

## PREDETERMINATION OF FLOODS

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*Abstract- Predetermination (or statistical prediction) can be defined as the announcement of the physical and statistical characteristics of a future event non-precisely located in time. So, it is quite different from forecasting, whose objective is to give the precise date of occurrence of a specified physical event. Predetermination will then be inseparable of probabilistic concepts such as the probability of occurrence of a given event or, equivalently, of its return period. About floods, one will estimate, for a given river cross-section, whether the probability that the discharge would exceed a given threshold or, symmetrically, the discharge which has a given probability of exceedance. Such estimations, the spirit of which is definitively different of that of PMP/PMF (Probable Maximum Precipitation or Flood), enable a rational approach of socioeconomic*

problems related to hydraulic works design and land use. The only available data has long been the instrumental discharge measurements, whose series are rather short, some decades in general, seldom reaching a century. More recently the so-called historical data have been used. They are compiled from old documents and supplemented by hydraulic studies and even geomorphologic or sedimentologic studies. Such data increase considerably the length of extreme events' time series available. There is still a great confusion about the statistical models to be used. The choice of such models is too often only dictated by the best graphical fit, while this choice is fundamental for data interpretation. Without a wide theoretical agreement, the results are still not so reliable. A very important standardisation effort has been done in the US, where, since the end of the sixties, the use of the Log-Pearson III distribution has been made compulsory for all federal projects, but the physical and even statistical arguments of this choice remain quite weak. All the efforts done to enlarge the data bases, as well as the theoretical developments based upon the physics of hydrological processes and especially their scale invariance, should nevertheless enable us to formalize in a near future the asymptotical behaviour of hydrological time series and to define effective predetermination methods in engineering.

Keywords: flood, predetermination, probability, return time, scaling, multifractal, Gumbel, Fréchet.

## 1. Introduction

Humankind has been concerned with floods due to the personal and material damages that these floods are likely to cause. Urgent concerns are related to timely flood alerts, hence specialized organisms were set up, e. g. the Central Service of Hydrometeorology and Support to the Flood Prediction (SCHAPI) in Toulouse (France), which use more or less complex models for flood prediction. The objective is then to forecast as precisely as possible which will happen in the next hours or days, e. g. water stage in given sites in order to be able to take appropriate decisions. Longer term concerns are related to engineering designs or to land-use planning, i.e. decisions that are beforehand taken and once for all. The notion of probability related to a given event is then fundamental to take rational decisions. Concerning engineering designs, e.g. a bridge opening or a spillway capacity, it effectively allows to assess the total cost related to a

given design choice. It is obtained by summing the cost of engineering works, which increases with the maximal flood intensity for which they are designed, and the damage expenses resulting from a flood exceeding this intensity. It is then possible to optimize this choice by minimizing the total cost (Tribus, 1972, Ulmo and Bernier, 1973). In the case of town and country planning, thanks to a usual probability expression, it is possible to map and to compare the hydrological risk and the vulnerability of land uses and human activities in zones liable to flooding and hence to develop a productive debate (e.g. (Gilard and Gendreau, 1998). Approaches based on the “maximal possible precipitation” or the “maximal possible discharge” (Guide des pratiques hydrologiques, 1994), which are fashionable in certain countries and which set out to estimate a hypothetical maximum of rainfall and streamflow (i.e., with a theoretical zero probability to exceed them), do not allow to rationally approach the problem of the town and country planning. Indeed, although they represent extreme events, they do not provide an associated probability, which will be useful for an economic valuation.

## 2. On Probability and Return Periods of Floods

Let us therefore discuss first the notions of probability and of return period. Let be a particular event, e. g. the discharge averaged over a given time step at a given point of a river exceeds  $1500 \text{ m}^3 \text{ s}^{-1}$  in the course of given year. The probability  $p$  of this event is a measure of the possibility of its occurrence and by convention it is represented by a number between 0, when it is impossible that event occurs and 1, when its realization is sure. The associated return period is defined the inverse of this probability:  $T = 1/p$ .

The return period is therefore only another way to state under an other form, which intends to be more colourful, the probability of an event at some point. In spite of his name, undoubtedly badly chosen, under no circumstance it refers to ideas of regularity or of periodicity and can even apply to events which never occurred and that will not perhaps occur in future. It is perfectly legitimate to be interested, particularly for safety studies, by millennium floods or floods or decamillennium floods (i.e. having a probability 0,001 and 0,0001 respectively to occur in the course of given year) of a river which did not exist five thousand years ago and that will not exist perhaps any more in ten thousand years, similarly to the industrial interest in the probability of faults which will not undoubtedly occur, because the minimization of their probability is an aspect of a policy of industrial security.

We have just spoken about the characterization of a flood by the discharge averaged over a given time step of time and it is on this basis that we shall speak in the following. You should not however forget that a flood is a complex phenomenon and that other variables can describe it, e. g. its instantaneous discharge peak (when the considered time step goes to zero), but also its duration (duration of being over a given threshold), its volume in the course of this duration, etc. These variables have their own statistics and there is no reason that a millennium flood with respect to its daily average is also a millennium flood with respect to its duration or its discharge. Also let us point out that each of these hydrological variables has its own socioeconomic relevance (e.g. height or duration of submersion) and that a specific approach may be necessary.

Ambiguity related to the notion of return period arises from the fact that until now probability estimates are mainly obtained, or even exclusively, with the help of analyses of time series. Taking back the above example, one can imagine to record a river discharge during a very large number of years. If during this  $N$  years, a given event occurs  $n$  time, its frequency  $f = n/N$  is a good estimate of its probability  $p$ . On the average, this event occurs  $p$  time a year and its return period is equal in  $T = 1/p$  years (if  $p$  is small enough), and it is possible to give a more concrete interpretation at the time of return: it is the average period between two occurrences of this event. This estimate procedure can be directly implemented to available time series, and meteorological and hydrological series are known to be comparatively short in general. Their length are often of the order of a few dozens years and seldom reach the century, therefore restrict return period estimates to a few dozens years.

Aware of limitations imposed by the shortness of time series, researchers tried to exploit other historical data or sediment data. It is for instance about observations recorded in local registers, which introduce the advantage of a precious time continuity. These recorded observations are often water stages, and it is necessary to make a detour by hydraulic models, which require to know the geometry of watercourses, and its possible past evolution in the course of time, to estimate discharges. They also tried to take advantage of sediments sometimes left by strong floods with the help of a flood reconstructed stratigraphy (Thorndycraft et al, 2002). These tasks are particularly delicate. They lean on data of various origins, which are often difficult to collect. They require collaborations between hydrologists, hydraulicians, historians and sedimentologists, but when undertaken they allow to considerably increase, sometimes by an order of magnitude, the length of observations concerning strong floods. These very long reconstructed series acutely pose the problem of their

stability and, throughout, that of the pertinence of the statistical analyses such as them are played.

Researchers worked out this question and they delivered coherent results: in Spain (Rio Llobregat, Ter), in France (Ardèche) or in central Europe (Elbe and Oder) (Barriandos et al, 2003, Lang et al, 2002, Mudelsee et al, 2003) on reconstructed series of several hundred of years when, in spite of proven climatic variations (small age of ice, for instance), it does not appear that the regime of the extreme floods changes. Regimes of the average or weak floods, i.e. with return periods not exceeding about twenty years, are much more sensitive to anthropogenic basin transformations, could, experienced fluctuations. The booming affirmations on the worsening of the successive floods as the consequence of an "enhancement" of the cycle of water are not therefore no empirical support, and it seems contrariwise very reasonable to rely, for XXIth century, upon a hypothesis of stability of the extreme floods.

### **3. Choice of the Statistical Law**

It is necessary to make hypotheses to extrapolate measured or reconstructed data to assess events of weak probability which were never recorded, i.e. to choose a statistical model, what is always a perilous exercise. Assuming that studied phenomenon obeys a given statistical law, whose parameters are adjusted with the available data: one supposes that this law remains valid for non recorded events. This apparently very simple operation in reality calls for a rather complex "cuisine", implying choices on the manner to attribute empirical probabilities to reconstructed or recorded events, or on the fitting method (e.g. method of moments or method of the maximum of likelihood, but there are many others and the imagination of the statistician hydrologists proved to be fecund in this field).

Estimates of probability and of return period depend of course on the adopted statistical law, whose choice is empirically justified only on its capacity to represent recorded events. There is a very large number of laws that were proposed and used, and the seemingly innocent selection turns out to have often extreme consequences: in a sensitivity study dedicated to an cost/benefit analysis of a height increase of a multipurpose dam on Ennepe in Ruhr (Tegtmeier et al, 1986), it was shown that the choice of the distribution law of floods was a decisive element of the choice of the economically optimal solution, a much more mattering element than parameters such as the water price or the flood damage evaluation.

The practice of the predetermination of floods developed since the beginning of the XXth century in a certain confusion, particularly with respect to used statistical laws. To take only an example, the HYFRAN

software developed by INRS-BE from Quebec ([http://www.inrs-eau.quebec.ca/activites/groupe/chaire\\_hydrol/chaire9.html](http://www.inrs-eau.quebec.ca/activites/groupe/chaire_hydrol/chaire9.html)), incidentally a very friendly software endowed with remarkable numerical and graphic tools, offers not less than 12 statistical laws in its "menu", with various fitting methods for each of them, but does not offer criteria of choice of the law to be used for a particular analysis. Results concerning the estimate of a flood of given recurrence can therefore greatly differ according to their analysts, who can be tempted to use their expertise "tricks" to provide the most suitable results to their "customer" expectations. It largely discredits this exercise. It is understandable that it is difficult to reproach to an administration or to an elected representative not to have protected his constituents against a millennium flood, but they would not excuse him for not having protected them against a fifty year flood and the former would "prefer" an analysis providing an "exceptional" return period...

To end this situation, the Water Resources Council (WRC) of the United States recommended, in a report submitted to the Congress in 1966 (WRC, 1966), to set out a uniform technique for determining flow frequencies, what was achieved next year by a team led by M.A. Benson, assisted by two statisticians (WRC, 1967). This team studied the application of six laws: gamma with two parameters / Gumbel / log-Gumbel (Fréchet) / log-normal / log-Pearson III / HAZEN, to ten long series (on the average 50 years) of annual maxima of the United States, chosen in various climatic and hydrologic conditions and with drainage areas ranging from some dozens to some dozens thousand square kilometers, but rejecting series including outliers (particular special events). Application consisted in estimating the floods of return period 2, 5, 10, 25, 50 and 100 years. It is finally log-Pearson III law which was selected for its stability. Since then its use is compulsory for all project of the American federal government.

However, one can question this choice. Database was very small and this sampling limitation was not taken into account. Why to systematically eliminate all outliers, which are genuine extreme events? No reasoning or physical argument was invoked to reinforce the choice of log-Pearson III law that nothing predisposes, statistically speaking, to be a law of extremes, contrary to the laws of Gumbel or Fréchet, (Embrechts et al, 1999, Galambos, 1978, Reiss and Thomas, 1997). If we can only congratulate the Water Resources Council for its efforts to undertake this normalization, we cannot be satisfied by its procedure and its choice. Here we are therefore at the dawn of the XXIth century.

#### 4. Some Physical Insights

The pure statistical approach developed in the course of the XXth century did not allow to the theoretician hydrologists and practitioners to agree on a corpus of knowledge and methods. Without agreement on the fundamentals, they spent a lot of time on details, to vainly sophisticate empirical probability estimations or fitting methods. If we do not want to neglect the role of the analysis of data, which stays and will remain indispensable, it would be perhaps timely to reinforce it by taking into account the physics of phenomena, that is to say hydrology as such, so often avoided and that is perhaps in fact the key of the problem!

The attribution of a probability to an event does not necessarily lean on a frequency analysis. Symmetries of objects or studied phenomena can also be used, alone or concurrently to the analysis of data. Let us take the example of throwing a dice, we can a priori claim that the probability to get "6", for instance, is equal to  $1/6$ , only because there are six possible results, which are of course equi-probable. No need, except to check if a dice is flawed, to throw it for eternity to empirically estimate the probability of every possible result. In that case, as for the games of chance in general, the probability of an event is a priori defined as the ratio of favourable cases (those to whom studied event is related) against the number of all possible cases. Is not it possible to use a similar reasoning to understand given properties of statistical distribution of discharges? We believe it and we recall that Navier-Stokes equations which govern fluid dynamics in a very general manner are scale invariant (Schertzer and Lovejoy, 1995).

This property should stem from the (unknown) partial differential equations governing rainfall and discharges that are non conservative integration in the space and time of rainfall (Hubert, 2001). From there, it is possible, by analysing hydrological series, to empirically check the relevance of scale invariance's, and, as a theoretical consequence, to draw the nature of statistical laws governing these time series and to tackle fundamental questions that had been heretofore neglected, e.g. the sampling role. While having expressly in mind the problem of the dependency of certain measurements on the scale of observation Mandelbrot (Mandelbrot, 1975, Mandelbrot, 1977) created the fractal geometry, by leaning on mathematical results forgotten or eclipsed by the beginning of the XXth century, that he developed and applied to numerous problems of natural sciences. He acknowledged, in a fractal (not integer) dimension, a link likely to link up a measure and a scale of measure to numerous geometrical objects likely to model natural objects. We must note given hydrological applications of this geometry, particularly for the description of hydrographic networks and basins (Bendjoudi and Hubert, 2002), but also



to characterize the temporal support of rainfall (Hubert and Carbonnel, 1989). These results however should not make us forget that the complex phenomena such as rainfall, and even more streamflow, do not come down to case or absence and that it is necessary to be concerned about the intensity of the rainfall or of discharge with respect to a given time scale.

The notion of intensity is in fact implicitly present in the definition of the reference threshold defining a geometric object, for instance the case of rain, and (Hubert et al., 1995) pointed out that the fractal dimension of the rain occurrence is a decreasing function of the reference threshold. This dimension dependency on the reference threshold, already noted by Schertzer and Lovejoy (Schertzer and Lovejoy, 1984) and by Halsey and al. (Halsey et al., 1986), lead for studies of this type to effectively go beyond, the notion of fractal (geometric) sets to that of multifractals.

## 5. Multifractal Approach and Critical Auto-Organization

Multifractal approach aims at linking up scale and intensity for cascade processes concentrating material and/or energy in smaller and smaller space-time domains (Lovejoy and Schertzer, 1986, Schertzer and Lovejoy, 1987). Multifractals models in which we were interested were first developed as phenomenological models of turbulence. They were conceived to reproduce in multiplying cascades the main properties (symmetries, conservation) of non linear equations (the Navier-Stokes equations) which govern the dynamics of this phenomenon. We have already pointed out above why it seemed to us legitimate to import these models in hydrology, but we will recall below that, in another context, the similarity of asymptotic behaviours of the rainfall and discharges was already postulated (Guillot and Duband, 1967) and lead to operational developments.

To consider a space and/or temporal field as multifractal corresponds to characterize it to be both multi-scale and multi-intensity. The stronger and stronger intensities of the field correspond to more and more extreme and rarer and rarer singularity, therefore associated to smaller and smaller fractal dimensions. Contrary to most of the models, all singularities, average ones as extreme ones, are generated by the same basic process. The theoretician does not have any more to add "by hand" (outliers), since they exist in germ in the average field and the experimenter does not have any more to laboriously categorize the extreme behaviour from more ordinary behaviours.

This a priori unusual link between extremes and average of a field can first be understood with the help of universal properties: although a multifractal field depends of an infinite number of parameters, alone a small



number of them can finally turn out to be relevant, the others being in a given way washed out by the repetition of an elementary phenomenon. The classical example of universality is the brownian motion, which is the universal attractor of any random walk whose infinitesimal step variance is finite. In the case of multifractal processes, we have rather to consider multiplying processes. A similar universal property was demonstrated for them and the multifractality index  $\alpha$  (between 0 and 2), which is proportional to the curvature radius of the codimension function of singularities around the average field, determines the distribution of extremes (Schertzer and Lovejoy, 1991).

This link can also be related up with the notion of critical auto-organization, as soon as we consider the behaviour of the field of rain on a big number of samples. In fact, from a certain critical singularity  $q_D$ , the observed intensity of the field is often much more important than that foreseen by a model taking into account only the scales larger than the scale of observation. It is due to the fact that not only small scale fluctuations are observable on a much larger scale, but that finally they pilot extremes on this scale. This link between microscopic and macroscopic are similar to phase transitions of conservative systems, where the correlation length diverges at a critical temperature. Here, it is the effective scale ratio that diverges.

Among the numerous implications of this "first-order multifractal phase transition", the most important is indeed the algebraic fall-off (that is to say slow) of the probability distribution of intensity beyond a certain level:  $\text{Prob}[X>x] \approx x^{-q_D}$ . It is important to note that this algebraic fall-off of a probability triggers the divergence of the statistical moments of orders greater than  $q_D$ . This divergence has numerous experimental and theoretical consequences, since the law of the large numbers does not apply any longer, hence the loss of ergodicity, the divergence of usual common estimators, sensitivity of estimates to the sample size, etc. Practical consequences of such an algebraic behaviour of the probability distribution are considerable, because algebraic laws drop immensely much more slowly than laws with exponential fall-off usually used for the determination of events of a given recurrence, which would then be considerably underestimated.

## 6. Scale Invariance Studies of Floods

The application of concepts of scale invariance to the river discharges is a prolongation of works of Hurst (Hurst, 1951), who was the first to put in evidence, starting from very practical concerns on reservoir design, long-range statistical dependency in discharge time series. Turcotte and Greene

(Turcotte and Greene, 1993) studied the frequency of the floods of ten American rivers. They characterized the scale invariance that they put in evidence for time scales from 1 to 100 years, with the help of the ratio of the century-years flood to the ten-year flood, equals in this approach framework to the ratio of the ten-year flood to the annual flood. This report varies from 2 to 8 about, and the authors relate these variations to climatic differences of the considered basins. Tessier et al. (Tessier et al., 1996) studied series of rains and discharge of 30 French basins whose surface range from 40 to 200 km<sup>2</sup>. They put in evidence a scale invariance from day to 30 years and observed a change of regime at about 16 days, which they relate to the synoptic maximum. The estimate from all data of the parameter  $q_D$  (critical order of divergence) is of the order of 3,2 for the time scales larger than 30 days, of the order of 2,7 for the time scales shorter than 16 days (with an important error bar). A more recent study of Pandey et al. (Pandey et al., 1998) dealt with 19 American basins, ranging from 5 km<sup>2</sup> to about two millions square kilometers (the later being that of Mississipi), totaling 700 station-years. They concluded to a multifractal behaviour for time scales ranging from 23 to 216 days. They also pointed out a change of regime about a few weeks. Their estimates of multifractal parameters are rather close to those of Tessier (Tessier et al., 1996), in particular they estimated the average of the critical parameter  $q_D$  to be about 3,1, but, contrary to Turcotte (Turcotte and Greene, 1993), they assign only to chance the dispersion of estimates related to various basins. Also let us note a study of Labat et al. (Labat et al., 2002) on karstic sources in the Southwest of France, which estimates the parameter  $q_D$  to be about 4, and the one that we accomplished on the discharge of Blavet in Brittany (Hubert et al., 2002), for which this parameter was estimated to be about 3. We shall signal finally a study of Tchiguirinskaia et al. (Tchiguirinskaia et al., 2002) concerning about 1500 gauging stations in the Arctic region, representing more than a million data, extracted from the database R-Artic Net (<http://www.r-arcticnet.sr.unh.edu>). This study offers a estimate of  $q_D$  of the order of 6 for the considered rivers, but it is especially interesting for expressly taking into account the seasonality of discharge series, which had unfortunately been neglected in most of the previous studies, what is likely to affect the quality of the scaling (scale invariance) put in evidence.

These studies, particularly the research of scale invariance and their range of scale, should be of course followed up on larger corpus of data, or even on all available databases, to conclude in a final manner on this point that, if it was established, would open, on the theoretical and practical levels, unprecedented perspectives on the mastery of scale effects that it is difficult to appreciate all its breadth.

## 7. Discussion and Perspectives

We shall not pretend, at the end of this short presentation of the multifractal approach for the river discharge analysis, that all problems over which stumbles the practice of the predetermination of floods are resolved, nor that all ambiguity is raised, but we think that scale invariance and the derived multifractal modelling constitute a foundation from which it is possible to advance fast. This hypothesis, formulated from theoretical considerations, got empirical support from analyses of time series, which show that asymptotically algebraic laws (Fréchet domain of attraction) give better account than asymptotically exponential laws (Gumbel domain of attraction). This finding is besides not new: it had been made by Morlat et al. (Morlat et al., 1956) who, as part of design studies of the Serre Ponçon dam, expressly rejected the Gumbel law in favor of Fréchet law. The estimates based on the latter are those that had finally been selected, a fact that seems to have been sometimes forgotten today. The insurers (Embrechts et al., 1999, Reiss and Thomas, 1997) are also much less cautious than many hydrologists to adopt statistical laws with algebraic fall-off. The coupling between theoretical developments and empirical analyses of data is particularly innovative in statistical hydrology, which had in fact only the graph fitting as compass for about a century. Works on discharges that we presented here are an extension of much more numerous works on multifractal analysis of rainfall (Hubert, 2001), which introduce, beyond climatic differences, a remarkable constancy of the parameter  $q_D$  with everywhere a value of the order of 3, and that already allow to envisage operational developments (Bendjoudi et al., 1997). If milestones were put down (Hubert et al., 2003, Tchiguirinskaia, 2002), a large amount of scientific work remains to be performed, particularly for setting out truly multifractal rainfall-discharge models, likely to fully monitor time-space scale effects, and to link up discharge statistics with those of the rainfall (as pre-represented by the GRADEX method (Guillot and Duband, 1967) for small basins), as well as with the basin characteristics, conciliating therefore determinist hydrology and statistical hydrology. A large amount of work remains to transfer these results into regulation and into engineering. This is particularly necessary for a time when all societies have to, and will even more owe, face an objective increase of hydrological risk (Neppel et al., 2003) due to an increased vulnerability, that the past practices of applied hydrology had indeed often underestimated.

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