ≈ 50, nearly 1000 satellite cloud pictures are analyzed, making a very solid case for scaling right through the meso-scale. In particular, since nearly two years of (fairly) systematically sampled data were used in, the outer scale of the cascade can be directly estimated even from scenes over relatively small areas (e.g. the ARM CART site in Oklahoma, 280×280 km). The resulting estimates of the outer scale ($\approx 20\,000$ km) are based on both weak and strong events in both IR and visible wavelengths and are sufficiently close to the size of the planet that it is likely that the atmospheric cascade does indeed start at planetary scales and passes right through the meso-scale.

Finally, it should be mentioned that usual spectral, fractal or multifractal analysis techniques effectively "wash-out" (by averaging) most of the (differential) horizontal anisotropy. If this directional averaging is justified at all, it is usually on the grounds of (presumed) statistical isotropy. However, the Coriolis (and other) forces induce strong differential stretching and rotation of structures. Although we do not expect this to break the scaling (the latter will no longer be self-similar, we will require GSI), it will generally lead to scaling breaks or logarithmically periodic oscillations in 1D sections.[42,43] This may explain the results of Barker and Davies[44] who used 2D spectral densities to show the strong anisotropy of two AVHRR pictures and then found slightly oscillatory 1D spectral cuts.[d]

1.2 Radiative Smoothing and the Scales <1 km

Theoretical considerations

Due to the lack of evidence for a break larger than ≈ 1 km (the inner scale of the AVHRR data), the search for characteristic scales has shifted to the

[b]Although real clouds are not passive scalars, at least some of their statistics are very close to passive scalars (their spectral exponents and the basic nonconservaton parameter H, see e.g. Lovejoy and Schertzer[11] and Davis et al.[41]

[c]Recall that scaling is a *statistical* symmetry principle which is only expected to hold on an infinite ensemble, it is almost broken on every single realization.

[d]Unfortunately, the authors did not perform 2D isotropic spectra (i.e. angle integration of the spectral density in fourier space) so that the scaling properties of the latter were not determined.

scales < 1 km which is smaller than that available from meteorological satellites. Since this range is also smaller than the meso-scale — it is in the classical turbulent "inertial" range — there are relatively abundant velocity and temperature spectra (especially at the corresponding time scales). In the horizontal, there is therefore not much doubt about the (roughly Kolmogorov[e]) scaling of the velocity and temperature fields down to the dissipation scale. Theoretically the situation for cloud radiances is still fairly straightforward: if, as a result of the turbulence, the cloud liquid water is scaling, then, since the corresponding radiative transfer equation has no characteristic scale, the radiance field should also be scaling. Various aircraft data have indeed found wide range liquid water scaling: 20 km to 10 m,[45] 330 km to down to 10 m,[11] and 13 km to[f] 5 m[46]). Since the photon effectively "integrates" the optical density over its path, the radiation field is thus a nonlinearly smoothed cloud field. *A priori*, since both the radiation process and cloud fields are both scaling, this smoothing occurs over a wide range of scales in a scaling manner. In a multifractal framework, the problem is thus to relate the singularities of the two fields; the simplest such relation being a (fractional) integration. The establishment of a connection between the singularities of the two fields is the aim of a series of papers.[47–49] In particular, the latter paper shows (with only relatively weak assumptions), that the transmitted radiative flux is an integral over a fractal flux tube.[g]

The primary theoretical objection to this scaling picture for the radiances has been voiced by Davis et al.[50] In effect, they pointed out that if clouds have characteristic vertical thicknesses, that this characteristic vertical cloud scale would break the scaling of the radiance field at roughly the corresponding horizontal distance. They have demonstrated this on both numerical cloud models, and by assuming classical (homogeneous field) photon diffusion. Some empirical support for this has come from 1D (time series) high resolare unknownare unknownution photometer cloud transmission data;[51] two stratiform clouds have indeed been shown to display breaks in the temporal spectra at scales which roughly correspond (using mean horizontal advection velocities) to the apparent mean cloud thickness. While there is not much doubt that imposition of a characteristic cloud thickness will indeed result in a characteristic horizontal radiance scale, we are rather more interested in characteristic scales in the cloud making *mechanism* (since it cannot be over-emphasized — scaling is only an ensemble symmetry). The implications, if *any*, to the problem of the scaling of cloud radiance *statistics* is therefore not at all obvious, and as the results below show, may not even extend to single cloud fields (i.e. the improved statistics of 2D spectra with respect to 1D spectra appear to be enough to eliminate the break even on individual cloud images). This is particularly true since one may presume (from 3D radar measurements of rain fields[34] or from the spectrum of the horizontal wind in the vertical direction discussed earlier, or from the statistics of vertically integrated passive scalar clouds,[52] that the vertical and horizontal cloud scalings are likely to be quite different (accounting for the differential stratification in the vertical). This means that the statistical relationship between the horizontal (L_x) and vertical (L_z) cloud extents will be of the form: $L_z = L_s(L_x/L_s)^{H_z}$ where L_s is the sphero-scale (the scale where structures are roughly isotropic), and H_z is an exponent < 1 (empirically $H_z \approx 0.22$ in rain,[34] ≈ 0.55 in the horizontal wind.[30] In stratiform clouds, L_s may be of the of the order of 10 m or less implying that the typical horizontal scale of clouds even only 1 km thick may be hundreds of kilometers (e.g. if $H_z < 0.5$). *A priori*, the (horizontal) radiative smoothing scale of an anisotropic multifractal with (roughly) 1 km thickness may therefore be far greater, being "smeared out" in a scaling way as the statistics are accumulated over a statistical ensemble of clouds with different thicknesses. Indeed, direct support for this lack of characteristic cloud thickness comes from recently published[53] temporal spectra of remotely sensed estimates of vertically integrated cloud liquid water: no significant breaks were observed over the range of about ten minutes to ten hours. These results suggest that neither the optical thickness nor the physical thickness has a characteristic value.

[e]To within multifractal intermittency corrections.

[f]Although the authors claimed that there was a real break at about 2–5 m, corresponding to some unknown fundamental cloud dynamics, the break is likely due to insufficient dynamical range of the sensor (see discussion below).

[g]This is a great improvement on the popular "Independent Pixel Approximation" which assumes that the integration is over a nonfractal, and hence much shorter, vertical column.

Empirical studies

Empirically, the scaling of cloud radiances at scales < 1 km is not well established, published studies having until now relied on non-meteorological satellites (primarily LANDSAT) which are designed to capture weak surface radiances, and frequently saturate when clouds are present. In addition to this technical problem, such data is very expensive (several thousand dollars a scene), prohibiting large scale studies; these problems undoubtedly explain the paucity of empirical studies < 1 km.

The first and most frequently cited paper in this scale range is that of Cahalan and Joseph[54] who fourier analyzed ten individual lines from a 1024×1024 subscene of a single LANDSAT cloud picture, concluding (on the basis of enhanced spectral smoothing observed over the last factor of four or so in scale, i.e. < 500 m), that this represented a fundamental cloud scale. More recently, Lovejoy et al.[27] have performed a 2D spectral analysis (much less sensitive to the effects of anisotropy than 1D spectra, and using the full data) of three LANDSAT (MSS) scenes (120 m resolution), finding evidence for a break at the extreme end (< 300 m), but that due to massive amounts of signal saturation that the results were inconclusive.[h] Davis et al.[50] have performed similar analysis of a single LANDSAT TM (30 m) cloud scene and found excellent scaling over the entire range except for a slight high frequency drop off over the extreme factor of two (which they however take as evidence for a real break), and Stanway[40] has performed a similar analysis on a single SPOT image (10 m), finding no evidence of any break (5 km to 10 m).

2. USING GROUND-BASED CLOUD PHOTOGRAPHY TO SEARCH FOR CHARACTERISTIC LENGTHS IN CLOUD RADIANCES

2.1 The Data

As this brief review makes clear, the empirical picture for clouds and cloud radiance characteristic lengths is still under debate probably primarily due to the relatively small amount of data which has been analyzed. In this paper, we use land-based photography to analyze cloud visible light transmittance. In this way, we are able to extend the study of their energy spectra to smaller scales than have been previously analyzed, over a large sample of data (i.e. many realizations). We then show that these energy spectra exhibit consistent scaling across all scales observed, and find empirical multifractal parameters for this energy field.

Thirty-eight cloud images were studied in order to statistically test for the existence of small scale scaling breaks covering the scale regime where scale breaks have been claimed: 30–500 m (Fig. 1). In order to achieve resolutions in the range of 1 km to under 1 m, land-based photographs were taken, digitally scanned and then analyzed by various scaling techniques (including isotropic energy spectra). The authors are aware of no comparable scaling studies using land-based photography; the closest being the 1D[51] transmittance series containing much less information than even a single one of the pictures studied here. The benefits of this method are not only the extremely high resolution (meters or less), but also the cost-effectiveness relative to the use of either satellite images or LWC data.

Photographs were taken using both 35 mm and 2 1/4 inch black and white film, in Montreal, shot near zenith near local noon. Cloud base heights were estimated from ceilometer data at nearby Dorval Airport, and combined with the camera lens characteristics were used to estimate the pixel size of the digitized images. However, since we did not find any strong breaks, the uncertainty in these heights ($\approx \pm 50\%$) although large, does not significantly affect our results. In addition, scenes where cloud base height was ill-defined were discarded. Finally, photographs with saturation greater than 10% of the pixels were also discarded (most scenes used had under 1% saturation). As well, all images were examined at high resolution for defects (e.g. scratches, dust), and those with such defects were also discarded. Note that for the graph of the ensemble average of energy versus wave number, such errors would tend to exaggerate scale breaks rather than tending to linearity, therefore the results themselves will show that these problems are unimportant.

2.2 Scaling Results

All digital images were scanned to 2048×2048 pixels using a commercial scanner with eight-bit

[h]Saturation — if occurring over enough pixels — can indeed artificially break the scaling.

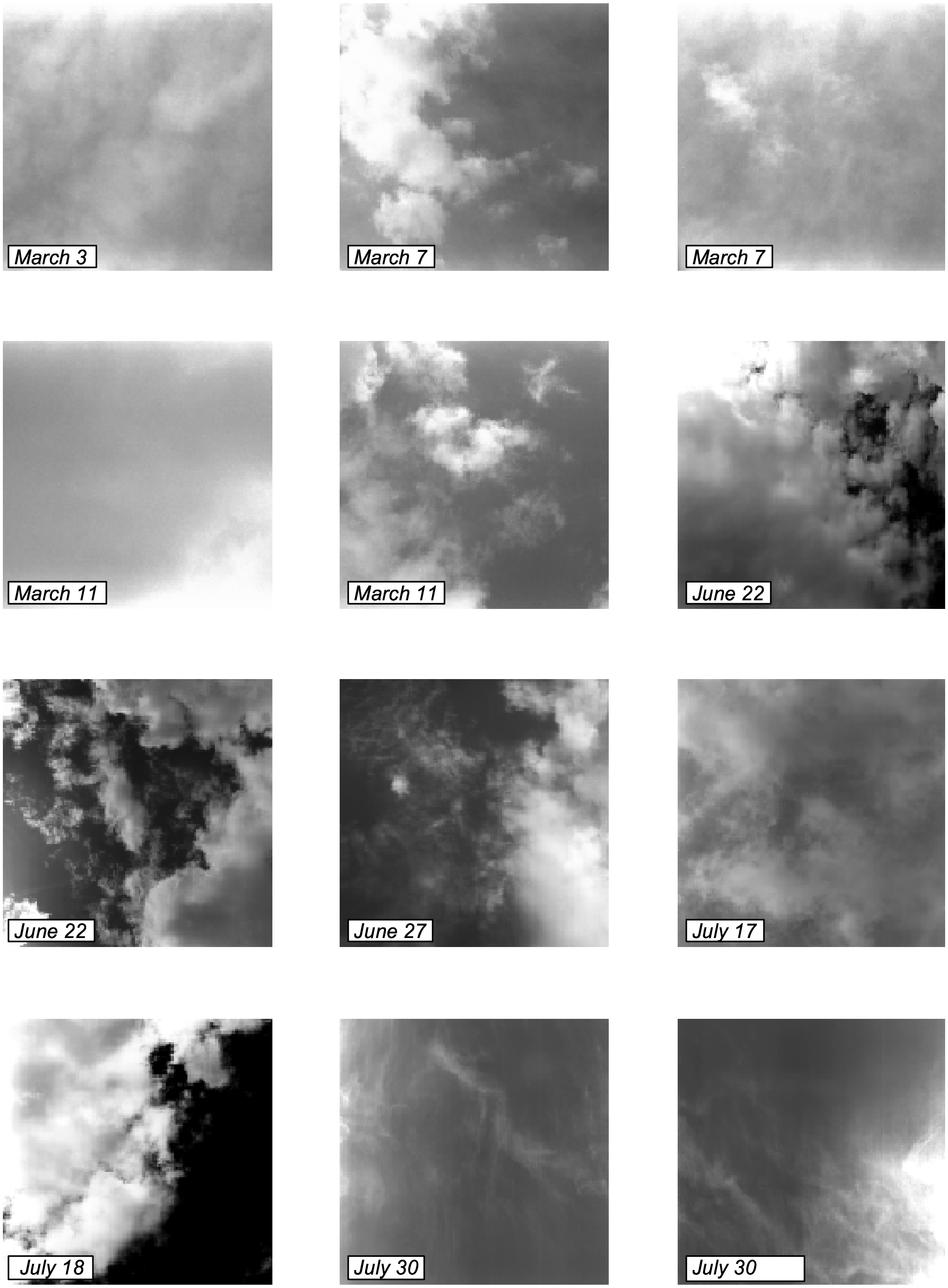
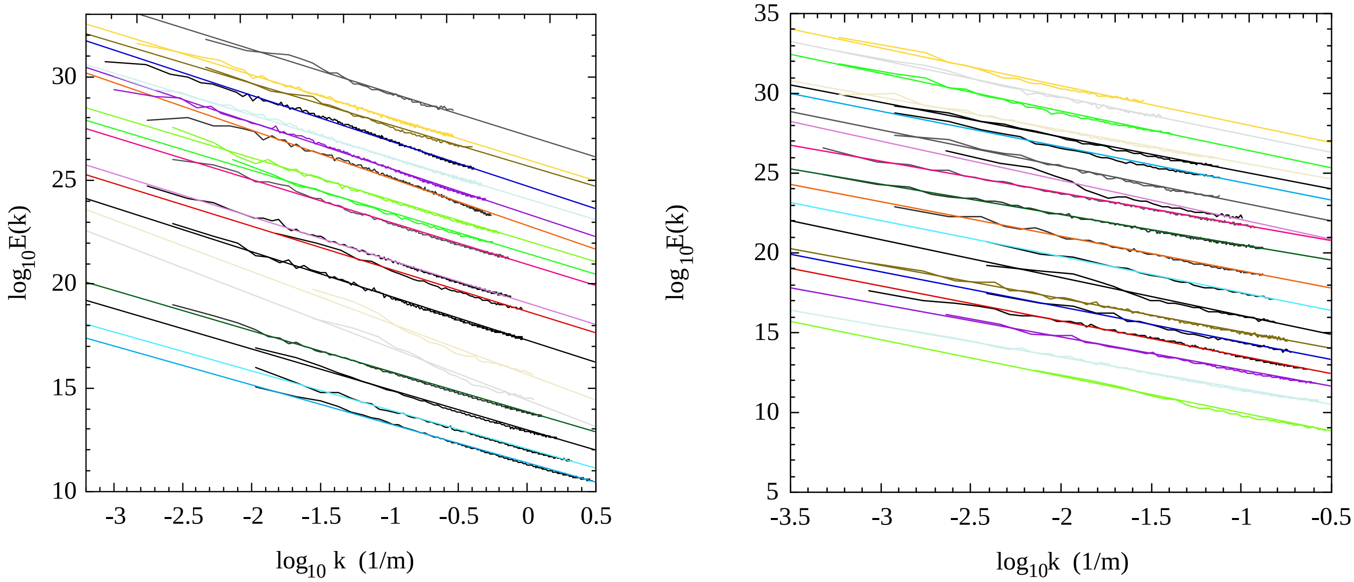


Fig. 1 Twelve of the 38 cloud pictures analyzed here — showing a wide diversity of cloud types.



(a) **(b)**

Fig. 2 Spectra of the **(a)** larger and **(b)** smaller scenes, separated in the vertical for clarity, with power law regressions shown.

dynamical range[i] (grayscale depth). This limited dynamical range effectively limits the possible range over which scaling can be observed, since at small enough scales neighboring pixels will generally have nominally identical values (due to this coarse "quantization"). A simple model for this quantization effect is that of a white noise of amplitude equal to the minimum distiguishable level differences; as expected, the spectra do flatten out at very high wave numbers. In geostatistical terminology, this is therefore a classical "nugget" effect usually observed in real space using variogrammes. However in wave number space, it is also quite easy to analyze and determine the wave number where this flattening will occur. For each image, the number of levels ("digital counts", DN) between the maximum and minimum values ($\Delta I_{\max}$; expressed in DN) gives an estimate of the low wave number ($k = 1$ corresponding to the entire scene size) standard deviation; hence if k_{cr} is critical wave number where flattening occurs, then $E(1)/E(k_{cr}) \approx (\Delta I_{\max})^2$. In practice, k_{cr} is easy to determine from $\log E(k)$ versus $\log k$ plots; starting at the low frequency end, one simply moves down to energy levels $E(k_{cr}) \approx E(1)/(\Delta I_{\max})^2$. Since generally, we find the spectrum to be scaling: $E(k) \approx k^{-\beta}$, we obtain:

$$k_{cr} \approx (\Delta I_{\max})^{2/\beta} \,. \tag{1}$$

In addition, since empirically, $\beta \approx 2$, and since $\Delta I_{\max}$ is usually not far from 2^8 (the maximum due to the eight-bit dynamical range), we typically obtain $k_{cr} \approx 256$ (in units of inverse pixels). In order to minimize this quantization effect, as well as the effect of other defects, pictures were therefore averaged to k_{cr} pixels across (roughly equivalent to as taking the picture at a lower pixel resolution). In Fig. 2, we show the resulting spectra arranged top to bottom in order of increasing absolute resolution (using the estimated cloud height). As can be seen, although the scaling need not hold on individual realizations, it is nevertheless surprisingly well respected; the main variations about power decay being displayed at the lowest wave numbers where each photo contains very few corresponding structures (i.e. the statistics become very poor).

We see fair scaling on each realization with small variability in the spectral exponent β; reminiscent of the surprisingly little scene-to-scene variability in scaling at much larger scales using AVHHR satellite data in Lovejoy et al.[27] Figure 3 shows the ensemble averaged energy spectra across all 38 scenes; only the range of scales over which at least five spectra were contributing to the average is shown since for the extremes, representing extremely large and small scale structures, few scenes contributed to the average. As can be seen, the central region,

[i]Technically the scanner had 12 bits but four were used internally to improve the signal; only eight were available for analysis.

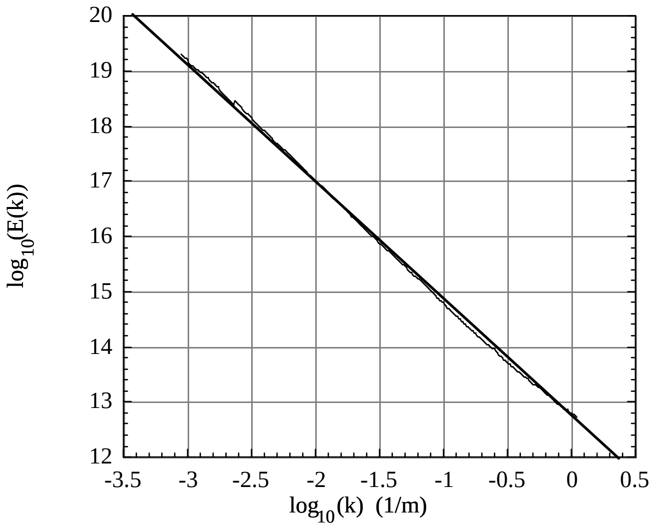


Fig. 3 Ensemble averaged power spectra (only the averages using spectra from at least five clouds are shown). The straight reference line has slope −2.10.

where most spectra contributed and the data is most meaningful as representing an "ensemble average", the scaling is excellent.

The power laws exponent estimated for the ensemble is $\beta \approx 2.10$. This is slightly higher than spectra obtained from satellite images with either ocean or land below, presumably reflecting the fact that the background field here is (nearly) totally smooth (sky). In the largest such surveys, Lovejoy et al.[27] obtained values of 1.67 (AVHRR, over ocean, 1.1–550 km) and 1.43 (AVHRR land[40]), see Table 1. In satellite images, the background for the clouds are earth and water, both of which exhibit their own multifractal (scaling) characteristics different from clouds.[55,56] In these images, background to the clouds is flat sky (i.e. to the limits of the resolution observed).

2.3 Multifractal Results

With these results showing scaling of the cloud energy spectra down to under 1 m, we then performed systematic multifractal analysis on the data to determine the exact type of scaling. Recall that whereas the general framework for scale invariant geometric sets of points is fractal sets, the general framework for scaling fields is multifractals involving an infinite hierarchy of fractal sets. If the intensity field at scale ratio λ (largest/smallest scale) is I_λ, then this means that:

$$I_\lambda = \phi_\lambda \lambda^{-H} \tag{2}$$

where ϕ_λ is the scale-by-scale conservative flux; H characterizes the distance of the observed I_λ from ϕ_λ. In the Fractionally Integrated Flux model (FIF[2,3] the linear scaling λ^{-H} corresponds to a fractional integral (power law filter) of order H, and ϕ_λ which is the direct result of a multiplicative cascade process has the following statistics:

$$\langle \phi_\lambda^q \rangle = \lambda^{K(q)} \tag{3}$$

where $K(q)$ is the multiscaling moment exponent function. Exploiting the existence of stable attractive multifractal processes, $K(q)$ can be characterized by the two universal multifractal parameters C_1, $0 \le \alpha \le 2$:

$$K(q) = \frac{C_1}{\alpha - 1}\left(q^\alpha - q\right). \tag{4}$$

These parameters were found using the technique of Double Trace Moments (DTM).[57] The DTM technique directly estimates α, C_1, through introduction of a scaling exponent $K(q,\eta)$ via the use of a second moment η, to which we raise the resolution field at the highest available resolution (Λ):

$$\langle (\phi_\Lambda^\eta)_\lambda^q \rangle = \lambda^{K(q,\eta)}. \tag{5}$$

The notation above indicates that the field at the highest available resolution Λ is raised to the η power; the result degraded to the intermediate resolution λ (obtained by either spatial averaging or by wavelets), and the average of the qth power taken. For universal multifractals, this yields a straightforward relation for α:

$$K(q,\eta) = \eta^\alpha K(q). \tag{6}$$

It should be noted that this analysis is carried out on the underlying conserved field ϕ. Since the observed I is related to the conserved cascade quantity ϕ by a fractional integration of order H, ϕ can be obtained by fractionally differentiating the measured field (filtering with k^H). Lavallée et al.[57] showed that if $H < 1$, it is sufficient to estimate ϕ from the (absolute value) of the finite difference approximation to the first derivative. In cases where the dynamical range is inadequate (such as here), this will lead to a potentially significant number of spurious exactly zero gradients (i.e. when neighboring field values have identical digital counts); even a small number of spurious values can greatly affect the low enough order statistics. Hence, following numerical studies discussed in Stanway,[40] we first fractionally integrated by a small amount

Table 1 A comparison of various multifractal radiance studies.

	α	C_1	H	β
Light transmitted through clouds (this paper)	1.77	0.061	0.61	2.10
Nebulae (star light transmitted through interstellar dust)[58]	1.96	0.038	0.67	2.26
AVHRR: clouds over land[40]	1.93	0.078	0.36	1.43
AVHRR: clouds over ocean[40 j]	1.83	0.095	0.21	1.67

(order 0.2), and then took absolute differences. The first step is a (32-bit) scaling smoothing operation which essentially eliminates all zero gradients; the second step eliminates the constant level corresponding to $k = 0$ in spectral space (the latter cannot be fractionally integrated by fourier techniques since the filter k^{-H} diverges for $k = 0$).

Equation (6) allows us to estimate α simply by plotting $\log K(q, \eta)$ versus $\log(\eta)$ for fixed q. Varying q will then improve our statistical accuracy. From this plot (Fig. 4) the slope yields α, and the intercept an estimate of C_1. Since for large enough $q\eta$, the statistics become poor (being eventually dominated by single large gradients), the curves eventually flatten off for large η. Similarly, for small enough η, the results become sensitive to problems measuring weak gradients (including round-off error in four-byte arithmetic); numerical investigation by Stanway,[40] shows that the optimum range for estimating C_1, α is the range $0.1 < (q\eta) < 1$.

Averaging the values from these graphs, we obtain, $\alpha = 1.77$, $C_1 = 0.061$. Since the spectrum is a second order statistic, it can be shown that for such processes, H can be derived simply from the values of β, α, and C_1, with β representing the spectral slope of the observed process.

$$H = (\beta - 1 + K(2))/2\,. \tag{7}$$

Using this relation and the above parameter estimates, we find $H = 0.61$ summarized in Table 1.

Table 1 shows that while C_1 and α are roughly the same for both transmitted and reflected radiances, the main difference being the value of H which is substantially lower for the reflected fields whether over ocean or land (although in the latter there is substantial scatter in the estimates from one cloud picture to the next, of the order of ± 0.2). There may indeed be a physical reason for the similitude of the radiances through atmospheric or interstellar clouds. In both cases, the scatterers are advected by turbulence (although in the latter, it is magneto-hydrodynamic turbulence), and in both cases, the scattering process is dominated by scattering (there is little absorption).

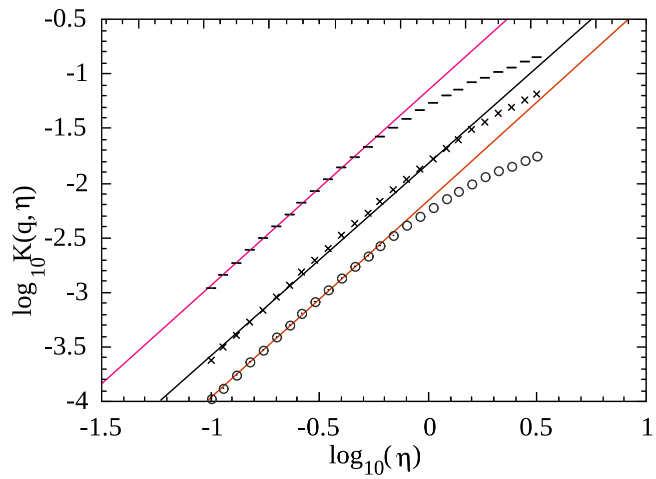


Fig. 4 Double trace moment for the ensemble of 38 scenes for $q = 0.5$, 1.1, 1,7 (the slopes fit over the range $0.1 < (q\eta) < 1$ are respectively: $\alpha = 1.76$, 1.79, 1.81).

3. CONCLUSION

The standard model of the atmosphere involves large scale isotropic 2D turbulent regime separated by a small scale isotropic 3D regime by a "mesoscale gap" somewhere in the vicinity of the presssure scale height ($\approx$ 10 km). With theoretical (scaling) and empirical advances seriously undermining this picture, attention has turned to smaller scales where

[j]The same data was analyzed by Tessier et al.[59] but without carefully dealing with the spurious zero gradients as discussed above, yielding a substantially lower α estimate, but similar C_1.

meteorological satellites do not operate: although the horizontal cloud liquid water field is empirically scaling down to meters or less, if clouds had a characteristic vertical thickness, then the radiation field would nevertheless yield a comparable horizontal scale.

In this study, systematic analyses of cloud radiance transmittance were carried out. The clouds analyzed were for scales from the 1 km range down to under 1 m. Using standard energy spectrum analyses — and carefully accounting for the limited dynamical range of the digitized images — we showed that even individual realizations exhibited good scaling throughout the ranges analyzed; the ensemble average spectrum had remarkably good scaling over the range of at least 1 km down to $\approx$ 1 m. This is strong support for the scaling of cloud liquid water densities in both horizontal and vertical directions.

When combined with similar systematic analyses of cloud reflectance taken from satellite images, it is likely that the atmosphere is scaling from global scales down to the sub-meter regime. This evidence is strong support of the Unified Scaling Model of the atmosphere. In addition to the range of scaling, we also investigated the type of scaling showing that the radiances are multifractal and characterized the latter by the corresponding multifractal universality classes.

Multifractal analyses were then applied to these fields to determine the parameters α, C_1 and H (which define the entire $c(\gamma)$ and $K(q)$ functions). Ensemble averaged results were similar to those found in previous studies of atmospheric fields. The high value for α in particular indicates strongly turbulent multifractal fields, where violent singularities will arise. These results give us a statistical basis for prediction of earth radiation variability extended beyond simple climatological averages, which do not account well for higher order statistics/extreme singularities.

A better understanding of the multifractal parameters of cloud light transmittance will be important in improved modeling of the earth's radiation budget, and is of fundamental importance in understanding the relation between cloud internal structure and light transmittance. This study provides further evidence for defining the values of these parameters at least within a small range. Because of the extreme cost efficiency of this method, it is hoped that the results of this study can quickly and independently be corroborated and extended.

REFERENCES

1. L. F. Richardson, *Weather Prediction by Numerical Process* (Cambridge University Press, republished by Dover, 1965, 1922).
2. D. Schertzer and S. Lovejoy, "Physical Modeling and Analysis of Rain and Clouds by Anisotropic Scaling of Multiplicative Processes," *J. Geophys. Res.* **92**, 9693–9714 (1987).
3. D. Schertzer, S. Lovejoy, F. Schmitt, Y. Chigirinskaya and D. Marsan, "Multifractal Cascade Dynamics and Turbulent Intermittency," *Fractals* **5**, 427–471 (1997a).
4. F. Schmitt, S. Lovejoy, D. Schertzer, D. Lavallée and C. Hooge, "First Estimates of Multifractal Indices for Velocity and Temperature Fields," *Comptes Rendus de l'Académie des Sciences de Paris*, *serie II*, **314**, 749–754 (1992b).
5. F. Schmitt, D. Schertzer, S. Lovejoy and G. Brunet, "Universal Multifractal Structure of Atmospheric Temperature and Velocity Fields," *Europhys. Lett.* **34**, 195–200 (1996).
6. F. Schmitt, D. Lavallée, D. Schertzer and S. Lovejoy, "Empirical Determination of Universal Multifractal Exponents in Turbulent Velocity Fields," *Phys. Rev. Lett.* **68**, 305–308 (1992a).
7. Y. Chigirinskaya, D. Schertzer, S. Lovejoy, A. Lazarev and A. Ordanovich, "Unified Multifractal Atmospheric Dynamics Tested in the Tropics. Part 1: Horizontal Scaling and Self-Organized Criticality," *Nonlinear Process. Geophys.* **1**, 105–114 (1994).
8. A. Lazarev, D. Schertzer, S. Lovejoy and Y. Chigirinskaya, "Unified Multifractal Atmospheric Dynamics Tested in the Tropics: Part II, Vertical Scaling and Generalized Scale Invariance," *Nonlinear Process. Geophys.* **1**, 115–123 (1994).
9. F. Schmitt, D. Schertzer, S. Lovejoy and Y. Brunet, "Empirical Study of Multifractal Phase Transitions in Atmospheric Turbulence," *Nonlinear Process. Geophy.* **1**, 95–104 (1994).
10. S. Lovejoy and D. Schertzer, "Multifractals and Rain," in *New Uncertainty Concepts in Hydrology and Water Ressources*, ed. Z. W. Kunzewicz (Cambridge University Press, 1995b), pp. 62–103.
11. S. Lovejoy and D. Schertzer, "How Bright is the Coast of Brittany?" in *Fractals in Geoscience and Remote Sensing*, ed. G. Wilkinson (Office for Official Publications of the European Communities, Luxembourg, 1995a), pp. 102–151.
12. P. Gabriel, S. Lovejoy, D. Schertzer and G. L. Austin, "Multifractal Analysis of Resolution Dependence in Satellite Imagery," *Geophys. Res. Lett.* **15**, 1373–1376 (1988).
13. S. Lovejoy and D. Schertzer, "Multifractals, Universality Classes and Satellite and Radar Measurements of Cloud and Rain Fields," *J. Geophys. Res.* **95**, 2021 (1990).

14. G. Salvadori, S. P. Ratti, G. Belli, S. Lovejoy and D. Schertzer, "Multifractal Objective Analysis of Seveso Ground Pollution," *Ann. Geophys.*, in press (1992).
15. Y. Chigirinskaya, D. Schertzer, G. Salvadori, S. Ratti and S. Lovejoy, "Chernobyl ^{137}Cs Cumulative Soil Deposition in Europe: Is It Multifractal?" in *Chaos, Fractals and Models 96*, eds. F. M. Guindani and G. Salvadori (Italian University Press, 1998), pp. 65–72.
16. D. Finn, B. Lamb, M. Y. Leclerc, S. Lovejoy, S. Pecknold and D. Schertzer, "Multifractal Analysis of Plume Concentration Fluctions in Surface Layer Flows," *J. Appl. Meteor.* **40**, 229–245 (2001).
17. S. Pecknold, S. Lovejoy and D. Schertzer, "Universal Multifractal Landscape Topography — Data Analysis and Simulations," *Ann. Geophys.* **14**, C633 (1996).
18. J. S. Gagnon, S. Lovejoy and D. Schertzer, "Multifractal Surfaces: The Example of Terrestrial Topography," submitted to *Phys. Rev. Lett.* (2002).
19. R. Fjortoft, "On the Changes in the Spectral Distribution of Kinetic Energy in Two-Dimensional, Nondivergent Flow," *Tellus* **7**, 168–176 (1953).
20. R. H. Kraichnan, "Inertial Ranges in Two-Dimensional Turbulence," *Phys. Fluids* **10**, 1417–1423 (1967).
21. I. van der Hoven, "Power Spectrum of Horizontal Wind Speed in the Frequency Range from 0.0007 to 900 Cycles Per Hour," *J. Meteorol.* **14**, 160–164 (1957).
22. N. Z. Pinus, "The Energy of Atmospheric Macro-Turbulence," *Izvestiya*, *Atmos. Oceanic Phys.* **4**, 461 (1968).
23. N. K. Vinnichenko, "The Kinetic Energy Spectrum in the Free Atmosphere for 1 Second to 5 Years," *Tellus* **22**, 158 (1969).
24. G. D. Nastrom and K. S. Gage, "A First Look at Wave Number Spectra from GASP Data," **Tellus 35**, 383 (1983).
25. D. Lilly and E. L. Paterson, "Aircraft Measurements of Atmospheric Kinetic Energy Spectra," *Tellus* **A35**, 379–382 (1983).
26. G. D. Nastrom and K. S. Gage, "A Climatology of Atmospheric Wavenumber Spectra of Wind and Temperature by Commercial Aircraft," *J. Atmos. Sci.* **42**, 950–960 (1986).
27. S. Lovejoy, D. Schertzer, P. Silas, Y. Tessier and D. Lavallée, "The Unified Scaling Model of Atmospheric Dynamics and Systematic Analysis in Cloud Radiances," *Ann. Geophys.* **11**, 119–127 (1993).
28. A. F. Tuck and S. J. Hovde, "The Hurst Exponent and Calipers Measures Applied *in situ* Aircraft Observations in the Lower Stratosphere and Upper Troposphere," *Geophys. Res. Abstracts* **1**(4), 772 (1999).
29. S. Lovejoy, D. Perrault-Joncas and D. Schertzer, "Fractal Aircraft Trajectories and Satellite Cloud Imagery: Direct Evidence for 23/9 Elliptical Dimensional Cascades," *Proceedings of the 9th European Turbulence Conference*, *Southhampton*, *UK*, *July 2002*.
30. D. Schertzer and S. Lovejoy, "The Dimension and Intermittency of Atmospheric Dynamics," in *Turbulent Shear Flow 4*, ed. B. Launder (Springer-Verlag, 1985a), pp. 7–33.
31. D. Schertzer and S. Lovejoy, "Generalised Scale Invariance in Turbulent Phenomena," *Physico-Chemical Hydrodynamics J.* **6**, 623–635 (1985b).
32. R. M. Endlich and R. L. Mancuso, "Objective Analysis of Environmental Conditions Associated with Severe Thunderstorms and Tornadoes," *Monthly Weather Rev.* **96**, 342–350 (1968).
33. S. I. Adelfang, "On the Relation Between Wind Shears over Various Intervals," *J. Atmos. Sci.* **10**, 138 (1971).
34. S. Lovejoy, D. Schertzer and A. A. Tsonis, "Functional Box-Counting and Multiple Dimensions in Rain," *Science* **235**, 1036–1038 (1987).
35. Z. S. She and E. Levesque, *Phys. Rev. Lett.* **72**, 336 (1994).
36. D. Schertzer, S. Lovejoy and F. Schmitt, "Structures in Turbulence and Multifractal Universality," in *Small-Scale Structures in 3D and MHD Turbulence*, eds. M. Meneguzzi, A. Pouquet and P. L. Sulem (Springer-Verlag, 1995), pp. 137–144.
37. S. Lovejoy, "Area Perimeter Relations for Rain and Cloud Areas," *Science* **187**, 1035–1037 (1982).
38. S. Lovejoy, D. Schertzer, J. D. Stanway, B. Watson and D. Sachs, "Multifractal Modelling and Analysis of Radiation in Clouds: 5000 km to 50 cm," in *7th Atmos. Rad. Meas.* (*ARM*) *Meeting*, ed. United States Department of Energy, DOE, San Antonio, 3/97, 1997.
39. S. Lovejoy, D. Schertzer and J. D. Stanway, "Direct Evidence of Planetary Scale Atmospheric Cascade Dynamics," *Phys. Rev. Lett.* **86**, 5200–5203 (2001).
40. J. D. Stanway, "Empirical Study of the Scaling of Cloud Radiances: 10 m to 5000 km," MSc. Thesis, McGill, Montreal, 1999.
41. A. Davis, A. Marshak, W. Wiscombe and R. Cahalan, "Scale Invariance of Liquid Water Distributions in Marine Stratocumulus. Part I: Spectral Properties and Stationarity Issues." *J. Atmos. Sci.* **53**, 1538–1558 (1996).
42. K. Pflug, S. Lovejoy and D. Schertzer, "Generalized Scale Invariance, Differential Rotation and Cloud Texture," *J. Atmos. Sci.* **50**, 538–553 (1993).
43. S. Pecknold, S. Lovejoy, D. Schertzer and C. Hooge, "Multifractals and the Resolution Dependence of Remotely Sensed Data: Generalized Scale Invariance and Geographical Information Systems," in *Scaling*

in Remote Sensing and Geographical Information Systems, ed. M. G. D. Quattrochi (Lewis, Boca Raton, Florida, 1997), pp. 361–394.

44. H. W. Barker, and J. A. Davies, "Cumulus Cloud Radiative Properties and the Characteristics of Satellite Radiance Wavenumber Spectra," *Remote Sens. Environ.* **42**, 51–64 (1992).
45. A. Davis, A. Marshak, A. Wiscombe and R. Cahalan, "Multifractal Characterization of Nonstationary and Intermittency in Geophysics: Observed Retrieved or Simulated," *J. Geophys. Res.* **99**, 8055–8072 (1994).
46. A. B. Davis, A. Marshak, H. Gerber and W. J. Wiscombe, "Horizontal Structure of Marine Boundary-Layer Clouds from cm to km Scales," *J. Geophys. Res.* (*Atmos.*) **D104**, 6123–6144 (1998).
47. S. Lovejoy, B. Watson, D. Schertzer and G. Brosamlen, "Scattering in Multifractal Media," in *Particle Transport in Stochastic Media*, ed. L. Briggs (American Nuclear Society, Portland, Oregon, 1995), pp. 750–760.
48. C. Naud, D. Schertzer and S. Lovejoy, "Fractional Integration and Radiative Transfer in Multifractal Atmospheres," in *Stochastic Models in Geosystems*, eds. W. Woyczynski and S. Molchansov (Springer-Verlag, 1996), pp. 239–267.
49. D. Schertzer, S. Lovejoy, F. Schmitt, C. Naud, D. Marsan, Y. Chigirinskaya and C. Marguerit, "New Developments and Old Questions in Multifractal Cloud Modeling, Satellite Retrievals and Anomalous Absorption," in *7th Conf. Atmos. Rad. Meas.* (*ARM*) *Meeting*, ed. United Department of Energy (San Antonio, 1997b), pp. 327–335.
50. A. Davis, A. Marshak, W. J. Wiscombe and R. F. Cahalan, "The LANDSAT Scale-Break in Stratocumulus as a Three-Dimensional Radiative Transfer Effect, Implications for Cloud Remote Sensing," *J. Atmos. Sci.* **54**, 241–260 (1997).
51. C. Savigny, O. Funk, U. Platt and K. Pfeilsicker, "Radiative Smoothing in Zenith-Scattered Skylight Transmitted to the Ground," *Geophys. Res. Lett.*, submitted (1999).
52. H. Gaonac'h, S. Lovejoy, D. Schertzer, K. St. Armand and J. Stix, "Direct Measurement of the Elliptical Dimension of Atmospheric Dynamics Using Remote Sensing of Volcanic SO_2 Gas," *Geophys. Res. Abstracts* **1**, 772 (1999).
53. K. Ivanova and T. Ackerman, "Multifractal Chacterization of Liquid Water in Clouds," *Phys. Rev.* **E59**, 2778–2782 (1999).
54. R. F. Cahalan and J. H. Joseph, "Fractal Statistics of Cloud Fields," *Monthly Weather Rev.* **117**, 261–272 (1989).
55. A. Laferrière and H. Gaonac'h, "Multifractal Properties of Visible Reflectance Fields from Basaltic Volcanoes," *J. Geophys. Res.* **104**, 5115–5126 (1999).
56. S. Lovejoy, D. Schertzer and Y. Tessier, "Multifractals and Resolution Independent Remote Sensing Algorithms: The Example of Ocean Colour," *Int. J. Remote Sens.* **22**, 1191–1234 (2001).
57. D. Lavallée, S. Lovejoy, D. Schertzer and P. Ladoy, Nonlinear Variability and Landscape Topography: Analysis and Simulation," in *Fractals in Geography*, eds. L. De Cola and N. Lam (Prentice-Hall, 1993), pp. 171–205.
58. J. Khoury, J. Sylvestre, S. Lovejoy, P. Bastien and D. Schertzer, "Multifractal Analysis of Scattered Light Field from Bipolar Nebulae," *Astrophys. J.*, submitted (1999).
59. Y. Tessier, S. Lovejoy and D. Schertzer, "Universal Multifractals: Theory and Observations for Rain and Clouds," *J. Appl. Meteorol.* **32**(2), 223–250 (1993).