# Structures in Turbulence and Multifractal Universality

Daniel SCHERTZER [1], Shaun LOVEJOY [2] and François SCHMITT [1]

[1] Laboratoire de Météorologie Dynamique (CNRS), Université Paris 6, 4 place Jussieu, 75252 Paris cedex 05, France

[2] Physics Department, McGill University, 3600 University Street, Montreal H3A2T8, Canada

**Abstract.** We show that a recent proposal for "log-Poisson" multifractality in turbulence is in fact a weak hypothesis of universality of turbulent cascades. By using the Lévy canonical measure, we relate this weak universality to the classical strong multifractal universality involving stable Lévy multifractal generators. Finally, using high Reynolds number atmospheric data, we show that for both weak and strong events, the data are inconsistent with Log-Poisson multifractality, whereas — when multifractal phase transitions are taken into account — it is extremely close to the strong universality over the entire range of singularities.

**Keywords.** Turbulence, universality, multifractals, singularities.

## 1 Introduction

A recent series of papers [1-4] consider new aspects of universality of scaling laws in fully developed turbulence. They discuss the particular case of multifractal universality ruled by Log-Poisson statistics [2, 4] originally motivated by filamentary structures [1]. Below we show that this suggestion is simply the continuous limit of the $\alpha$-model [5], which was a pedagogical but hardly realistic model. Therefore, we argue that a priori it will be less relevant than existing proposals for strong multifractal universality ([6, 7]; see also [8] for simulations). On the contrary, the classical strong universality when combined with multifractal phase transitions [9] fits the entire spectrum of observable singularities extremely well [10, 11, 12].

## 2 Universality

### 2.1 The general framework

Mathematically, an infinite number of parameters is generally necessary to specify a multifractal process. This is because the hierarchy of singularities $\gamma$ can have arbitrary (convex and increasing) co-dimensions $c(\gamma)$ — or equivalently arbitrary (convex) scaling moment functions $K(q)$ of the orders of moments $q$ (we have used the codimension/Cramer function multifractal formalism [6, 13]). Taking $L$ the outer scale, $\ell$ the scale of observation $\lambda = L/\ell$ as the scale ratio, then $c, K$ are defined:

In *Small scale structures in 3D hydro and MHD turbulence*, M. Meneguzzi, A. Pouquet and P.L. Sulem, eds., Springer, Lecture notes in physics vol.462, 1995, pp.137-144.

$$\Pr\left(\varepsilon_\lambda \geq \lambda^\gamma\right) \approx \lambda^{-c(\gamma)} \ ; \ <\left(\varepsilon_\lambda\right)^q> \approx \lambda^{K(q)} \tag{1}$$

where "Pr" indicates "probability" and the angle brackets ensemble averages.

Unless only a few of the infinite number of parameters are physically relevant (determining the "universality" classes) such cascades would be unmanageable either theoretically or empirically. It is therefore unfortunate that after simplistic claims on lognormal universality in turbulent cascades e.g. [14] there had been several attempts to deny universality for multifractals [15-17]. The technical difficulty is that although the fundamental singular small scale limit prevents iterations of the process to smaller scales from approaching a universal limit (see [18] for discussion) this in no way contradicts the general idea of universality. In the following we will distinguish two types of universality both of which can be illustrated with random walks.

*Strong universality* e.g. [19] refers to *stable* and *attractive* walks (for i.i.d. steps $\Delta X_i$) under some rescaling and/or recentring (with $b(N)$ and $a(N)$), i.e.:

$$\sum_{i=1}^{N} \frac{\Delta X_i - a(N)}{b(N)} \xrightarrow[N\to\infty]{} Y \tag{2}$$

For elementary steps with finite variance the process converges to a Brownian motion, whereas for divergent variance (i.e. critical order of divergence of moments $\alpha \leq 2$: all $q \geq \alpha$: $<|\Delta X|^q> = \infty$) it converges [20] to Lévy stable laws ($L_\alpha$, for which $b(N) \propto N^{1/\alpha}$; $a(N) \propto N$).

In constrast, a *weaker* type of *universality* exists, not requiring stability and attractivity (under renormalized) sums, but rather the property that a finite step can be always decomposed into a sum of $N$ more elementary i.i.d. steps involving a limited number of parameters, i.e.:

$$\forall\ N, \exists\ \Delta X_{N,i}\ :\ Y = \sum_{i=1}^{N} \Delta X_{N,i} \tag{3}$$

where Y has an infinitely *divisible law*. Note that we call this weaker universality because $L_\alpha$ satisfies (3). It is easy to derive ([21] for brief arguments and [22] for more mathematical ones) a common expression for their second (Laplace) characteristic function (cumulant generating function) $K(q)$ by considering "homogeneous random" sums instead of deterministic ones, i.e. a Poisson distribution (parameter $p$) of jumps of intensity distributed by a "Lévy canonical measure" $dF(x)$, which needs not be a probability measure (the recentring term $qx$ is only needed when this measure is too singular at the origin):

$$<e^{qY}> = e^{K_Y(q)} \ ; \ K_Y(q) = p\int_0^\infty (e^{-qx} - 1 + qx)dF(x) \tag{4}$$

Strong universality corresponds to (recentring needed for $\alpha \geq 1$):

$$dF = c_\alpha \frac{dx}{x^{\alpha+1}} \ ; \ 0 \leq \alpha \leq 2 \tag{5}$$

## 2.2 Strong universality in multifractals

Instead of considering only the iteration of the process down to smaller and smaller scales, one can first consider interactions of this process over a finite range of scales $\Lambda$ with *larger and larger numbers of its replicas, and then* seek the limit $\Lambda \to \infty$, i.e. (with possible combination): (i) "nonlinear mixing" of these processes:

multiplication of independent, identically distributed processes on the same scales, (ii) "scale densification" of the process: introducing more and more intermediate scales. In both cases, multiplying processes ( $\varepsilon$ ) corresponds to adding generators ( $\Gamma$ ) defined as:

$$\varepsilon_\Lambda \approx e^{\Gamma_\Lambda} \tag{6}$$

Taking the power $1/b(N)$ of $\varepsilon_\lambda^N$ corresponds to rescaling $\Gamma_\lambda^N$ by $1/b(N)$ as in (2). Equations (4) and (5) yield the strong universal scaling functions $K(q)$ and $c(\gamma)$, (where $\frac{1}{\alpha}+\frac{1}{\alpha'}=1$ and $C_1$ $(=c_\alpha \frac{\Gamma(2-\alpha)}{\alpha(\alpha-1)})$ is the codimension of the mean field ($q$=1); and its singularity at the same time) [6,7]:

$$c(\gamma)=C_1\left(\frac{\gamma}{C_1\alpha'}+\frac{1}{\alpha}\right)^{\alpha'} ;\ K(q)=\frac{C_1}{\alpha-1}(q^\alpha-q) \tag{7}$$

### 2.3 Weak universality in multifractals

It seems reasonable that one must seek weaker types of universality only when there is a failure of strong universality (no renormalization of the generator). In this case the generator and its iterates are only loosely related; they no longer involve rescaling and/or recentring. The α-model [5] is the canonical (binomial) model generated by a (Bernouilli) two-state generator $\gamma$ on elementary discrete step scale ratio $\lambda_1$:

$$\begin{aligned} dP(\gamma) &= \lambda_1^{-c}\delta_{\gamma-\gamma^+} + (1-\lambda_1^{-c})\delta_{\gamma+\gamma^-};\ \gamma^+,\gamma^- \geq 0 \\ \lambda_1^{K(q)} &= \lambda_1^{q\gamma^+-c} + \lambda_1^{-q\gamma^-}(1-\lambda_1^{-c}) \end{aligned} \tag{8}$$

Canonical means conservation on ensemble averages $\langle\varepsilon_\lambda\rangle=\langle\varepsilon_1\rangle$ for any $\lambda$; $\gamma^+$ is the upper bound of singularities; $c$ ( $\equiv c(\gamma^+)$ ) is its codimension and can be chosen rather arbitrarily; $\gamma^-$ is the lower bound of singularities and is constrained so that the ensemble average $<(\lambda_1)^\gamma>=1$; the (monofractal) β-model [23] is recovered for $\gamma^-=\infty$, $\gamma^+=c=C_1$. Whereas the central limit theorem was used [24] to show that the (renormalized) nonlinear mixing of (discrete) α-models leads to a (continuous) lognormal multifractal process, one may consider on the contrary the classical Poisson limit, by using a smaller and smaller elementary step ( $\lambda_{1/N}=\lambda_1^{1/N}\to 1$; $N\to\infty$) of the binomial law (of occurrences of weak eddies) with parameter $p=N\left(1-(\lambda_{1/N})^{-c}\right)\approx cLog(\lambda_1)$, and correspondingly a (simple) Poisson process identical for $\varepsilon_\lambda$ to the model considered in [1, 2, 4]; (8) yields in this limit:

$$\begin{aligned} dF &= \delta_{x+\gamma^- Log\lambda_1} ;\ \gamma^+=(1-\lambda_1^{-\gamma^-})c \\ K(q) &= q\gamma^+ + (\lambda_1^{-q\gamma^-}-1)c \equiv q\gamma^+ - c + \left(1-\frac{\gamma^+}{c}\right)^q c \\ c(\gamma) &= \left(1-\frac{\gamma^+-\gamma}{c\gamma^-}\left(1-\log\frac{\gamma^+-\gamma}{c\gamma^-}\right)\right)c \quad \gamma\leq\gamma^+;\ c(\gamma)=\infty\ \ \gamma^+<\gamma \end{aligned} \tag{9}$$

Assuming (non fractal, $D=1$) filament-like structures for the highest order singularity and homogeneous eddy turn over times, She and Leveque [1] selected: $c=2$, $\gamma^{+}=2/3$, and mean conservation yields $\lambda_1^{\gamma^-}=\frac{3}{2}$. In MHD turbulence, considering current sheets extreme events and also homogeneous eddy turn over times, one selects [25] $c=1$, $\gamma^{+}=1/2$. These choices are obviously questionable.

## 3. Theoretical and observable bounds on singularities

For normal and Lévy ($\alpha>1$) generators (the corresponding processes are inaccurately termed "log normal" and "log-Lévy") there are no bounds on $\gamma$, as is generally the case for canonical processes. On the contrary, micro-canonical conservation (i.e. per realization) of the flux of energy — e.g. the microcanonical version of the α-model called the "p-model" [26] —, imposes $\gamma \le D$ (the dimension of space). For $D=1$, it is the celebrated inequality (expressed via $K(q)$) of Novikov [27] who in fact imposed microcanonical conservation by considering, instead of the flux of energy, the dissipation. In the inertial range — especially in the limit of infinite Reynolds numbers — the relevance of the dissipation is questionable, especially since it is bounded by volume integration.

Frisch [16] argued for a physical bound to singularities due to the finite speed of sound, whereas Schertzer et al. [28] considered both incompressible Navier-Stokes equations (without any characteristic velocity, infinite speeds of sound) and the physical issues of compressible turbulence involving compressibility effects. The corresponding hypersonic gradients are of course beyond the scope of incompressible Navier-Stokes equations.

On the contrary, in a series of papers [9] we argue that not only do unbounded singularities pose interesting problems of observation and estimation, but are a requisite to the introduction, via first order multifractal phase transitions, of a non classical Self-Organized Criticality (SOC), which is often desirable in order to explain the phenomenology of extreme events.

For SOC singularities $\gamma \ge \gamma_D$ ($\gamma_D \ge D$ being the critical singularity of transition to SOC), the observed singularities (empirically bounded by $\gamma_s$, the maximum reachable singularity in the samples studied) has a codimension different from the theoretical one given by (7):

$$\begin{aligned} c(\gamma) &= q_D\gamma - K(q_D) \quad \gamma \ge \gamma_D = K'(q_D) \\ K(q) &= \gamma_s q - c(\gamma_s) \quad q \ge q_D \\ \gamma_s &= \gamma_D + \frac{c(\gamma_s)-c(\gamma_D)}{q_D} \end{aligned} \tag{10}$$

The observed codimension for SOC singularities ($\gamma \ge \gamma_D$) follows the tangent instead of the theoretical parabola-like codimension, which means that the probability distribution of these extreme events has an algebraic fall-off. Consequently there is a divergence of higher order moments $q \ge q_D$ for infinite samples. However, because of the finite size of empirical datasets, estimated $K(q)$ for $q \ge q_D$ are also linear in $q$, of slope $\gamma_s$, given by (10); when the number of samples increases, $\gamma_s \to \infty$. There is a priori no compelling reason that $\gamma^{+} < D$ in either the Log-Poisson or α-model:

indeed, the α-model was developed to illustrate the generality of divergence of moments for multifractal fields, its basic parameter $\alpha$ being $q_D \approx (c-D)/(\gamma^+ - D)$ (>1 for $c > \gamma^+$).

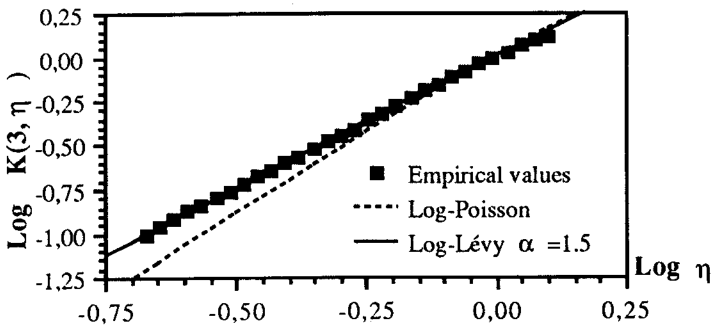


Fig. 1 Double trace moment estimate of the energy flux: $K(3,\eta)$ vs. $\eta$ in a log-log plot, where $K(q,\eta) = K(q\eta) - qK(\eta)$. In this figure, the log-Poisson model yields a slope ($=\alpha$) of 2 (due to analycity), whereas empirical values yield $\alpha \approx 1.5$.

## 4. Empirical Evidence

Here we use velocity measurements recorded 25 m above ground, over a pine forest in south-west France, sampling at $\omega = 10$ Hz. We analyzed 22 profiles of duration 55 minutes each: they present a $\omega^{-5/3}$ scaling over frequencies from about $\omega/1000$ to $\omega/2$ (see [12]). Here we present direct and precise estimates using "extended self-similarity" (ESS) [29,30] which is useful in accurately estimating structure functions scaling exponents; it has been used to compute the first 10 integer moments, and to compare them with theoretical models [1,2,4]. We prefer here to estimate a (near) continuous empirical curve $\zeta(q)$ for real $q$, taking absolute values of the increment:

$$\left\langle (\Delta V_\lambda)^q \right\rangle = \left\langle |V(x+L/\lambda) - V(x)|^q \right\rangle = \left\langle (\Delta V_L)^q \right\rangle \lambda^{-\zeta(q)} \quad (11)$$

The widely accepted Kolmogorov refined hypothesis [31] relates the scaling singularities of the wind velocity increments $\gamma_V$ to those of the energy flux, as well as to their respective scaling functions:

$$\gamma_V = \frac{\gamma - 1}{3};\ \zeta(q) = \frac{q}{3} - K\left(\frac{q}{3}\right) \quad (12)$$

The Double Trace Moment technique [32] has been widely used to test the non analyticity of $K(q)$ ($\alpha$<2). The data at the highest resolution is raised to the power $\eta$ and then the corresponding (trace-) moment is estimated. One obtains a scaling function $K(q,\eta)$ ($K(q,1) = K(q)$) which at least for small $\eta$ scales like $\eta^\alpha$ ($\alpha$=2 in case of analytical $K(q)$). This technique applied [11] to our data (Fig. 1) clearly yields $\alpha \approx 1.5$, instead of $\alpha$=2 for the Log Poisson model and α-model (see theoretical curves). One may note that similar values for $\alpha$ have been found in numerical simulations of Navier-Stokes caricatures [33].

We can on the contrary test the high order singularities directly with the help of the structure functions: Fig. 2 displays empirical estimates of $\zeta(q)$; $q = 0-15$ for N=2 and 22x32=704 realizations, the empirical values of Benzi [30] as well as the theoretical estimates of the Log Lévy and Log Poisson (rather identical to the α-model). We also estimate $C_1 = K'(1) = 1 - 3\zeta'(3) \approx 0.15$.

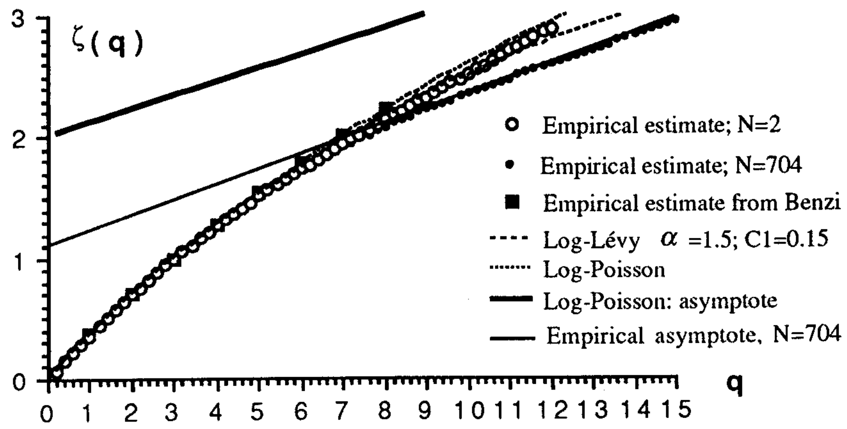


**Fig. 2** Empirical values of the structure function scaling exponent $\zeta(q)$ — for 2 and 704 realizations, and Benzi's estimates [30] —, compared to log-Poisson and log-Lévy models. For large order moments, empirical estimates follow straight lines; the log-Poisson asymptote does not fit the empirical estimates, whereas the Lévy generator is consistent with the change of slope of the asymptote for 2 and 704 realizations.

First we see in Fig. 2 that the empirical estimate for 2 or 704 realizations are very close to Benzi's estimates for integer moments for $q<8$. Furthermore, for the same range of $q$ the universal multifractals fit very well the empirical results. Up until moments order $\approx q \approx 6$, it is not possible to distinguish between the different models using the structure function scaling exponent; Fig. 1 shows that indeed other analysis techniques such as the double trace moment, are very useful to discriminate between different universal models.

But for larger order moments, there are visible discrepancies with the $\zeta(q)$ estimates. We see also that the estimates for 2 and 704 realizations deviate significantly from each other for moments order $q \geq q_D \approx 7 \pm 0.5$, and that for these values $\zeta(q)$ is linear as predicted by (10) and (12) (with $-\gamma_{s,V}$=0.19 and 0.124 and $c(\gamma_{s,V})$=0.62 and 1.11 for 2 and 704 realizations respectively). We may note that in Log-Poisson model there is also a linear asymptote which is reached for very large order moments: $\zeta(q) = \frac{1}{9}q + 2$. This behaviour is clearly not compatible with our data: for 2 realizations, the asymptotic slope is too large, and if we increase the number of realizations, it decreases, getting smaller than 1/9 (if we had more realizations, but even here for 704 realizations the intercept of the asymptote is clearly too small to be compatible with Log-Poisson extreme events). We finally note that the critical moment $q_D = 7 \pm 0.5$ is the order of divergence of moments previously estimated with different methods [12]. We predict here that all structure functions scaling exponent are linear for moments larger than this critical value, with a slope which decreases with the number of realizations as more and more SOC structures are analyzed (see [34] for meteorological implications).

## 5. Conclusions

There have been many attempts to give special importance to different basic structures in turbulence. The multifractal approach has the advantage that such structures need no longer be input in an ad hoc way; they are generated automatically by the cascade — the singularities. However, without any other considerations, the multifractal approach would be useless since it would involve an infinite number of parameters. The classical (strong) universality hypothesis for multifractality uses statistical and physical arguments to reduce the number of parameters to a small finite number, but without any ad hoc assumptions about structures. We argued that strong universality should not be abandoned unless other compelling theoretical or empirical reasons can be given. We have evaluated a recent proposal based on the possible special role of filament-like strucutres which corresponds to a weak form of multifractal universality which is a continuous limit of the $\alpha$-model, sharing the latter's unattractive assumption of an upper bound on the singularities. We show empirically, that the weak universality is quite incompatible with the data for both weak and strong events, whereas the strong universality is extremely well respected by the data.

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