Isotropising anisotropic cyclic cosmologies

Chandrima Ganguly

DAMTP, University of Cambridge

Seminar, McGill University
6th October, 2017

How do we get a bounce?

- Coming out of the contracting phase the Hubble rate H is negative.
- $H > 0$ in the expanding phase
- So in the transition or 'bounce' phase, $H = 0$ and

$$\dot{H} = \frac{k}{a^2} - \frac{1}{2}(\rho + P)$$

- If the spatial curvature k is 0, then for $\dot{H} > 0$ *and* $H = 0$, we must have $\rho + P < 0$ (NEC violation)
- If we have positive spatial curvature, we can have a bounce, In the closed radiation FRW universe, exact solutions show this but need a NEC violating field to have the bounce occur at non-zero volume.

J.D.Barrow and Christos G.Tsagas, CQG Vol. 26, No. 19 (2009)

Do the most general cyclic universes isotropise?

- Closed FRW universe with ordinary matter or dust shows oscillatory behaviour
- Simple solutions in these scenarios have been found

Do the most general cyclic universes isotropise?

- Closed FRW universe with ordinary matter or dust shows oscillatory behaviour
- Simple solutions in these scenarios have been found

Bouncing models need to solve flatness, isotropy and homogeneity problems

J.D.Barrow, M.P.Dabrowski, MNRAS, 275, 850 − 862, 1995, J.D.Barrow and C.G.Tsagas, CQG,26,19,2009, P.W.Graham *et al.*, JHEP 1402, 029, 2014

We focus on the isotropy problem and split it up into 2 regimes of interest.

We focus on the isotropy problem and split it up into 2 regimes of interest.

- In the contracting branch, on approach to the singularity, or in the case of non-singular cosmologies, on approach to the bounce

We focus on the isotropy problem and split it up into 2 regimes of interest.

- In the contracting branch, on approach to the singularity, or in the case of non-singular cosmologies, on approach to the bounce
- Over a large period of oscillations with increasing expansion maxima

We focus on the isotropy problem and split it up into 2 regimes of interest.

- In the contracting branch, on approach to the singularity, or in the case of non-singular cosmologies, on approach to the bounce
- Over a large period of oscillations with increasing expansion maxima

A simple example of ekpyrosis

- The metric

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2$$

- Friedmann equation: $3H^2 = \sigma^2 + \rho_{matter}$,
- The shear evolves as,

$$\dot{\sigma}_{\alpha\beta} + 3H\sigma_{\alpha\beta} = 0$$

- ρ_{matter} should evolve as V^{-n}, $n \gg 2$

J. Khoury, B.A. Ovrut, P. J. Steinhardt and N.Turok, 2001, J. High Energy Phys. 11(2001)041

Bianchi Class A cosmologies

- The generalised metric

$$ds^2 = dt^2 - h_{ab} d\omega^a d\omega^b$$

- Having an isotropic ultra stiff field of density ρ with equation of state $p = (\gamma - 1)\rho$, such that $\gamma > 2$

The phase plane system

- We introduce

$$\begin{aligned}\sigma_+ &\equiv \tfrac{1}{2}(\sigma_{22}+\sigma_{33}),\\ \sigma_- &\equiv \frac{1}{2\sqrt{3}}(\sigma_{22}-\sigma_{33}).\end{aligned}$$

- Write EFE in terms of expansion normalised variables

$$\Omega \equiv \frac{\rho}{3H^2}, \quad \Sigma^2 \equiv \frac{\sigma^2}{3H^2}, \quad K \equiv -\frac{{}^{(3)}R}{6H^2}.$$

The phase plane system looks like...

- Einstein equations of the form $\mathbf{x}' = \mathbf{f}(\mathbf{x})$
- subject to the Friedmann constraint $\mathbf{g}(\mathbf{x}) = 0$
- where the state vector $\mathbf{x} \in \mathbb{R}^6$ is given by
$$\{H, \underbrace{\Sigma_+, \Sigma_-}_{\text{shear components}}, \underbrace{N_1, N_2, N_3}_{\text{spatial curvature variables}}, \Omega\}$$

- The fact that the matter is ultra stiff $\gamma > 2$ is used and

- A no-hair theorem can be proved for all Bianchi types, I-VIII as well as IX(separately)

Cosmic no-hair theorem

All initially contracting, spatially homogeneous, orthogonal Bianchi Type I-VIII cosmologies and all Bianchi type IX universes sourced by an ultra-stiff fluid with an equation of state such that $(\gamma - 2)$ is positive definite, collapse into an isotropic singularity, where the sink is a spatially flat and isotropic FRW universe.

J.E.Lidsey, CQG, 23, 3517,(2005)

Why include anisotropic stress?

- Bouncing models of the universe, such as ekpyrotic scenarios or LQC models claim isotropisation occurs at early times. But this isn't true on addition of anisotropic stress.
- Interaction rates of particles

$$\Gamma = \sigma n v \sim g\alpha^2 T$$

- To remain in equilibrium, $\Gamma > H$
- Before isotropisation, anisotropic universe expands faster
- Harder to maintain equilibrium

Decoupled collisionless particles free stream and exert anisotropic stresses.

Anisotropic stresses in a Bianchi I universe

We go back to our simple flat anisotropic universe and add anisotropic pressures in.

- Friedmann equation

$$3H^2 = \sigma^2 + \rho_{matter},$$

- The shear evolves as,

$$\dot{\sigma}_{\alpha\beta} + 3H\sigma_{\alpha\beta} = \underbrace{\boxed{\mu\mathcal{P}_{\alpha\beta}}}_{\text{anisotropic stress}}$$

The equation for the shear isn't homogeneous and we can't say straight away that an ultra stiff field will be able to dominate over it.

Anisotropic stresses in Bianchi Class A

- Resort to the expansion normalised variables and introduce $Z \equiv \frac{\mu}{3H^2}$ where μ is the anisotropic pressure field energy density with EOS, $p_i = (\gamma_i - 1)\mu$ and $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3 > \gamma$

- try to perform stability analysis on the state vector $\mathbf{x} = \{H, \Sigma_+, \Sigma_-, N_1, N_2, N_3, \Omega, Z\}$

- Linearise expansion normalised EFE around the FL point

$$\Sigma_+ = 0,\ \Sigma_- = 0,\ N_1 = 0,\ N_2 = 0,\ N_3 = 0,\ \Omega = 1,\ Z = 0$$

Stability analysis with anisotropic pressures:the results

We find the following eigenvalues

- $\frac{3}{2}(2-\gamma)$ of multiplicity 2
- $\frac{3\gamma-2}{2}$ of multiplicity 3
- $3(\gamma-\gamma_\star)$ of multiplicity 1
- Using the condition $\gamma_\star > \gamma > 2$, FL equilibrium point stability cannot be determined

Stability analysis with anisotropic pressures:the results

We find the following eigenvalues

- $\frac{3}{2}(2-\gamma)$ of multiplicity 2
- $\frac{3\gamma-2}{2}$ of multiplicity 3
- $3(\gamma-\gamma_\star)$ of multiplicity 1
- Using the condition $\gamma_\star > \gamma > 2$, FL equilibrium point stability cannot be determined

We can no longer determine the stability of the FL point and can't prove a no hair theorem like before.

Bianchi Type IX: What it is and why we use it

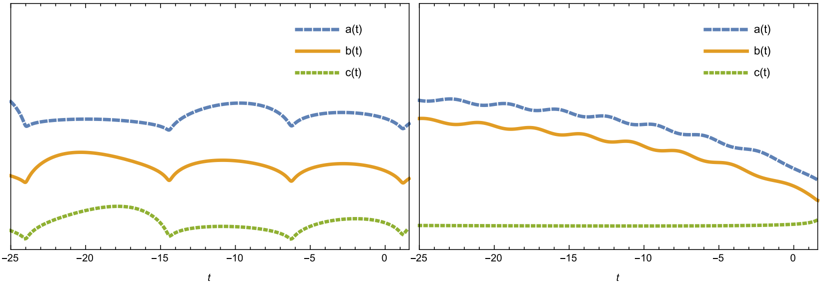
- It is the most general closed homogeneous universe, describable by ODEs
- It has the closed FRW universe as its isotropic sub-case
- It has expansion anisotropy and anisotropic 3-curvature(which has no Newtonian analogue)
- The 3-curvature can change sign through the course of its evolution and is positive when the model is closest to isotropy.
- On approach to $t \to 0$, in an open interval $0 < t < T$, exhibits chaotic Mixmaster oscillations, however oscillations become finite in number even if $t \to t_{Pl}$ on the finite interval $t_{Pl} < t < T$ excluding $t \to 0$.

We have a Bianchi Type IX universe with

- an isotropic pressure field with energy density ρ which follows the equations of state $p = (\gamma - 1)\rho$ and is effectively NEC violating, to bring about a non-singular bounce
- Anisotropic pressure field with energy density μ and $p_i = (\gamma_i - 1)\mu$ with $i = 1, 2, 3$, such that $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3$ and $\gamma_\star > \gamma$
- Choose initial conditions satisfying the Friedmann constraint

Scale factor evolution

Figure: Scale factors with isotropic ghost field and with fields with anisotropic pressures respectively

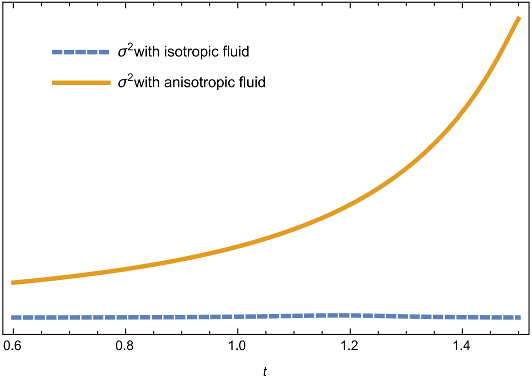


- The scale factors with just an isotropic pressure ghost field bounce and start to expand.
- The scale factors with the anisotropic pressure field included seem to contract towards a singularity.

Evolution of the shear

If we look at the evolution of the shear, we find,

Figure: Evolution of σ^2 with time



Bouncing cosmologies and the flatness problem

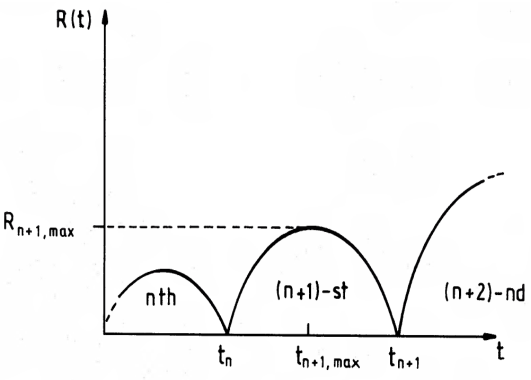
- Simple models of bouncing universes such as matter+ radiation closed FRW incorporated increasing radiation entropy to increase expansion maxima from cycle to cycle
- Universe seemed to approach flatness
- Suitable candidate for the current day universe?

Question

Would an anisotropic, bouncing cosmological model under similar increasing radiation entropy from cycle to cycle undergo isotropisation simultaneously with approach to flatness?

Present day flatness can perhaps be achieved by diluting the curvature with increasing volume

Figure: Scale factor with increasing entropy of radiation in closed FRW

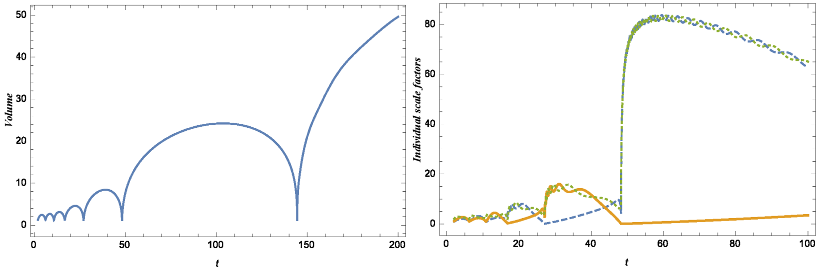


J.D.Barrow, M.P.Dabrowski, MNRAS, 275, 850 – 862, 1995

The scale factors with increasing radiation entropy

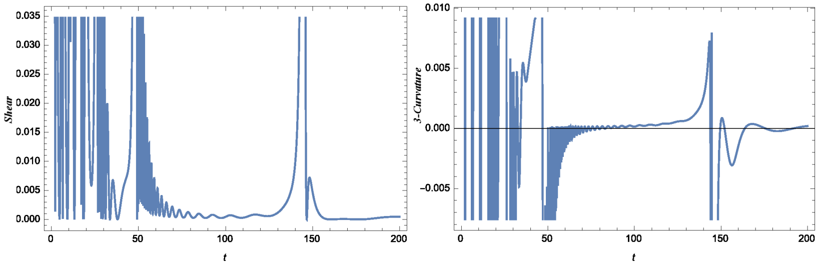
Increasing entropy of radiation in Bianchi IX

Figure: Evolution of volume scale factor and individual scale factors respectively



Let's see how the shear and the 3-curvature behave

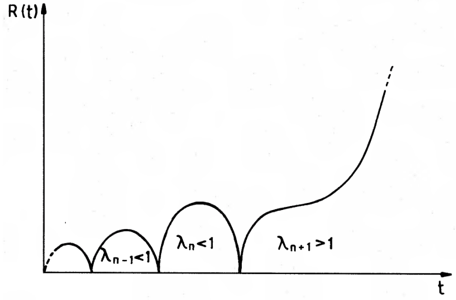
Figure: Evolution of σ^2 and ${}^{(3)}R$ respectively



Cosmological constant in an oscillating FRW model

When the cosmological constant starts to dominate, the isotropic model stops oscillating and instead of recollapsing enters a de Sitter exponential expansion

Figure: Adding a cosmological constant to the oscillating, closed FRW model

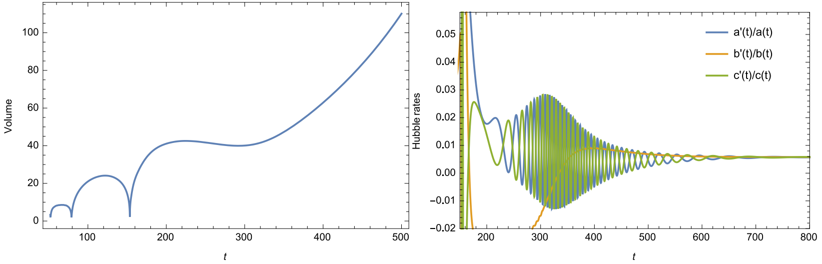


J.D.Barrow, M.P.Dabrowski, MNRAS, 275, 850 — 862, 1995

The scale factors with increasing radiation entropy

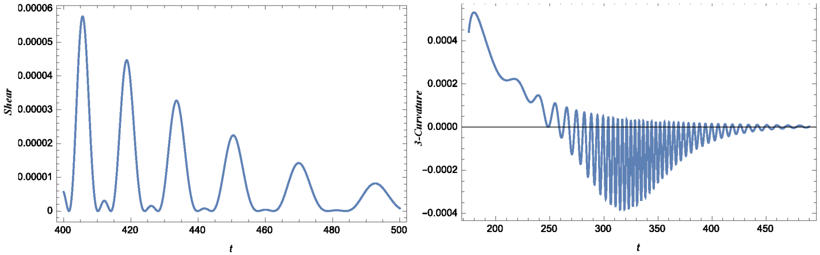
The volume scale factor and hence the individual scale factors evolve through a series of oscillations with increasing maxima until the cosmological constant starts to dominate and they expand exponentially

Figure: Evolution of volume scale factor and individual Hubble rates from left to right



Let's see how the shear and the 3-curvature behave

Figure: Evolution of σ^2 and ${}^{(3)}R$ respectively



Summary I

- In the initially contracting Bianchi Class A models, in the presence of ultra-stiff anisotropic stresses, FL is no longer an attractor in the asymptotic past
- In the Bianchi IX equations, including an ultra stiff anisotropic pressure field causes the scale factors to contract towards a collapse near the singularity.
  They bounce with only an isotropic ghost field present.
- The shear remains small and nearly constant in the isotropic case but increases without bound when the anisotropic pressure field is included.

Summary II

- By future evolving the model, we find that with radiation entropy increase, the height of the scale factor maxima increases, but the shear and the curvature oscillate and do not decrease to indicate isotropisation at any time.
- On adding the cosmological constant to the analysis, at the point of cosmological constant domination, the scale factors stop oscillating and undergo exponential expansion.
- The shear and the curvature tensors oscillate as before and then under cosmological constant domination, they fall to smaller and smaller values

So the takeaway message...

Near the singularity...

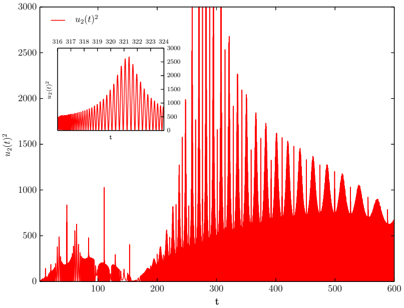
Including anisotropic stress, does not always result in isotropisation near the singularity, even if the anisotropic stress field is ultra-stiff on average

On future-evolving the system..

On evolving the system into the future, isotropisation does not occur as the shear keeps oscillating with the oscillations of the volume scale factors. On adding a cosmological constant, the shear and curvature fall to very small values

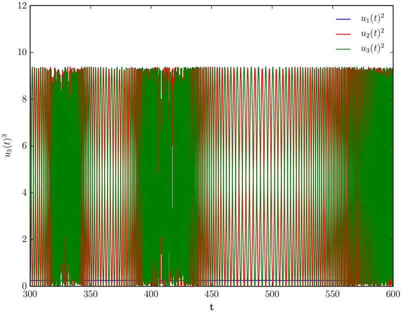
The effect of non comoving velocities with entropy increase

Figure: Evolution of the square of one of the spatial velocity components



The effect of non comoving velocities after cosmological constant domination

Figure: Evolution of the square of one of the spatial velocity components



The effect of non comoving velocities, in brief

- On imposing momentum and angular momentum conservation, the spatial components of the velocities fall to smaller values with an increase in entropy density and vice versa
- On addition of cosmological constant, bounces cease, expansion tends to the quasi dS asymptote and velocities tend to oscillate with a constant amplitude, while one of them tends to a constant value.

Thank you

References

- CG and John D.Barrow, 2017, arXiv:1710.00747
- John D. Barrow and CG, 2017, arXiv:1705.06647(Accepted in Int. Journal Mod. Phys. D)
- John D. Barrow and CG, 2017, Physical Review D **95**,083515
- John D. Barrow and CG, 2015, Class. Quantum Grav., Vol. 33, No. 12
- Dynamical Systems in Cosmology, edited by J. Wainwright and G.F.R. Ellis, Cambridge University Press
- V.A. Belinskii, I.M. Khalatnikov and E.M. Lifshitz, 1970, Adv. Phys. Vol. 19, No. 525
- V.A. Belinskii and I.M. Khalatnikov, 1972, Zh. Eksp. Teor. Fiz Vol. 63, No. 1121
- A.G. Doroshkevich, V.N.Lukash and I.D.Novikov, 1973, Zh. Eksper. Teor. Fitz, 64, No. 1457 – 1474

Definition: *Bianchi models are spatially homogeneous cosmologies admitting a three-parameter local group G_3 of isometries that act simply transitively on spacelike hypersurfaces Σ_t.*

$$ds^2 = dt^2 - h_{ab} d\omega^a d\omega^b$$

where $d\omega^a = \frac{1}{2} C^a_{bc} \omega^b \wedge \omega^c$ and C^a_{bc} are the structure constants of the Lie algebra G_3 As $C^a_{(bc})=0$, there are 9 independent components, and

$$C^a_{bc} = n^{cd} \epsilon_{dab} + \delta^c_{[a} A_{b]}$$

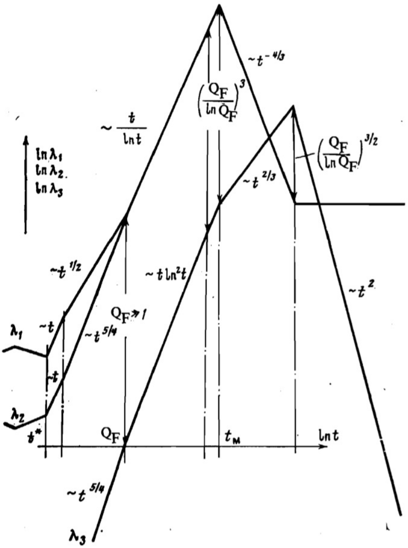
where n_{ab} is a symmetric 3×3 matrix, and $A_b = C^a_{ab}$ is a 3×1 vector.

Using the Jacobi identity, $C^e_{d[a} C^d_{bc]}$, we have $n^{ab} A_b = 0$. Choose $A_b = (A, 0, 0)$ and $n_{ab} =$diag$[n_1, n_2, n_3]$, to get,

$$n_1 A = 0$$

If $A = 0$, Bianchi Class A models, and if $A \neq 0 (n_1 = 0)$, Bianchi Class B.

- We define the unit timelike vector field **u** perpendicular to the group orbits and the projection tensor h_{ab}
- $u_{a;b} = \sigma_{ab} + \omega_{ab} + \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b$
- We have specialised to cases where the total stress tensor(isotropic+anisotropic) is diagonal
- We can write EFE as $\mathbf{x}' = \mathbf{f}(\mathbf{x})$. The functions $\mathbf{f}(\mathbf{x})$ are homogeneous of degree 2
- System is invariant under scale transformation $\tilde{\mathbf{x}} = \lambda \mathbf{x}$ and $d\tilde{t}/dt = \lambda$
- so we can introduce dimensionless variables, as well as because the variables in their current form diverge close to the big bang and tend to zero at late times in ever-expanding models
- Things evolve wrt the scale factor, so it seems natural to normalise wrt the Hubble rate



A.G. Doroshkevich, V.N.Lukash and I.D.Novikov, 1973, Zh. Eksper. Teor. Fitz, 64, No 1457 – 1474

- We have ρ and μ for isotropic and anisotropic pressure fields which follow the equations of state $p = (\gamma - 1)\rho$ and $p_i = (\gamma_i - 1)\mu$ with $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3$ and $\gamma_\star > \gamma$
- the 3 scale factors in the 3 directions are expressed as,

$$a(t) \equiv \mathrm{e}^{\alpha(t)},\ b(t) \equiv \mathrm{e}^{\beta(t)},\ c(t) \equiv \mathrm{e}^{\delta(t)}$$

- Define

$$\begin{aligned} x &\equiv \alpha'(t) - \beta'(t), \\ y &\equiv \alpha'(t) - \delta'(t), \\ H &\equiv \frac{1}{3}\left(\alpha'(t) + \beta'(t) + \delta'(t)\right). \end{aligned}$$

- Choose initial conditions satisfying the Friedmann constraint for the variable system

$$\{x, y, H, \alpha, \beta, \delta, \rho, \mu\}$$

The setup

- The generalised metric

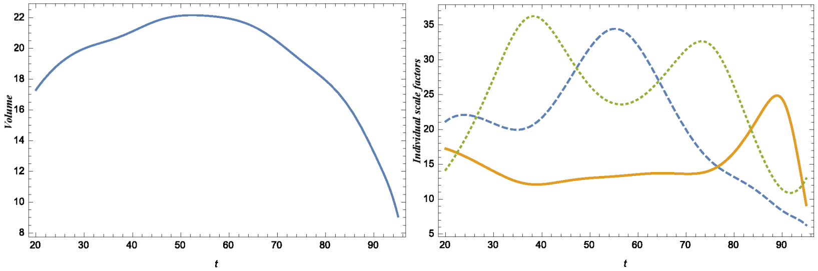
$$ds^2 = -dt^2 + h_{ab} d\omega^a d\omega^b$$

- Having isotropic ultra stiff ghost field of density ρ with equation of state $p = (\gamma - 1)\rho$
- and anisotropic pressure ultra stiff field of density μ with equation of state $p_i = (\gamma_i - 1)\mu$
- with $\gamma_\star = (\gamma_1 + \gamma_2 + \gamma_3)/3$ and $\gamma_\star > \gamma$

The Simple Harmonic Universe

- simple model of an oscillating universe
- Ingredients: Closed FRW, "domain wall matter" i.e. matter which obeys an equation of state $p = -(1/3)\rho$ and a negative cosmological constant

The scale factors with increasing radiation entropy



Let's see how the shear and the 3-curvature behave