L'utilisation du scaling pour la modelisation et prévision du changement climatique anthropique et naturelle

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Ouranos
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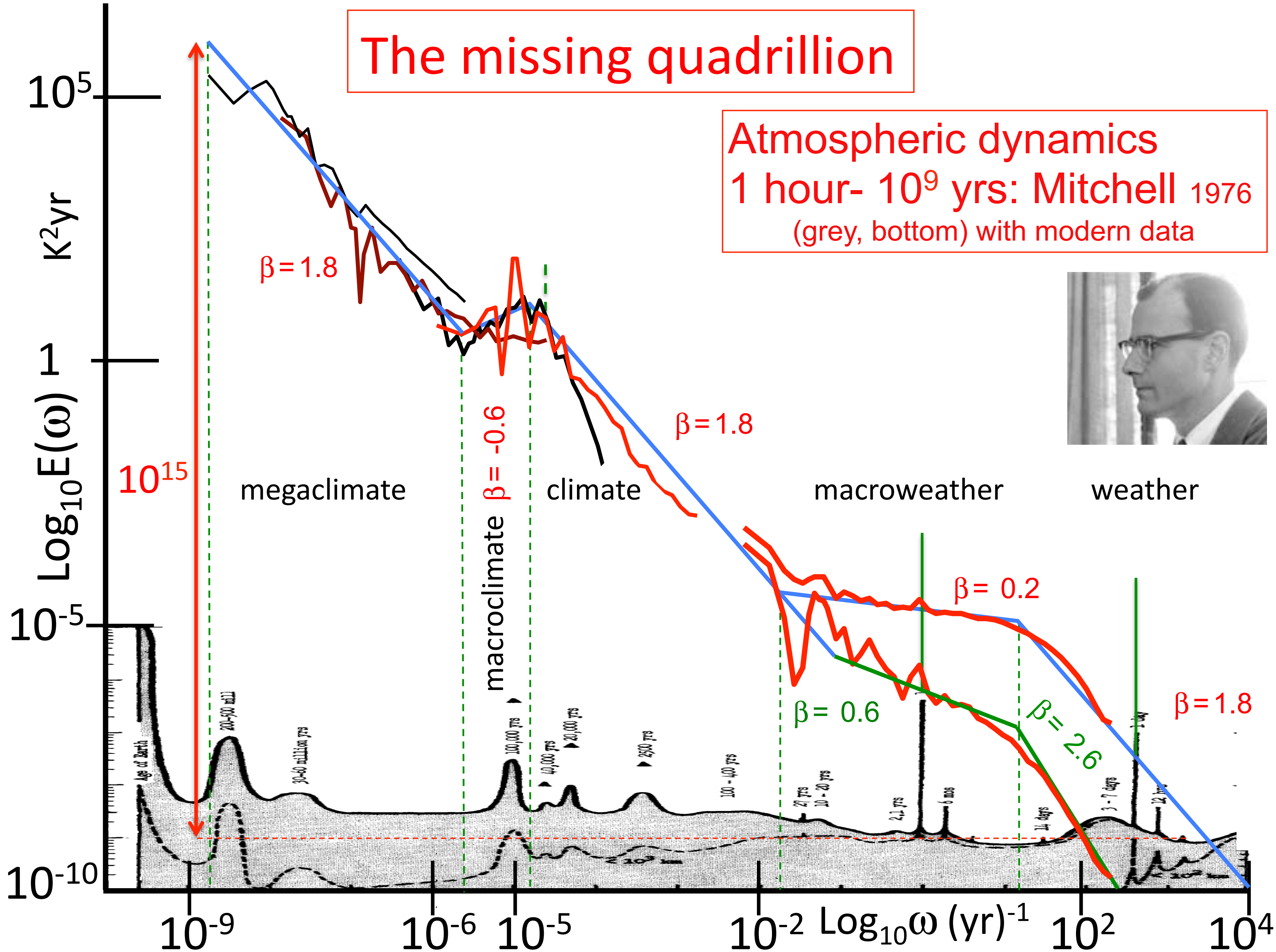
From the age of the earth to the viscous dissipation scale: 4.5×10^9 years - 1 ms:

20 orders of magnitude in time

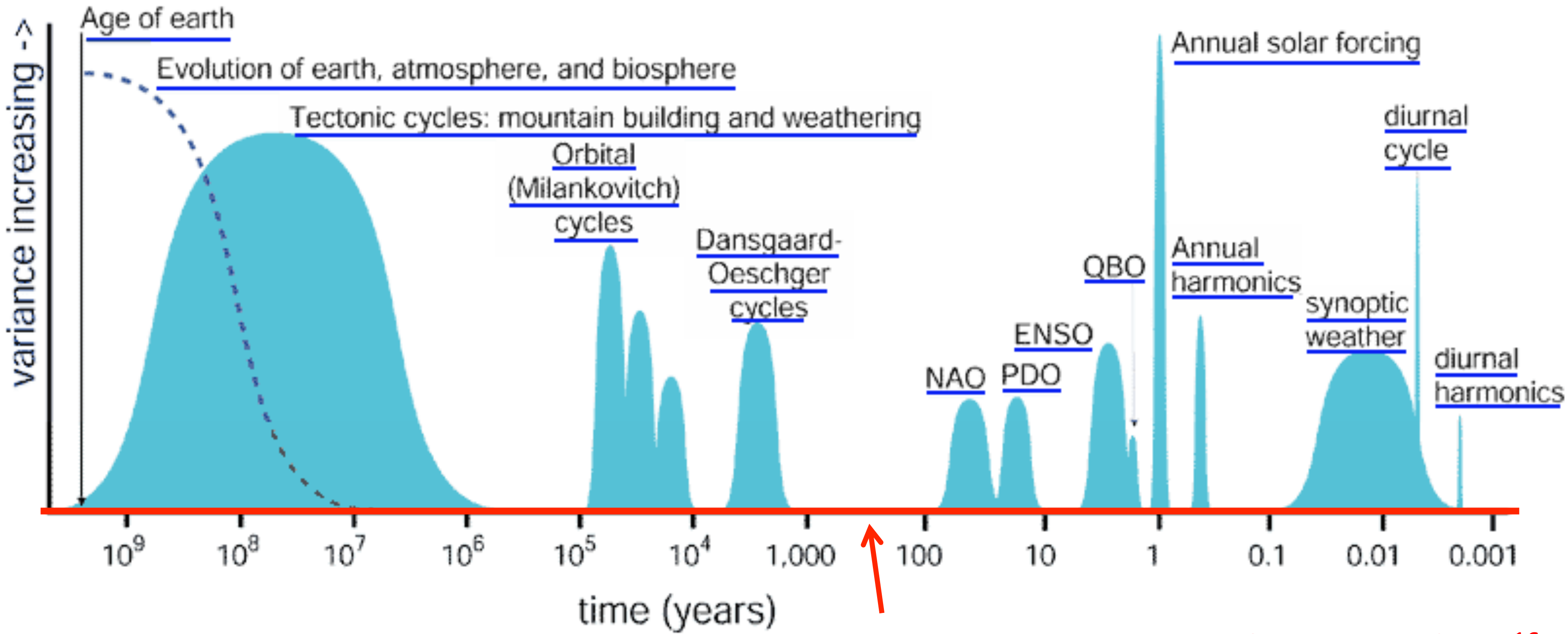
In space: the size of the planet to viscous dissipation scales:

10 orders of magnitude



The missing quadrillion
Atmospheric dynamics
1 hour- 10^9 yrs: Mitchell 1976
(grey, bottom) with modern data
K²yr
Log10E(ω)
10^5
1
10^-5
10^-10
10^15
β=1.8
β= -0.6
β=1.8
β= 0.2
β= 0.6
β= 2.6
β=1.8
megaclimate
macroclimate
climate
macroweather
weather
10^-9
10^-6
10^-5
10^-2
Log10ω (yr)^-1
10^2
10^4

The NOAA NCDC Paleoclimate data site graph (inspired by Mitchell)



The background is totally flat: error of ≈10^{16}

The explanation of the figure:

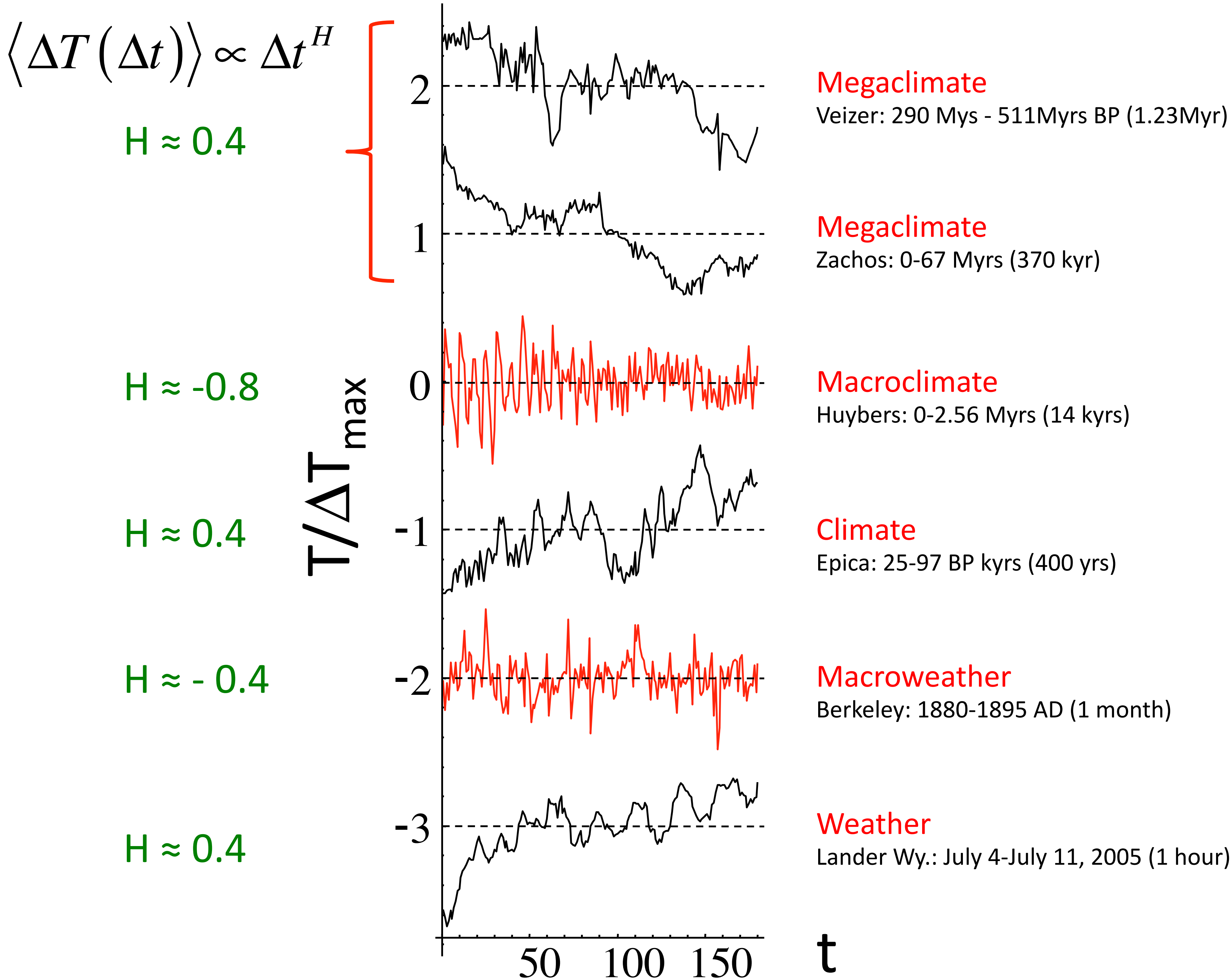
"… figure is intended as a mental model to provide a general "powers of ten" overview of climate variability, and to convey the basic complexities of climate dynamics for a general science savvy audience."

The site assures us that just "because a particular phenomenon is called an oscillation, it does not necessarily mean there is a particular oscillator causing the pattern. Some prefer to refer to such processes as variability."

How to understand the variability?

- Scaling, scale invariance:

$$\left\langle \Delta T\left(\Delta t\right)\right\rangle = \langle \varphi \rangle \Delta t^{H}$$



$\langle \Delta T(\Delta t) \rangle \propto \Delta t^H$
H ≈ 0.4
H ≈ -0.8
H ≈ 0.4
H ≈ - 0.4
H ≈ 0.4
T/ΔT max
2
1
0
-1
-2
-3
50
100
150
t
Megaclimate
Veizer: 290 Mys - 511Myrs BP (1.23Myr)
Megaclimate
Zachos: 0-67 Myrs (370 kyr)
Macroclimate
Huybers: 0-2.56 Myrs (14 kyrs)
Climate
Epica: 25-97 BP kyrs (400 yrs)
Macroweather
Berkeley: 1880-1895 AD (1 month)
Weather
Lander Wy.: July 4-July 11, 2005 (1 hour)

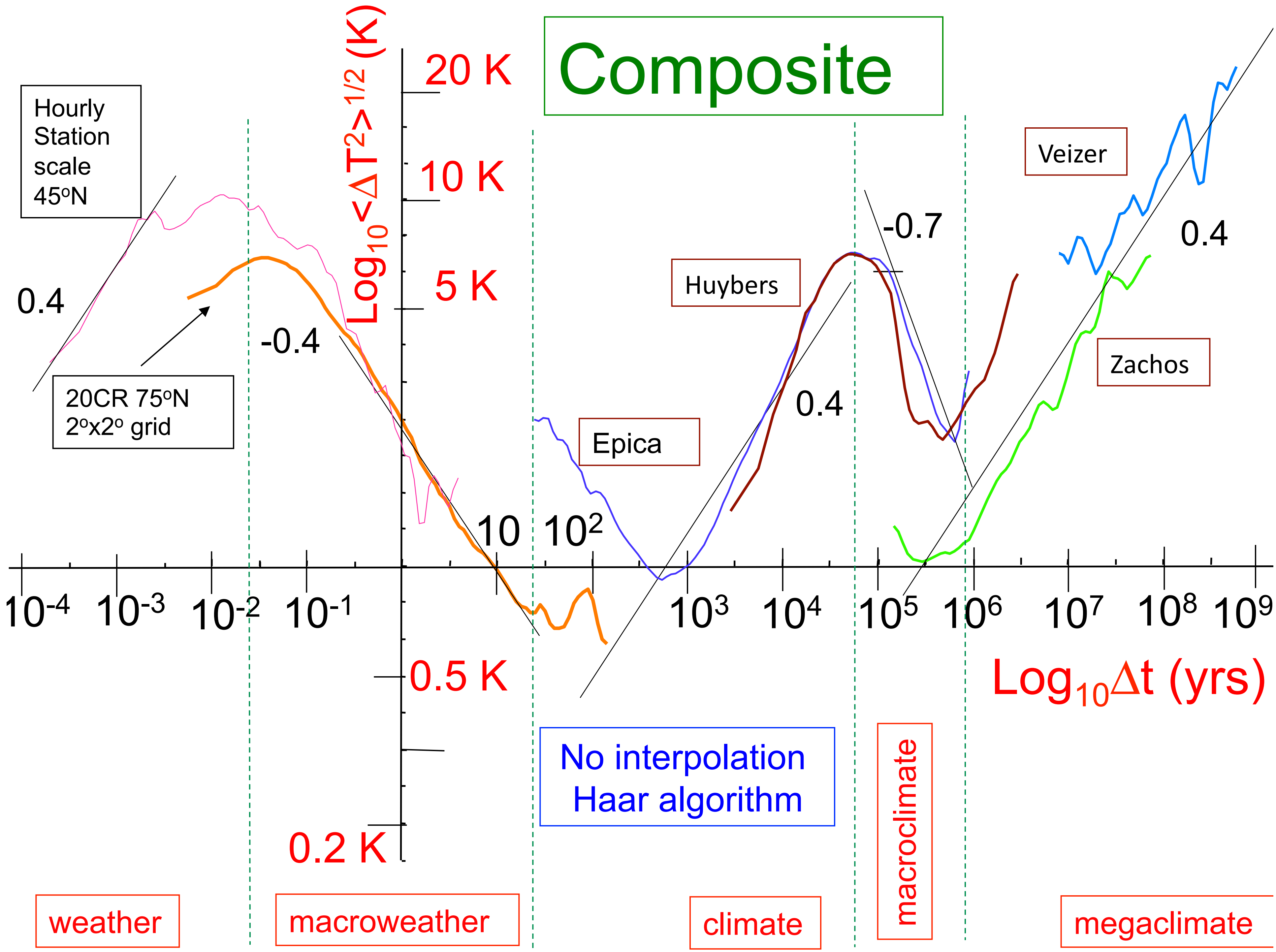
Difference, Anomaly, Haar fluctuations

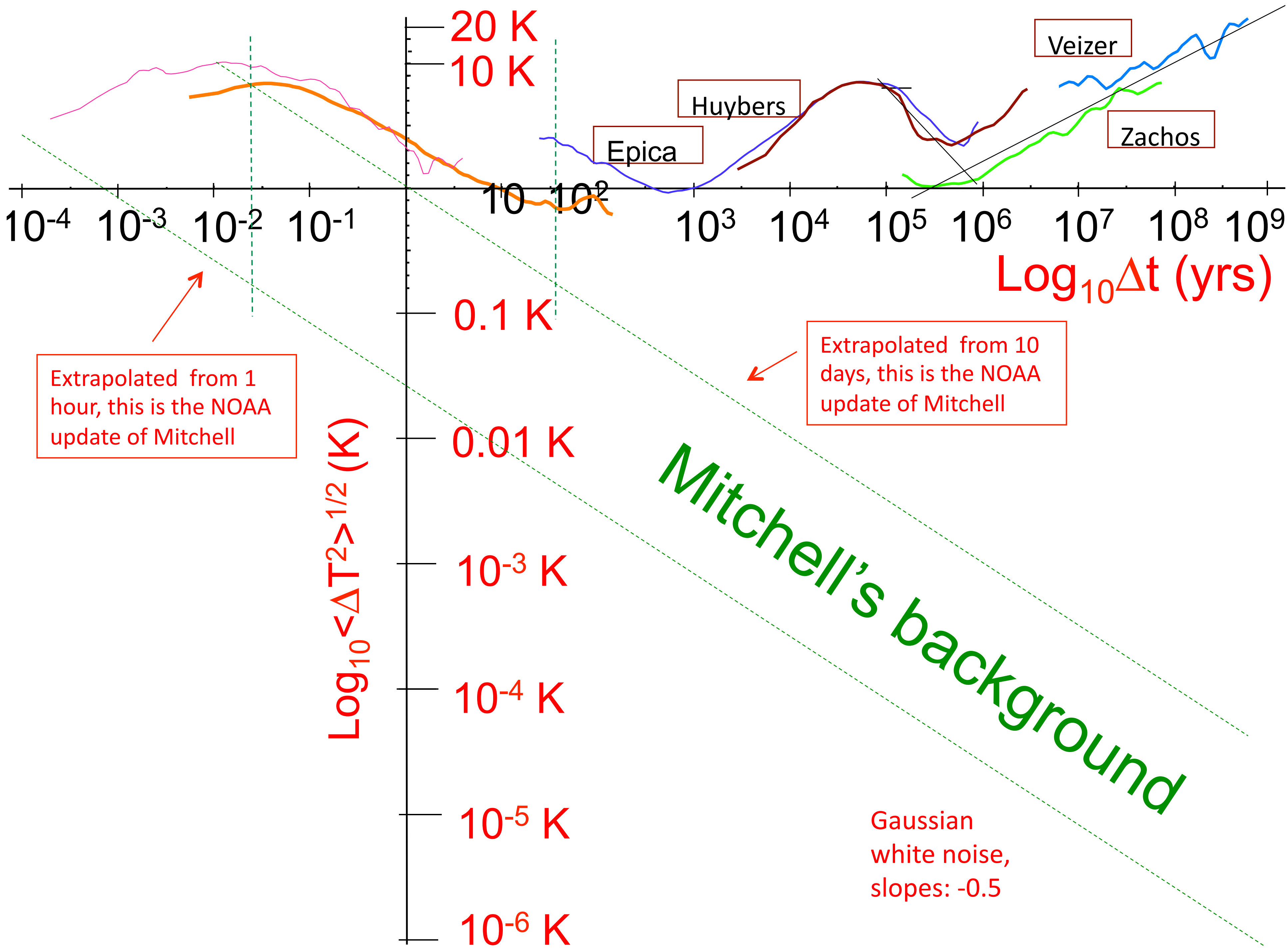
Differences: The difference in temperature between t and t+Δt

Anomaly: The average of the temperature (with overall mean removed) between t and t+Δt

Haar: The difference between the average of the temperature from t and t+Δt/2 and from t+Δt/2 and t+Δt

Relations:
When $1 > H > 0$: Haar ≈ difference
When $0 > H > -1$: Haar ≈ tendency


Composite
Hourly Station scale 45ºN
20CR 75ºN 2ºx2º grid
Log10<ΔT2>1/2 (K)
20 K
10 K
5 K
0.5 K
0.2 K
0.4
-0.4
-0.7
0.4
0.4
Epica
Huybers
Veizer
Zachos
10
102
10-4 10-3 10-2 10-1
103 104 105 106 107 108 109
Log10Δt (yrs)
No interpolation Haar algorithm
weather
macroweather
climate
macroclimate
megaclimate


20 K
10 K
0.1 K
0.01 K
10-3 K
10-4 K
10-5 K
10-6 K
10-4 10-3 10-2 10-1 10 102 103 104 105 106 107 108 109
Log10Δt (yrs)
Log10<ΔT2>1/2 (K)
Veizer
Huybers
Epica
Zachos
Extrapolated from 1 hour, this is the NOAA update of Mitchell
Extrapolated from 10 days, this is the NOAA update of Mitchell
Mitchell's background
Gaussian white noise, slopes: -0.5

Standing Mitchell on his head

Mitchell

Narrow scale range processes are the most important, the continuum background is unimportant

Mitchell standing on his head

Wide scale range "continuum" processes are the most important, the other processes are perturbations

The attribution problem
and
anthropogenic warming

Proving the truth of Anthropogenic Global Warming

Diminishing returns

- In its AR5 report last September, the IPCC upgraded the AR4's (2007) qualification "*likely*" to conclude that it is "*extremely likely* that human influence has been the dominant cause of the observed warming since the mid-20th century".

"*extremely likely*" = 95-100% confidence

- Climate sensitivity: 1.5 – 4.5 ^{o}C

Unchanged since 1979

- Disproving natural global warming

Relatively easy due to an asymmetry

-No theory can ever be proven beyond "reasonable doubt" but a single decisive experiment can effectively *disprove* one.

Requires no numerical models, needs Nonlinear Geophysics

"A mephiticly ectoplasmic emanation from the forces of darkness"
– Viscount Christopher Monckton of Brenchley describing the Climate Dynamics paper

Natural variability as a perturbation to anthropogenic change

Anthropogenic

Measurement error: ≈±0.03K

$$T_{globe}(t) = \lambda_{2xCO2,eff} \log_2\left(\rho_{CO_2}(t) / \rho_{CO_2,pre}\right) + T_{natural}(t) + \varepsilon(t)$$

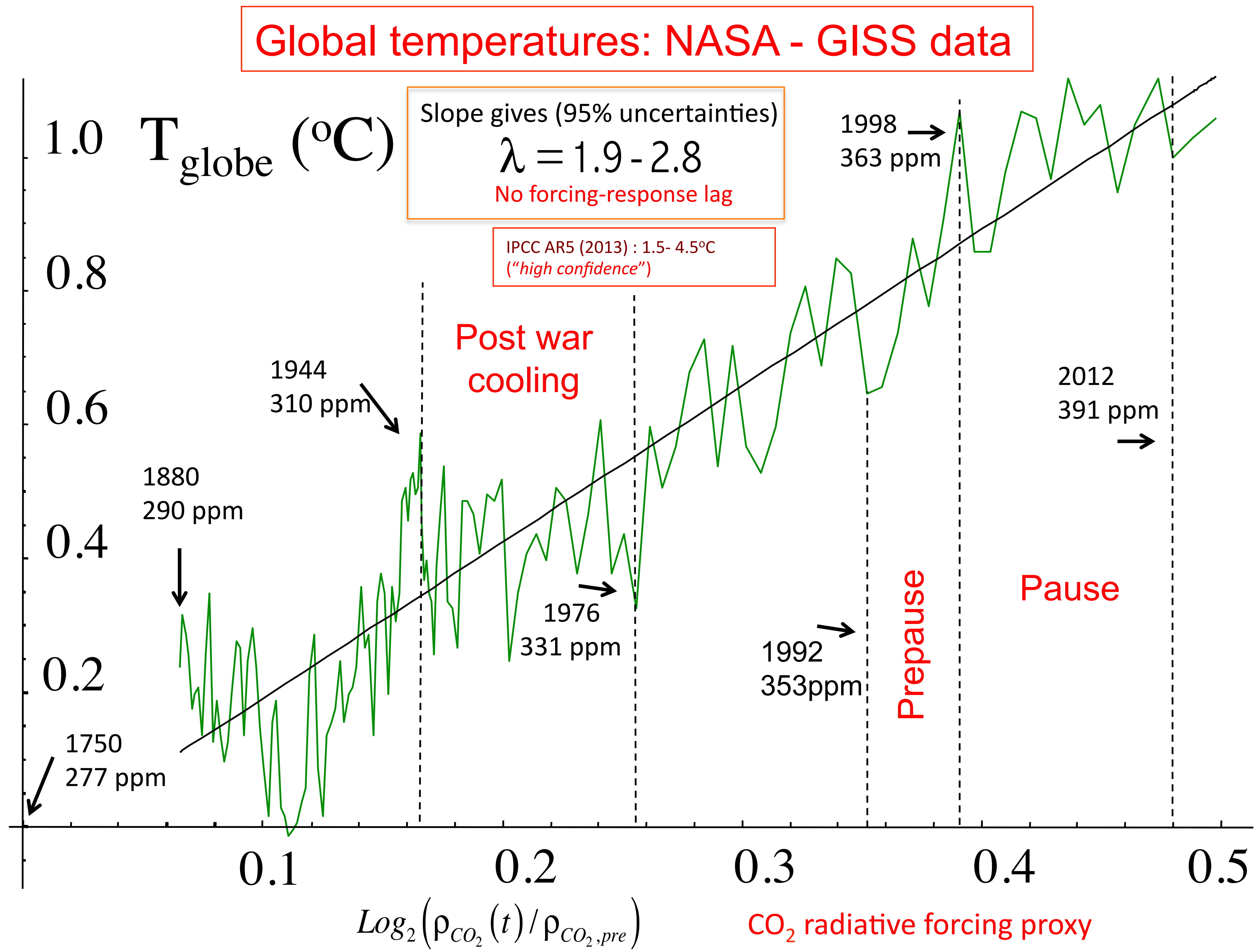
Effective climate sensitivity (K/(W/m²))

Proportional to CO_2 radiative forcing (W/m²)
Linear **Surrogate** for all anthropogenic forcings (determinstic)

Small fluctuations due to natural variability (stochastic).
Includes responses to solar, volcanic and other natural forcings.

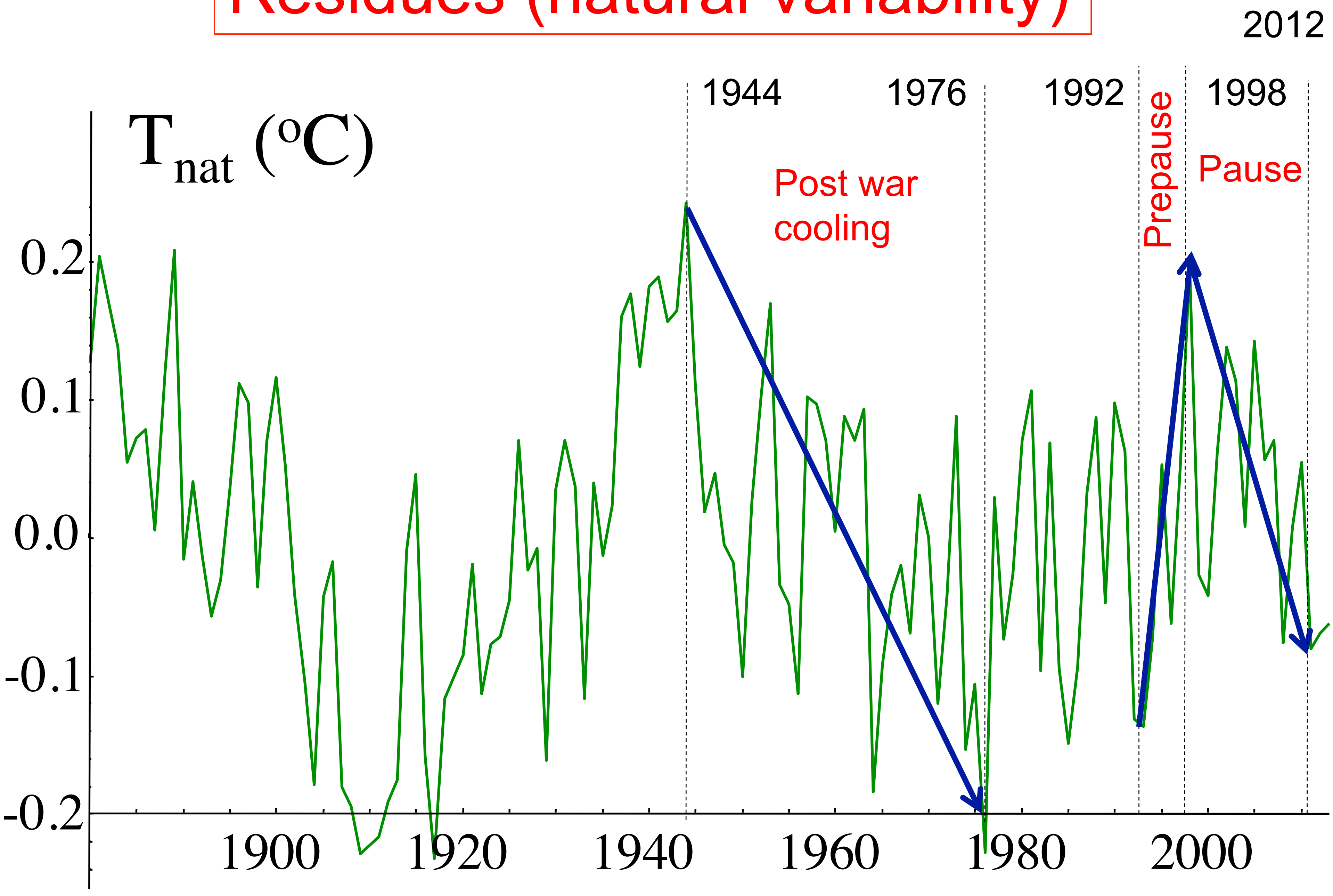
CO_2 forcing as surrogate for all anthropogenic effects

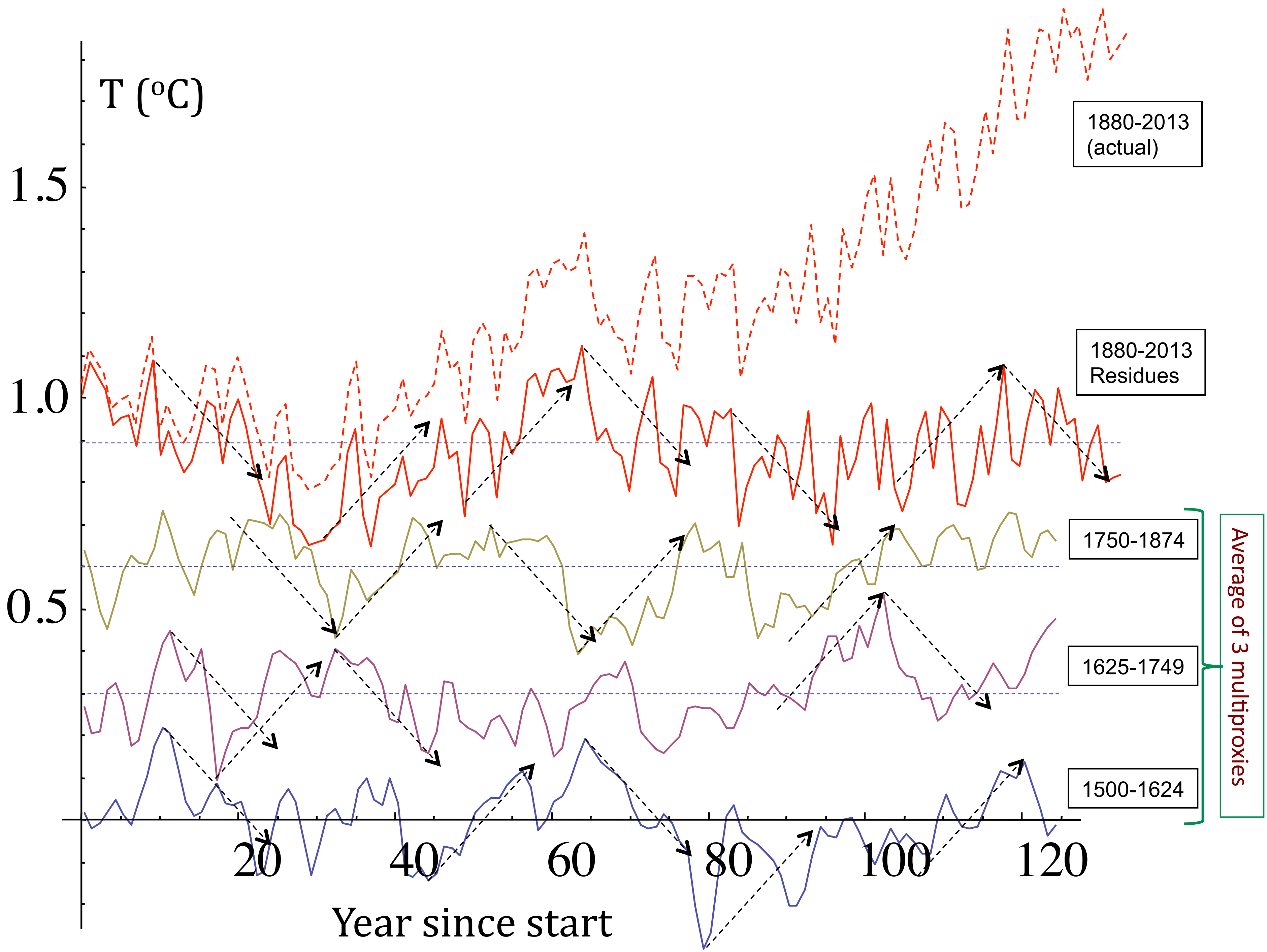
Roughly: you double the global economy, you double the emissions, land use and other changes, you double the effects



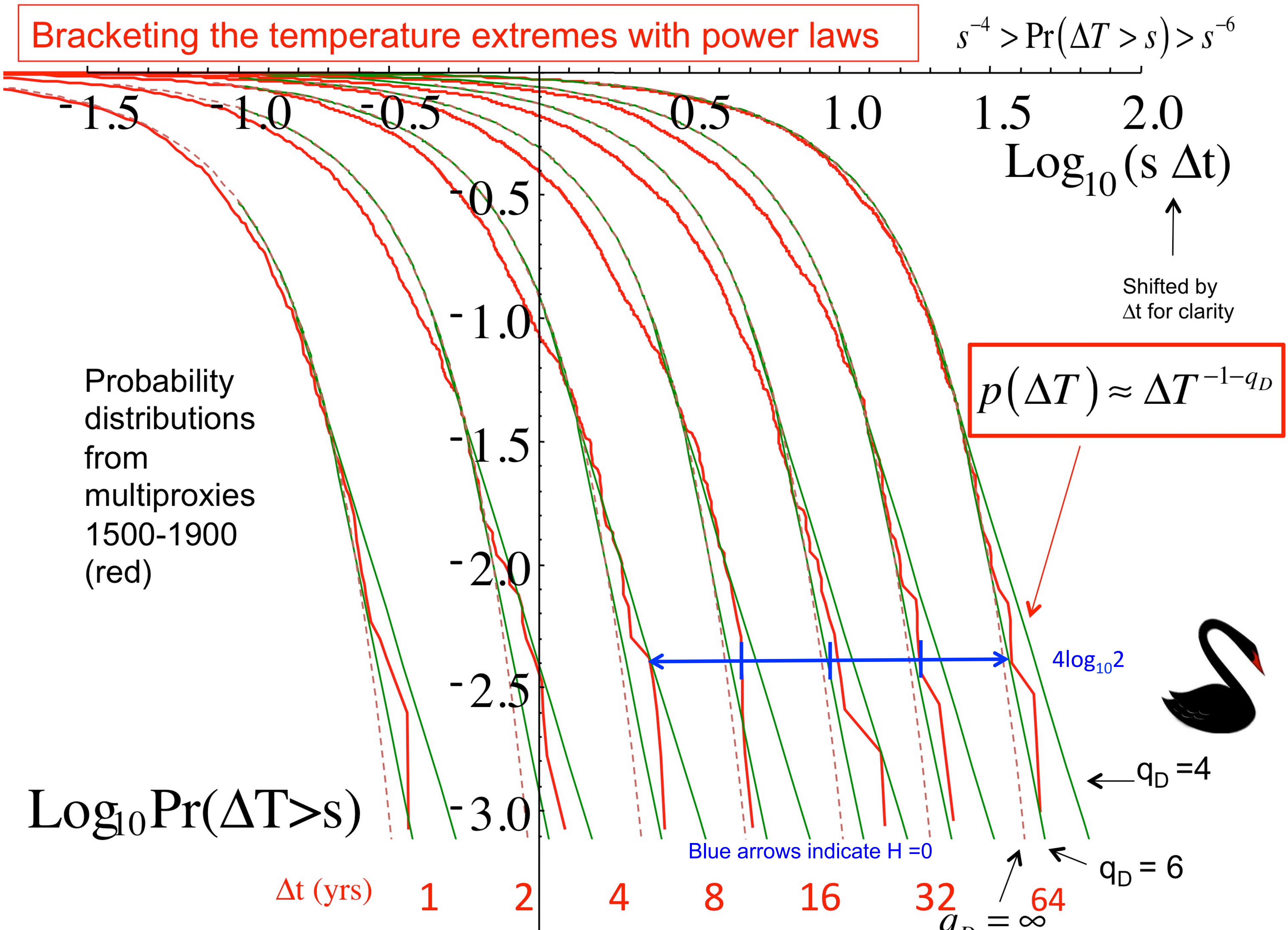
Global temperatures: NASA - GISS data
T_{globe} (°C)
Slope gives (95% uncertainties)
$\lambda = 1.9 - 2.8$
No forcing-response lag
IPCC AR5 (2013) : 1.5- 4.5°C
("high confidence")
1.0
0.8
0.6
0.4
0.2
1998
363 ppm
1944
310 ppm
Post war
cooling
2012
391 ppm
1880
290 ppm
1976
331 ppm
1992
353ppm
Prepause
Pause
1750
277 ppm
0.1
0.2
0.3
0.4
0.5
$Log_2(\rho_{CO_2}(t)/\rho_{CO_2,pre})$
CO_2 radiative forcing proxy

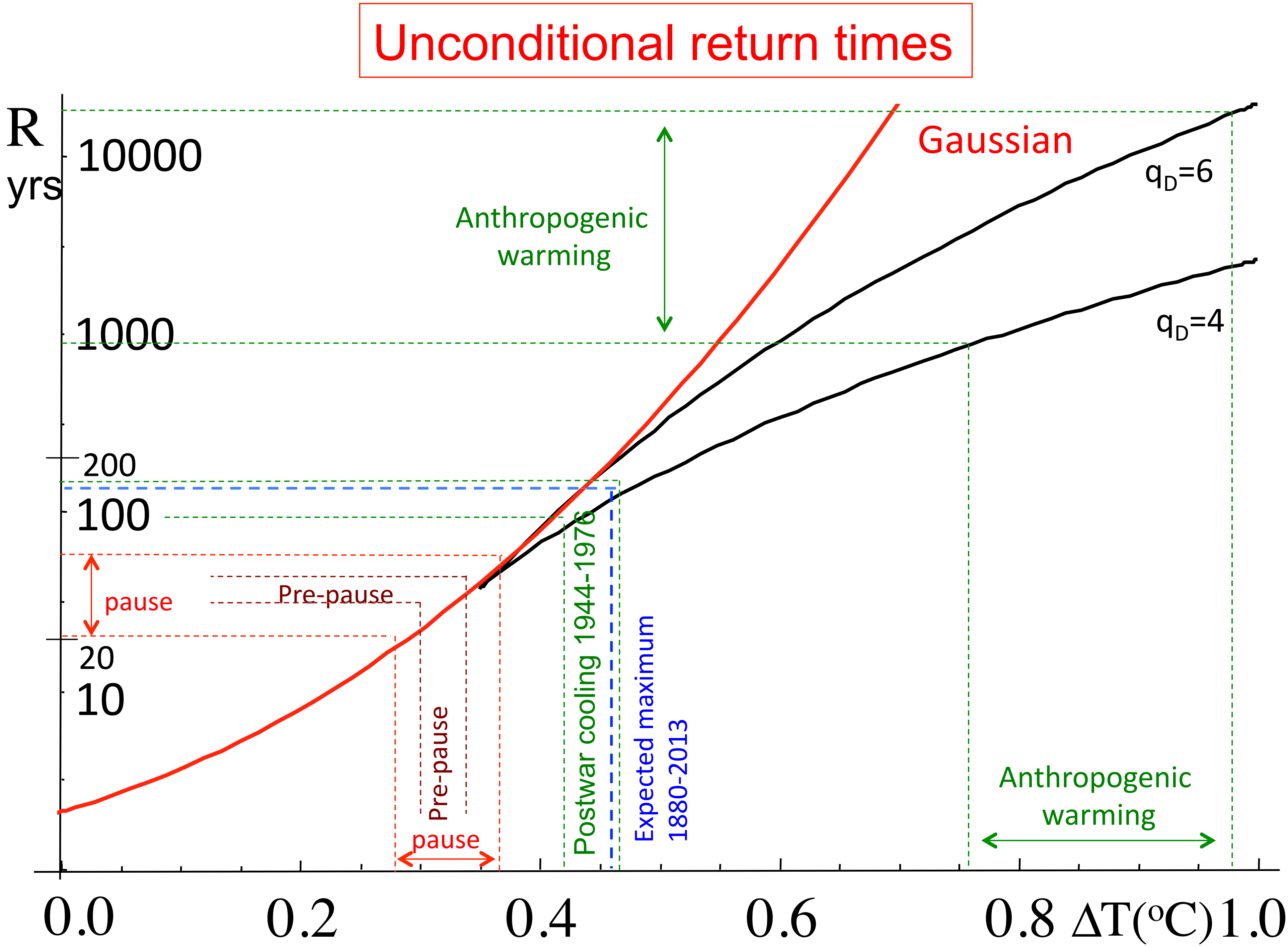
Residues (natural variability)





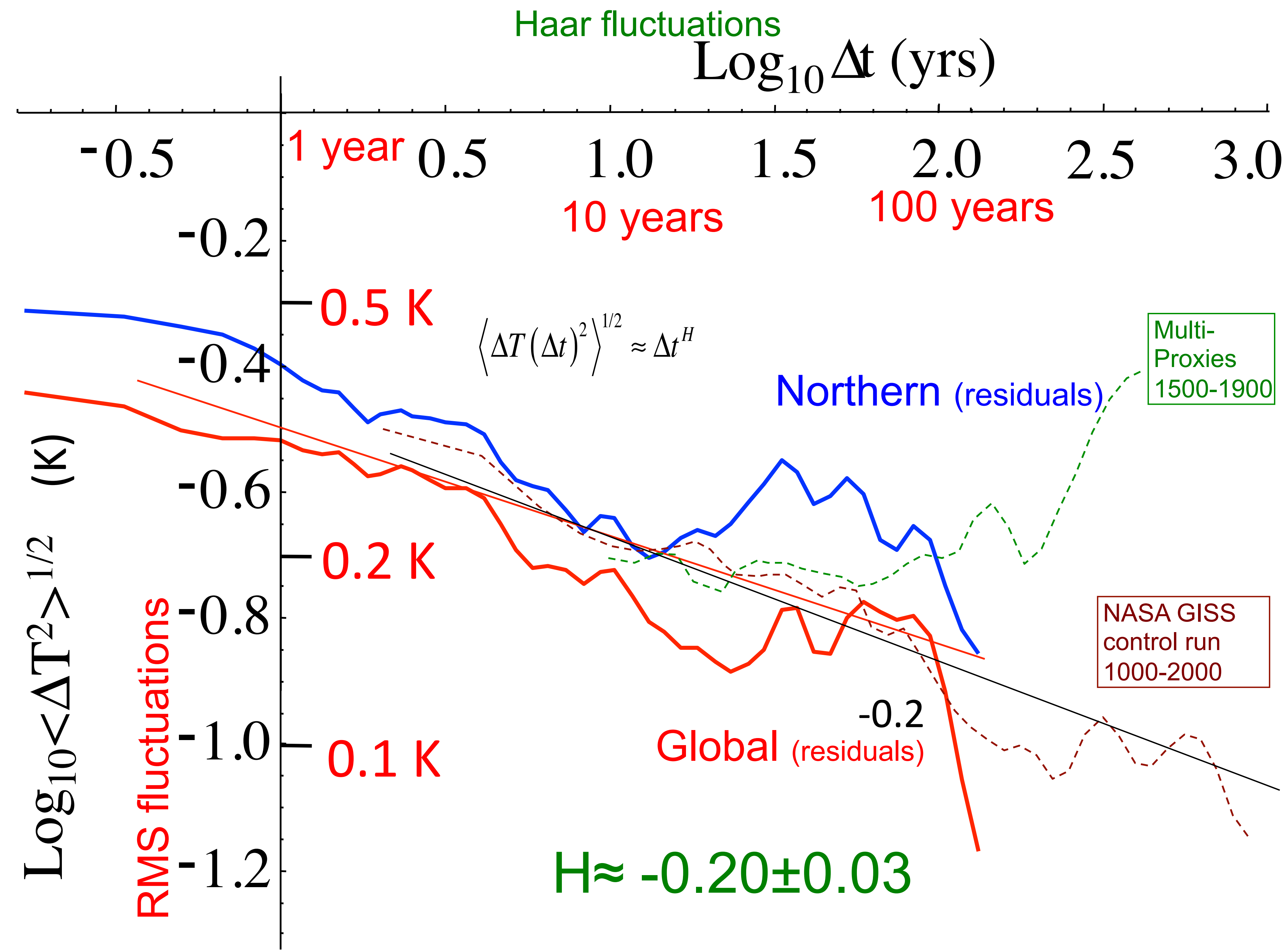
T (°C)
1.5
1.0
0.5
20
40
60
80
100
120
Year since start
1880-2013 (actual)
1880-2013 Residues
1750-1874
1625-1749
1500-1624
Average of 3 multiproxies


Bracketing the temperature extremes with power laws
$s^{-4} > \Pr(\Delta T > s) > s^{-6}$
-1.5
-1.0
-0.5
0.5
1.0
1.5
2.0
$\mathrm{Log}_{10}(s\ \Delta t)$
Shifted by Δt for clarity
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
Probability distributions from multiproxies 1500-1900 (red)
$p(\Delta T) \approx \Delta T^{-1-q_D}$
$4\log_{10}2$
$q_D = 4$
$\mathrm{Log}_{10}\Pr(\Delta T > s)$
Blue arrows indicate H =0
$q_D = 6$
Δt (yrs)
1
2
4
8
16
32
64
$q_D = \infty$



Unconditional return times
R
yrs
10000
1000
200
100
20
10
Gaussian
q_D=6
q_D=4
Anthropogenic warming
pause
Pre-pause
Postwar cooling 1944-1976
Expected maximum 1880-2013
Anthropogenic warming
0.0
0.2
0.4
0.6
0.8
ΔT(ºC)
1.0

How well are the natural and anthropogenic variabilities separated?



Haar fluctuations
Log10 Δt (yrs)
-0.5
1 year
0.5
1.0
1.5
2.0
2.5
3.0
10 years
100 years
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5 K
0.2 K
0.1 K
$\left\langle \Delta T(\Delta t)^2 \right\rangle^{1/2} \approx \Delta t^H$
Northern (residuals)
Multi-Proxies 1500-1900
NASA GISS control run 1000-2000
-0.2
Global (residuals)
H≈ -0.20±0.03
Log10<ΔT²>^1/2 (K)
RMS fluctuations

Accuracy of Hindcasts
(RMS global T variability)

Comparison of standard deviations with Smith et al 2007 and Laepple 2007, Newman 2013

	1 year	5 year anomalies	9 year anomalies
Without assimililation (Smith) 1983 -2004	0.132	0.106	0.090
With DePresSys (Smith) 1983 -2004	0.105	0.066	0.046
GFDL CM2.1 (initialized yearly)	0.11		
CMIP5 multimodel ensemble (Doblas-Reyes et al 2013)		0.06 (0.095 when not initialized)	
Laepple 1983 -2004	0.106	0.059	0.044
Pre-industrial Multiproxies (1500-1900)	0.112	0.105	0.098
Residues (1880-2013)	0.109	0.077	0.070

Historical hindcasts with data assimilation →

CMIP5 →

CMIP3+bias corrections →

Pre-industrial variability →

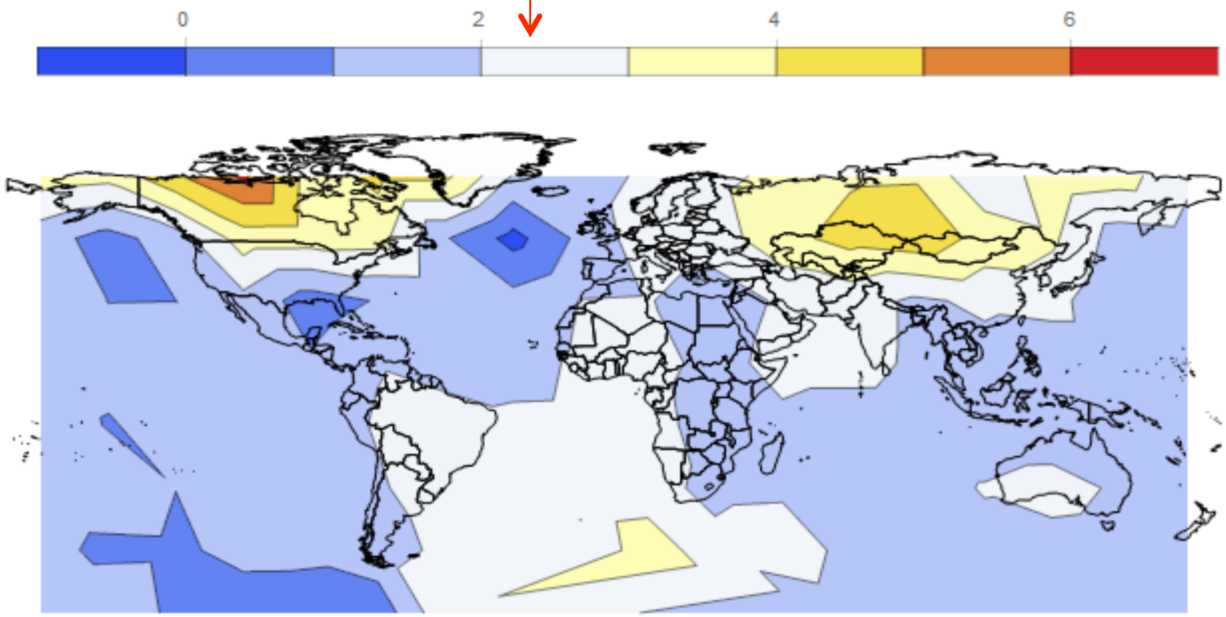
Industrial epoch estimated natural variability (one parameter) →

a) The residues are the same as hindcast errors
b) The same as the pre-industrial multproxies

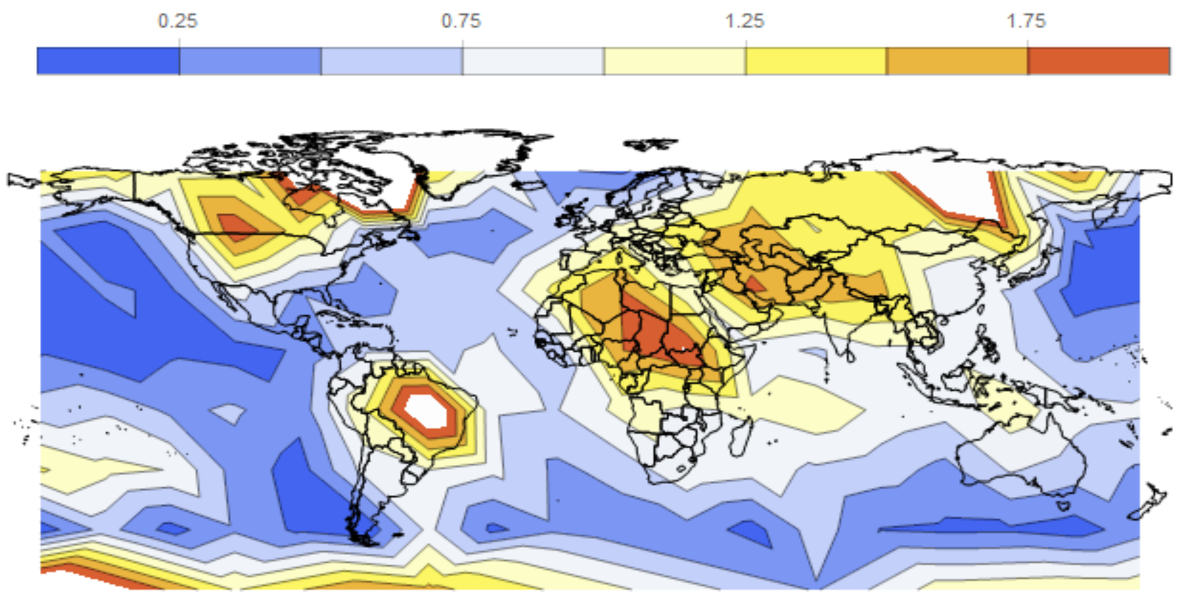
Spatial distrubution of effective climate sensitivity

Global mean: 2.33

sensitivity



uncertainty



At 20º resolution, using HADCRUtem, NOAA, NASA data since 1880, uncertainty from the differences between data sets

Global Climate Models as random number generators

scales <≈ 10 days prediction =initial value problem
(weather prediction)

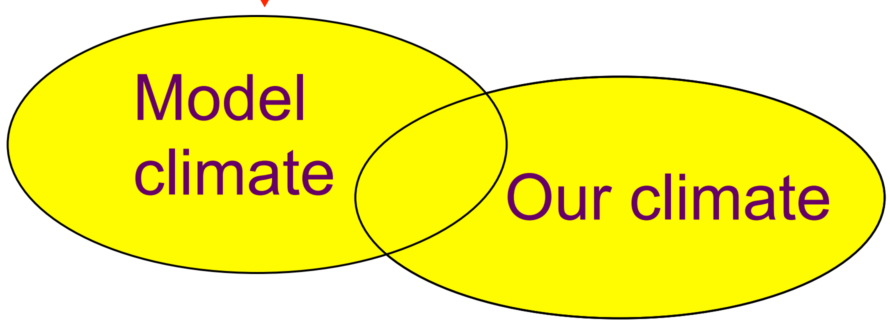
↓

"butterfly effect"

↓ "Brute force"

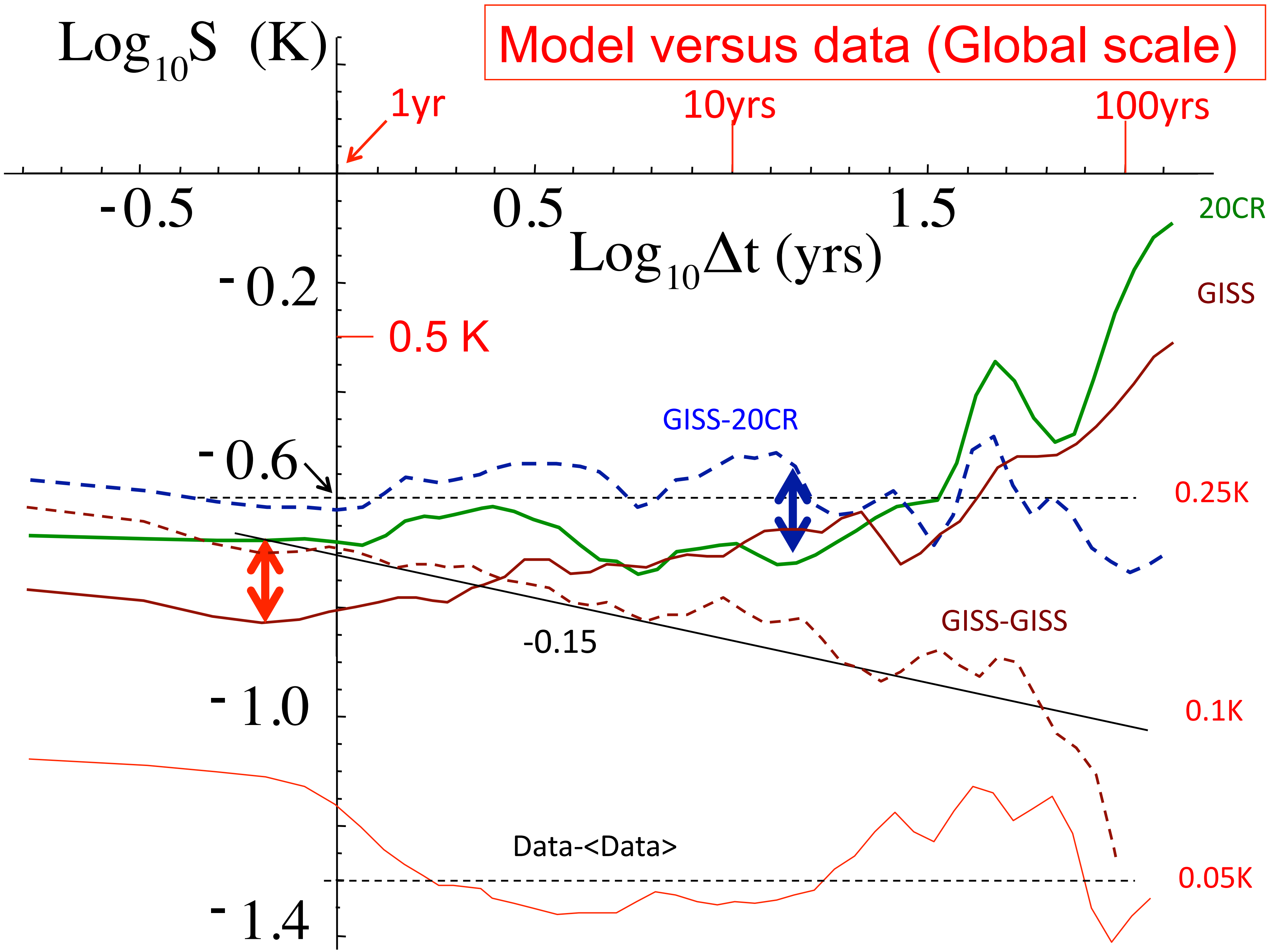
Weather systems generated by GCMs = random weather noise… but not fully realistic

Averages: slow convergence to ↓

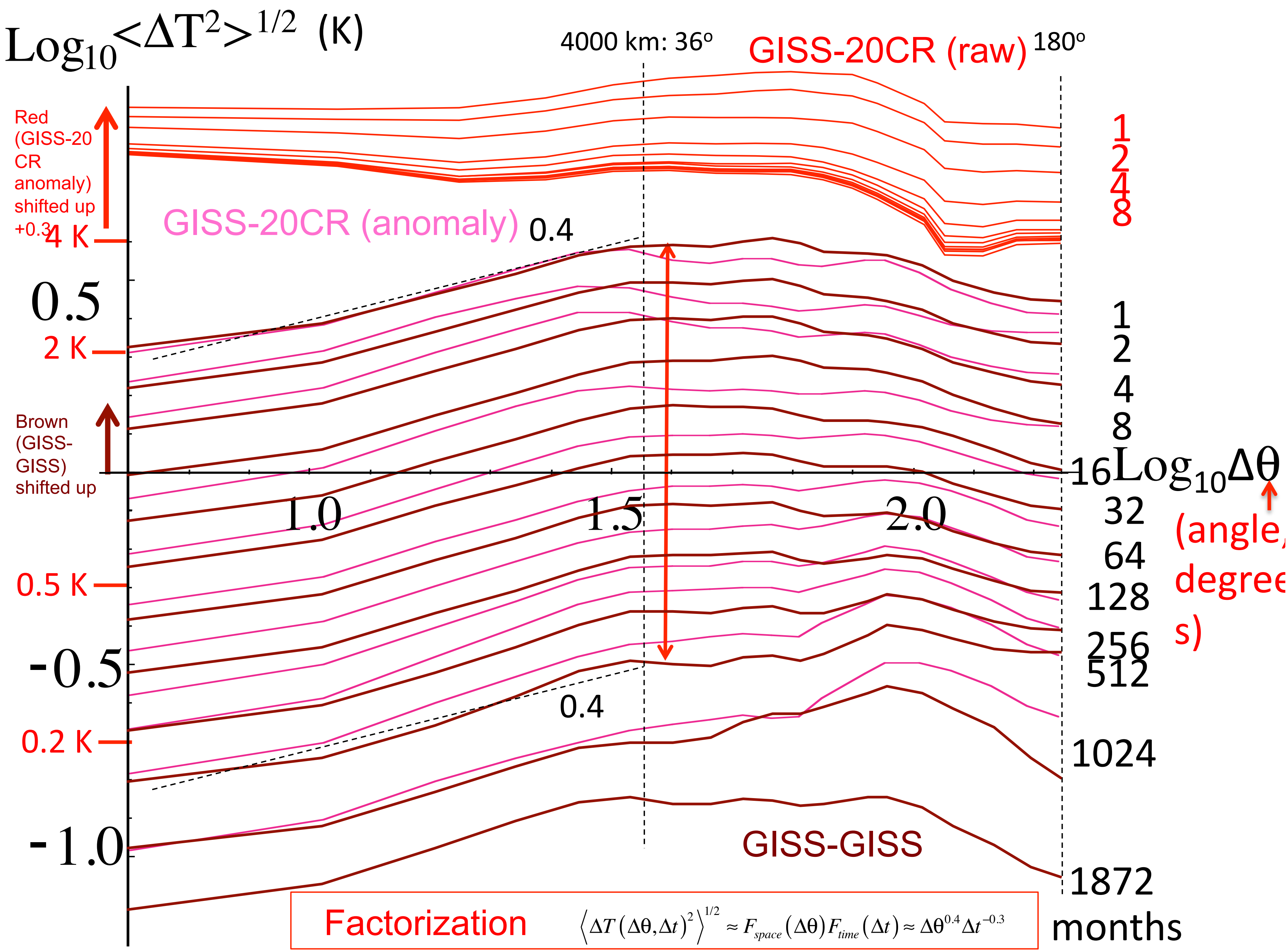


← High level scaling laws generate realistic noise

Converge of GCM's to their climate, to our climate



Model versus data (Global scale)
Log10S (K)
1yr
10yrs
100yrs
-0.5
0.5
1.5
Log10Δt (yrs)
20CR
GISS
-0.2
0.5 K
GISS-20CR
-0.6
0.25K
GISS-GISS
-0.15
-1.0
0.1K
Data-<Data>
0.05K
-1.4



$Log_{10}<\Delta T^2>^{1/2}$ (K)
4000 km: 36°
GISS-20CR (raw)
180°
Red (GISS-20 CR anomaly) shifted up +0.3
4 K
2 K
0.5 K
0.2 K
Brown (GISS-GISS) shifted up
GISS-20CR (anomaly)
0.4
0.5
-0.5
-1.0
1.0
1.5
2.0
$Log_{10}\Delta\theta$
(angle, degrees)
1
2
4
8
16
32
64
128
256
512
1024
1872
months
GISS-GISS
Factorization
$\left\langle \Delta T(\Delta\theta,\Delta t)^2 \right\rangle^{1/2} \approx F_{space}(\Delta\theta) F_{time}(\Delta t) \approx \Delta\theta^{0.4}\Delta t^{-0.3}$

Predictability and Stochastic Forecasting (conditional expectations)

Linear Inverse Modelling (LIM) paradigm versus scaling paradigm for macroweather

The most accurate global, annual forecast of temperatures: not from GCM's but from stochastic models!

LIM (scalar version)

$$\left(\frac{d}{dt}+\tau^{-1}\right)T(t)=\gamma(t)$$

weather frequency: $\approx$(10 days)$^{-1}$

Gaussian forcing

At low frequencies, d/dt≈0

$$T(t)\approx\tau\gamma(t)$$

T is white noise (zero memory)

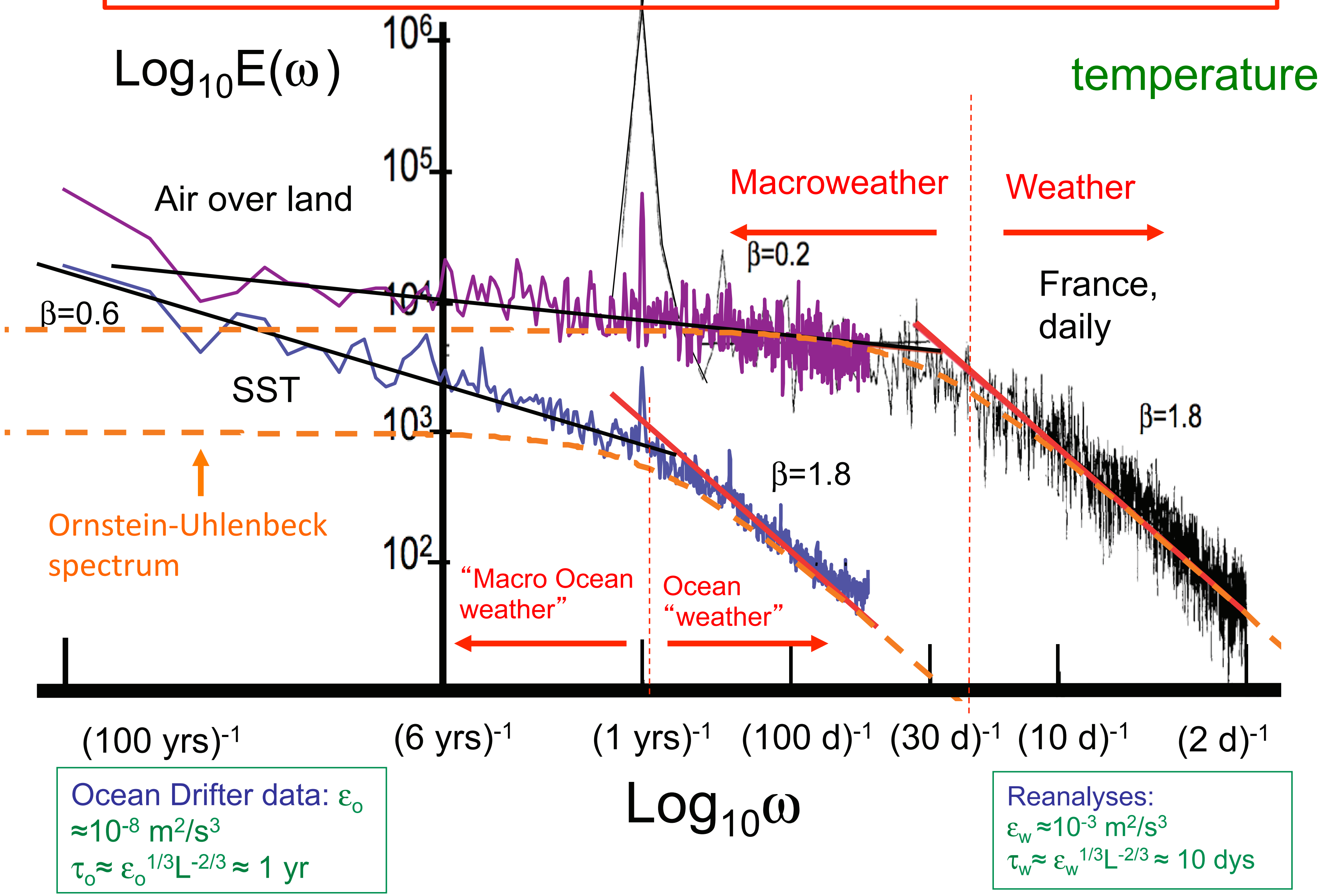
the spectrum
$$E_T(\omega)\approx\frac{\tau^2\sigma_\gamma^2}{(\tau\omega)^2+1}$$

Orenstein-Uhlenbeck processes

spectral exponents β_l =0 (low), β_h = 2 (high)

$$E(\omega)\approx\omega^{-\beta}$$

Macroweather,
Macro "ocean weather"



A stochastic scaling model

Scaling Linear Inverse Model (SLIM)

$$\frac{d^{H+1/2}}{dt^{H+1/2}}\left(\tau^{-1}+\frac{d}{dt}\right)T=\gamma(t)$$

Extra fractional order differentiation

smoothing operator

$$\gamma_\tau(t)=\left(\tau^{-1}+\frac{d}{dt}\right)^{-1}\gamma(t)$$

$$=\int_{-\infty}^{t}e^{-(t-t')/\tau}\gamma(t')dt'$$

$\gamma_\tau=\gamma$ smoothed over scales smaller than τ

Hence:

$$\frac{d^{H+1/2}}{dt^{H+1/2}}T(t)=\gamma_\tau(t)$$

$$E_T(\omega)\approx\omega^{-(2H_l+1)}E_{\gamma_\tau}(\omega)=\omega^{-(2H_l+1)}\frac{\sigma_\gamma^2\tau^2}{1+(\tau\omega)^2}$$

Low frequency limit

with exponents $\beta_l=2H_l+1$, $\beta_h=2H_l+3$.

SLIM: Extension of Fractional Brownian Motion (fBm) to -1/2<H<0

$$\frac{d^{H+1/2}}{dt^{H+1/2}} T(t) = \gamma_\tau(t)$$

← Scalar SLIM model

Solution

Fractional integral of order H+1/2:

$$T = I_{H+1/2}\gamma_\tau$$

$$I_{H+1/2} = \frac{d^{-(H+1/2)}}{dt^{-(H+1/2)}}$$

$$T(t) = \Theta(t)t^{-(1/2-H)} * \gamma_\tau$$

Heaviside singularity Smoothed noise

$$T(t) = \int_{-\infty}^{t} (t-t')^{-(1/2-H)} \gamma_\tau(t')dt'; \quad -1/2 < H < 0$$

τ = the resolution of smoothing

Some properties of SLIM

-1/2<H<0

Autocorrelation

$$R(\Delta t) = \langle T(\Delta t) T(0) \rangle = A\left(\frac{\Delta t}{\tau}\right)^{2H} - \ldots$$

$$-1/2 < H < 0$$

$$A = \sigma_T^2 2^{-2H} \frac{U}{(1+2H)}$$

where:

$$U = -\frac{2H\Gamma(-H)\Gamma\left(\frac{3}{2}+H\right)}{\sqrt{\pi}}$$

$$\sigma_T^2 = \langle T(t)^2 \rangle = \frac{\sigma_\gamma^2}{(-2H)}\tau^2; \quad H < 0$$

The spectrum

$$E(\omega) = \left\langle \left|\widetilde{T(\omega)}\right|^2 \right\rangle \approx \omega^{-\beta}; \quad \beta = 1 + 2H$$

Power law spectra, autocorrelations

Forecasts and limits to predictability

Conditional Expectation forecast

$$T_{p,cond}(t) = \int_{-\infty}^{0} (t-t')^{-(1/2-H)} \gamma_{\tau}(t')\,dt'; \quad -1/2 < H < 0$$

$$\sigma_{ET}^2(t) = \left\langle E_T(t)^2 \right\rangle = \sigma_T^2 \left(1 - \left(\frac{t}{\tau} \right)^{2H} \right); \quad -1/2 < H < 0$$

Limits to predictability= power law

Forecast Skill

Definition

S_k = Skill= Fraction of variance explained by the forecast:

$$S_k(t,\tau)=\frac{\sigma_{T,\tau}^2-\sigma_{ET,\tau}^2}{\sigma_{T,\tau}^2}$$

Temperature forecast error variance

Temperature variance

Process averaged at resolution τ

Temperature forecasts ($t>\tau$):

t = forecast horizon
τ = resolution

$$S_K(t,\tau)\approx\frac{(1/2+H)^2(1+H)2^{2H+2}}{U}\left(\frac{t}{\tau}\right)^{2H};\quad t>>\tau;\quad -1/2<H<0$$

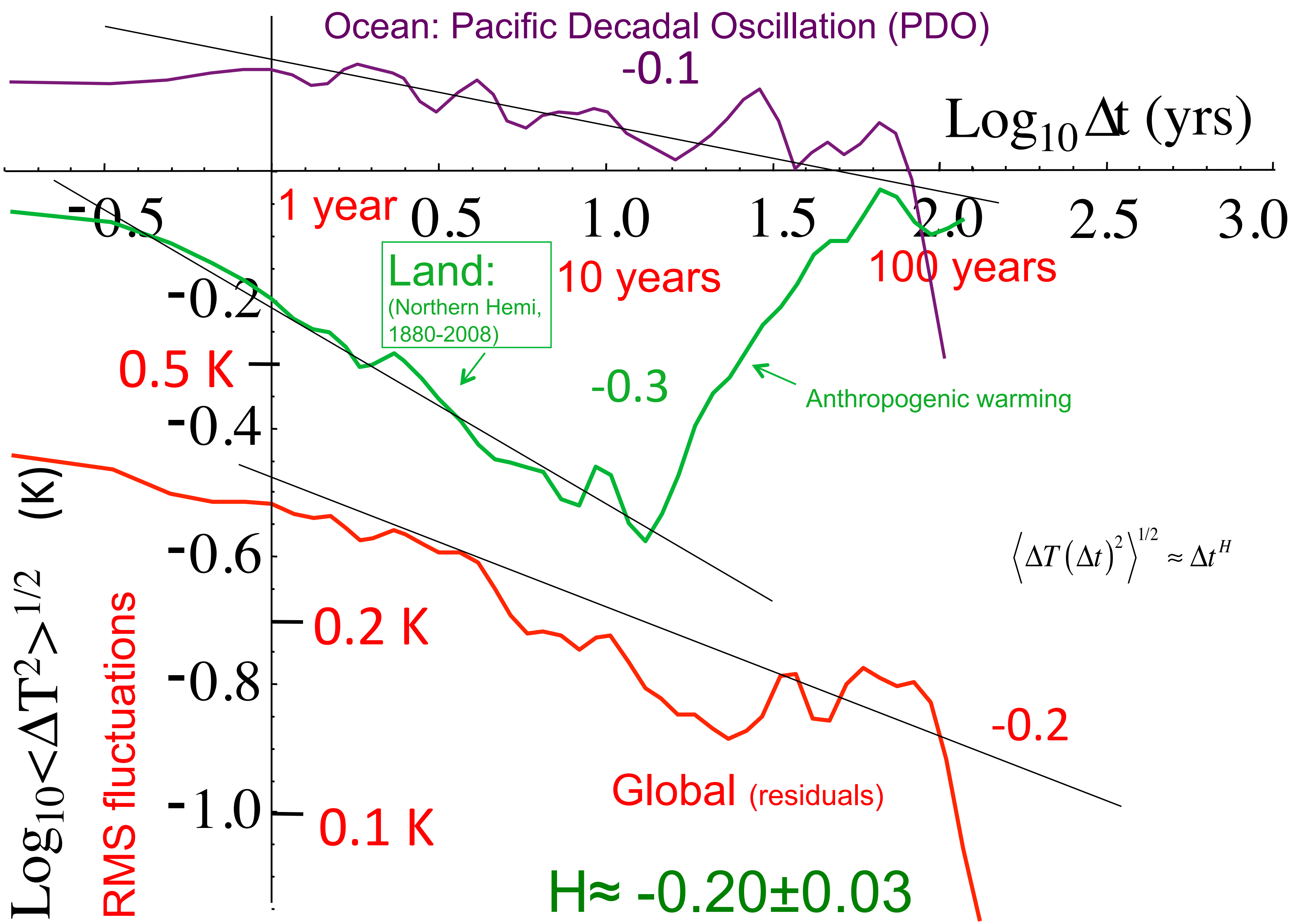
Anomaly forecasts ($t=\tau$):

Constant skill!

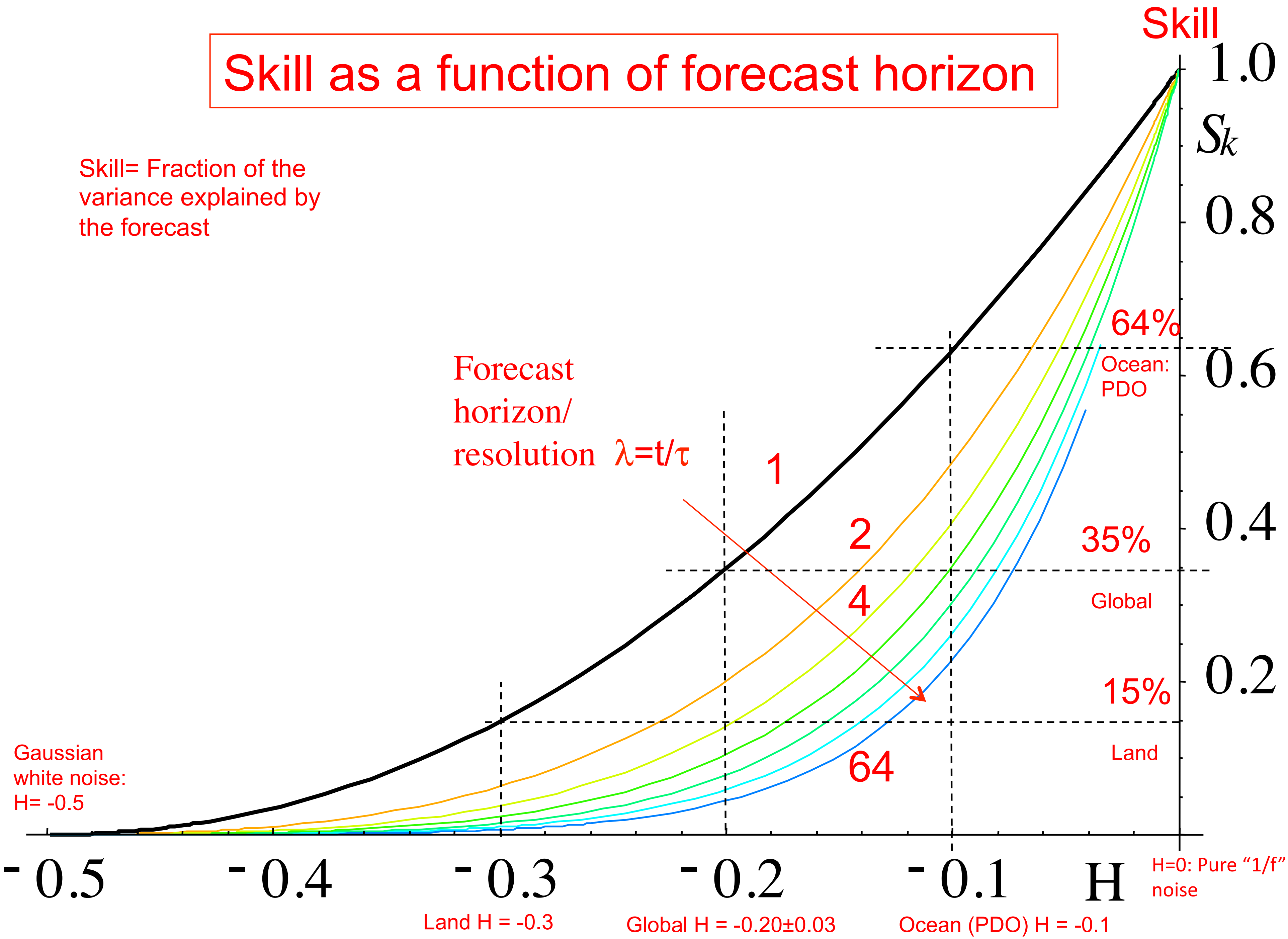
$$S_k(t,\tau)=1-\frac{(-H)2^{2H+1}}{U};\quad -1/2<H<0$$

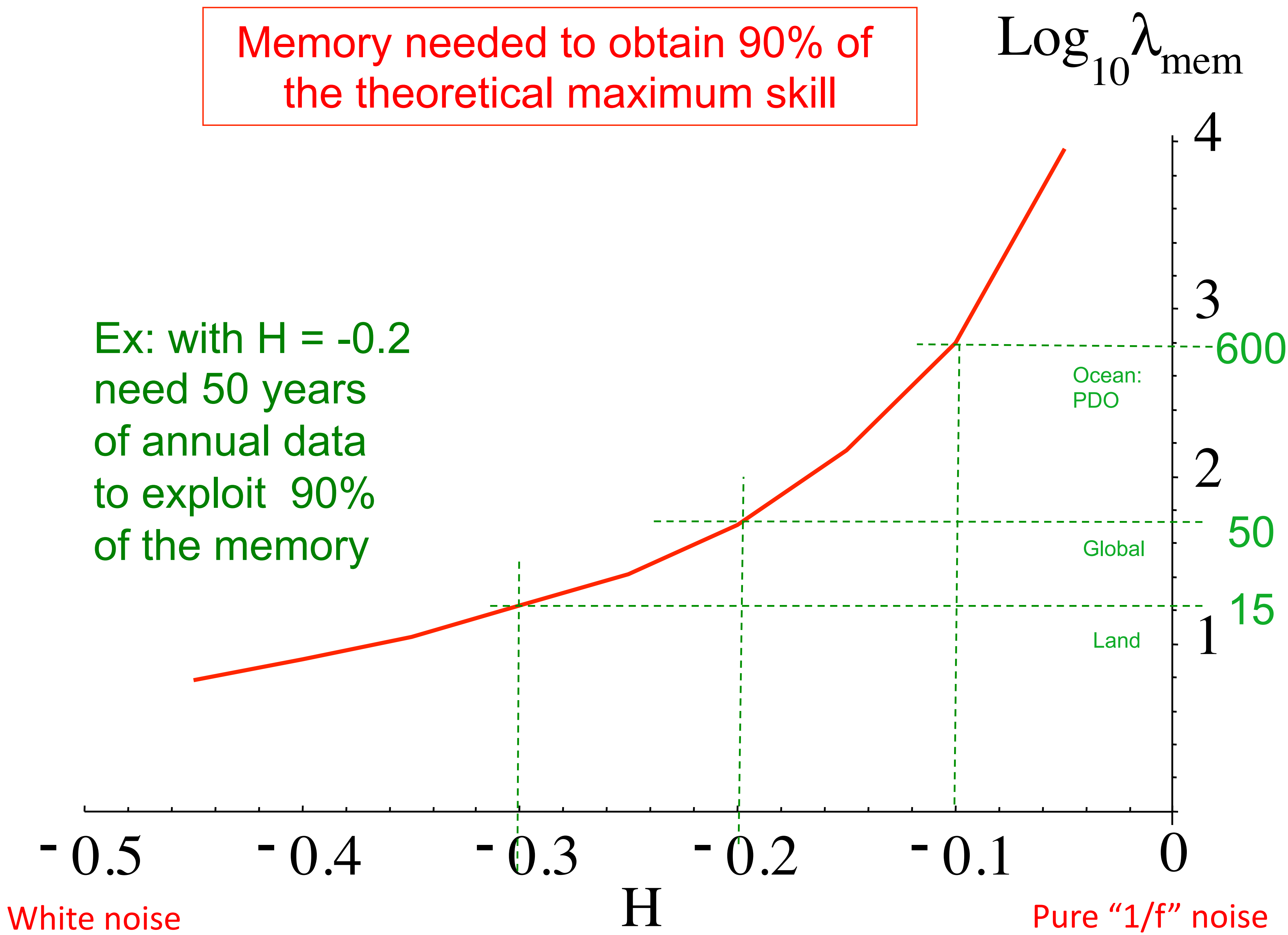
$$U=-\frac{2H\Gamma(-H)\Gamma\left(\frac{3}{2}+H\right)}{\sqrt{\pi}}$$

$U\approx1$; $-1/2<H<0$



Ocean: Pacific Decadal Oscillation (PDO)
-0.1
$\text{Log}_{10}\Delta t$ (yrs)
-0.5
0.5
1.0
1.5
2.0
2.5
3.0
1 year
10 years
100 years
Land:
(Northern Hemi, 1880-2008)
-0.2
-0.4
-0.6
-0.8
-1.0
0.5 K
0.2 K
0.1 K
-0.3
Anthropogenic warming
$\left\langle \Delta T(\Delta t)^2 \right\rangle^{1/2} \approx \Delta t^H$
-0.2
Global (residuals)
$\text{Log}_{10}\langle\Delta T^2\rangle^{1/2}$ (K)
RMS fluctuations
H≈ -0.20±0.03

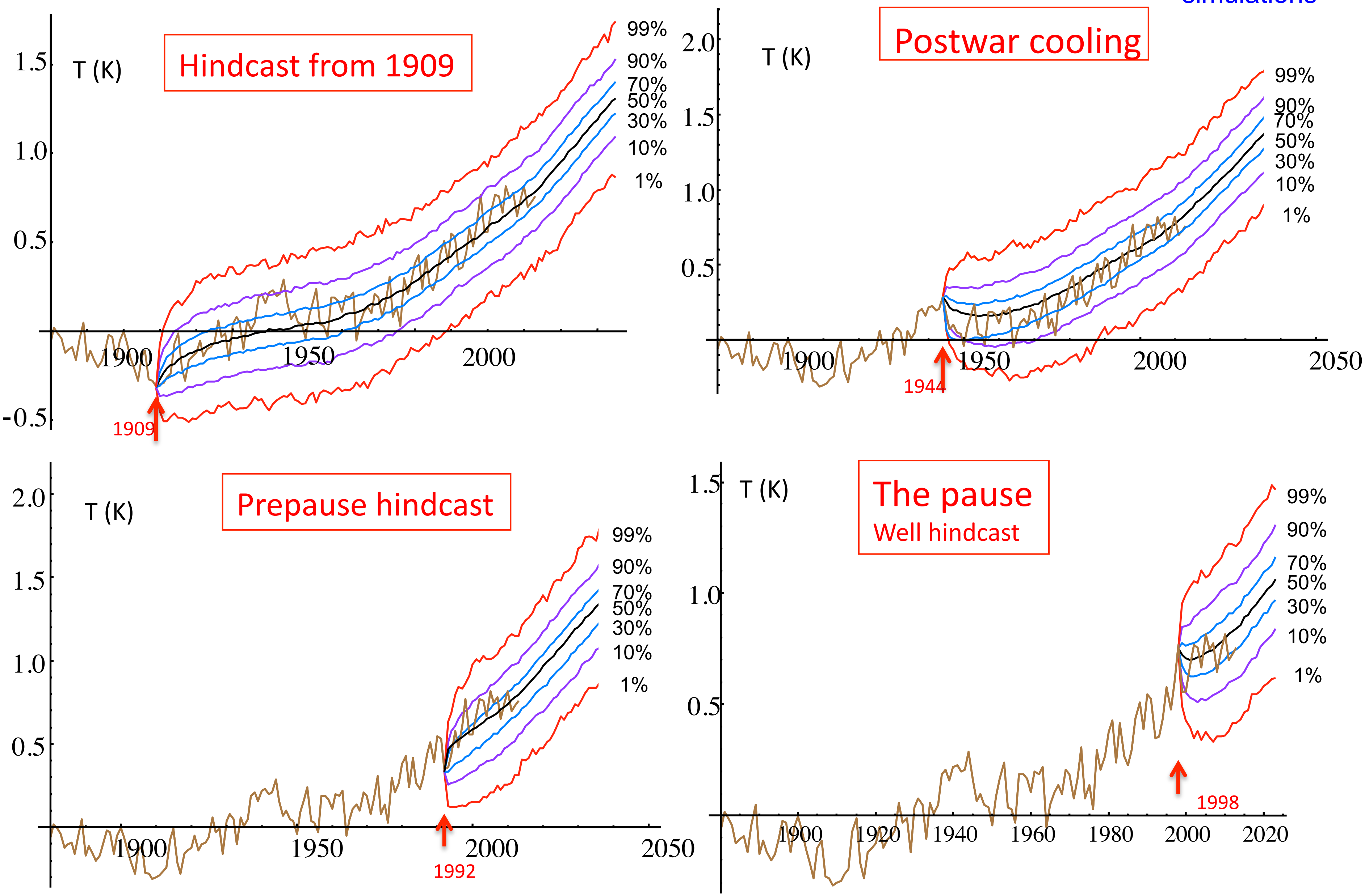

Skill as a function of forecast horizon
Skill
1.0
S_k
0.8
0.6
0.4
0.2
Skill= Fraction of the variance explained by the forecast
Forecast horizon/ resolution λ=t/τ
1
2
4
64
64%
Ocean: PDO
35%
Global
15%
Land
Gaussian white noise: H= -0.5
-0.5
-0.4
-0.3
-0.2
-0.1
H
H=0: Pure "1/f" noise
Land H = -0.3
Global H = -0.20±0.03
Ocean (PDO) H = -0.1



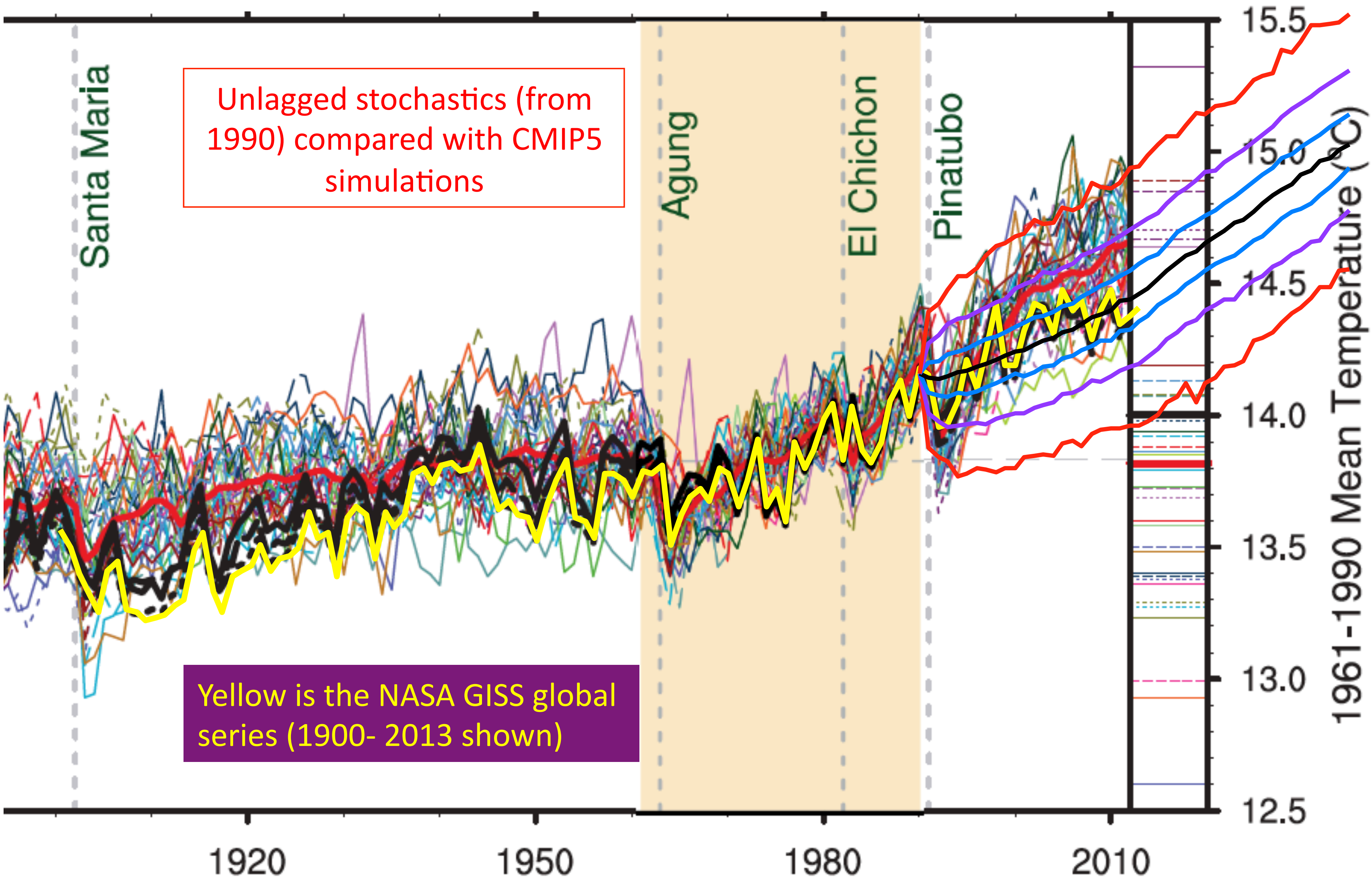
Memory needed to obtain 90% of the theoretical maximum skill
$Log_{10}\lambda_{mem}$
Ex: with H = -0.2 need 50 years of annual data to exploit 90% of the memory
4
3
2
1
600
Ocean: PDO
50
Global
15
Land
-0.5
-0.4
-0.3
-0.2
-0.1
0
H
White noise
Pure "1/f" noise

With anthropogenic contribution

1000 (conditional) simulations



Observed and CMIP5 mean surface temperature

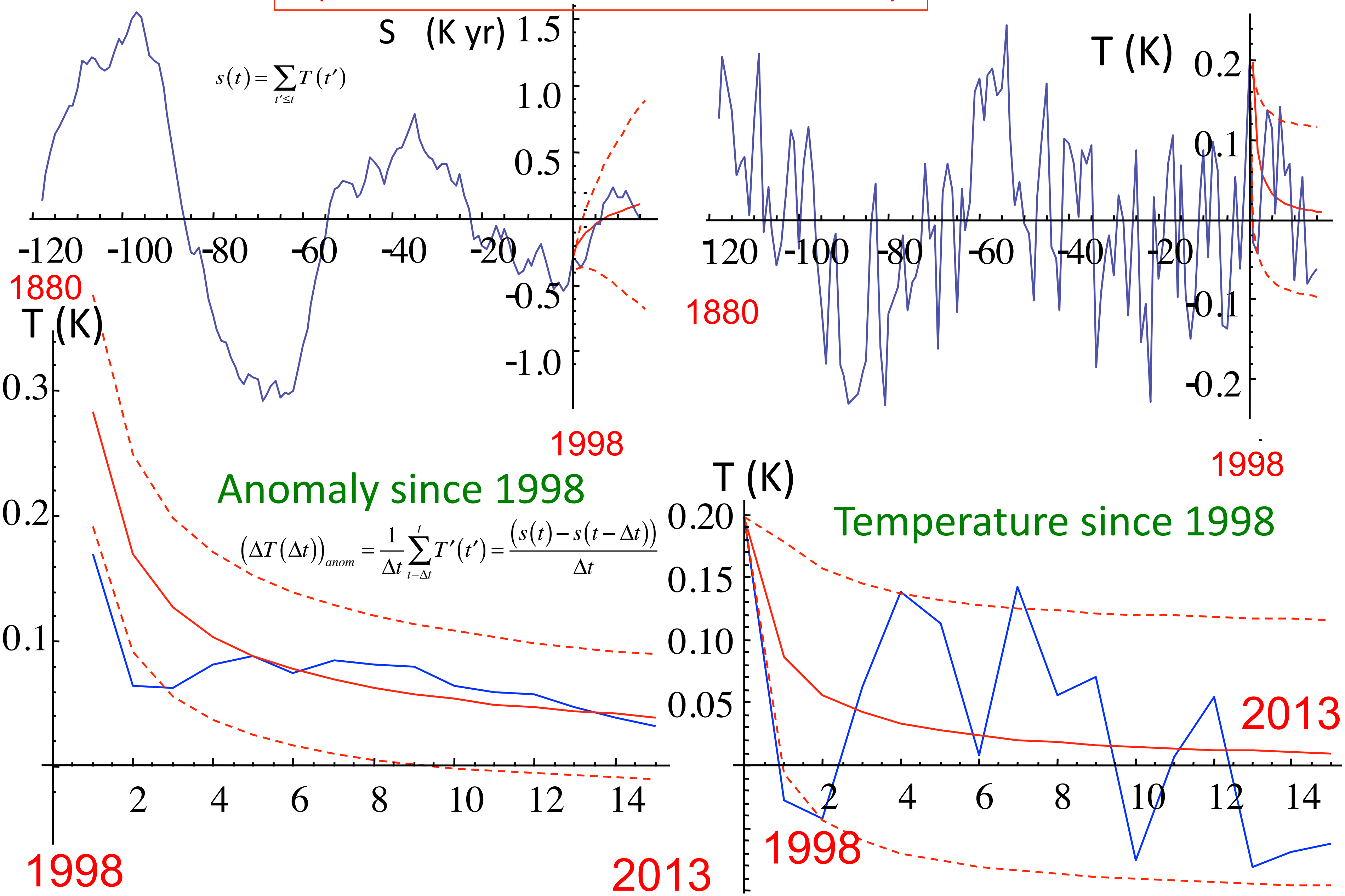


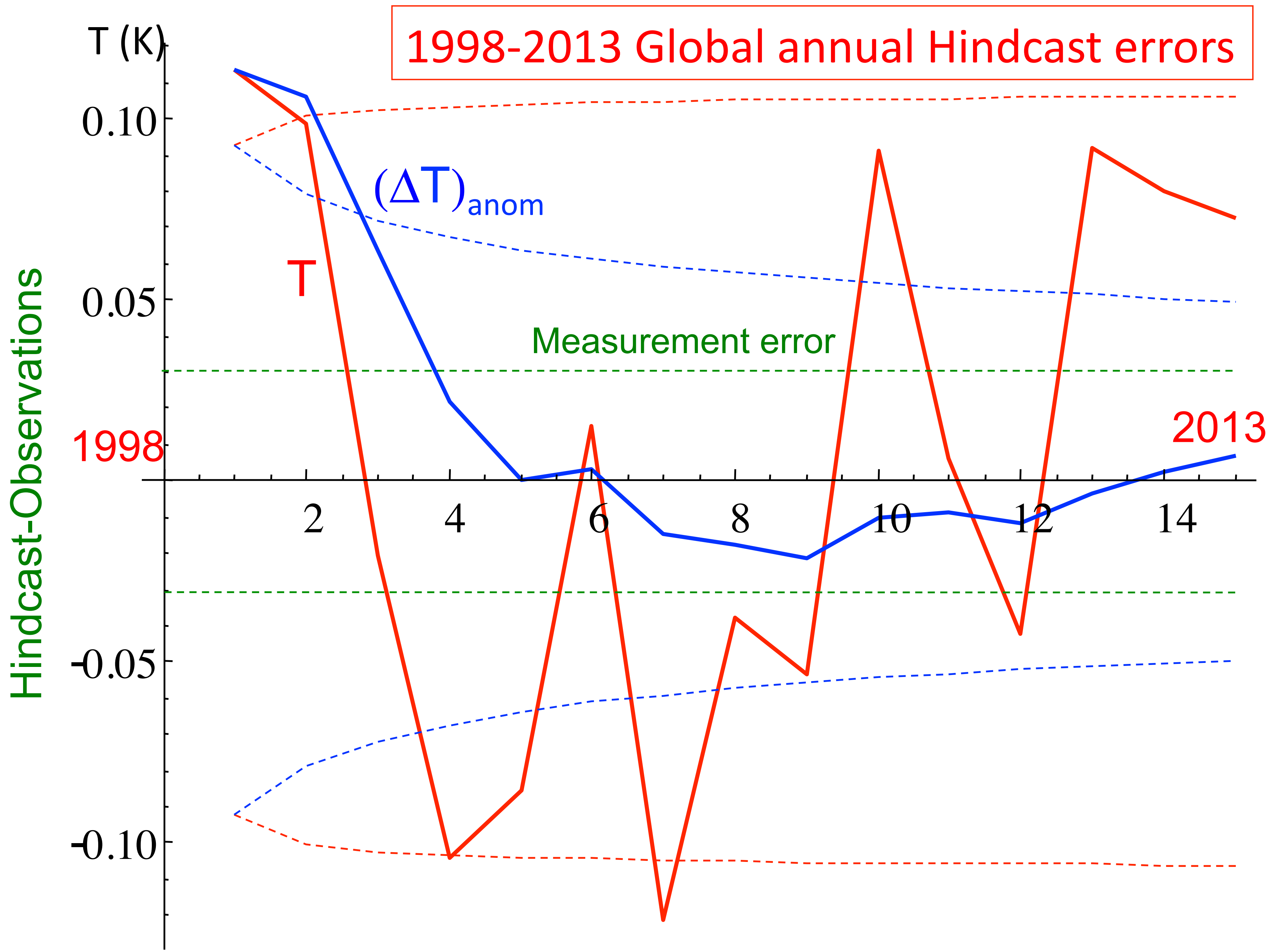
Hindcasting the Pause

(Global mean annual T since 1998)

Red= forecast

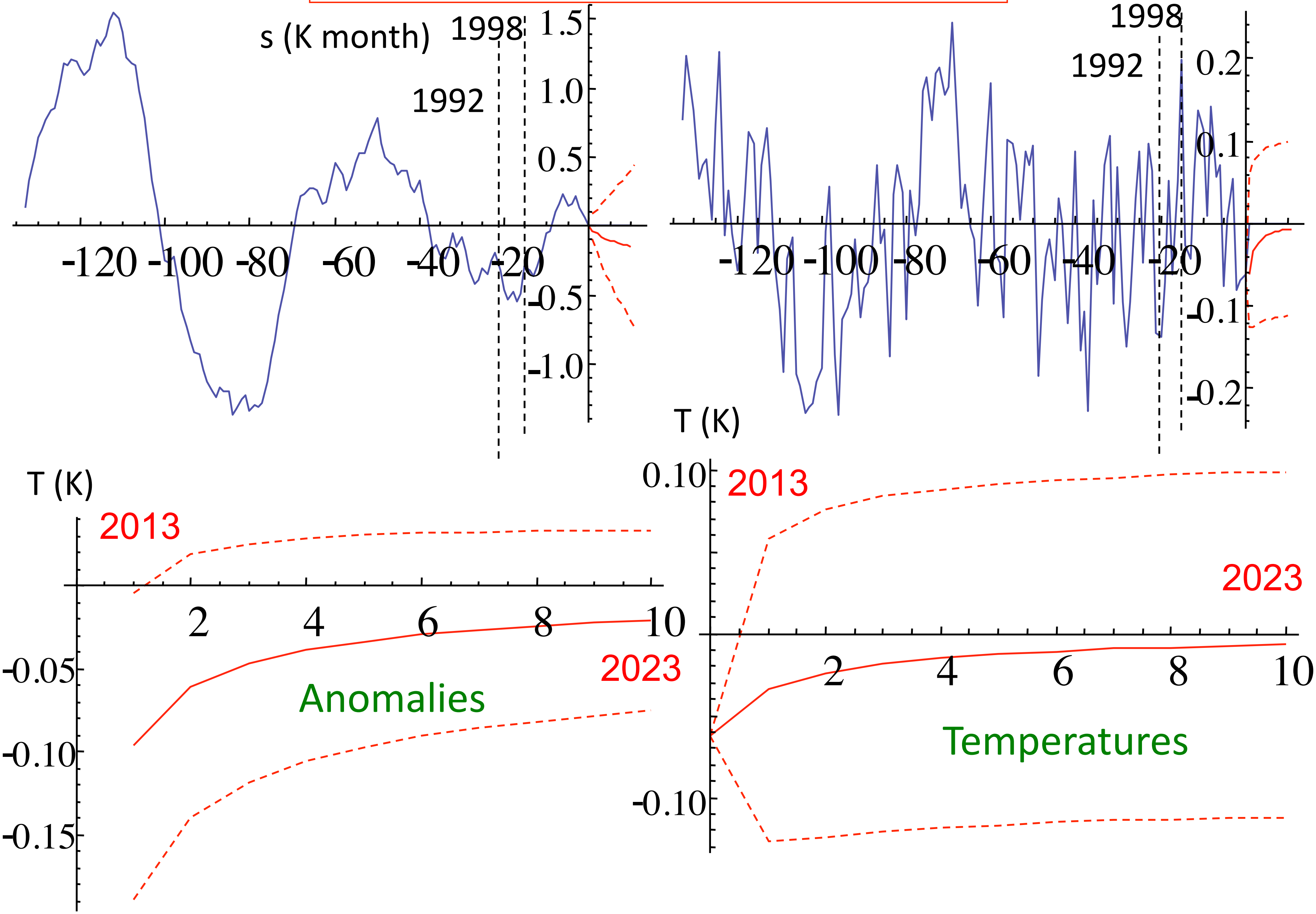
Blue= observations



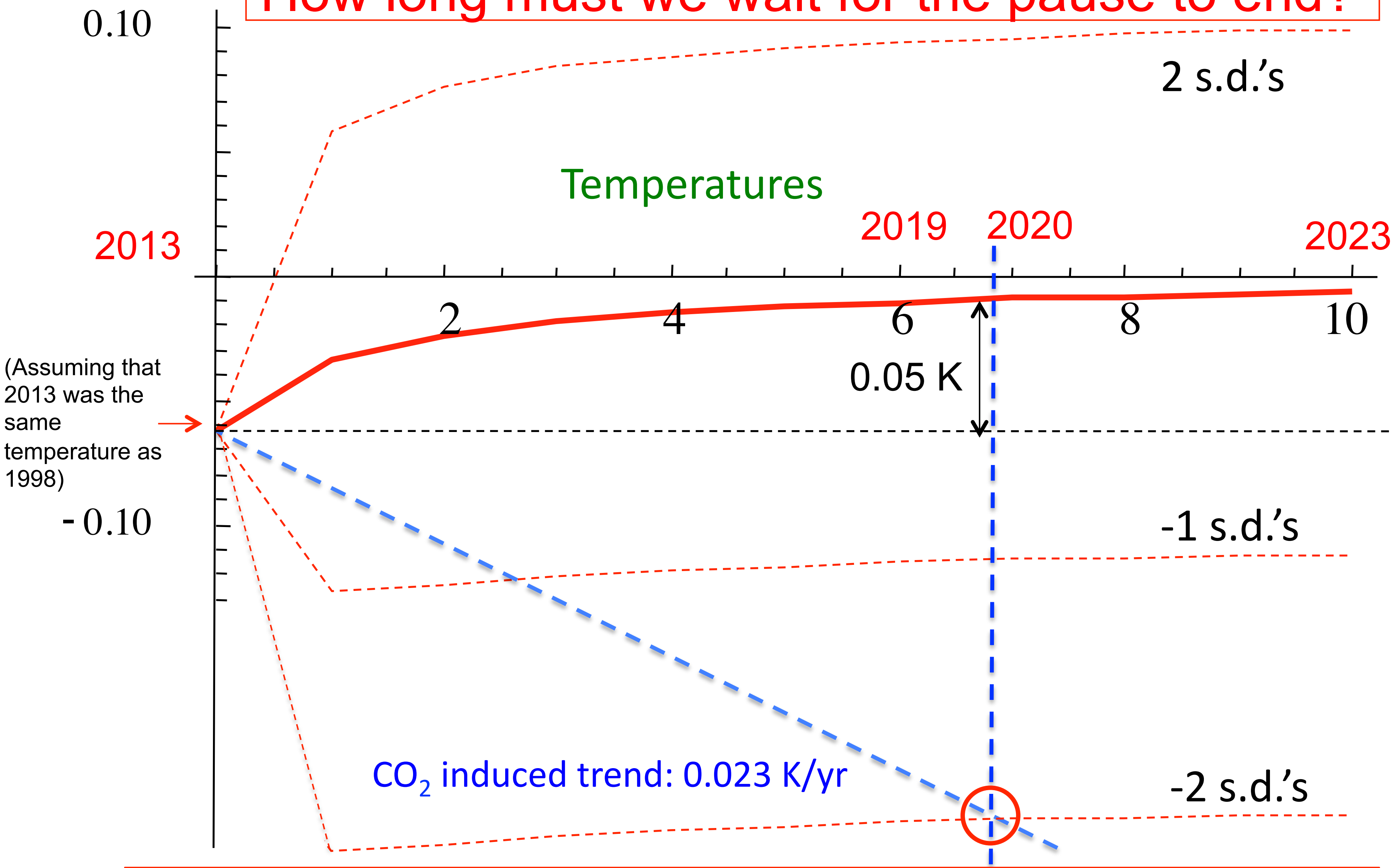


1998-2013 Global annual Hindcast errors
T (K)
0.10
0.05
-0.05
-0.10
Hindcast-Observations
$(\Delta T)_{anom}$
T
Measurement error
1998
2013
2
4
6
8
10
12
14

The next 10 years: Global: 2014-2023



How long must we wait for the pause to end?

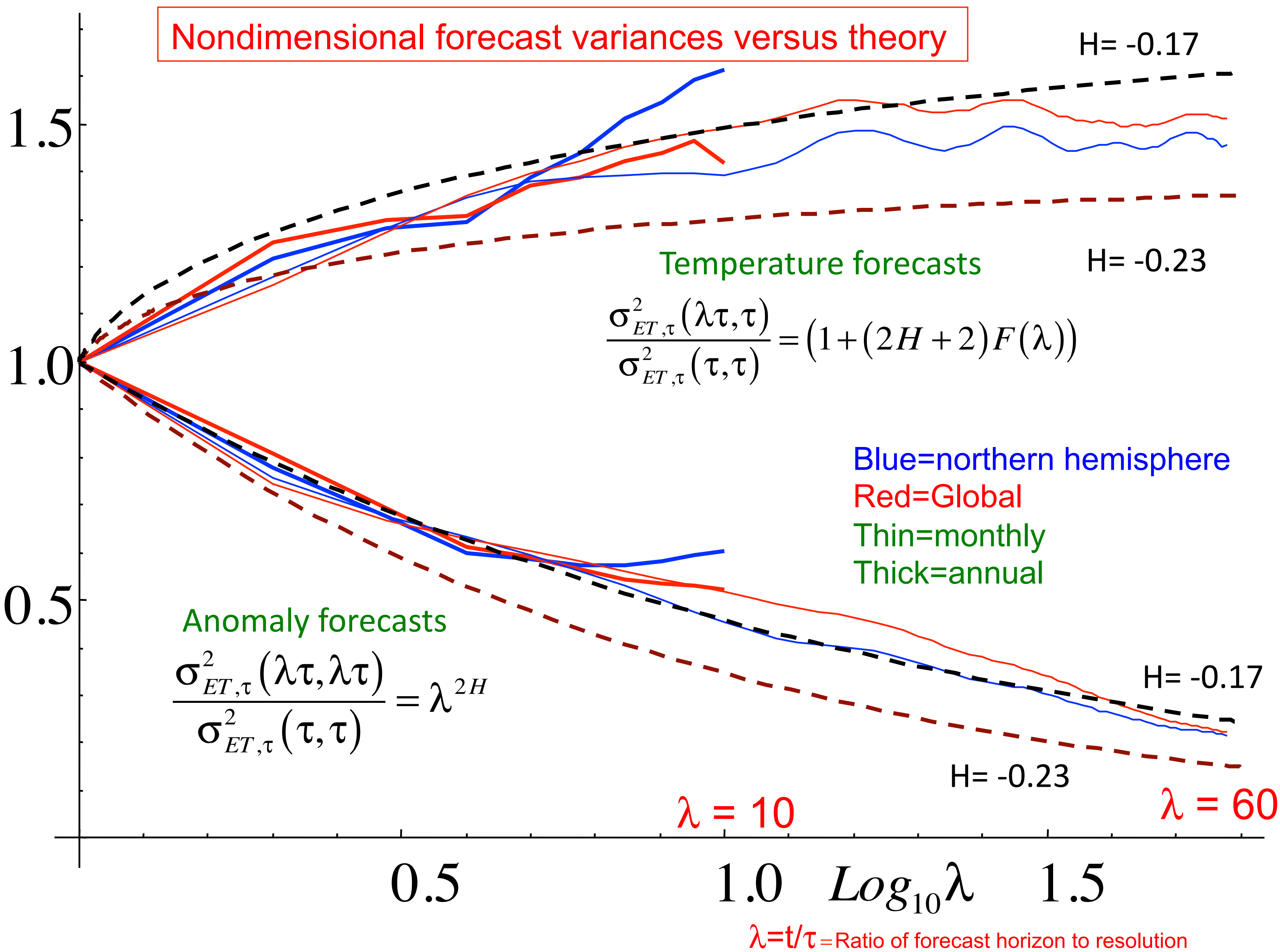


Current Anthropogenic increase: $\frac{d \log_2 CO_2}{dt} \approx 0.010 / yr$ $\frac{dT}{dt} \approx 0.023 / yr$

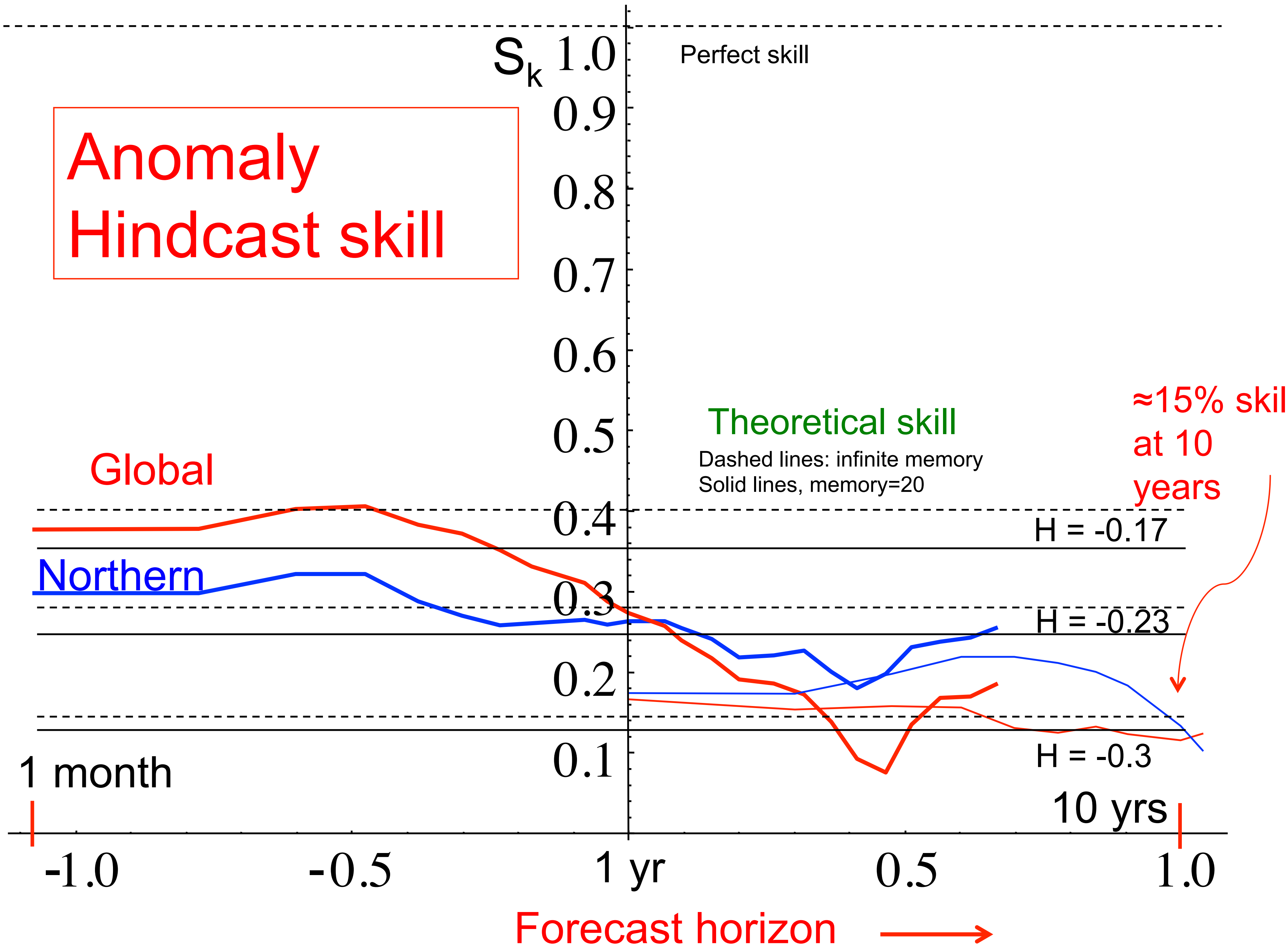
Forecast for 2023: +0.05±0.10K (natural)+0.23±0.02 K (anthropogenic) = 0.28±0.11 K above 2013

Forecast Skill
(theory- empirical comparison)

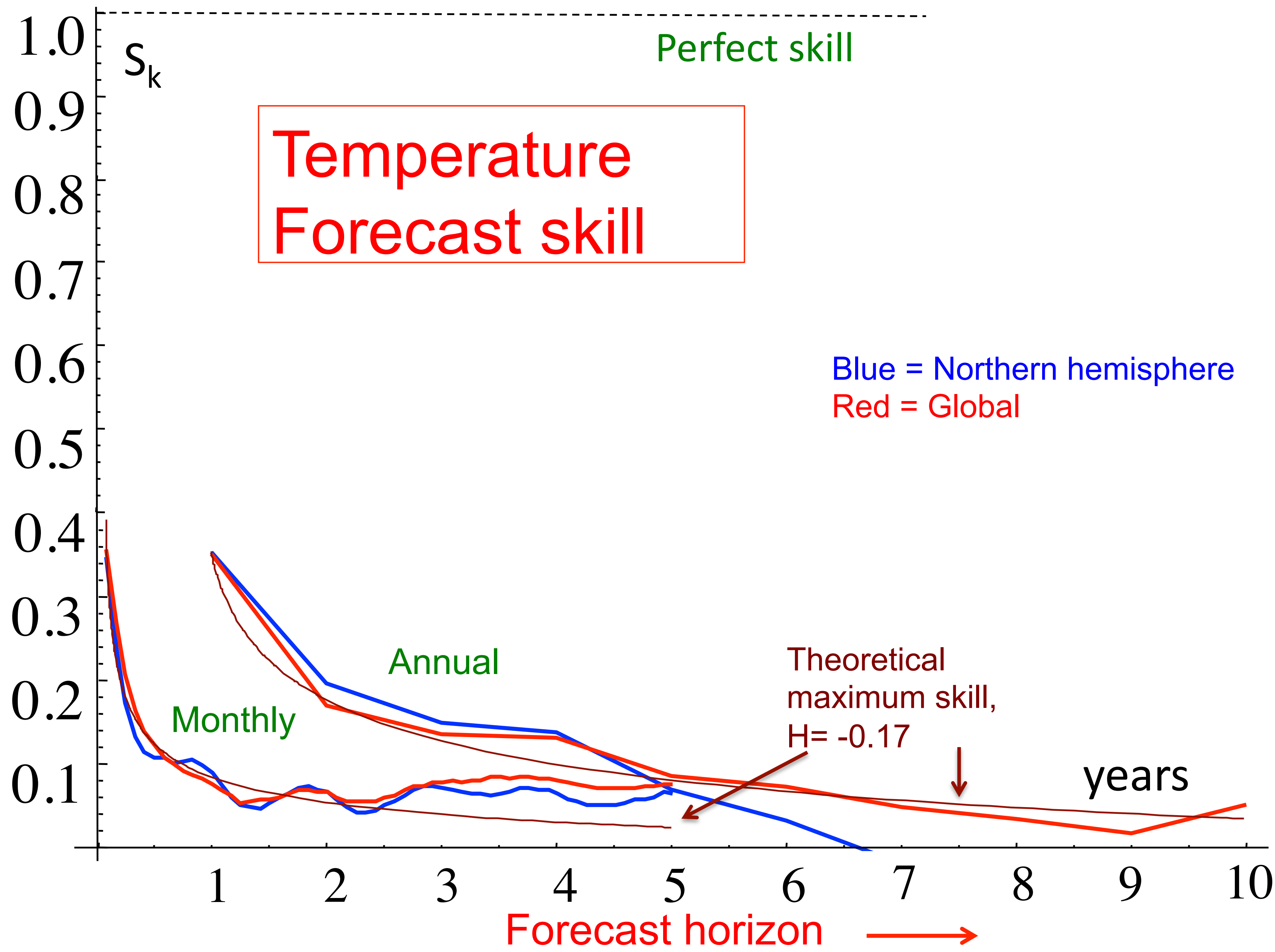
a) One parameter λ_{eff} to remove anthropogenic effects.
b) One parameter (H) to make forecasts.



Nondimensional forecast variances versus theory
H= -0.17
H= -0.23
Temperature forecasts
$$\frac{\sigma^2_{ET,\tau}(\lambda\tau,\tau)}{\sigma^2_{ET,\tau}(\tau,\tau)} = \left(1+(2H+2)F(\lambda)\right)$$
Blue=northern hemisphere
Red=Global
Thin=monthly
Thick=annual
Anomaly forecasts
$$\frac{\sigma^2_{ET,\tau}(\lambda\tau,\lambda\tau)}{\sigma^2_{ET,\tau}(\tau,\tau)} = \lambda^{2H}$$
H= -0.17
H= -0.23
λ = 10
λ = 60
1.5
1.0
0.5
0.5
1.0
$Log_{10}\lambda$
1.5
λ=t/τ =Ratio of forecast horizon to resolution



Anomaly
Hindcast skill
S_k
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Perfect skill
Theoretical skill
Dashed lines: infinite memory
Solid lines, memory=20
≈15% skill
at 10
years
Global
Northern
H = -0.17
H = -0.23
H = -0.3
1 month
10 yrs
-1.0
-0.5
1 yr
0.5
1.0
Forecast horizon


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
S_k
Perfect skill
Temperature
Forecast skill
Blue = Northern hemisphere
Red = Global
Annual
Monthly
Theoretical
maximum skill,
H= -0.17
years
1
2
3
4
5
6
7
8
9
10
Forecast horizon

Forecast Skill Summary

Summary

a) Predictability limits: Temperature forecast skill decays algebraically not exponentially

b) Skill depends only on the ratio of the forecast horizon to resolution. Consequence: anomaly forecasts with resolution= forecast horizon have constant skill

c) There is a theoretical maximum skill (here about 35%)

d) There is a theoretical maximum correlation (forecast with observations, here max $\rho \approx 0.6$)

Accuracy of Hindcasts

Comparison of standard deviations with Smith et al 2007 and Laepple 2007, Newman 2013

	1 year	5 year anomalies	9 year anomalies
No assimililation (Smith) 1983 -2004	0.132	0.106	0.090
With DePresSys (Smith) 1983 -2004	0.105	0.066	0.046
GFDL CM2.1 (initialized yearly)	0.11		
CMIP5 multimodel ensemble (Doblas-Reyes et al 2013)		0.06 (0.095 when not initialized)	
Laepple 1983 -2004	0.106	0.059	0.044
Pre-industrial Multiproxies (1500-1900)	0.112	0.105	0.098
Residues (1880-2013)	0.109	0.077	0.070
LIM (Stochastic, 20 eigenmodes, >100 parameters Newman 2013)	0.085	0.128 (temps)	0.155 (temps) (c.f. residue of linear trend: 0.163)
SLIM (one parameter, Stochastic 1880-2013)	0.092	0.071 (anomaly) 0.102 (temperature)	0.067 (anomaly) 0.105(temperature)

Effect of stochastic memory

Regional climate forecasting: Space-time: LIM versus SLIM

General linear form respecting space-time factorization

i, j indices of grid points, spatial location

$$T_i(t) = \sum_j \int_{-\infty}^{t} G_{ij}(t-t')\gamma_j(t')dt'$$

Independent unit Gaussians

$$\left(\frac{d}{dt} + \underline{\underline{B}}\right)\underline{T}(t) = \underline{\gamma}(t)$$

Spatial coupling coefficients = teleconnections

Long range memory

LIM

$$G_{ij}(t) = \left(e^{-\underline{\underline{B}}t}\right)_{ij}$$

$$G_{ij}(t) = A_{ij}t^{-(1/2-H_j)}$$

Exponential versus power law (scaling)

Linear Inverse Modelling (LIM)

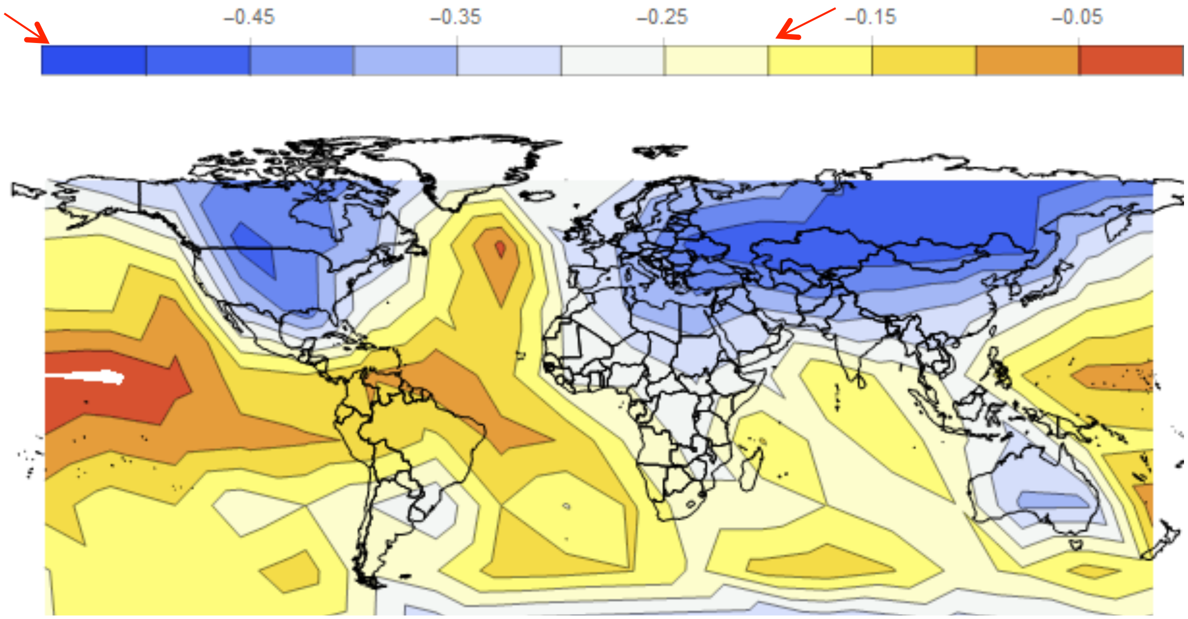
Scaling Linear Inverse Modelling (SLIM)

White noise: $\beta=0$ ($H=-1/2$)

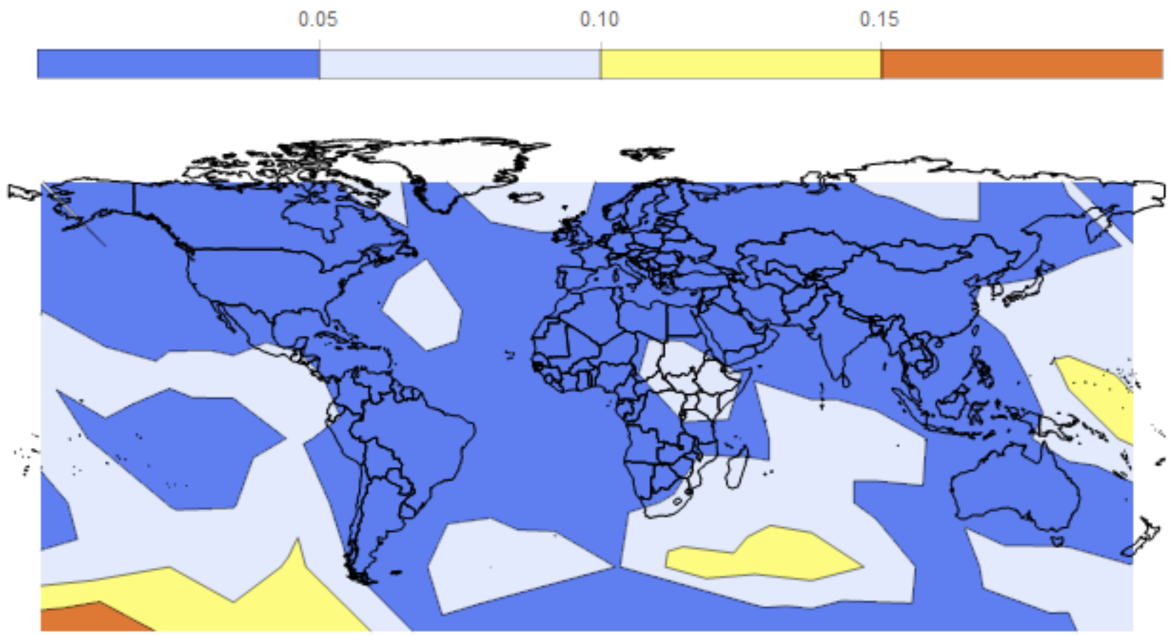
Global mean: $\beta=0.6$ ($H=-0.3$)

(low frequency Orenstein-Uhlenbeck processes)

Mean H



Uncertainty H



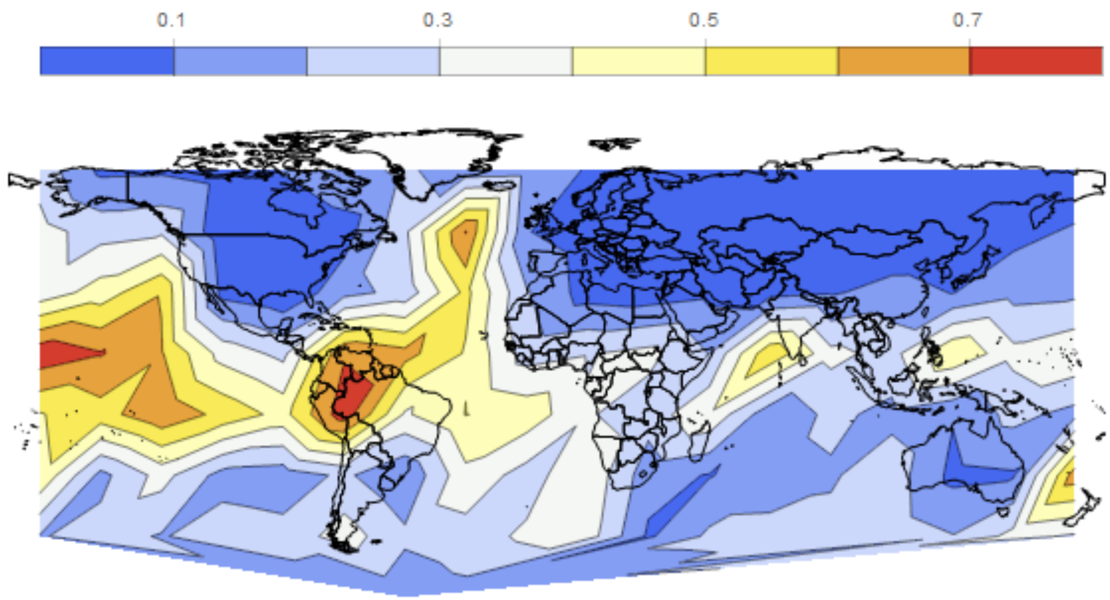
At 20° resolution, using HADCRUtem, NOAA, NASA data since 1880, uncertainty from the differences between data sets

Forecast Horizon =1 forecast skill

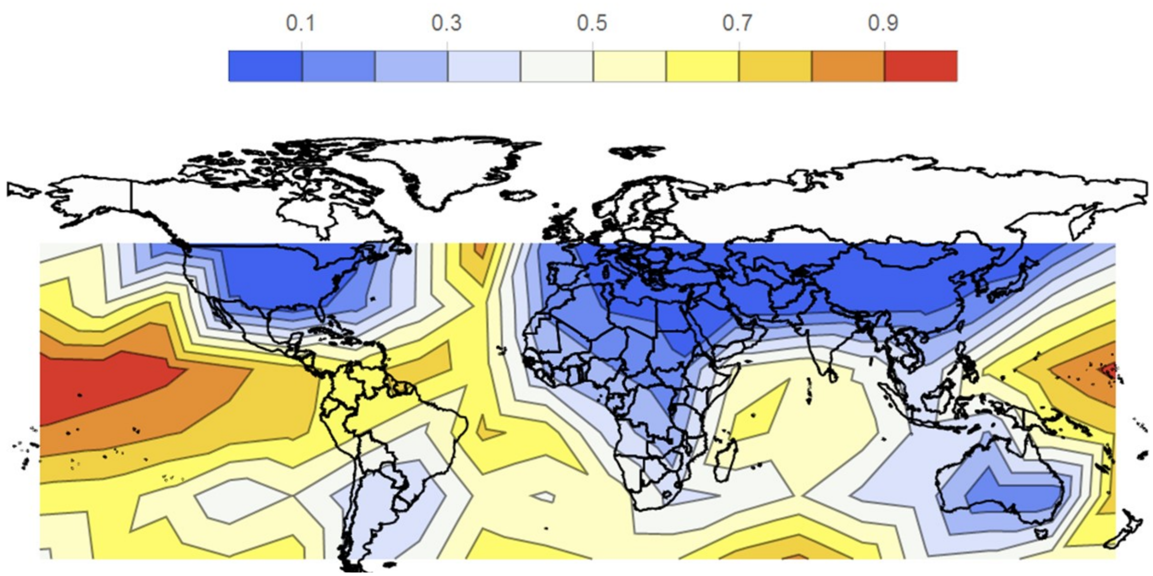
(e.g. 1 yr forecast for data 1 year resolution)

Skill= Fraction of variance explained from time series only

Only temporal correlations used



With some tele-connections



Conclusions (1)

1. Using scaling fluctuation analysis to characterizing the climate by its type of variability: expect macroweather not climate

2. The need for GCM-free approaches:
a) their climate not ours,
b) disarming climate skeptics
c) Using statistical hypothesis testing to rule out natural variability

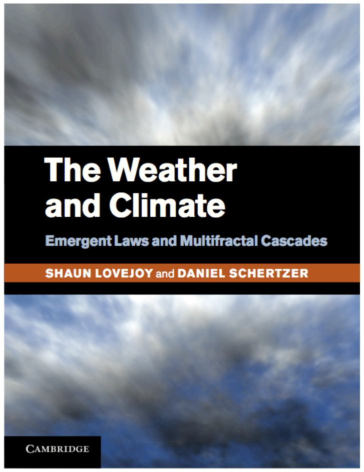
3. Anthropogenic warming dominates macroweather at about 10 years rather than about 100 years (preindustrial).

4. The total anthropogenic warming is about 0.85°C, for CO_2 doubling, 3.08±0.58°C, GCM's: 1.5-4.5°C (1979-2013).

5. The probability that the warming since 1880 is natural is <1% (most likely <0.1%).

6. The "pause" has a return period of 20-50 years, the post war cooling ≈ 125 years: not surprising.

Conclusions (2)



-Simple one parameter ($\lambda_{CO2x2,eff}$ ≈2.33 K/doubling) gives accurate "unconditional" hindcasts.

-Simplest scaling model (one parameter, H = -0.20±0.03) gives 1 year global hindcasts better than GCM's (±0.092 K), nearly as good as LIM model with > 100 parameters.

-Algebraic predictability limits of natural variability: Lyanpunov $\longrightarrow \infty$

-Hindcast skill for temperature anomalies is nearly the highest theoretically possible.

-Extensions to Regional climate forecasting: Stochastic Linear Inverse Modelling (SLIM)