Beyond multifractal phenomenology of intermittency: nonlinear dynamics and multifractal renormalization

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1. Introduction

Intermittency is one of the most challenging and presumably also one of the most frustrating problems for many applications: nonlinear dynamical phenomena associated with a large number of degrees of freedom, in particular turbulence, display an intimate and complex coupling of order (cosmos/quiescence) and disorder (chaos/activity). Until now, this phenomenon has remained out of the scope of renormalization techniques, in spite of the development of powerful analytical tools since the Quasi Normal Approximation (Milliontchikov, 1941), including the Direct Interaction Approximation (Kraichnan 1959) and numerous related analytical "closure" techniques (e.g. Leslie 1973, Lesieur 1990 for reviews) as well as application of the Renormalization Group (Forster et al. 1977). While these analytic attempts have yielded some insight into the structure of the Navier-Stokes equations and the first basic feature of turbulence - its scaling (notwithstanding some fundamental difficulties in deriving the correct scaling law) - they have been totally unable to handle its intermittency (e.g. Frisch et al. 1979). It is becoming increasingly clear that this second feature is neither secondary nor second order.

On the other hand, a paradigm going back at least since Richardson's famous poem (Richardson 1922), the paradigm of turbulent cascades has recently led to a deeper understanding of the phenomenology, the analysis and modeling of the intermittency in turbulence with the help of stochastic multiplicative cascades. Indeed, wild probability distributions are rather a direct outcome of these processes, which have been subject to numerous recent developments especially with the emergence of multifractal notions. We emphasize the fact that they are rather generic multifractal processes, and discuss in particular a rather large class of them: the Fractionaly Integrated Flux (FIF) models. On the contrary, although it was less and less explicitly stated, analytical/renormalization theories have remained more or less quasi-normal and have therefore been unable to deal with probability distributions as wild as those of a log-normal or algebraic type, not to mention the simple (and in fact fractal) idea of puffs of activity inside of puffs of activity (Batchelor and Townsend, 1949).

Nevertheless, the cascade paradigm itself relies on the hand-waving original Richardson arguments and until now the corresponding processes have only taken into account a single symmetry of the Navier-Stokes equations: its invariance under rescaling. We are faced with

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a paradoxical situation: a deeper and deeper understanding of the phenomenology of intermittency, but a looser contact with the structure of the Navier Stokes equations. Contrary to a rather pessimistic recent viewpoint (e.g. Frisch 1995) the aim of this paper is to show that there is a way of bridging up the gap between analytical renormalization techniques and multiplicative processes via multifractal renormalization.

2. Renormalizing techniques and nonlinearity

2.1 A General framework

In order to discuss in a general manner the renormalization of (quadratic) nonlinear equations, among which Navier-Stokes equations for fluids mechanics is rather prototypical, it is important to write them in a rather compact and abstract manner, such as:

$$G_0^{-1}(1,2)u(2) = f(1) + P(1,3,4)u(3)u(4) \qquad (1)$$

where, following many authors (in particular Herring 1965, Martin et al. 1973), the generalized indices correspond not only to internal indices (e.g. coordinates of a vector or tensor field), but also denote space and time variables. We furthermore extend the Einstein convention of implicit summation over repeated indices with an (implicit) index summation for spatial and temporal variables corresponding to a (generalized) convolution, as illustrated below. Just before becoming more explicit, let us point out that the importance of the non linearity of Eq. 1 (in particular for Navier Stokes equations) can be more easily perceived with the help of Feynman type diagrams as illustrated in Fig. 1 where the symbols correspond respectively to:

-  represents the field $u(1) = u_{i_1}(\underline{x}_1, t_1)$ (which is the velocity field for turbulence)
- O represents the (external) forcing term $f(1) = f_{i_1}(\underline{x}_1, t_1)$, which is rather indispensable in order to maintain a "quasi-equilibrium" since the system is dissipative.
- represents the (linear) propagator or Green's function $G_0(1,2) = \delta u(1)/\delta f(2)$ which corresponds to the inverse of the linear differential equation corresponding to the l.h.s. of the Eq. 1 (the notation δ, when preceding a function, indicates functional variation). It corresponds to the infinitesimal response ($\delta u(1) = \delta u_{i_1}(\underline{x}_1, t_1)$) to an infinitesimal (Dirac) forcing placed on the r.h.s. of Eq.1 (the nonlinear term being suppressed): $\delta f_{i_1}(\underline{x}_1, t_1) = \delta_{i_1,i}\delta(\underline{x}_1 - \underline{x}, t_{1-t})$. $\delta_{i,j}, \delta(\underline{x}, t)$ are respectively the Kronecker and the Dirac functions, the latter being the unity for the convolution):

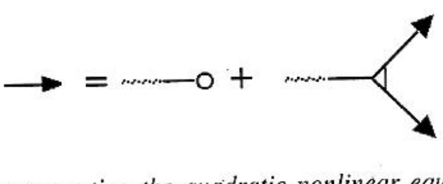

Fig. 1. Diagram representing the quadratic nonlinear equation (Eq.1), the corresponding symbols are explained in the text. The evolution of a given structure results from nonlinear interactions between pairs of others.

$$G_0^{-1}(1,2)G_0(2,3) = \delta(1,3) \equiv \delta_{i_1,i_3}\delta(\underline{x}_1 - \underline{x}_3, t_1 - t_3) \quad (2)$$

For fluid flows (ν: being the viscosity) the (diagonal) propagator and its inverse ($H(t)$ denoting the Heaviside function) are the following:

$$G_0^{-1}(2,1) = (\frac{\partial}{\partial t_2} - \nu\Delta_2)\delta_{i_1,i_2}\delta(\underline{x}_2 - \underline{x}_1, t_2 - t_1) \; ; \; G_0(2,3) = (4\pi\nu|t_2 - t_3|)^{-\frac{3}{2}} e^{-\frac{x_{i_2}.x_{i_2} - x_{i_3}.x_{i_3}}{4\nu|t_2 - t_3|}} H(t_2 - t_3) \quad (3)$$

Obviously, the (stochastic) infinitesimal response function $\hat{G}$ of the full nonlinear equation will be of far more importance:

$$\hat{G}(2,3) = \delta u(2) / \delta f(3) \quad (4)$$

but much more complex; it satisfies the following (exact equation) equation (illustrated in Fig. 2):

$$\hat{G}(2,3) = G_0(2,3) + 2G_0(2,4)P(4,5,6)\hat{G}(5,3)u(6) \quad (5)$$

- $\lessdot$.represents the "vertex" P which corresponds to the kernel of the nonlinear interaction. In the case of incompressible turbulence, the vertex corresponds to the solenoidal projection of the advection, i.e. under its symmetrical form for the last two (generalized) indices:

$$2P(1,2,3) = -[\delta_{i_1,i_2} - \nabla_{i_1}\nabla_{i_2}\Delta^{-1})\nabla_{i_3} + (\delta_{i_1,i_3} - \nabla_{i_1}\nabla_{i_3}\Delta^{-1})\nabla_{i_2}]\delta(\underline{x}_1 - \underline{x}_2, t_1 - t_2)\delta(\underline{x}_1 - \underline{x}_3, t_1 - t_3) \quad (6)$$

With these elements, the fundamental problem caused by nonlinearity of Eq. 1 is immediately transparent, as soon as the ratio of the amplitude of the non linear term versus the (linear) propagator becomes large (in turbulence this corresponds to the Reynolds number). Indeed, if one iterates the diagram of Fig. 1, one obtains a Von Neuman series (illustrated in Fig. 3), which is divergent because it contains terms of higher and higher order nonlinearity. The same difficulty occurs when considering the infinitesimal response function, i.e. when iterating the diagram displayed in Fig. 2.

2.2 *A Common Structure and corresponding stochastic models:*

Renormalizing techniques are singular perturbative techniques which circumvent the divergence of this series by proceeding to partial resumming. Corresponding to these partial resummations, appears the notion of "dressed" or renormalized quantities, e.g. vertex, forc-

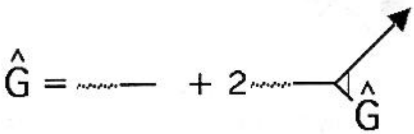

***Fig. 2.** Diagram of the corresponding (stochastic) infinitesimal response function, which satisfies the (exact) equation Eq.5.*

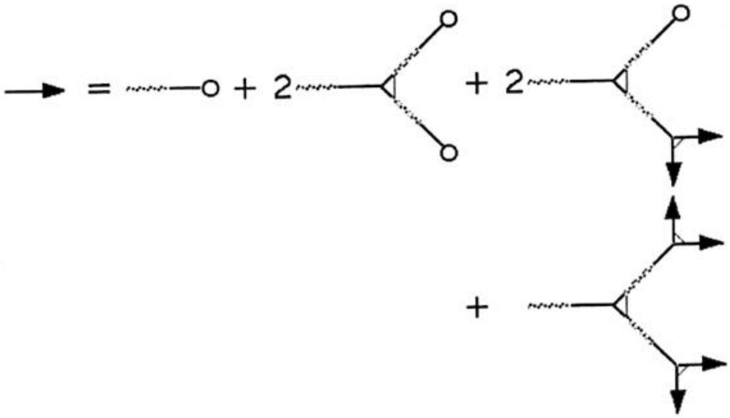

Fig. 3. Diagram of the first terms of the Von Neuman's series, corresponding to the iteration of the diagram displayed in Fig. 1.

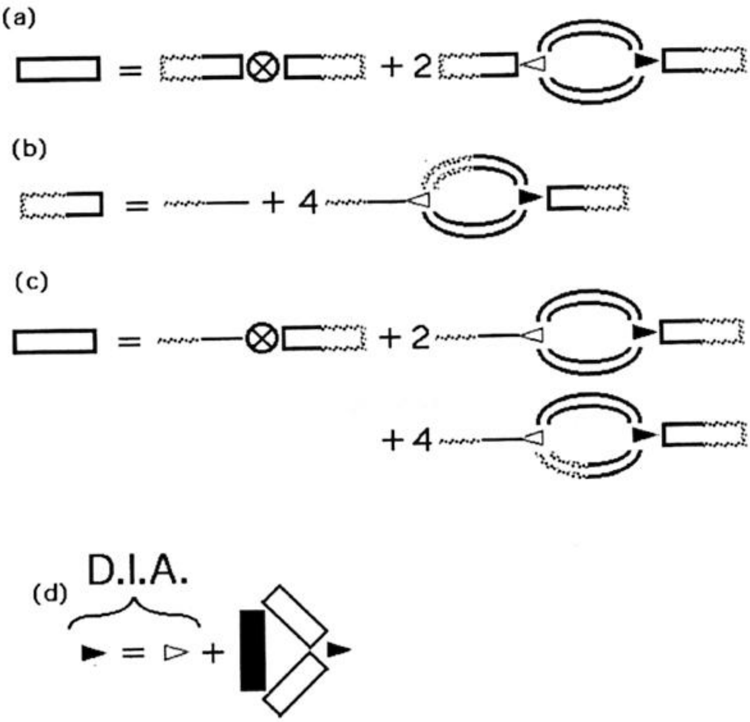

Fig. 4. *From top to bottom the renormalization of respectively the correlator of the field (a, c), of the propagator (b) and the vertex (c). The latter is omitted in the Direct Interaction Approximation. The equivalence of the diagrams (a) and (c) are equivalent due to (b). The different symbols are explained in the text.*

ing, propagator (as displayed in Fig. 4) which contrary to the original "bare" ones are dressed by the interactions over which the resuming acts. The pioneering work of Wyld (1961) in deriving these renormalized quantities was already based on these ideas. However the unique use of diagrammatic techniques can be dangerous due to the cumbersome work of counting correctly the number of equivalent diagrams. For instance, Wyld considered a (symmetric) double renormalization of the vertex, i.e. only renormalized vertices in diagrams (a-c), instead of the asymmetry displayed in Fig. 4 which was first noticed by Lee (1965). This asymmetry is on the contrary obtained in a rather straightforward manner with the help of functional derivatives (see next section).

A rather general result of the renormalizing techniques, irrespective of their particularities, is that they yield a common structure which could be called "renormalized propagator/renormalized forcing". Indeed, both terms correspond (e.g. Forster et al (1976) for Renormalizing Group, Herring et al. (1982) for closure techniques, Chigirinskaya et al (1996, 1997, 1998) for the space-time extensions of shell- models) to the leading contributions - "internal" damping and forcing- to the evolution of a given scale, from scales quite smaller through nonlocal interactions:

$$G_R^{-1}(1,2)u(2) = f_R(1) \tag{7}$$

where $G_R(1,2) = \delta v(1)/\delta f_R(2)$ is the (effective) renormalized propagator (represented by [dashed-solid box] in the following diagrams). It corresponds to some average of the (stochastic) bare infinitesimal response function $\hat{G}$ (Eq.4 and f_R is the renormalized (internal) forcing ($\otimes$ represents the correlation of the external forcing):

$$f_R(1) = P_R(1,2,3)u_\perp(2)u_\perp(3) \tag{8}$$

P_R being the renormalized vertex (represented by ▶ in the following diagrams) and $u_\perp$ is a projection of the original field u on a given stochastic sub-space. As further discussed below, the main failure of the renormalization techniques as developed up until now is related to quasi-normal assumptions which corresponds to taking $u_\perp$ as the gaussian projection of the field u, and furthermore not to perform any renormalization of the vertex (i.e. considering $P_R \equiv P$). In this case, $u_\perp$ is merely a gaussian field having the same covariance as u: (<.> denotes the ensemble average)

$$U(1,2) = \langle u(1)u(2) \rangle = \langle u_\perp(1)u_\perp(2) \rangle \tag{9}$$

which is represented by ▭ in the corresponding diagrams. As soon as the renormalized propagator G_R is defined this yields a stochastic model whose correlation is given by (see diagram (a) of Fig. 4):

$$U(1,2) = G_R(1,3)F(3,4)G_R(4,2) + G_R(1,3)P(3,4,5)U(4,6)U(5,7)P(8,6,7)G_R(8,2) \tag{10}$$

the Direct Interaction Approximation (Kraichnan, 1959) corresponds to the further approximation of G_R (see diagram (c) of Fig. 4)

$$G_R(1,2) = G_0(1,2) + 2G_0(1,3)P(3,4,5)G_R(4,6)U(5,8)P(6,7,8)G_R(7,2) \tag{11}$$

which we will further discuss after getting the necessary statistical tools in the next sections.

2.3 *The renormalization of the vertex and nongaussian statistics of the field:*

In order to obtain the statistics of the field u, one has to consider (Hopf, 1952) the first (Z) and second (K) characteristic functionals of an arbitrary function η (which will not explicit most of the time), which are respectively the moment and cumulant generating functionals:

$$Z(\eta) = < e^{\eta(1)u(1)} > = e^{K(\eta)} \tag{12}$$

the moments and cumulants of order η being defined respectively as the n^{th} order functional derivative of respectively Z) and (K) (strictly speaking the limit $\eta \to 0$ must be considered, although not in the intermediate steps):

$$< u(1)u(2)..u(n) >_\eta = \frac{\delta^n Z}{\delta\eta(1)\delta\eta(2)...\delta\eta(n)} \quad \{u(1)u(2)..u(n)\}_\eta = \frac{\delta^n K}{\delta\eta(1)\delta\eta(2)...\delta\eta(n)} \tag{13}$$

in many respects the cumulants are more fundamental than the moments, in particular any moment of order n corresponds to the sum of all products of cumulants of whose sum of orders is n :

$$< u(1)u(2)..u(n) >_\eta = Z(\eta) \sum_{\bigcup\{i_m..j_m\}=\{1,2...n\}} \{u(i_1)..u(j_1)\}_\eta \{u(i_2)..u(j_2)\}_\eta ... \{u(i_l)..u(j_l)\}_\eta \tag{14}$$

Equation 12 is the Laplace transform of a probability functional; using Eq. 1 and the Laplace transform of a derivative we obtain the following equation of evolution for the moments generating function:

$$G_0^{-1}(1,2)\frac{\delta Z}{Z\ \delta\eta(2)} = \frac{\delta Z_f}{Z_f\ \delta\eta(1)} + P(1,3,4)\frac{\delta^2 Z}{Z\ \delta\eta(3)\delta\eta(4)} \tag{15}$$

where Z_f is the moment generating functional for the forcing term f, with associated cumulants $\{f(1)f(2)...f(n)\}_\eta$. Due to the decomposition of moments into cumulants (Eq.14), we can rewrite this equation as:

$$G_0^{-1}(1,2)\{u(2)\} = \{f(1)\} + P(1,3,4)[\{u(3)u(4)\} + \{u(3)\}\{u(4)\}] \tag{16}$$

by differentiating with respect to η, one obtains for the second order statistics:

$$G_1^{-1}(1,2)\{u(2)u(1')\} = \{f(1)f(1')\} + P(1,3,4)\{f(3)f(4)f(1')\} \tag{17}$$

where the propagator $G_1(1,2)$ takes into account the first cumulant (if not zero):

$$G_1(2,1') = G_0(2,1') + 2\ G_0(2,1)P(1,3,4)\{u(4)\}G_1(3,1') \tag{18}$$

By differentiating a second time Eq.16 (and still using the decomposition of moments into cumulants, Eq.14), one obtains (assuming furthermore gaussianity of the forcing):

$$G_1^{-1}(1,2)\{u(2)u(1')u(1'')\} = P(1,3,4)[\{u(3)u(4)u(1')u(1'')\} + 2\{u(3)u(1')\}\{u(4)u(1'')\}] \qquad (19)$$

when quasi-gaussianity is assumed (or the projection on gaussian fields is considered as being exact):

$$\{u(3)u(4)u(1')u(1'')\} = 0 \qquad (20)$$

then the work is almost done (!): one has to replace (see below) the bare propagators (G_0 and G_1) by their renormalized counterpart(s) (G_R). For the more general case, one has to define a renormalized vertex in the following way:

$$2[P_R(1,3,4) - P(1,3,4)]\{u(3)u(1')\}\{u(4)u(1'')\} = P(1,3,4)\{u(3)u(4)u(1')u(1'')\} \qquad (21)$$

Eq. 10 is therefore modified into:

$$U(1,2) = G_R(1,3)F(3,4)G_R(4,2) + G_R(1,3)P(3,4,5)U(4,6)U(5,7)P_R(8,6,7)G_R(8,2) \qquad (22)$$

as illustrated by the corresponding diagram (a) in Fig. 4. One can check that some seemingly more abstract expressions for the renormalization of the vertex, in particular those given by Martin et al. 1973 (e.g., their Eq. 3.14) are in fact equivalent to this one. However, in order to proceed to this comparison, one has to consider some details of the renormalization of the propagator.

2.4 The renormalization of the propagator and non gaussian statistics of the adjoint field:

The renormalization of the propagator is best understood in a self consistent manner by introducing the "adjoint field" $\tilde{f}$ of f, in a similar manner (although with important differences which will be discussed elsewhere), as done by Martin et al. (1973) and De Dominicis (1976). The adjoint field $\tilde{f}$ corresponds (in the sense of Lie algebra) to an inner derivative:

$$\hat{G}(2,3) \equiv \frac{\delta u(2)}{\delta f(3)} = u(2)\hat{f}(3) \qquad (23)$$

This allows one to rewrite the average propagators as a correlators, in particular, we have for the renormalized propagator:

$$G_R(2,3) \equiv <\frac{\delta u(2)}{\delta f_R(3)}> = <u(2)\hat{f}_R(3)> \qquad (24)$$

The renormalization of propagators and correlators is therefore similar and performed on equal footing. More precisely, the procedure outlined in the previous section must be applied *mutatis mutandis* to the "vector" $\phi(1)=(u(1),\hat{f}(1))$ instead of the unique component ($u(1)$). However, it is important to note that this similarity in fact implies an asymmetry on the resulting renormalizations. Indeed, the rather obvious asymmetry between the correlators and propagators (diagrams (a) and (b) of the Fig. 4)is easily explained by the strong asymmetry of generalized correlator (the property $<\hat{f}\hat{f}>=0$ will be discussed elsewhere) and its inverse (written with some shorthand notations):

$$\{\phi(1)\otimes\phi(2)\}=\begin{bmatrix} U(1,2) & G_R(1,2) \\ -G_R(1,2) & 0 \end{bmatrix};\quad \{\phi\otimes\phi\}^{-1}=\begin{bmatrix} 0 & -G_R^{-1} \\ G_R^{-1} & G_R^{-1}UG_R^{-1} \end{bmatrix} \tag{25}$$

On the other hand, one needs to introduce, as done in Eq. 20, a second renormalized vertex P_R^G, in order to factorize the forth order cumulant ($\{\hat{f}uuu\}$)into a convolution of two second order cumulants ($\{\hat{f}u\}$ and $\{uu\}$)which yields analogously to Eq. 22:

$$G_R(1,2)=G_0(1,2)+2G_0(1,3)P(3,4,5)G_R(4,6)U(5,8)P_R{}^G(6,7,8)G_R(7,2)_0 \tag{26}$$

Therefore, the DIA involves a second assumption of quasi-gaussianity, rather less explicit than the first one, which corresponds to the quasi-gaussianity of the adjoint field. This is presumably the source of the major trouble of DIA, since its renormalized propagator introduces violation of (random) Galilean invariance (Kraichnan, 1971).

3. Strong non gaussianity and the Fractionaly Integrated Flux (FIF) models

In a sharp contrast to the difficulties of introducing non gaussian statistics into renormalizing procedures, multifractal models yield naturally such statistics. After recalling some of the salient features of FIF models which represent a large class of multifractal models, we will the discuss how to use them in order to implement strongly non gaussian (internal) forcing in renormalizing procedures.

3.1 Some fundamental features of FIF models.

The *static* version FIF models have became popular for simulations of clouds (Wilson et al. 1991, Naud et al 1996) and other geophysical fields (Pecknold et al. 1993), and their dynamic versions have been more recently developed for studying turbulence and rainfield predictability (Marsan et al. 1996, 1997, Schertzer et al. 1997).

Stochastic multifractal processes originated from the phenomenological assumption (e.g. Yaglom 1966) that in turbulence the successive cascade steps define independent fractions of the flux F transmitted to smaller scales and that a cascade from the scale ratio $\lambda=\frac{L}{\ell}$ (L) being the outer scale, ℓ the scale corresponding to scale ratio λ) to the scale ration $\Lambda=\frac{L}{\ell'}=\lambda\lambda'$ corresponds to a rescaling (by a contraction T_λ of scale ratio λ; $T_\lambda(f(\underline{x}))=f(T_\lambda(\underline{x}))$; and in the isotropic case $T_\lambda(\underline{x})=\frac{\underline{x}}{\lambda}$) of a cascade from ratio 1 to λ', i.e. is a multiplicative group ($\overset{d}{=}$ means equality in distribution):

$$F_{\Lambda=\lambda\lambda'}\overset{d}{=}F_\lambda.T_\lambda(F_{\lambda'})\ (\forall\lambda,\lambda'\geq 1) \tag{28}$$

This implies a similar group property for the statistical moments, therefore the following scaling law:

$$< F_{\lambda\lambda'}{}^q >= \lambda'^{K(q)} < F_\lambda{}^q > \quad (\forall q, \forall\lambda, \lambda' \geq 1) \tag{29}$$

where the scaling function $K(q)$ is the cumulant generating function of the infinitesimal generator of the group (Eq. 28 and by a Mellin transform (Schertzer and Lovejoy, 1993) one obtains the corresponding scaling law for the probability distribution:

$$\Pr\{F_\lambda \geq \lambda^\gamma\} \propto \lambda^{-c(\gamma)} \tag{30}$$

where $c(\gamma)$ is a statistical codimension (Schertzer and Lovejoy, 1987, 1992) often called the "Cramer" function (Oono 1989, Mandelbrot 1991). The two scaling functions are related by the celebrated Legendre transform (Parisi ad Frisch 1985):

$$K(q) = \max_\gamma \{q\gamma - c(\gamma)\} \tag{31}$$

Fluxes generated by multiplicative processes which are continuous in scale are obtained by fractional integration of a 'sub-generator' which is a white noise ($\gamma_{0,\lambda}$) limited to resolution λ:

$$F_\lambda = e^{\Gamma_\lambda} \quad g^{-1} * \Gamma_\lambda = \gamma_{0,\lambda}(\underline{x}, t) \tag{32}$$

and g is a scaling (retarded) Green's function of fractional (codimension) order h (and with corresponding dimension D_h):

$$T_\lambda g = \lambda^{D_h} g; \; D_h = d - h \tag{33}$$

Eq. 32 corresponds to a generalized diffusion equation (Cheskin et al., 1995). In case of strong universality (Schertzer and Lovejoy 1997) the sub generator it a Levy stable noise of Levy index α:

$$D_h = \frac{d}{\alpha}; \; h = \frac{d}{\alpha'}; \; \frac{1}{\alpha} + \frac{1}{\alpha'} = 1 \tag{34}$$

in order to assure the logarithmic divergence of the cumulant generating function which yields the scaling law of the moments (Eq. 29).

A component of the velocity field (u) or the concentration field of a scalar FIF model corresponds to the fractional integration with the help of another (retarded) Green's function (G) of the a th power of a flux F^a

$$G^{-1} * u_\lambda = F_\lambda^a \tag{35}$$

G has a similar scaling behavior to g (Eq. 33), however with a different order H and corresponding dimension of integration $D_H = d - H$. We will discuss below the different possibilities. One may note that the extension of FIF to vector fields has been considered (Schertzer and Lovejoy, 1995). We have now some tools in order to begin to revisit the renormalization techniques.

3.2 Preliminary steps towards a multifractal renormalization technique.

In spite of the fact we stressed the interest of treating in a similar way the strong non gaussianity of both the adjoint of the forcing field and the field itself, a preliminary step corresponds to dealing with an average renormalized propagator obtained mostly on phenomenological considerations, concentrating our attention on the renormalized forces. For instance, we can consider the following average propagator, corresponding to a scaling eddy-viscosity:

$$G_R^{-1} = \frac{\partial}{\partial t} + (-\Delta)^{\frac{1-H_t}{2}} \tag{36}$$

where $H_t \neq 0$ measures the scaling anisotropy between time and space, and can be often fixed on phenomenological or dimensional arguments, e.g. due to the conservation of the flux of energy in turbulence $H_t = \frac{1}{3}$. On the other hand, not only the Eqs 5, 35 are similar, but they become equivalent by choosing the following fractional power of convolution:

$$G(\underline{x},t) \propto G_R^{\left(\frac{H}{1-H_t}\right)*}(\underline{x},t) \tag{37}$$

which corresponds to a mere power law relationship for their Fourier transforms:

$$\tilde{G}(\underline{k},\omega) \propto \tilde{G}_R^{\frac{H}{1-H_t}}(\underline{k},\omega) \tag{38}$$

we obtain on the other hand:

$$\hat{f}_R(\underline{x},t) \propto G_R^{-1} * G * \hat{F}_\lambda^a \tag{39}$$

One may note that in turbulence: $H_t = H = a = \frac{1}{3}$.

As a first result we obtained dynamical models based on the renormalized propagator/renormalized forcing discussed in section 2.2. However, a second question arises: a priori not all these models could satisfy the nonlinearity of the basic equation, in particular as stated in the Eq. 8. In other words, the adequacy of the FIF model chosen to introduce strongly nonlinear forcing could be tested by a fixed point procedure: does the resulting field u, obtained by Eq. 5, yields (by Eq. 8) a similar forcing?

4. Conclusions

We first discussed some issues related to the renormalizing techniques, and in particular the fundamental role of strongly nongaussian statistics. We emphasized that, despite some abstract formulations of it, the long-standing question of the renormalization of the vertex is directly related to this question of statistics. On the other hand, we rendered more explicit the fact that the renormalization of the propagator involve a similar question, which is often hidden in the formalisms, whereas it could be even more acute.

These considerations, as well as the recent development of dynamical multifractal models, help us to consider how to bridge up the gap between the failure of the present renormalizing techniques and a better understanding of the phenomenology of the intermittency. In particular, we point out a fixed point procedure to test the relevant modeling of strongly non gaussian renormalized forcing.

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