Towards a T-dual cosmology

Guilherme Franzmann

McGill University
In collaboration with: Robert Brandenberger, Renato Costa and Amanda Weltman[1]

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Outline

- The Concordance Model and Inflation
- Problems with Inflation
- Supergravity and T-duality
- String Gas Cosmology
- Introduction to Double Field Theory (DFT)
- $O(D, D)$ Cosmological Completion
- Summary and Future Directions

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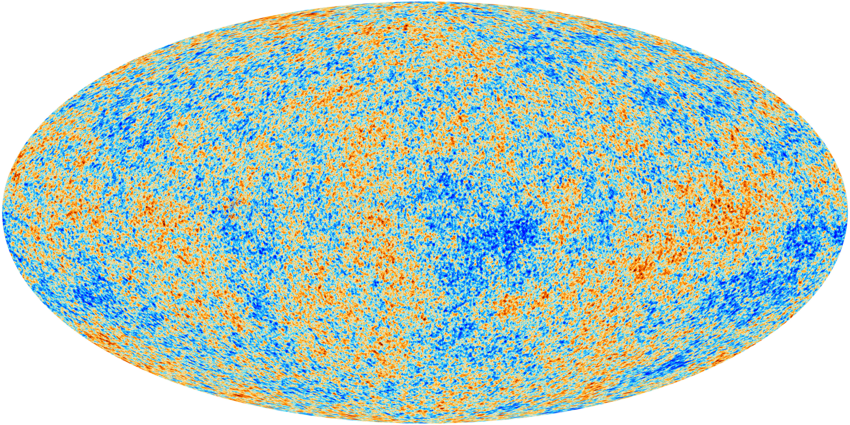

Black Body Radiation with average $T \sim 2.7$ K and fluctuations of order $\Delta T/T \sim 10^{-5}$ (Planck Collaboration 2013)

ΛCDM Model

- With only 6 parameters is able to explain all the current cosmological data
- Among these parameters, two are related to the initial fluctuations that gave rise to the CMB

$$P(k) = \Delta_R^2 k^{n_s - 1},$$

where $\Delta_R^2 \sim 2.5 \times 10^{-9}$ and $n_s \sim 0.9667$. The small value of $n_s - 1$ encodes the almost scale-invariance of the power spectrum

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Inflation

- First causal scenario that yields such physics by invoking **a phase of quasi-de Sitter expansion**
- It predicts an almost scale-invariant power spectrum for the adiabatic fluctuations with a red tilt (Chibisov, Mukhanov 1981)
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Problems with Inflation

- Several issues:
  - non-fundamental scalar field
  - trans-planckian problem
  - eternal inflation and multiverse
  - **singularity** (A. Borde and A. Vilenkin '94)
- Standard Cosmological Model remains incomplete
- Singularities are classical, thus finding a good QG theory should do it: **string theory**[2]
- Strings allow for new degrees of freedom and introduce new symmetries/dualities

[2] No convincing embedding of inflation in string theory so far, e.g. swampland conjectures (Vafa et al 2018)

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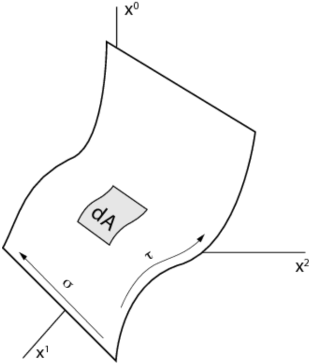
Look! A string!

$$S_{string} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}\gamma^{ab}g_{\mu\nu}(X)\,\partial_a X^\mu \partial_b X^\nu$$



$\{x^\mu\}$: spacetime coordinates (target space), $\{\tau, \sigma\}$: worldsheet coordinates

How SUGRA was born?

- We could have added a topological term and a 2-form field:

$$S_{\phi,b} = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{\gamma}\left[i\epsilon^{ab}b_{\mu\nu}(X)\,\partial_a X^\mu \partial_b X^\nu + \alpha'\phi(X)\,R^{(2)}\right]$$

- Weyl-inv. classically ($T^a_a = 0$), but contain QM anomalies:

$$T^a_a = -\frac{1}{2\alpha'}\beta^g_{\mu\nu}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta^b_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\phi R^{(2)}$$

- Using RG flow:

$$\beta^g_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \frac{1}{4}H_{\mu\lambda\omega}H_\nu{}^{\lambda\omega} + \mathcal{O}\left(\alpha'^2\right)$$

$$\beta^b_{\mu\nu} = -\frac{1}{2}\nabla^\omega H_{\omega\mu\nu} + \nabla^\omega\phi H_{\omega\mu\nu} + \mathcal{O}\left(\alpha'^2\right)$$

$$\beta^\phi = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\phi + \alpha'\nabla_\omega\phi\nabla^\omega\phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \mathcal{O}\left(\alpha'^2\right)$$

where $H_{\mu\nu\rho} \equiv 3\partial_{[\mu}b_{\nu\rho]}$

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$$S_{SUGRA} = \int d^D x \sqrt{g} e^{-2\phi} \left[R + 4 (\partial\phi)^2 - \frac{1}{12} H_{ijk} H^{ijk} \right]$$

- Symmetries:
  - Diffeomorphisms: $L_\lambda g_{ij} = \lambda^k \partial_k g_{ij} + g_{kj} \partial_i \lambda^k + g_{ik} \partial_j \lambda^k$
  - Gauge: $b_{ij} \to b_{ij} + \partial_i \tilde{\lambda}_j - \partial_j \tilde{\lambda}_i$
- Equations of motion:

$$R_{ij} - \frac{1}{4} H_i{}^{pq} H_{jpq} + 2 \nabla_i \nabla_j \phi = 0$$

$$\frac{1}{2} \nabla^p H_{pij} - H_{pij} \nabla^p \phi = 0$$

$$R + 4 \left(\nabla^i \nabla_i \phi - (\partial\phi)^2 - \frac{1}{12} H^2 \right) = 0$$

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T-duality 101

T-duality is a symmetry of string theory relating winding modes in a given compact space with momentum modes in another (dual) compact space.

- Mass spectrum of a closed string on a circle of radius R:

$$M^2 = \left(N + \tilde{N} - 2\right) + p^2 \frac{l_s^2}{R^2} + w^2 \frac{R^2}{l_s^2}$$

- The mass spectrum is invariant under:

$$\begin{cases} \frac{R}{l_s} & \leftrightarrow \frac{l_s}{R} \\ p & \leftrightarrow w \end{cases}$$

- For $M = 0$, then $\{p, w\} = 0$, $\left\{N, \tilde{N}\right\} = 1 \Rightarrow \{\phi, g_{\mu\nu}, b_{\mu\nu}\}$[3]

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String Gas Cosmology[4]

- We consider a thermodynamical gas of closed strings. Since we know the string's spectrum, we can write

$$\rho = \frac{1}{a^{D-1}} \sum_s N_s E_s$$

$$E_s^2 = \left(N + \tilde{N} - 2\right) + p^2 \frac{l_s^2}{a^2} + w^2 \frac{a^2}{l_s^2}$$

where $s = \left\{p, w, N, \tilde{N}\right\}$

- The pressure is given by,

$$p = -\frac{\partial \left(\rho V\right)}{\partial V} = -\frac{1}{D-1} a^{1-D} \sum_s \frac{N_s}{l_s^2} \left(-\frac{l_s^2}{a^2} n^2 + \frac{a^2}{l_s^2} w^2\right)$$

too complicated!

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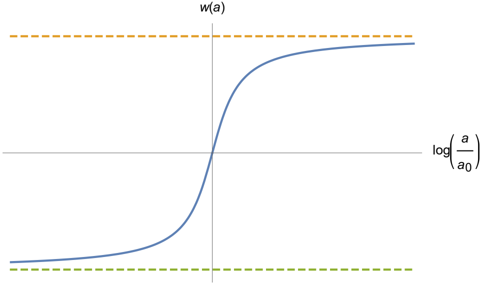
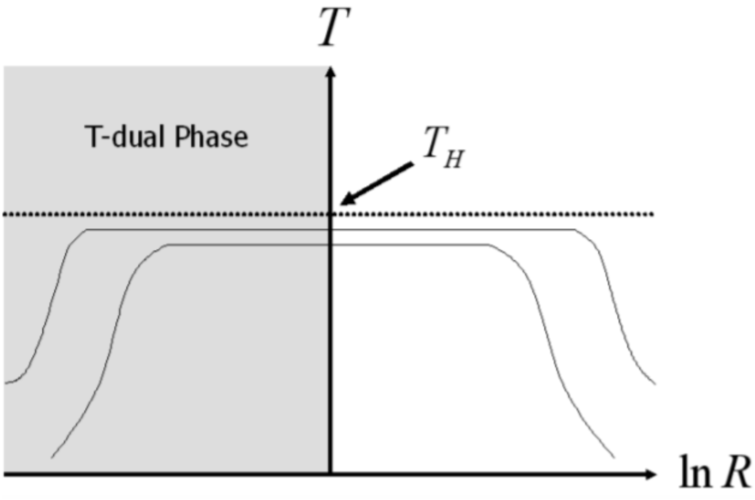
small box ($a \ll l_s$)	self-dual ($a \sim l_s$)	large box ($a \gg l_s$)
$\omega = -1/(D-1)$	$\omega = 0$	$\omega = 1/(D-1)$

$$\omega(a) = \frac{2}{\pi(D-1)} \arctan\left(\beta \ln\left(\frac{a}{a_0}\right)\right)$$



Note that $\omega\left(a^{-1}\right) = -\omega(a)$.



T
T-dual Phase
T_H
ln R

It remains a kinematical proposal, with no dynamics accounting for such picture of the early universe. Supergravity is not enough and still singular (Veneziano, Gasperini 2002).

T-duality and double space

- QM in a box:

$$|x\rangle = \sum_p e^{ipx}|p\rangle, \qquad p \in \mathbb{Z}$$

- Since the winding modes are dual to momentum modes through T-duality, one could argue for the existence of the following operator:

$$|\tilde{x}\rangle = \sum_w e^{iw\tilde{x}}|w\rangle, \qquad w \in \mathbb{Z}$$

- Thus, string states in general could be seen as point particles propagating in a **doubled space**

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- **Objective:** To T-dual covariantize SUGRA
- **Idea:** to implement T-duality as a manifest symmetry of a field theory

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Qualitative Understanding

- Double coordinates: for compact dimensions, momentum and winding modes. Momenta are dual to y^m, windings dual to $\tilde{y}_m$ (new coordinates)
- **In the second quantization (String Field Theory) this is not even an option**
- Formally, DFT has the following coordinate dependence:

$$X^M = (\tilde{x}_\mu, \tilde{y}_m, x^\mu, y^m)\,, \qquad \tilde{x}_\mu : \text{aesthetic}$$

  $m = 1, ..., n$ for compact and $\mu = 1, ..., d$ for non-compact dimensions
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- The T-duality group associated to string toroidal compactification on T^n is $O(n,n)$. We enhance this symmetry to the full duality group $O(D,D)$
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"Gold Standard" Model

$$S = -\frac{1}{4\pi}\int d\sigma d\tau \left(\eta^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j G_{ij} + \epsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j B_{ij}\right)$$

- Define

$$G_{ij} = \begin{pmatrix} \hat{G}_{ab} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} \hat{B}_{ab} & 0 \\ 0 & 0 \end{pmatrix}$$

  Define also,

$$\hat{E}_{ab} = \hat{G}_{ab} + \hat{B}_{ab}$$

- Note that

$$\hat{E}' = h(\hat{E}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \hat{E} \equiv (a\hat{E} + b)(c\hat{E} + d)^{-1}$$

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- The Hamiltonian:

$$H_{string} = \frac{1}{2} Z^t \mathcal{H}(\hat{E}) Z + N + \bar{N}$$

$$Z = \begin{pmatrix} w^i \\ p_i \end{pmatrix}, \quad \mathcal{H}(\hat{E}) = \begin{pmatrix} \hat{G}^{-1} & -\hat{G}^{-1}\hat{B} \\ \hat{B}\hat{G}^{-1} & \hat{G} - \hat{B}\hat{G}^{-1}\hat{B} \end{pmatrix}$$

with w^i, $p_i \in \mathbb{Z}$ with $p_i = n/R$ and $w^i = mR/l_s^2$

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$$L_0 - \bar{L}_0 = 0 = N - \bar{N} - p_i w^i$$

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- Therefore, h preserves the metric η, so $h \in O(D, D)$
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Double space and generalized fields

- How to represent d.o.f. as T-dual object?
- Consider the generalized metric defined as

$$\mathcal{H} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix}, \qquad g(x), b(x)$$

$$\mathcal{H} \in O(D,D), \quad \mathcal{H}^{MN} = \eta^{MP} \mathcal{H}_{PQ} \eta^{QN}, \quad \mathcal{H}_{MP} \mathcal{H}^{PN} = \delta_M^N$$

- The dilaton appears together with g,

$$e^{-2d} = \sqrt{g} e^{-2\phi}$$

defining a $O(D,D)$-scalar

- **Note: det $\mathcal{H} = 1$!**

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Where do fields live?

- Fundamental representation of $O(D, D)$ has dimension $2D$, but only D coordinates
- Fix this by introducing new coordinates $\tilde{x}_i$, so that the generalized coordinates are

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Is that even possible?

- No, we cannot just double our space and leave it like that
- **Section Condition:**

$$\boxed{\eta^{MN}\partial_M\partial_N(\ldots) = 0}$$

trivial-unique solution: $\tilde{\partial}(\ldots) = 0$ (or any $O(D,D)$ rotation of it). This section is called supergravity frame

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Generalized Lie Derivative

- Gauge + diff.

$$\xi^M = \left(\tilde{\lambda}_i, \lambda^i\right)$$

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$$\mathcal{L}_\xi e^{-2d} = \partial_M \left(\xi^M e^{-2d}\right)$$
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The Action

$$S = \int dX e^{-2d} \mathcal{R}$$

$$\begin{aligned}\mathcal{R} \;=\; & \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \\ & +4\mathcal{H}^{MN}\partial_M\partial_N d - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d \\ & -\partial_M\partial_N\mathcal{H}^{MN}\end{aligned}$$

i) terms up to 2nd order derivatives; ii) recover SUGRA in the supergravity frame; iii) respect the gauge symmetries

DFT with single time

- For cosmological background w/ a single time parameter, the DFT's EOM = SUGRA's. Any solution from SUGRA can be embedded into DFT space, given

$$dS^2 = -dt^2 + \mathcal{H}_{MN} dX^M dX^N = -dt^2 + a^2(t)\, d\vec{x}^2 + a^{-2}(t)\, d\tilde{x}^2$$

- This has been considered[7] and singularities cannot be avoided
- Is it possible to do something else?

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DFT with single time

- For cosmological background w/ a single time parameter, the DFT's EOM = SUGRA's. Any solution from SUGRA can be embedded into DFT space, given

$$dS^2 = -dt^2 + \mathcal{H}_{MN} dX^M dX^N = -dt^2 + a^2(t)\, d\vec{x}^2 + a^{-2}(t)\, d\tilde{x}^2$$

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Interlude: on clocks and rods

- We live in space and time, and all the measurements can be made in terms of clocks and rods[8]
- If there is a constant speed for all the possible observers (SR), thus we only need either rods or clocks, since light rays follow null geodesics:

$$\Delta s^2 = 0 \Rightarrow \Delta t^2 = \frac{\Delta x^2}{c^2}$$

- However, if the world is made of closed strings, we could have used winding modes for building our rods, such that

$$\Delta \tilde{x} = \frac{l_s^2}{\Delta x}$$

where $\alpha^{'}$ is the string tension

[8] G.A. Matsas, V. Pleitez, A. Saa,, D. A.T. Vanzella: arXiv:0711.4276

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$$|\Delta\tilde{x}| = \left|\tilde{c}\Delta\tilde{t}\right|$$

  Thus, it is also natural to propose a "winding-clock" that is dual to the momentum-one by,

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DFT's EOM with 2-time parameters

- From previous argument, the DFT EOMs with double-time in vacuum are[9]:

$$2\bar{\phi}'' - \bar{\phi}'^2 - (D-1)\tilde{H}^2 + 2\ddot{\bar{\phi}} - \dot{\bar{\phi}}^2 - (D-1)\,H^2 = 0$$
$$(D-1)\,\tilde{H}^2 - \bar{\phi}'' - (D-1)H^2 + \ddot{\bar{\phi}} = 0$$
$$\tilde{H}' - \tilde{H}\bar{\phi}' + \dot{H} - H\dot{\bar{\phi}} = 0$$

- In the presence of matter[10]

$$2\bar{\phi}'' - \bar{\phi}'^2 - (D-1)\tilde{H}^2 + 2\ddot{\bar{\phi}} - \dot{\bar{\phi}}^2 - (D-1)\,H^2 = 0$$
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where $p\left(t,\tilde{t}\right)$ and $\rho\left(t,\tilde{t}\right)$, in principle

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SUGRA frame: large box

- For a constant dilaton, EOMs are

$$2\left(\tilde{H}' + \dot{H}\right) + D\left(\tilde{H}^2 + H^2\right) = 0$$

$$\left(\tilde{H}^2 - H^2\right) + \left(\tilde{H}' - \dot{H}\right) = \frac{1}{2(D-1)} G\rho$$

$$\left(\tilde{H}' + \dot{H}\right) + (D-1)\left(\tilde{H}^2 + H^2\right) = \frac{G}{2} p$$

- For SUGRA frame, only t-dependence should be relevant, given the $\tilde{t}$—was introduced exactly to tackle the winding modes

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It implies,

$$w \equiv \frac{p}{\rho} = \frac{1}{D-1}$$

Thus,

$$\rho(a) \propto a^{-D}$$
$$a(t) = a_0 t^{2/D}$$

This is a radiation-like solution, as we had before in SUGRA with constant dilaton.

Winding-frame (small box)

- Winding modes dominate: $\tilde{t}$-dependence is kept,

$$2\tilde{H}' + D\tilde{H}^2 = 0$$

$$\tilde{H}^2 + \tilde{H}' = \frac{1}{2(D-1)} G\rho$$

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- Implying EOS:

$$w = -\frac{1}{D-1}$$

- This is the EOS one would have for only winding modes!

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The Friedmann-like equation has a minus sing,

$$\tilde{H}^2 = -\frac{G}{(D-2)(D-1)}\rho$$

Considering $\tilde{H}$ complex ,

$$a\left(\tilde{t}\right) = \tilde{A}\left(\tilde{t}\right) e^{i\theta(\tilde{t})}$$

s.t.,

$$\tilde{H}_{\tilde{A}}^2 - \theta'^2 = -g\rho_0 \tilde{A}^{-D}\cos(D\theta)$$
$$2\tilde{H}_{\tilde{A}}\theta' = g\rho_0 \tilde{A}^{-D}\sin(D\theta)$$

where $g \equiv G/(D-2)(D-1)$. For $\theta = \pi/D$, the 2nd equation vanishes and the 1st equation gives:

$$\boxed{a\left(\tilde{t}\right) = \tilde{a}_0 \tilde{t}^{2/D} e^{i\pi/D}}$$

Interpretation

- For momenta:

$$a_m(t) = a_0 t^{2/D} e^{i\theta_m}$$

  where $\theta_m = 0$.

- Thus,

$$\begin{cases} a_m(t) & = a_0 t^{2/D} \\ a_w(\tilde{t}) & = \tilde{a}_0 \tilde{t}^{2/D} e^{i\pi/D} \end{cases}$$

- Momentum's and winding's scale factor are dual:

$$a_m \to a_w^{-1}$$

- The solutions are dual given,

$$\boxed{t \to \tilde{t}^{-1} e^{-i\pi/2}}$$

- This is a Wick rotation of the reciprocal of the time coordinate. Since θ is dynamical, this rotation happens dynamically.

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General case,

$$\tilde{H}_{\tilde{A}}^2 - \theta'^2 = -g\rho_0 \tilde{A}^{-D} \cos(D\theta)$$
$$2\tilde{H}_{\tilde{A}}\theta' = g\rho_0 \tilde{A}^{-D} \sin(D\theta)$$

The solutions are,

$$\theta(\tilde{t}) = \pm\frac{2}{D} \arccos\left[\left(\frac{\tilde{A}}{\tilde{A}_0}\right)^{-D/2}\right]$$
$$\tilde{A}(\tilde{t}) = \left[\tilde{A}_0^D + \frac{D^2}{4} g\rho_0 \tilde{t}^2\right]^{1/D}$$

Note that for large $\tilde{t}$ limit,

$$\tilde{A}(\tilde{t}) \rightarrow \tilde{t}^{2/D}$$

and

$$\theta(\tilde{t}) \rightarrow \pm\frac{2}{D} \arccos\left(\frac{1}{\tilde{t}}\right) \xrightarrow[\tilde{t}\rightarrow\infty]{} \pm\frac{\pi}{D}.$$

Hence, deep in the winding regime the oscillations cease to exist. This shows that the temporal duality appears due to the dynamics of our solutions.

DFT's EOM with 2-time parameter

Dual-clock:

$$|\Delta\tilde{t}| = \left|\frac{\alpha'^2}{c\tilde{c}\Delta t}\right|$$

Physically there is a single clock. When only winding or momentum modes are cheap, the existence of a unique time coordinate is clear. Around the self-dual radius, we need a prescription

(Physical Clock Constraint)

$$\boxed{|\tilde{t}| \to \frac{1}{|t|}}$$

Physical Clock Constraint

$$\tilde{t} \to \frac{1}{t} \qquad\qquad \frac{d}{d\tilde{t}} \to -t^2 \frac{d}{dt}$$

Physical time is defined as,

$$dt^2 + d\tilde{t}^2 \to \boxed{dt^2 \left(1 + \frac{1}{t^4}\right) \equiv dt_p^2}$$

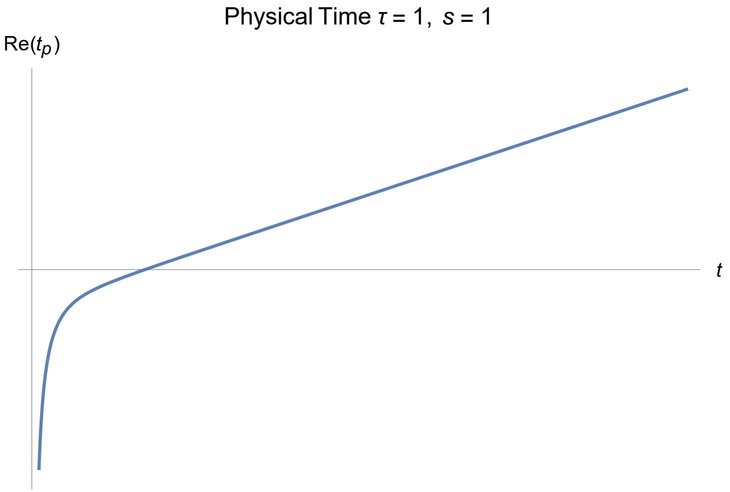
As $t \to 0$,

$$dt_p \sim \frac{1}{t^2} dt \quad \to \quad t_p \sim -\frac{1}{t}$$

so that $t_p \to -\infty$. When $t \to \infty$, then

$$dt_p \sim dt \quad \to \quad t_p \sim t$$

so that $t_p \to \infty$. Thus, $t_p \in (-\infty, \infty)$.



Physical clock definition: $dt_p \equiv \sqrt{1 + \frac{1}{t^4}}dt.$

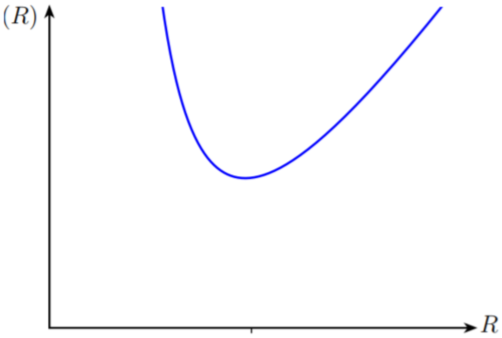


FIG. 9: Physical length (vertical axis) as a function of the coordinate length (horizontal axis).

Dynamics

- Equations of motion in terms of the physical time,

$$2\ddot{\bar{\phi}}_p - \dot{\bar{\phi}}_p^2 - (D-1)\,H_p^2 = \boxed{\frac{4}{t_p}\dot{\bar{\phi}}_p \sigma \left(1-2\sigma^2\right)}$$

$$-(D-1)H_p^2 + \ddot{\bar{\phi}}_p = \frac{1}{2}e^{\bar{\phi}}\bar{\rho}\boxed{\sigma^2\left(\frac{1-\sigma^2}{1-2\sigma^2}\right)} + \boxed{\frac{2}{t_p}\frac{\sigma}{1-2\sigma^2}\left(1-2\sigma^2+2\sigma^4\right)\dot{\bar{\phi}}_p}$$

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- Asymptotically ($|t_p| \to \pm\infty$),

$$2\ddot{\bar{\phi}}_p - \dot{\bar{\phi}}_p^2 - (D-1)\,H_p^2 \to 0$$

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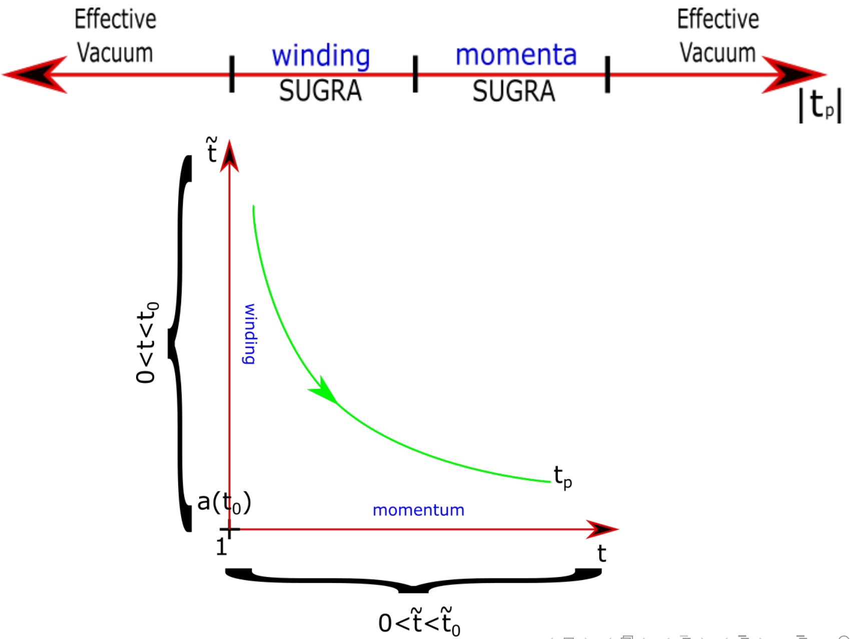

Effective
Vacuum
winding
SUGRA
momenta
SUGRA
Effective
Vacuum
$|t_p|$
$\tilde{t}$
$0<t<t_0$
winding
t_p
$a(t_0)$
momentum
1
t
$0<\tilde{t}<\tilde{t}_0$

$O(D,D)$ Cosmological Completion[11]

- SUGRA and matter:

$$S = \int d^D x \sqrt{g} e^{-2\phi} \left[R + 4 (\partial\phi)^2 - \frac{1}{12} H_{ijk} H^{ijk} \right] + \int d^D x \sqrt{g} \mathcal{L}_m$$

- $O(D,D)$ completion in the supergravity frame:

$$S = \int d^D x \sqrt{g} e^{-2\phi} \mathcal{L}_{SUGRA} + \int d^D x \sqrt{g} \boxed{e^{-2\phi}} \mathcal{L}_m$$

[11] Stephen Angus, Kyoungho Cho, GF, Jeong-Hyuck Park, Shinji Mukohyama (in preparation)

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$$3H^2 - 6H\dot{\phi} + 2\dot{\phi}^2 = e^{2\phi}\rho$$

$$\dot{H} + 4H\dot{\phi} - 2\dot{\phi}^2 = -e^{2\phi}(\rho - p) + \boxed{\frac{T_{(0)}}{2}}$$

$$\ddot{\phi} + 3H\dot{\phi} - 2\dot{\phi}^2 = -\frac{e^{2\phi}}{2}(\rho - 3p) + \boxed{\frac{T_{(0)}}{2}}$$

Imposing the dilaton to be constant, one recovers Friedmann equations for any matter content!

Summary and Future Directions

- Cosmological Standard Model still lacks a clear picture of the early universe
- Inflation albeit successful presentes many conceptual problems
- If string theory is the correct quantum gravity theory, T-duality is key for understanding early stages of the Universe
- Double Field Theory may provide a better description of the background for string cosmology as well perturbations
- Recent results using DFT and extensions could result in a non-singular picture of the early universe
- Non-commutativity of closed string theory
- Lots of work to do!

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Thank you!