Chernobyl 137CS cumulative soil deposition in Europe: is it multifractal?

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1. Introduction

Contrary to standard deterministic modeling (Israel et al., 1990a,b) for handling extreme variability in geophysics we apply approaches which recognize that the scale invariance is a (widely respected) dynamical symmetry principle and that nonlinear scale invariant processes generally lead to multifractal fields. Much evidence now exists (e.g. Lovejoy and Schertzer, 1990, 1995) showing that the wind, rain and other atmospheric fields are multifractal over most of the meteorologically significant range of space and time scales. Furthermore, multifractality is generally associated (see below) with a qualitatively distinct "Self-Organized Critical" (Schertzer et al. 1993, Schertzer and Lovejoy, 1994, 1997) extreme behaviour.

In this paper we discuss the influence of geophysical turbulence transport on the complexity of the Chernobyl radio nuclides contamination distribution. By proceeding to an empirical estimate of the universal multifractal exponents and corresponding critical exponents of ^{137}Cs soil contamination we show that the distribution of Chernobyl fallout is indeed both multifractal and displays SOC structures. We argue that this complex structure of "hot spots" at all scales and all intensities displayed by the Chernobyl fall-out plays a fundamental role for its understanding, especially for risk assessment and monitoring.

2. Chernobyl accident data base

The data base analyzed in this paper was available in the framework of INTAS grant collaboration and it consists the mean values of ^{137}Cs cumulative soil deposition. The data were collected in 1986 by more than 31 different Organizations of former Soviet Union, in order to constitute a public data bank. The final database contains 11605 points in Ukraine with the geographical coordinates (e.g., longitude and latitude), for every point of measurement the archived data values were averaged over the number of individual measurements.

On the Fig. 1 we present a map display of this ^{137}Cs cumulative soil deposition data set with values range from 0.9 to $6*10^5$ KBq/m^2. One must note the fractal/multifractal nature of the measurement points and the need to take it into account, as done by Salvadori et al. (1994) for dioxin measurements around Seveso and by Tessier et al (1994) for in-situ rain

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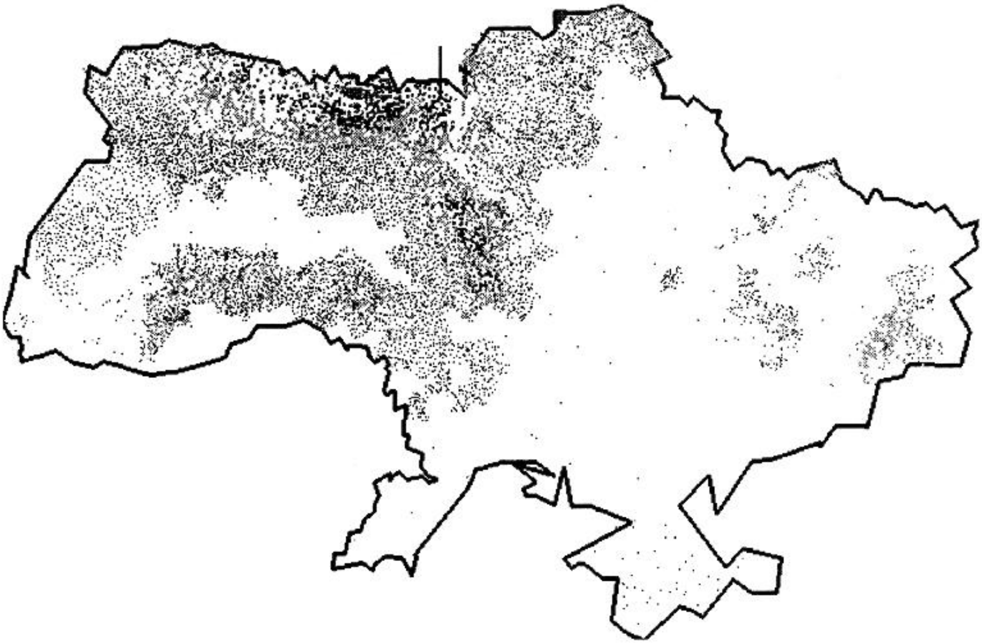

Fig. 1. *Distribution of 11605 points of* ^{137}Cs *cumulative soil deposition measurements on the territory of Ukraine. The values range from 0.9 to* $6*10^5$ kBq/m^2.

measurements. The source corresponding to Chernobyl has been decreased by a factor 3 in order to enhance the "intermittent" features of the radioactive concentration around it. "Leopard's skin" distribution of fallout "hot spots", also observed for other data sets displayed in table 1 (Salvadori et al. (1994b) and Schertzer et al. (1995)), undoubtedly results from the extreme variability of the various geophysical processes, especially the nonlinear transport and scavenging processes (associated with the wind and rain fields).

3. Universal multifractal approach

The multifractal approach to turbulent transport is based on the fundamental scale symmetry property of the nonlinear (e.g. Navier Stokes and passive advection) equations. The simplest way of understanding how transport variability occurs over a very large range of scales is to suppose that the same type of elementary process acts at each relevant scale (from the large scale down to the small viscous scale): this generically yields a multifractal cascade process.

In the case of a stochastic multifractal field ε_λ observed at different scale ratios $\lambda=L/l$, where L is the outer scale and l is the scale of observation, the statistics of the field can be described in the framework of the codimension multifractal formalism (Schertzer and Lovejoy 1987, 1989, 1992, Oono 1989, Mandelbrot 1991) either in terms of probability distributions or statistical moments, involving respectively the codimension function ($c(\gamma)$) of the order of singularities γ and scaling function ($K(q)$) of the moments order q:

$$\Pr(\varepsilon_\lambda \geq \lambda^\gamma) \approx \lambda^{-c(\gamma)} \;;\quad \langle \varepsilon_\lambda{}^q \rangle \approx \lambda^{K(q)} \tag{1}$$

$c(\gamma)$ and K(q) are dual for the Legendre transform (Parisi and Frisch, 1985):

$$c(\gamma) = \max_q (q\gamma - K(q)); \quad K(q) = \max_\gamma (q\gamma - c(\gamma)) \tag{2}$$

The only constraints that must be respected by the two functions $c(\gamma)$ and K(q) are that they should be both convex, and $c(\gamma)$ should be an increasing function.

Due to the existence of stable and attractive multifractal processes under rather general circumstances, mixing of different multifractal processes may lead to universal processes (Schertzer and Lovejoy, 1987, 1997) which depend on very few relevant aspects of the initial processes. Indeed -up to a critical order discussed below- these universal multifractal processes have codimension and moments scaling functions ruled by only three exponents, e.g. for the moments scaling function:

$$K(q) + qH = \frac{C_1}{\alpha - 1}(q^\alpha - q) \tag{3}$$

the three basic universal exponents being:
- the Hurst exponent H: it corresponds to the order of fractional integration over a conservative flux and is related the degree of non conservation.
- the mean singularity C_1, i.e. those contributing to the mean field, measures the fractality/sparseness of the mean field, it corresponds at the same time to the codimension of the mean field.
- the Lévy index α determine the extent of multifractality, it is indeed the Levy index a of the generator of the process and is proportional to curvature radius of the codimension function around the mean singularities, $R_c(C_1\text{-}H) = 2^{3/2}\, C_1\alpha$.

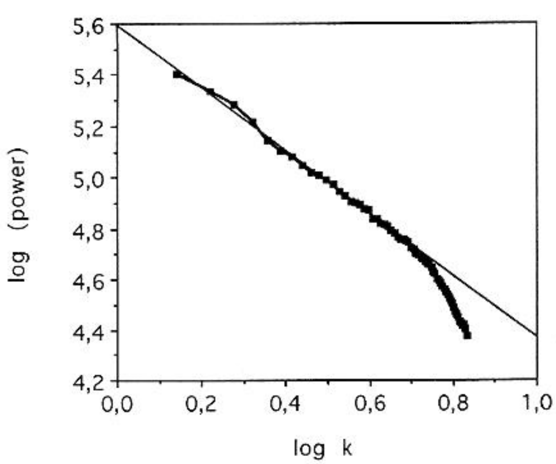

Fig. 2. *The power spectrum of subset (9836 points) of sparsely distributed* [137]*Cs soil deposition measurements displayed on Fig. 1. The straight line indicates scaling regime with slope* β*=1.2.*

4. An empirical estimate of the universal multifractal exponents

The spectrum of fluctuations of ^{137}Cs soil deposition resulting from Chernobyl accident (Fig. 2) was first computed in order to have an estimate of the range of the scaling regime, and its slope $\beta = 1.2$ will be used in order to estimate the exponent H.

Universal multifractal exponents C_1 and α where estimated with the help of (Lavallée et al. 1992, 1993) the Double Trace Moment technique (DTM). Indeed, we may first consider the normalized η powers of the field ε_λ. Obviously, ε_λ^η will have a moment scaling function $K(q,\eta)$:

$$\left\langle \left({\varepsilon^{(\eta)}}_\lambda \right)^q \right\rangle \approx \lambda^{K(q,\eta)}; \quad K(q,\eta) = K(q\eta) - q \cdot K(\eta) \tag{4}$$

The DTM indeed will be ruled by the scaling exponent $K(q,\eta)$ (Eq.4) until a critical moment order q_D.

On the Fig. 3 we present the DTM curves for the order of moments q=1.5 and 2.0 from which one can estimate $\alpha = 1.67$ (as a clop of corresponding straight lines) and C_1=0.42.

Due to the Fourier transform $2H + K(2) = \beta$-1, which with the help of Eq. 3 and the estimates of α and C_1, yields the following estimate: $H = .47$.

5. Multifractal phase transition: the high order moment behaviour is dominated by the rare high concentrations

For high threshold (s>>1) of intensity, which due to the Legendre transform (Eq. 2) correspond to high order statistics (q>>1), multifractals generically (Schertzer et al. 1993, Schertzer and Lovejoy, 1994, 1997) have a power law probability distribution function which has been taken (Bak et al., 1987) as a basic feature of Self Organized Criticality (SOC):

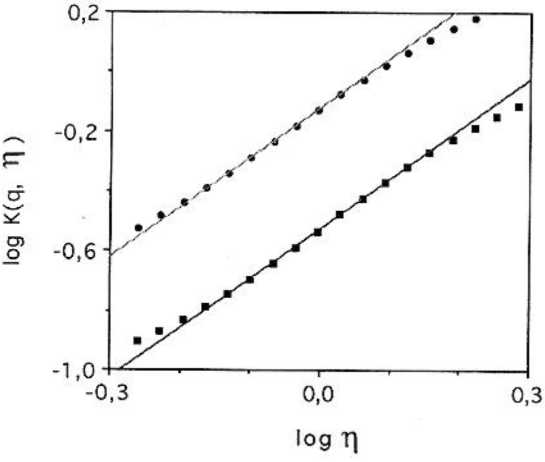

Fig. 3.The Double Trace Moment curves for ^{137}Cs soil deposition measurements with q=2.0 (top) and q=1.5 (bottom). Corresponding estimate of universal multifractal parameters is: α =1.67 and C_1=0.42

$$\Pr(X \geq s) \approx s^{-q_D} \quad (s >> 1) \Leftrightarrow < X^q >= \infty \quad \text{for } q > q_D \tag{5}$$

where Pr denotes the probability of a multifractal field X, q_D is a critical exponent which depends on the dimension of space (D) over which the multifractal is averaged. Fluctuations of the field (amplitudes of avalanches in the original model of S.O.C.) following Eq. 5 can be extremely large; they have no characteristic value. As indicated, this "hyperbolic" (algebraic) fall-off of the probability distribution is equivalent to divergence of statistical moments of order greater than q_D.

The analogy (e.g. Tel, 1988; Schuster, 1988) between multifractal exponents and thermodynamic variables was made using the following correspondences (Schertzer et al. 1993): the singularity order (γ, $C(\gamma)$) is the analogue of (energy, entropy), whereas (q, $K(q)$) is the analogue of (inverse of temperature, thermodynamic potential), the scale ratio is the analogue of the correlation length. And indeed, the first order multifractal transition corresponds to the fact that for a finite q_D and corresponding γ_D, the effective scale ratio will diverge as the correlation length for thermodynamic phase transitions. Indeed, the scale of observation becomes irrelevant since the spatial averaging (necessary to obtain coarse grained observables) is no longer able to smooth out these singularities. These apparently macroscopic observables will therefore be microscopically determined, depending crucially on the small scale details (e.g. hot spots). The observed singularities in fact correspond to the huge ratio of scale of activity of the multifractal processes, rather than the (much smaller) ratio corresponding to the scale of observation. As a consequence, it can be shown that K(q) becomes linear, but with a slope determined by the maximal observed singularity ($\gamma_d > \gamma_D$) (hence a first order discontinuity/phase transition, see Schertzer and Lovejoy 1992, 1994 and Schertzer et al. 1993).

On the Fig. 4 we presented $c(\gamma)$ calculated from the probability distribution, the slope of the asymptote ($\gamma \geq \gamma_D$) of the resulting curves gives us q_D=1.7. One may note (see Table 1) that this determination of q_D is rather consistent with observed algebraic fall-off of the probability distribution for previously analyzed Chernobyl fall-out data base.

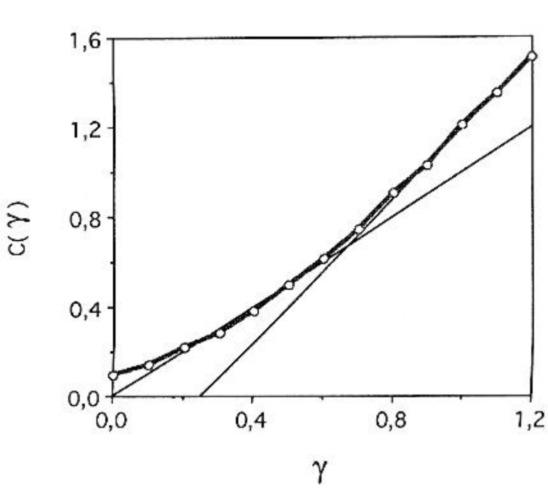

Fig. 4. *The codimension of the ^{137}Cs soil contamination on 9836 pixels covering territory of Ukraine around Chernobyl. The linear asymptotic behavior corresponds to q_D=1.7. The bisectrix yields the estimate C_1=0.42, which is in good agreement with previous results, see table 1.*

6. Discussion and perspectives: risk assessment and monitoring

Since the radioactivity of the Chernobyl cloud is transported by aerosols, which are advected by atmospheric dynamics, it was then of interest to study the influence of atmospheric turbulence on the dispersion of the Chernobyl cloud.

Data base	Number of points	Universal multifractal parameters		Critical moment
		α	C_1	q_D
R.E.M. Chernobyl fallout (Salvadori et al, 1994)	4000	-	-	2.15
300 km radius around Chernobyl (Schertzer et al, 1995)	16384	1.5	0.43	2.7
100 km radius around Chernobyl (Schertzer et al, 1995)	148	1.55	0.4	2.2
This paper	9836	1.67	0.42	1.7

Tab. 1. *The characteristics of different Chernobyl fallout data base analyzed in the framework of universal multifractals and multifractal phase transition.*

The empirical estimates of the universal multifractal and corresponding critical exponents confirm that the highly intermittent field of ^{137}Cs soil contamination is the result of a non classical SOC transport mechanism. We therefore have a clear framework in order to study the structures of distribution of fallout "hot spots" as stochastic self organized critical structures. Indeed, we first define structures by the order of the singularity of their flux (scale by scale and intensity by intensity), i.e. filtering out the rest of the field having flux singularities smaller than a given order of singularity. Self organized critical structures are then those having avalanche-like fluxes, i.e. corresponding to singularities higher than the critical γ_D. Such critical structures ("hot spots") at all scales and at all intensities displayed by the Chernobyl fall-out play a fundamental role for risk assessment and monitoring. Indeed, the probability distribution of the doses accumulated by random walkers traveling within a multifractal distribution of contamination can not be characterized by the usual mean dose. One has to consider that the random walk dimension D_w and the moment scaling function $K(q)$ of contaminated area defines a critical moment q_{D_w} after which the probability distribution function of accumulated doses will have an algebraic falloff: the probability of having the dose 10 times larger than given dose will by only $10^{-q_{D_w}}$ times smaller.

Acknowledgments

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