From Preprints IUTAM Symposium on turbulence and chaotic phenomena in fluids. 5-10 Sept., 1983 / Kyoto, Japan

ON THE DIMENSION OF ATMOSPHERIC MOTIONS. D.Schertzer et S.Lovejoy, EERM/CRMD, Paris

Introduction:

The classical scheme of atmospheric motions (e.g. Monin (1972), considers the large scale as two-dimensional, and the small scale as three-dimensional. Between these scales a "dimensional transition" is expected to occur; possibly in conjunction with a "meso-scale gap" (Van der Hoven (1957). This transition, if it were to occur would be likely to have fairly drastic conseqeuences because of the significant qualitative differences between turbulence in two and three dimensions (e.g. Kraichnan (1967): the all important stretching and folding of vortex tubes cannot occur in two dimensions.

Recently, Schertzer and Lovejoy (1983) have examined the considerable body of theoretical and empirical evidence supporting the view that no fundamental length scales occur between an inner dissipation scale of the order of centimeters, and an outer "external" scale near planetary sizes. They argue that in this "scaling" regime, that two basic aspects of atmospheric behaviour are characterised by the exponents H, α. The scaling exponent H is defined by:

$$\Delta X(\lambda\Delta x) \overset{d}{=} \lambda^{H}\Delta X(\Delta x) \quad , \quad \lambda > 1$$

which relates fluctuations ΔX in field X at large scales $(\lambda\Delta x)$ and at small scales (Δx). "$\overset{d}{=}$" means equality of probability distributions. (For a finite variance ΔX, $H = 2\beta + 1$ where $-\beta$ is the exponent of the power spectrum). The hyperbolic exponent α characterised the frequency of occurence of extreme fluctuations (the intermittency), defined by:

$$\Pr(\Delta X' > \Delta X) \sim (\Delta X / \Delta X^{*})^{-\alpha}$$

for the probability of a random fluctuation $\Delta X'$ exceeding a fixed ΔX, and ΔX^{*} characterises the amplitude of the fluctuations. A phenomenological turbulence model proposed by Mandelbrot (1974) predicts hyperbolic distributions for $\bar{\epsilon}$, the average energy flux.

Lovejoy (1981) and Schertzer and Lovejoy (1983) found empirically: $\alpha_R \sim 5/3$, $\alpha_\epsilon \sim 5/3$, $\alpha_v \sim 5$, $\alpha_{\ln\theta} \sim 10/3$, $\alpha_\phi\, 1$ (for rainrate R, horizontal velocity v, log potential temperature $\ln\theta$ and the flux of buoyancy force variance, ϕ). In the following, we examine these two aspects in greater detail: a) the scaling which is quite different in the horizontal and the vertical leads to the introduction of an elliptical dimension $D_{el} = 23/9 \sim 2.56$ to characterise the unbroken anisotropic scaling, b) the hyperbolic intermittency, which we show may be a universal feature of turbulence- recent wind tunnel experiments also suggest $\alpha_v \sim 5$.

Vertical Structure and anisotropic scaling

Perhaps the most serious objection to the hypothesis of scaling behaviour in the atmosphere arises from the special role of the vertical axis. Indeed, there has been a deluge of papers based on non-scaling techniques which reject implicitely a priori any possibility of vertical scaling (e.g. "one point closures"). In what follows, it will be apparent that this rejection has had un-

unfortunate consequences.

The vertical structure plays a key role for the following reasons

i) The gravity field defines a direction at every point.

ii) The atmosphere is globally stratified.

iii) It has a well-defined thickness (exponential decrease of the mean pressure) :

iv) The fundamental sources of disturbances are the vertical shear and the buoyancy force (e.g; the Kelvin-Helmholz instability) .

In recent years, experiments using thousands of Jimspheres (Endlich et Al (1969) , Adelfang (1971), Van Zandt (1982)' and radiosondes (Schertzer and Lovejoy (1983) ,' have found evidence for a contiuous, unbroken vertical scaling of the horizontal wind field up to distances of at least 16km. Schertzer and Lovejoy (1983) showed that these results can be accounted for if, after Bogliano (1959), and Obukhov (1959), ϕ is taken as the fundamental parameter governing the vetical structure. In this case, dimensional analysis yields:

$$\Delta v(\Delta z) \overset{d}{=} \bar{\phi}(\Delta z)^{1/5} \Delta z^{H_v} \quad , \quad H_v = 3/5.$$

If we compare this with the Kolmogorov scaling known to hold in the horizontal ($\Delta v(\Delta x) \overset{d}{=} \bar{\epsilon}(\Delta x)^{1/3} \Delta x^{H_h}$, $H_h = 1/3$), then the small scale structure differs from the large not only in size (a similarity transformation) ,' but also by a stretching transformation. This may be written:

$$\Delta v(\underline{\underline{G}}_\lambda \Delta \underline{r}) \overset{d}{=} \lambda^{H_h} \Delta v(\Delta \underline{r}) \quad \text{with}$$

$$\underline{\underline{G}}_\lambda = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{H_z} \end{bmatrix} \quad , H_z = H_h/H_v = 5/9$$

This transformation, which applies to the statistical properties of the atmosphere, increases the average volume of an eddy by the factor: $\lambda \cdot \lambda \cdot \lambda^{H_z} = \lambda^{D_{el}}$, $D_{el} = 2 + H_z = 23/9 = 2.56$. Atmospheric motions are therefore never flat ($D_{el} = 2$), nor the same in all directions ($D_{el} = 3$) ; but always display aspects of both according to a well defined fractal (scaling) geometry. An immediate physical consequence is that the number of eddies $N(\ell)$ of horizontal size ℓ is:

$$N(\ell) \sim \ell^{-D_{el}}$$

This anisotropic scaling has lead to the introduction of several new notions e.g. the "sphero-scale" (the scale at which average eddies are isotropic) , and "stochastic stratification" which expresses the fact that this anisoptropy implies that atmospheric fields are on average more and more statified at larger and larger scales (see Schertzer and Lovejoy (1983) for a further discussion) .

The Universality of the divergence of moments

Schertzer and Lovejoy (1983) showed by direct determination of the probability distributions that $\alpha_v \sim 5$ in the atmosphere. In order to see if this is a general property of turbulence, below we analyse wind tunnel data from Anselmet et Al(1983) , which shows clear evidence that the moments diverge at order approximately 5, as required if $\alpha_v \sim 5$.

Anselmet et Al (1983) 'use high Reynolds number wind tunnel experiments to statistically estimate $\zeta_e(p)$ defined by:

$$\langle \Delta v(\Delta x)^p \rangle = \langle (v(x+\Delta x) - v(x))^p \rangle = \Delta x^{\zeta_e(p)}$$

$\zeta_e(p)$ was statistically estimated by the statistic $\zeta_s(p)$ defined by:

$$(1/n)\sum_{i=1}^{n} \Delta v_i^p(\Delta x) = \Delta x^{\zeta_s(p)}$$

where the Δv_i are the n experimental Δv's. By assuming i) unbroken scaling parameter $H=1/3$, and ii) hyperbolic intermittency, $\alpha \sim 5$, we obtain excellent theoretical agreement with their experimental results (see fig. 1). The theoretical calculation of $\zeta_s(p)$ proceeds as follows:

$$\Delta x^{\zeta_s(p)} = \Delta v^{*p}\left(\sum_{i=1}^{n} \varphi_i\right)/n$$

where $\varphi_i = (\Delta v_i/\Delta v_i^*)^p$ is a random variable, unit amplitude such that $\Pr(\varphi' > \varphi) \sim \varphi^{-\alpha/p}$. Scaling implies $\Delta v^* \sim \Delta x^H$, thus

$$\Delta x^{\zeta_s(p)} = \Delta x^{Hp}\left(\sum_{i=1}^{n} \varphi_i\right)/n$$

For $p<\alpha$, $\lim_{n\to\infty}(1/n)\sum_{i=1}^{n}\varphi_i \to \bar{\varphi}$ (= mean = constant); and thus $\zeta_s(p) = pH$ for $p<\alpha$. However, the situation is quite different for $p \geq \alpha$, because the mean $\bar{\varphi}$ is infiniteand the standard theory of stable processes (e.g. Feller (1971)) yields $\lim_{n\to\infty}(1/n)\sum_{i=1}^{n}\varphi_i \to n^{p/\alpha - 1}$. Thus:

$$\Delta x^{\zeta_s(p)} = \Delta x^{Hp}\, n^{p/\alpha - 1}$$

In order to evaluate $\zeta_s(p)$ we must use the fact that n is the number of independent experimental points and typical set-ups yield $n \sim 1/\Delta x$. Thus:

$$\Delta x^{\zeta_s(p)} \sim \Delta x^{p(H-1/\alpha)+1}$$

$$\Rightarrow \quad \zeta_s(p) = p(H-1/\alpha)+1 \quad \text{for } p \geq \alpha.$$

Fig. 1 shows a very clear break from the line pH for $p>5$, and indicates $5<\alpha<6$. The value $\alpha \sim 5$ may thus be a universal feature of turbulence. This shows that the breakdown of the relationship $\zeta_s(p) = pH$ at $p \sim 5$ noted by Frisch (1983) is not due to a breakdown of the scaling (a "broken symmetry"), but is due to an unbroken scaling coupled with the divergence of high moments.

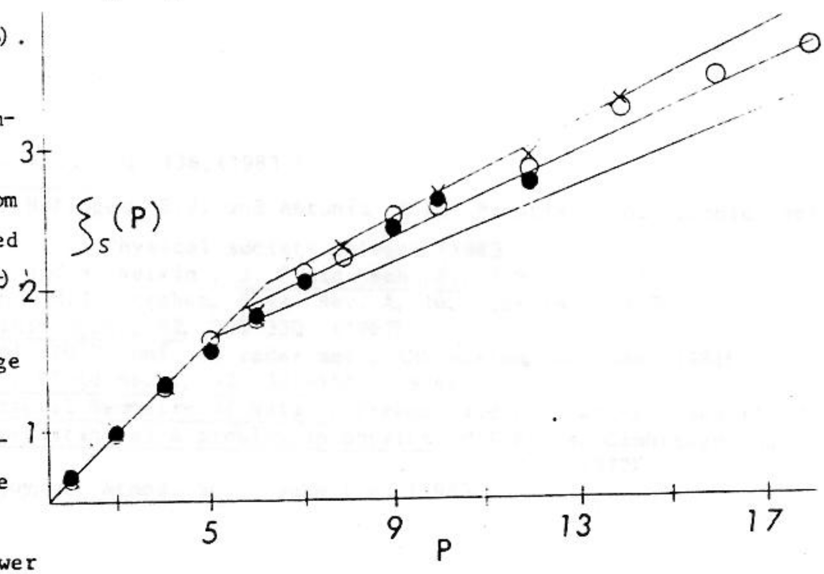

Fig. 1: Graph of $\zeta_s(p)$. Experimental points are taken from Anselmet et Al (1983). Open circles are from $R_\lambda = 852$ (jet), closed circles, $R_\lambda = 536$ (jet), X's, $R_\lambda = 513$ (duct). Note the sharp change in behaviour for $p>5$. The three functions shown are the theoretical curves for $\alpha = 5, 5.5, 6$ (lower to upper respectively), assumming $H=1/3$.

The dimension(s) of the support(s) of turbulence:

Schertzer and Lovejoy (1983) discussed the extension of Mandelbrot's (1974) phenomenological turbulence model to the anisotropic case by using elliptical dimensions instead of ordinary fractal dimensions. The basic ingredient of this model is the random function $W(\lambda)$ which is a "curdling operator"- it distributes the flux of non-linear energy into sub-eddies by a cascade of steps λ^n. Specifying all the moments of W is sufficient to uniquely determine the intermittency properties of the model. The dimension $D_s(1)$ describes the dimension of the active regions of the field (for any arbitrary level of activity). Similary, $D_s(h)$ can be defined to characterise the active regions of the field ϵ^h. Its value is given by:

$$\langle W^h\rangle = \lambda^{(h-1)(D_{el}-D_s(h))}$$

The "β-model" (Frisch et Al (1978) corresponds to the trivial case of $D_s(h)=D_s(1)$ for all h- this model can therefore not be used to study the divergence of moments. In the "α-model" described in Schertzer and Lovejoy (1983), $D_s(h)$ is a decreasing function of h which tends to the limit D_∞. Averages of ε^h taken over sets dimension $D_A < D_{el}-D_s(h)$ diverge.

Hyperbolic Renormalisation:

The introduction of W in the preceeding model may be understood as a phenomenological "renormalisation of the vertex" (or of the non-linear interactions), by taking the non-direct interactions of the smaller wave-numbers into account during the decimation process (as described in Forster et Al (1978)), but from large to small wavenumbers). (Symbollically: $l \to \lambda l$, $v \to \lambda^H v$, the vertex $P \to \lambda^{-1} WP$). Work is in progress is assess the validity of this kind of "hyperbolic renormalisation".

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