From Color Glass Condensate to Plasma (via the Glasma)

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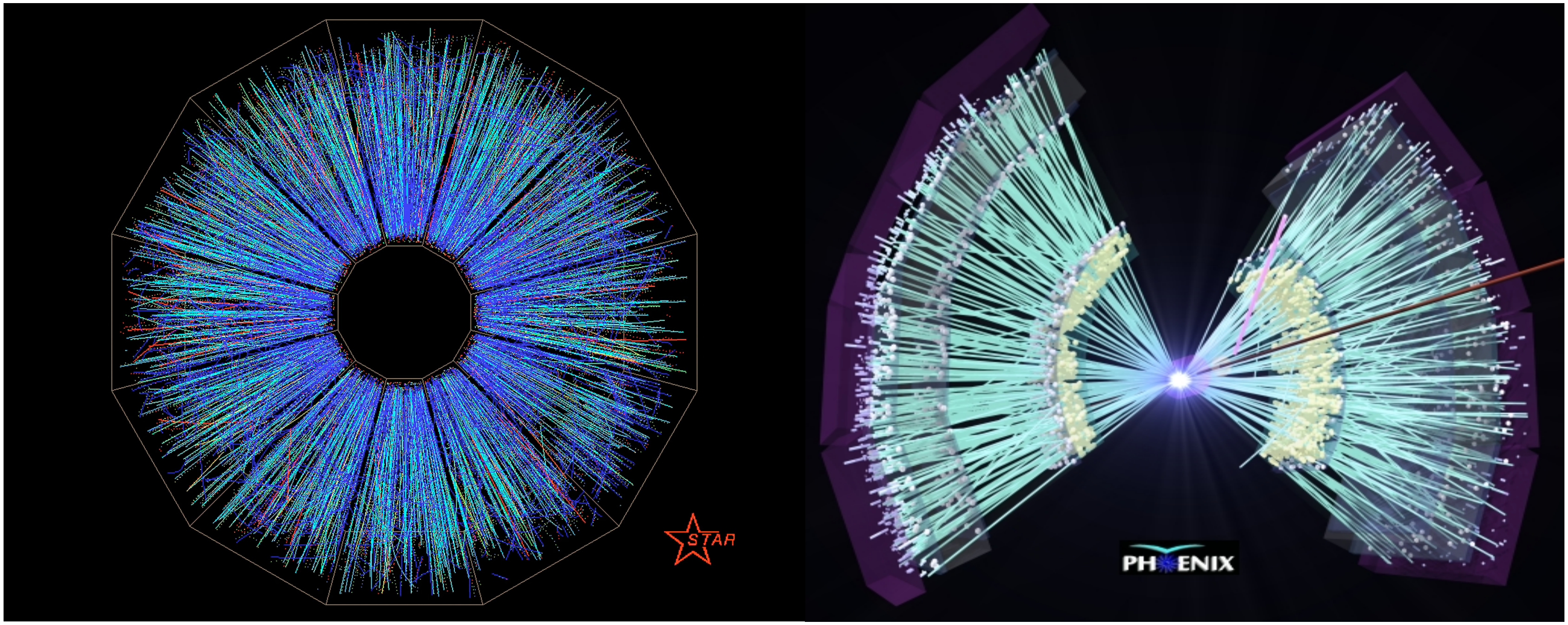
Outline of lectures

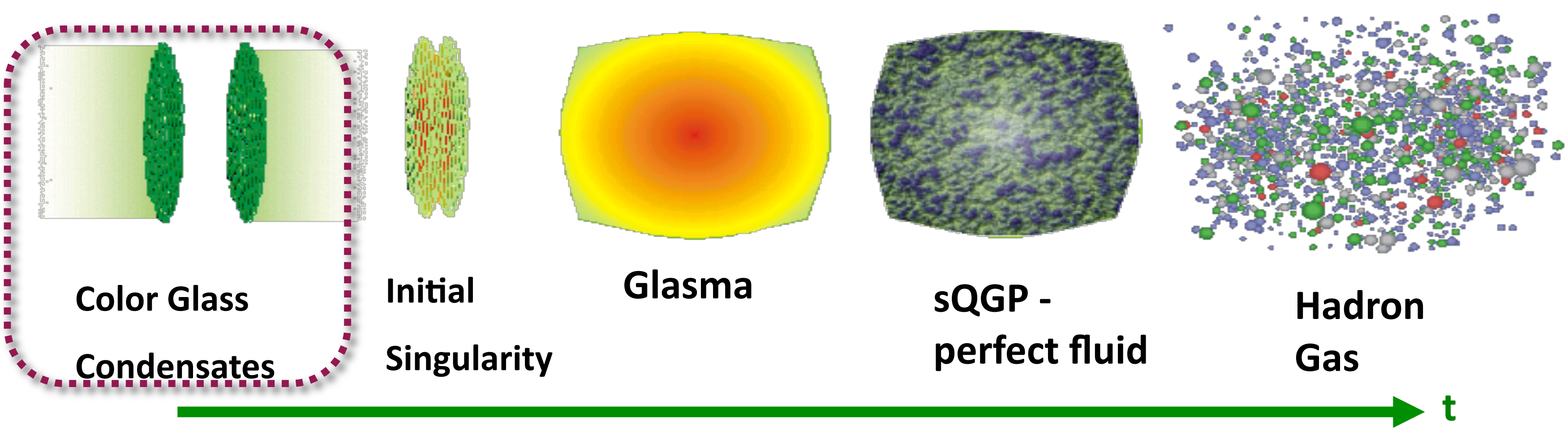
- **Lecture I: Gluon Saturation and the Color Glass Condensate**

- **Lecture II: Quantum field theory in strong fields. Factorization and the Glasma**

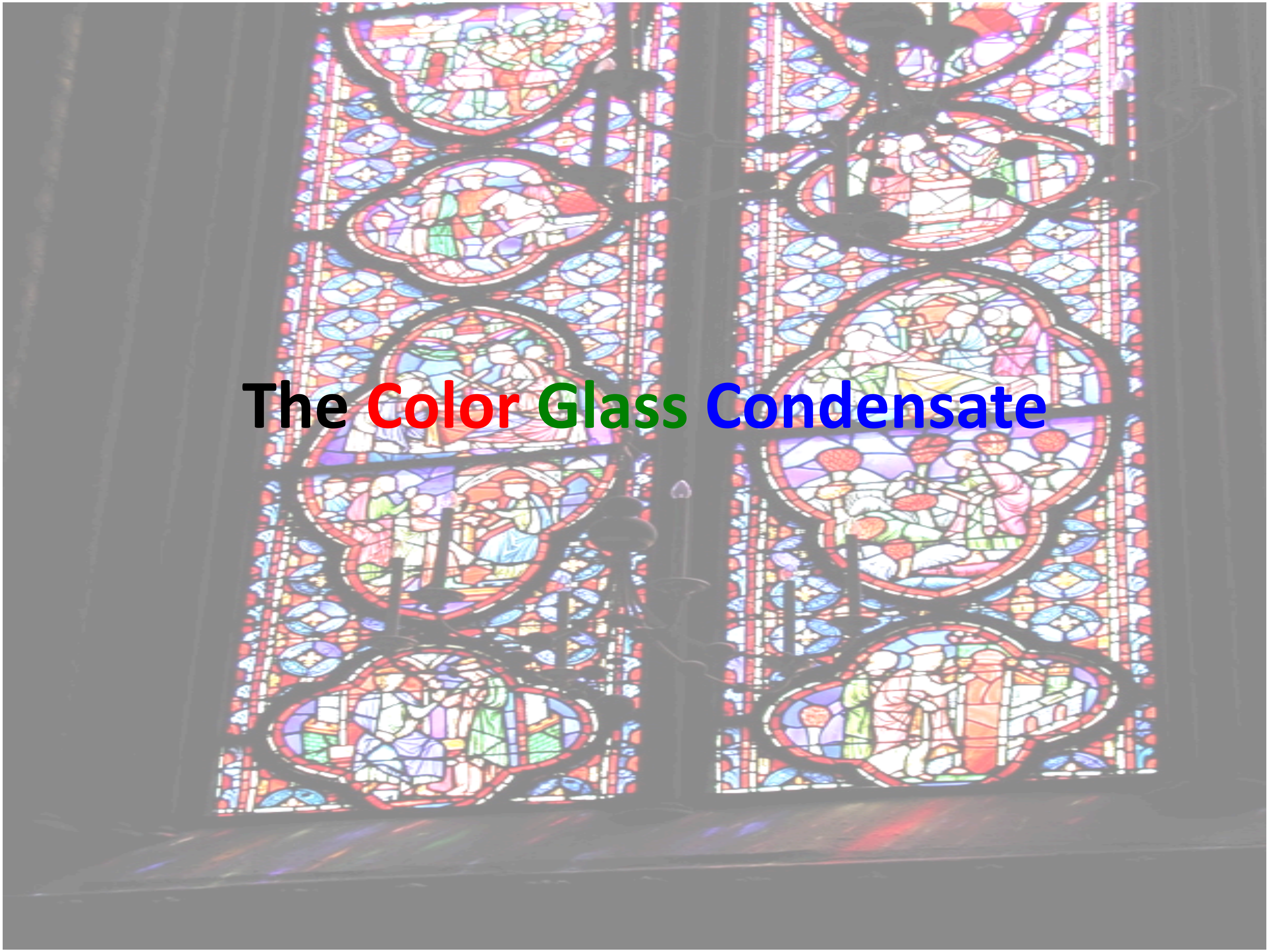
- **Lecture III: Quantum field theory in strong fields.**
  Instabilities, the spectrum of initial quantum fluctuations, decoherence, hydrodynamics, B-E condensation & thermalization

What does a heavy ion collision look like ?

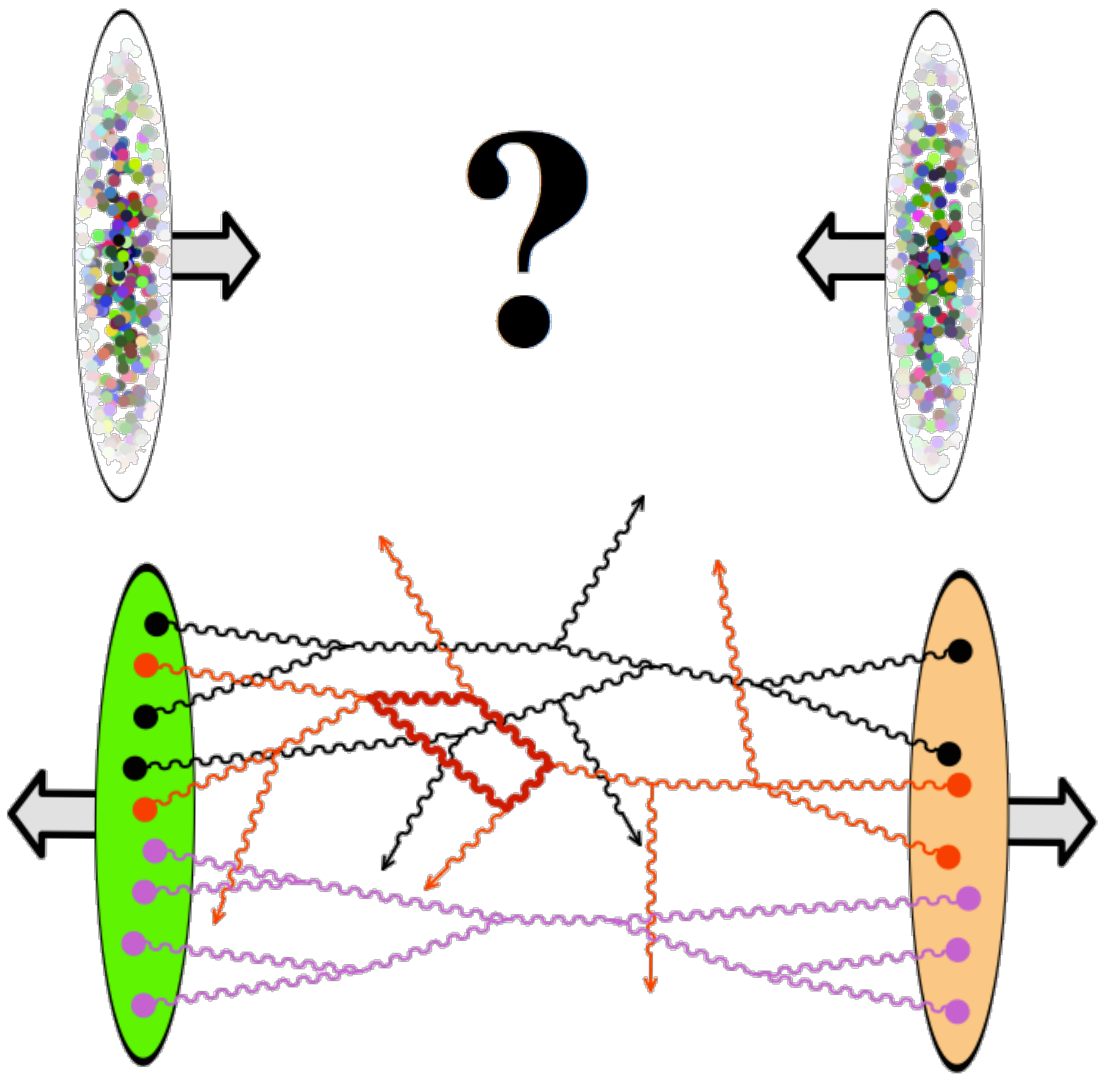




The Color Glass Condensate



The big role of wee gluons



The big role of wee glue

D. Nucleus-Nucleus Collisions at Fantastic Energies

(Nucleus-Nucleus Collisions at Fantastic Energies)

Before leaving this subject it is fun to consider the collision of two nuclei at energies sufficiently high so that in addition to the fragmentation regions, a central plateau region can develop. Let us consider a central collision of a relatively small nucleus, say carbon, with a big one, say lead. Let us look at this collision in a center-of-mass frame for which the rapidities of both of the nucleus projectiles exceeds the critical rapidity. In such a frame they both possess the fur coat of wee-parton vacuum fluctuations. In such a central collision we see that the collision initially occurs between the fur of wee partons in each of the projectiles. Therefore the number of independent collisions will be of order of the area of overlap of the two projectiles; namely the cross-sectional area of the smaller nucleus.

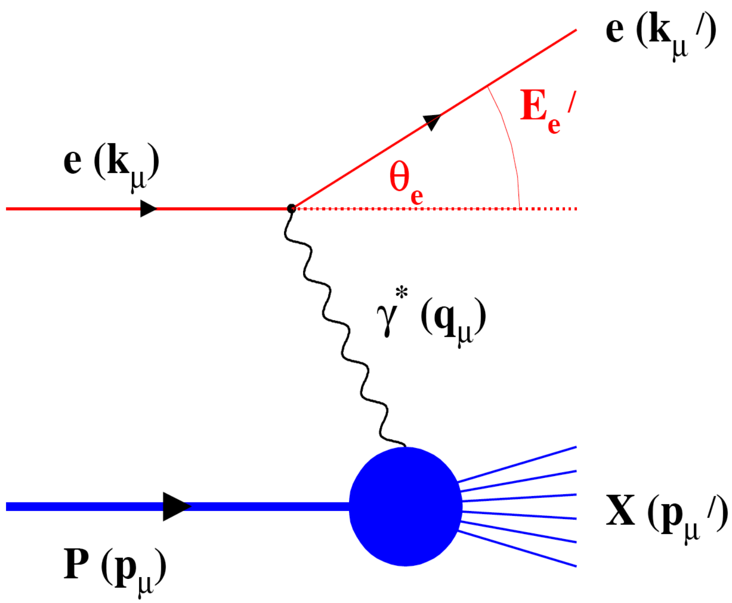
At LHC, ~14 units in rapidity!

Bj, DESY lectures (1975)

The big role of wee glue

- What is the role of wee partons ? ✔
- How do the wee partons interact and produce glue ? ✔
- Can it be understood *ab initio* in QCD ? ✔

The DIS Paradigm



$$Q^2 = -q^2 = -(k_\mu - k_\mu')^2$$

$$Q^2 = 4E_e E_e' \sin^2\left(\frac{\theta_e'}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\left(\frac{\theta_e'}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

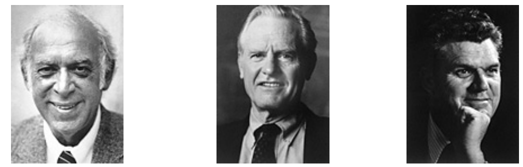
Measure of momentum fraction of struck quark

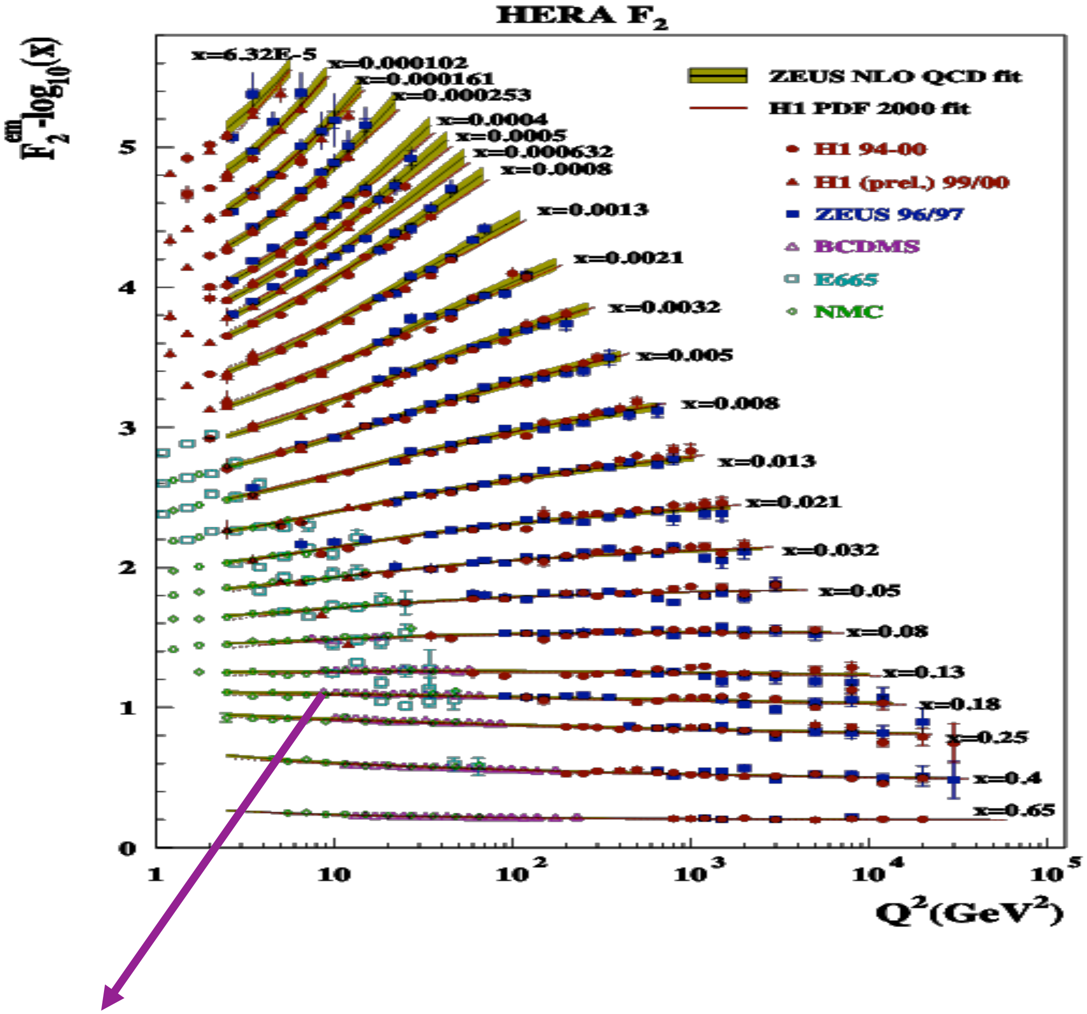
$$\frac{d^2\sigma^{eh\to eX}}{dxdQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

quark+anti-quark mom. dists. (F_2)

gluon mom. dists (F_L)

Nobel to Friedman, Kendall, Taylor

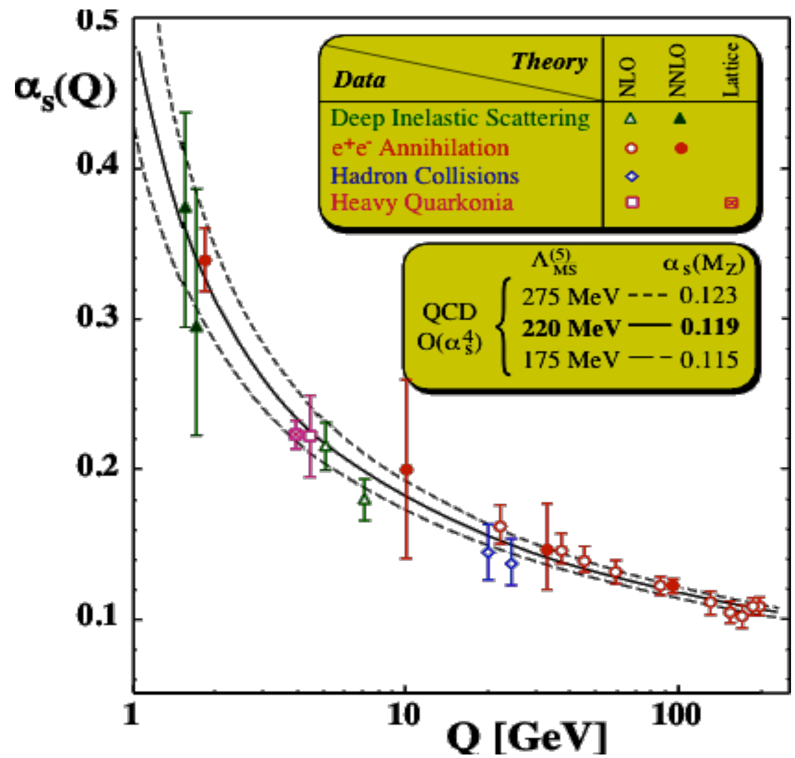




Bj-scaling: apparent scale invariance of structure functions

Puzzle resolved in QCD…

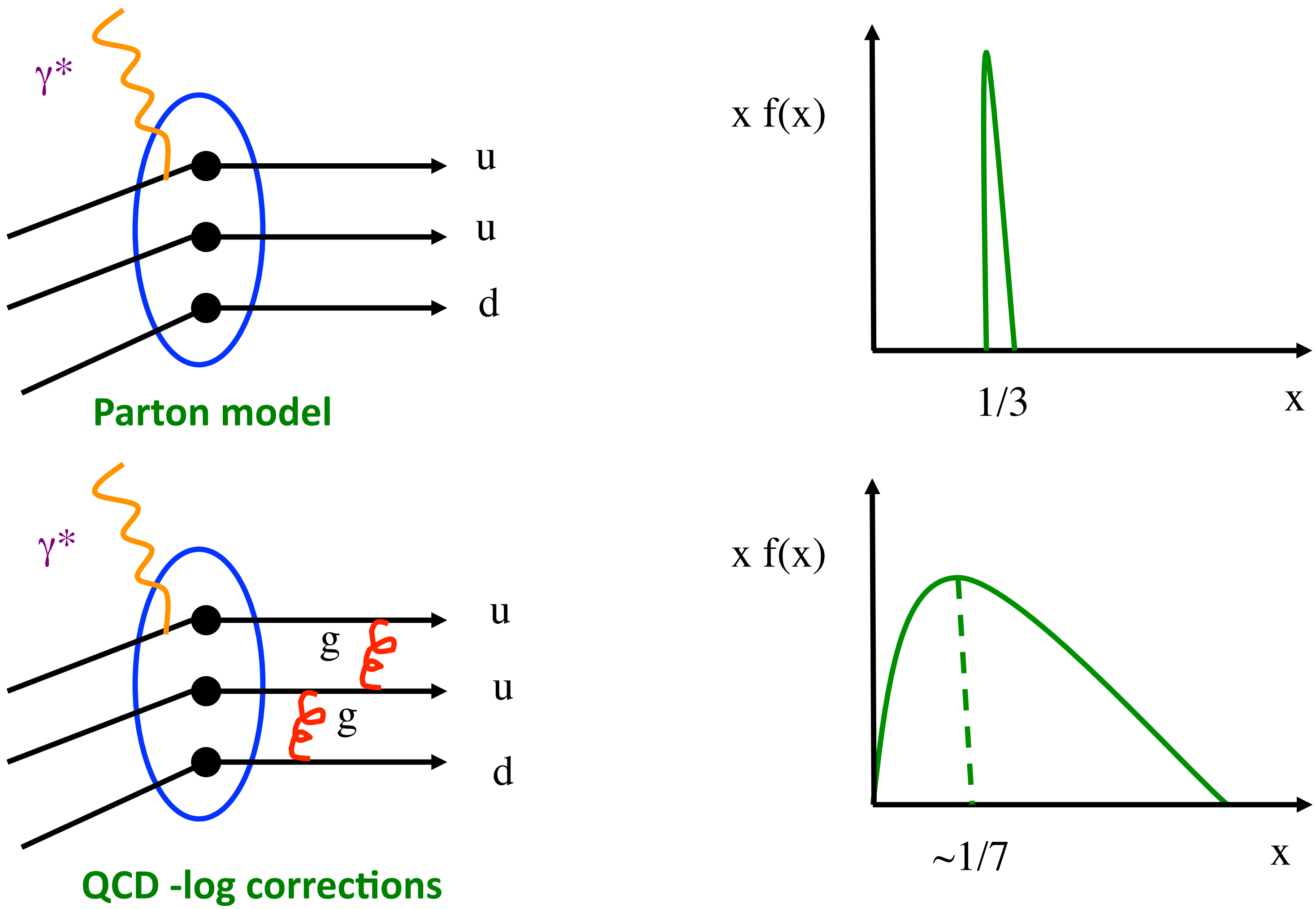
Gross, Wilczek, Politzer

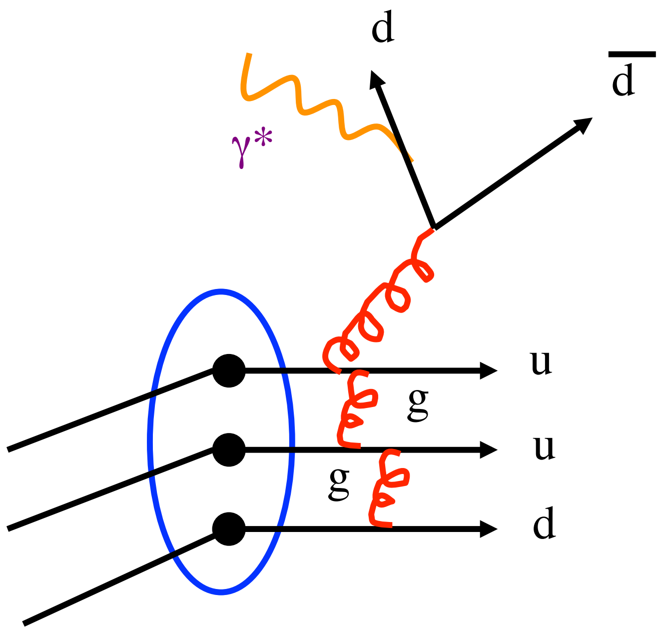


QCD ≠ Parton Model
Logarithmic scaling violations

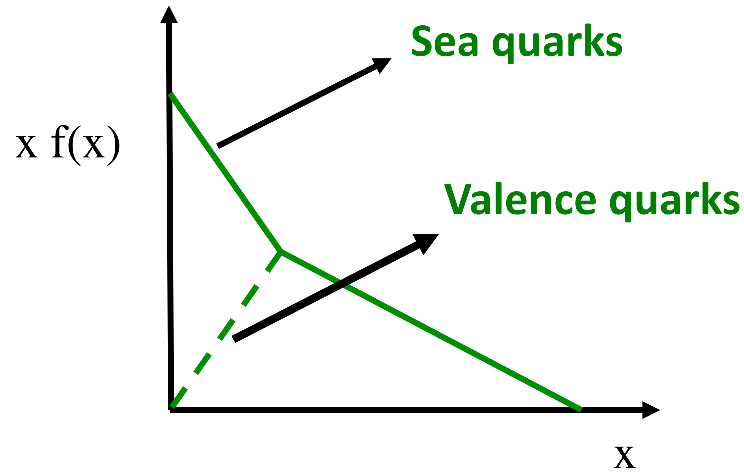
$$F_2(x, Q^2) = \sum_{\substack{q=u,c,t \\ d,s,b}} e_q^2 \left(x\, q(x, Q^2) + x\, \bar{q}(x, Q^2)\right)$$

The proton at high energies



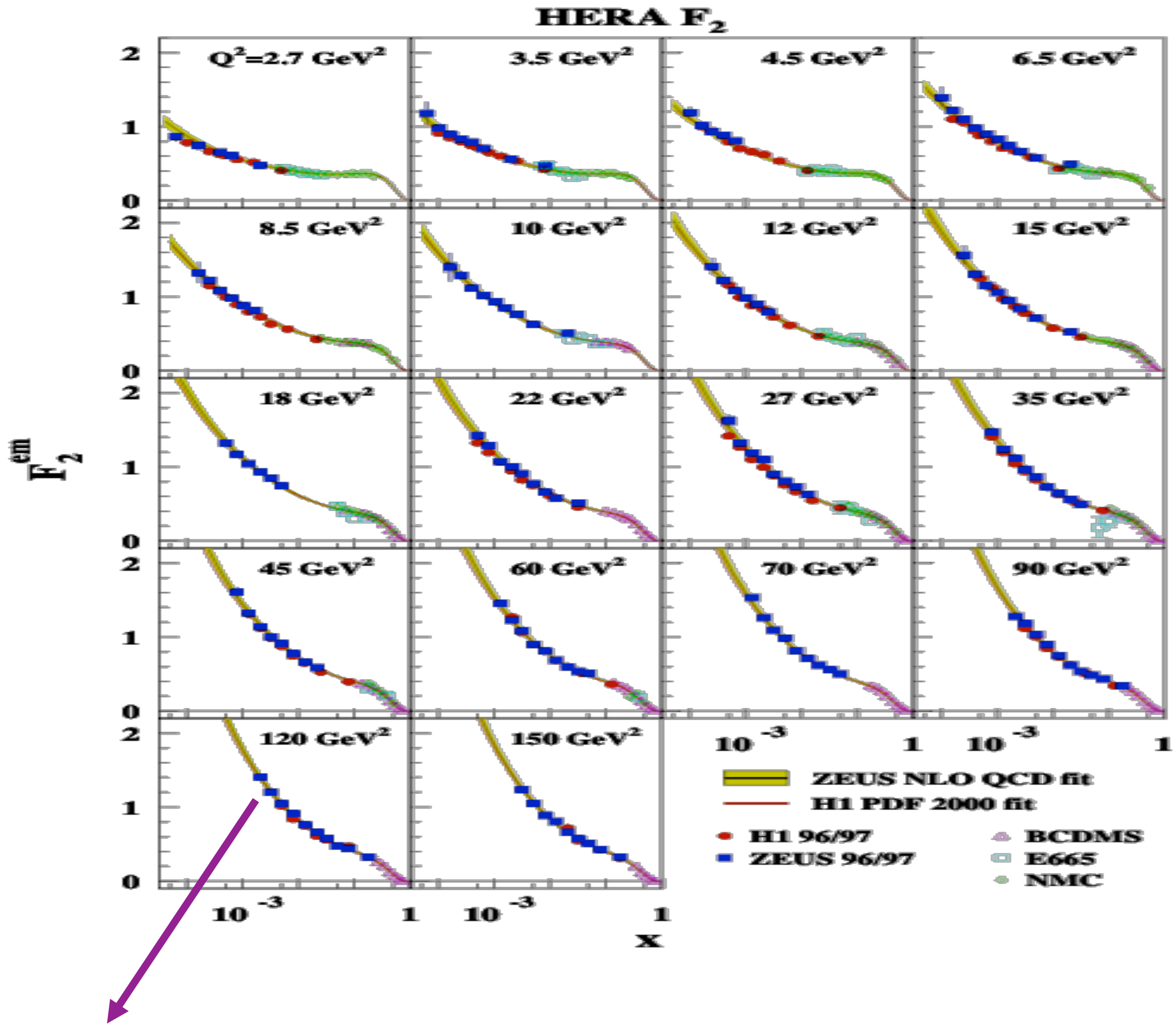


“x-QCD”- small x evolution



$$\int_0^1 \frac{dx}{x}\,(xq(x) - x\bar{q}(x)) = 3$$ → **# of valence quarks**

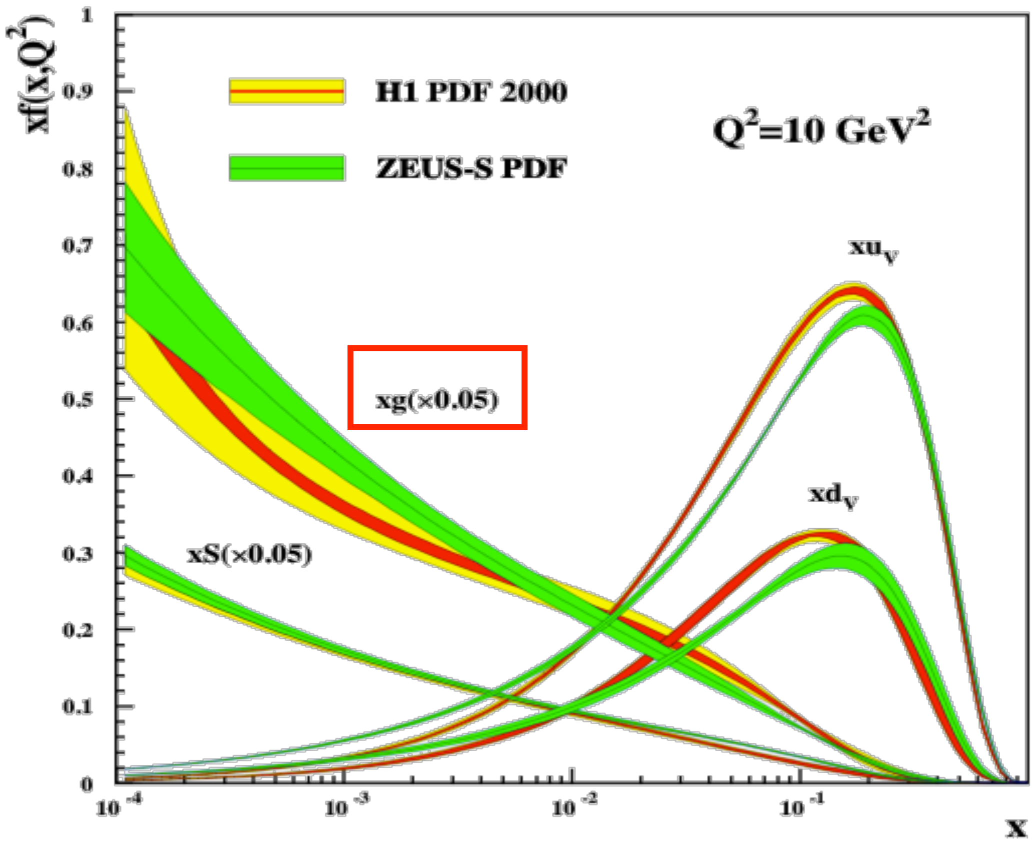
$$\int_0^1 \frac{dx}{x}\,(xq(x) + x\bar{q}(x)) \to \infty$$ → **# of quarks**



Structure functions grow rapidly at small x

Where is the glue ?

partons / unit rapidity



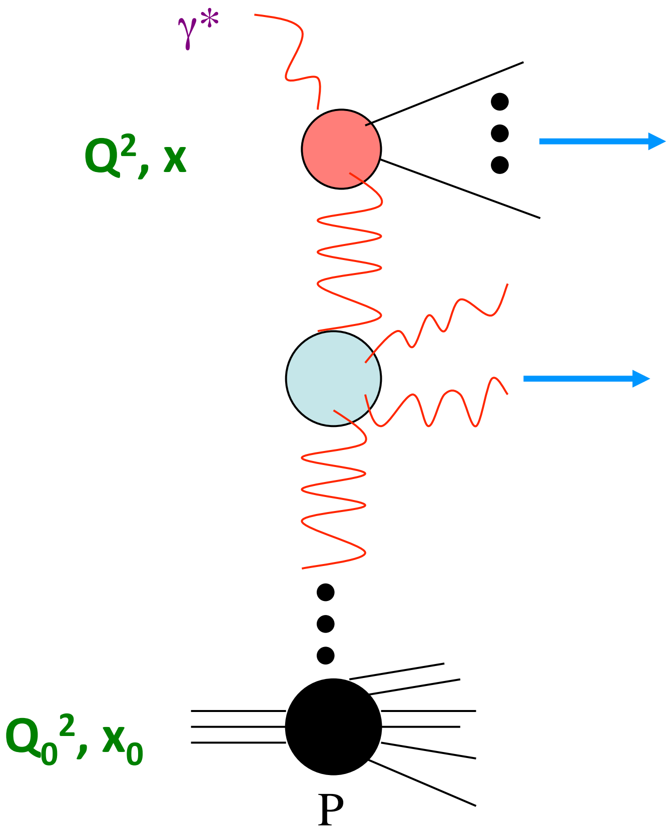
For x < 0.01, proton dominated by glue-grows rapidly
What happens when glue density is large ?

The Bjorken Limit

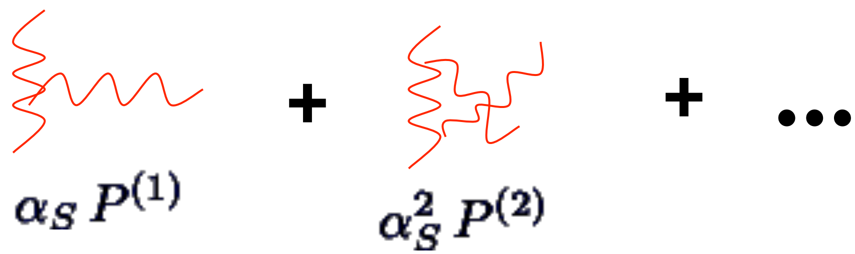
$$Q^2 \to \infty\,;\, s \to \infty\,;\, x_{\mathrm{Bj}} \approx \frac{Q^2}{s} = \text{fixed}$$

- Operator product expansion (OPE), factorization theorems, machinery of precision physics in QCD

Structure of higher order perturbative contributions in QCD



$$C_2(x, \alpha_S) = \delta(1-x) + \alpha_S C_2^{(1)}(x) + \alpha_S^2 C_2^{(2)} + \cdots$$



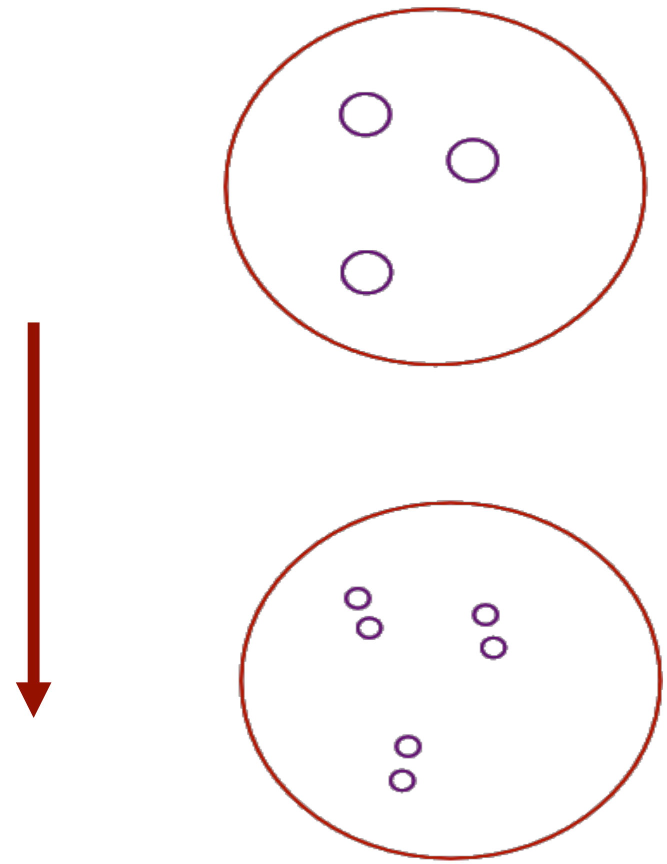
+ higher twist (power suppressed) contributions…

- Coefficient functions C - computed to NNLO for many processes
- Splitting functions P - computed to 3-loops

Resolving the hadron…

Ren.Group-DGLAP evolution
(sums large logs in Q^2)

Increasing Q^2



Phase space density (# partons / area / Q^2) decreases
- the proton becomes more dilute…

BEYOND pQCD IN THE Bj LIMIT

- Works great for inclusive, high Q^2 processes
- Higher twists important when $Q^2 \approx Q_S^2(x)$
- Problematic for diffractive/exclusive processes
- Formalism not convenient to treat shadowing, multiple scattering, diffraction, energy loss, impact parameter dependence, thermalization…

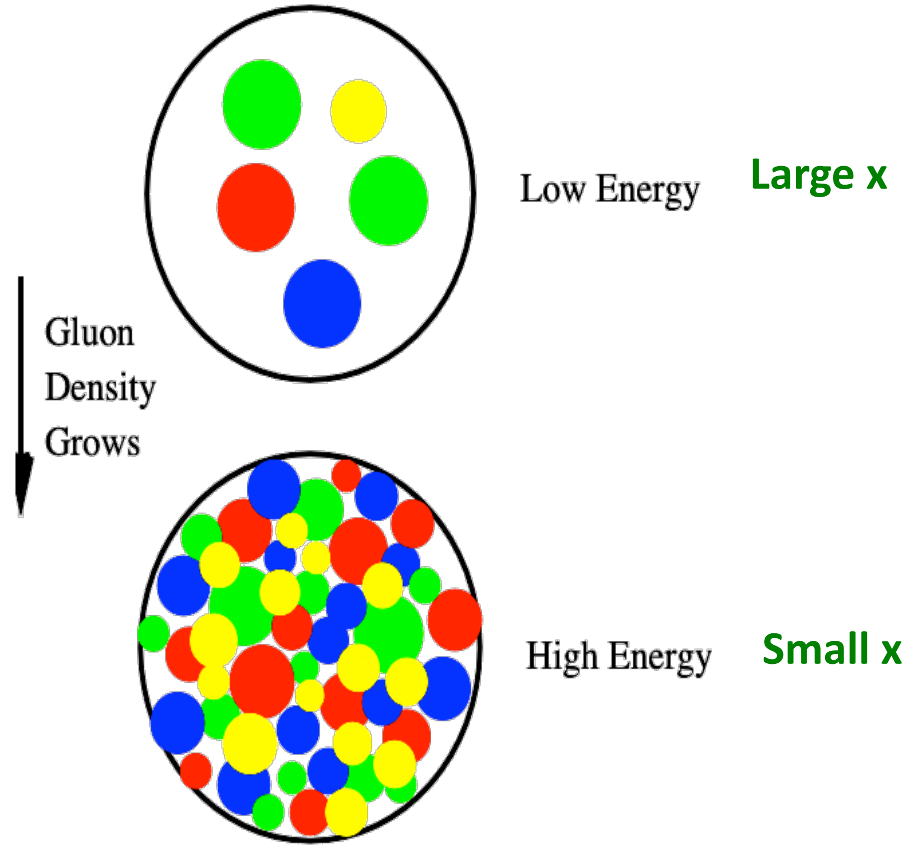
The Regge-Gribov Limit

$$x_{\mathrm{Bj}} \to 0;\ s \to \infty;\ Q^2 (>> \Lambda^2_{\mathrm{QCD}}) = \mathrm{fixed}$$

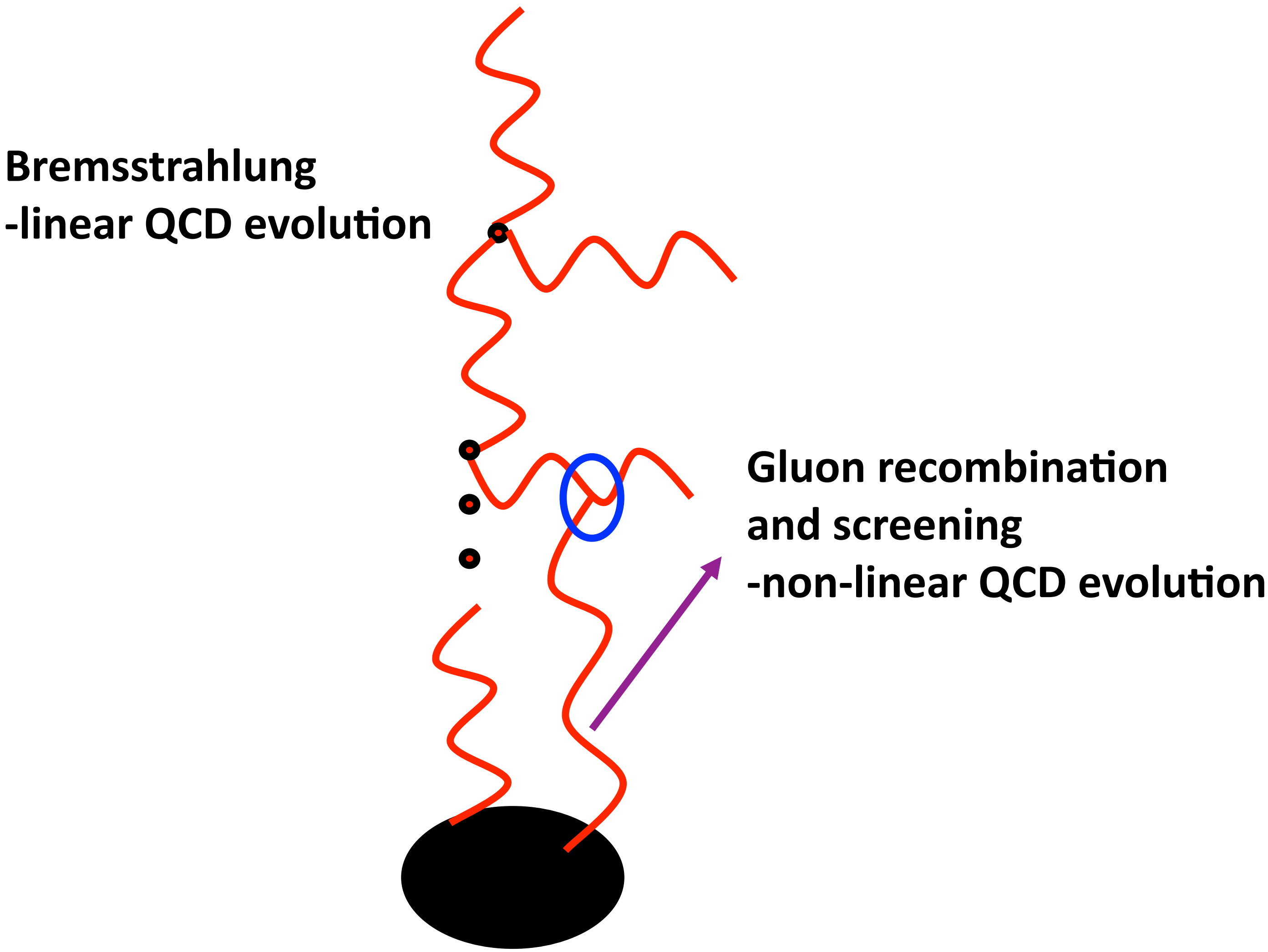
- **Physics of strong fields in QCD, multi-particle production,**
  Novel universal properties of QCD ?

Resolving the hadron…

Ren.Group-BFKL evolution
(sums large logs in x)



Gluon density saturates at phase space density $f = 1/\alpha_s$
- strongest (chromo-) E&M fields in nature…



Proton becomes a dense many body system at high energies

Parton Saturation

Gribov,Levin,Ryskin
Mueller,Qiu

- Competition between attractive bremsstrahlung and repulsive recombination and screening effects

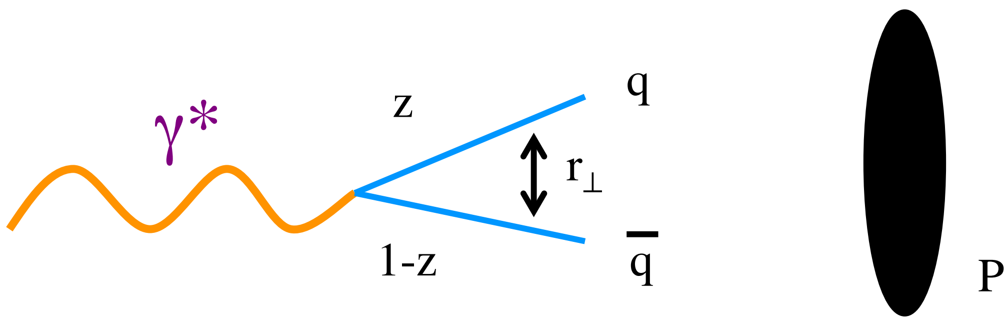
Maximum phase space density (f = $1/\alpha_S$) =>

$$\frac{1}{2(N_c^2-1)}\,\frac{x\,G(x,Q^2)}{\pi R^2\,Q^2} = \frac{1}{\alpha_S(Q^2)}$$

This relation is saturated for

$$Q = Q_s(x) >> \Lambda_{\mathrm{QCD}} \approx 0.2\ \mathrm{GeV}$$

Parton Saturation:Golec-Biernat --Wusthoff dipole model



$$\sigma_{\mathrm{T,L}}^{\gamma^*,P} = \int d^2 r_\perp \int dz\, |\psi_{\mathrm{T,L}}(r_\perp, z, Q^2)|^2\, \sigma_{q,\bar{q},P}(r_\perp, x)$$

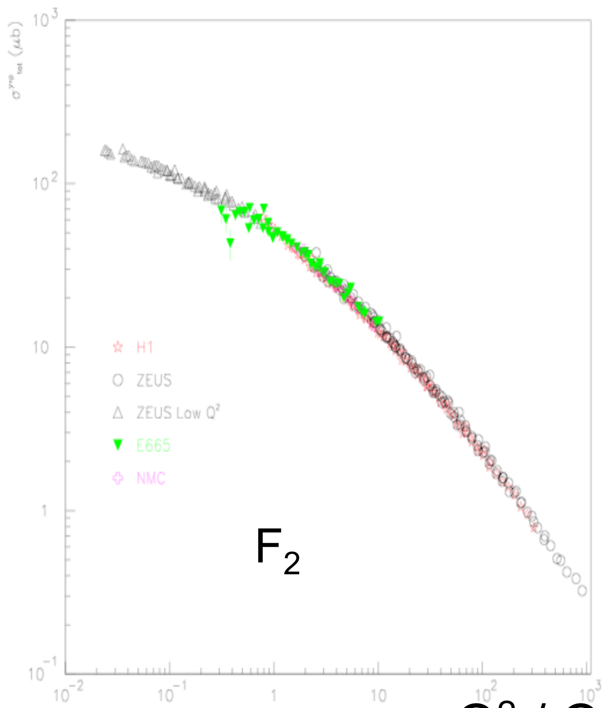
$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0\, \left[1 - \exp\left(-r_\perp^2\, Q_s^2(x)\right)\right]$$

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$$

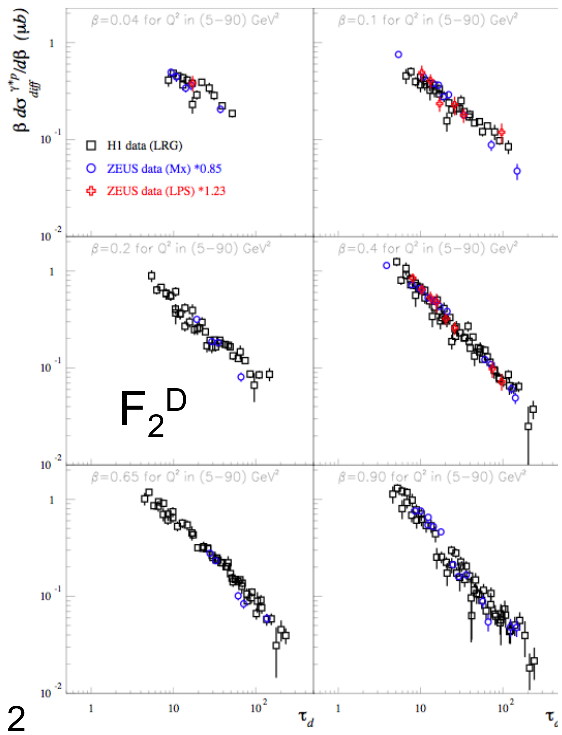
Parameters: Q_0 = 1 GeV; λ = 0.3; x_0 = 3* 10^{-4} ; σ_0 = 23 mb

Evidence from HERA for geometrical scaling

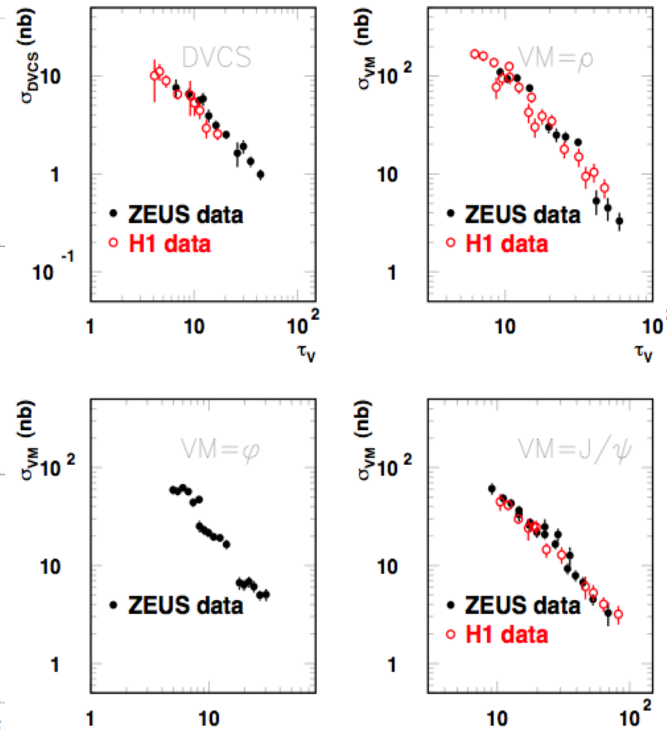
Golec-Biernat, Stasto,Kwiecinski



$\tau = Q^2 / Q_S^2$



τ_D



VM, DVCS τ_V

Marquet, Schoeffel hep-ph/0606079

- **Scaling seen for F_2^D and VM,DVCS for same Q_S as F_2**

Gelis et al., hep-ph/0610435

Nuclear Physics B307 (1988) 34–60
North-Holland, Amsterdam

VIRTUAL PAIR CREATION IN A STRONG BREMSSTRAHLUNG FIELD: A QED model for parton saturation

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Received 10 March 1988

Virtual pair creation in a strong, virtual, bremsstrahlung field is considered in QED as a model for parton saturation. In a weak field the virtual pair density increases quadratically in the external field, however, at large values of the field the number density becomes independent of the strength of that field. A similar effect is found in scalar electrodynamics.

1. Introduction

At small values of the Bjorken-x-variable parton (quark and gluon) number densities are expected to grow rapidly [1]. However, when, say, the gluon distribution in a hadron, $xG(x, Q^2)$, reaches a value as large as $Q^2 r^2/\alpha$, with r the radius of the hadron, these gluons are so densely packed that one expects scattering and annihilation of partons to become important, thus limiting the ultimate number density to be of the size indicated above [1, 3].

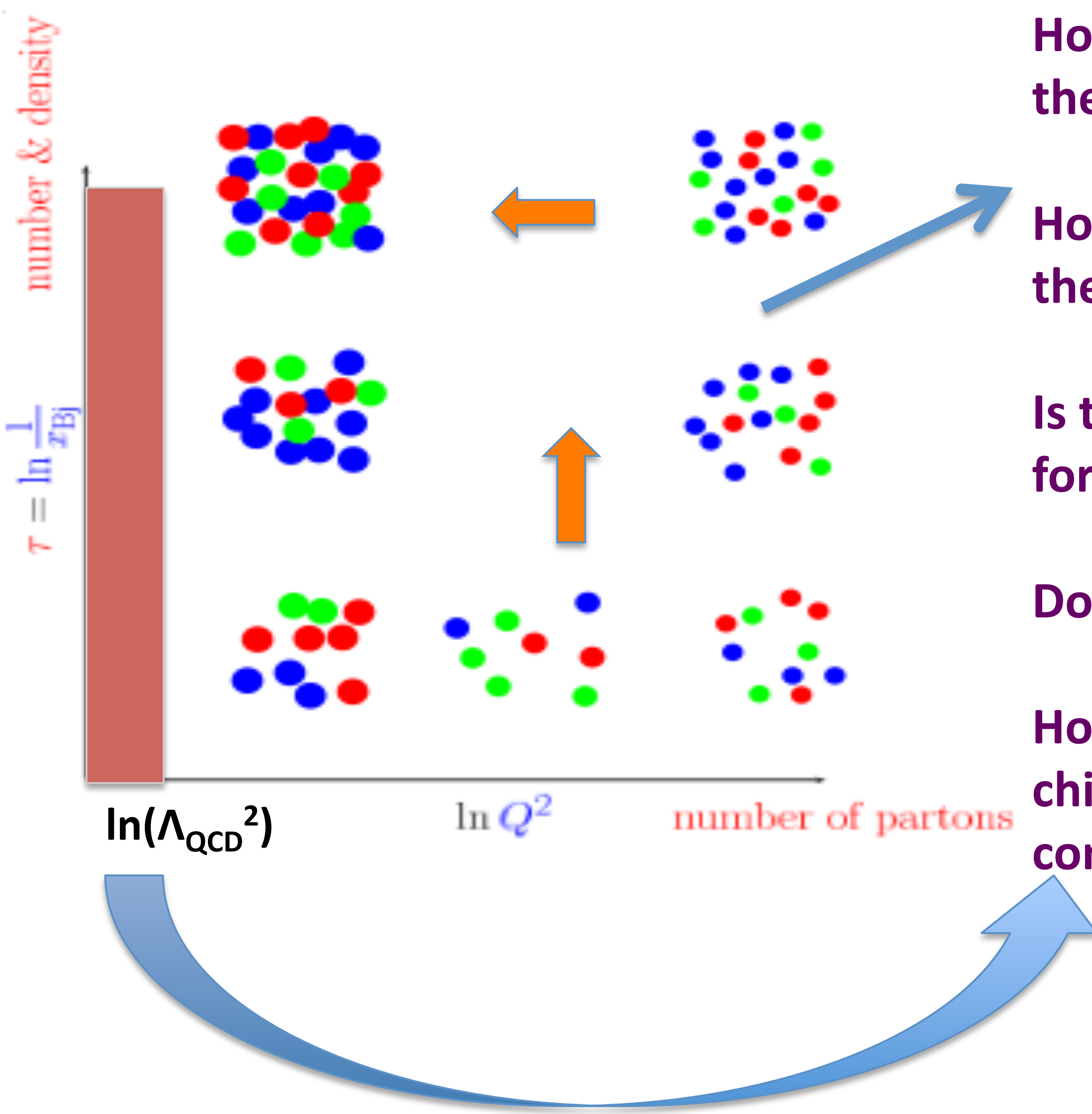
This high density quark-and-gluon system is a most fascinating regime of QCD. On the one hand, if $Q^2 \geq 1$ GeV^2 the coupling, $\alpha(Q^2)$, is small and the usual non-perturbative condensates are unimportant while, on the other hand, the system is strongly interacting because of the high parton densities. That is, this regime of weak coupling but large numbers of partons is a new regime of QCD. Such a high-density parton system occurs in a number of different high-energy processes. (i) In deeply inelastic scattering one can directly measure such high-density systems at small x using the virtual photon as a probe [1, 3]. (ii) In the very early stages of a heavy ion collision such a system is produced over a large transverse area [4]. (iii) Two-jet correlations in high-energy reactions can trigger on local hot spots [5], high parton density regions which are smaller than the radius of a normal hadron.

So far, it has not been possible to theoretically study this high density, non-equilibrium, regime of QCD directly. Lowest order gluon recombinations have been

[1] Work supported in part by the Department of Energy and NSF Grant PHY82-17853, supplemented by NASA.

High energy QCD as a many body system

Many-body dynamics of universal gluonic matter



How does this happen ? What are the right degrees of freedom ?

How do correlation functions of these evolve ?

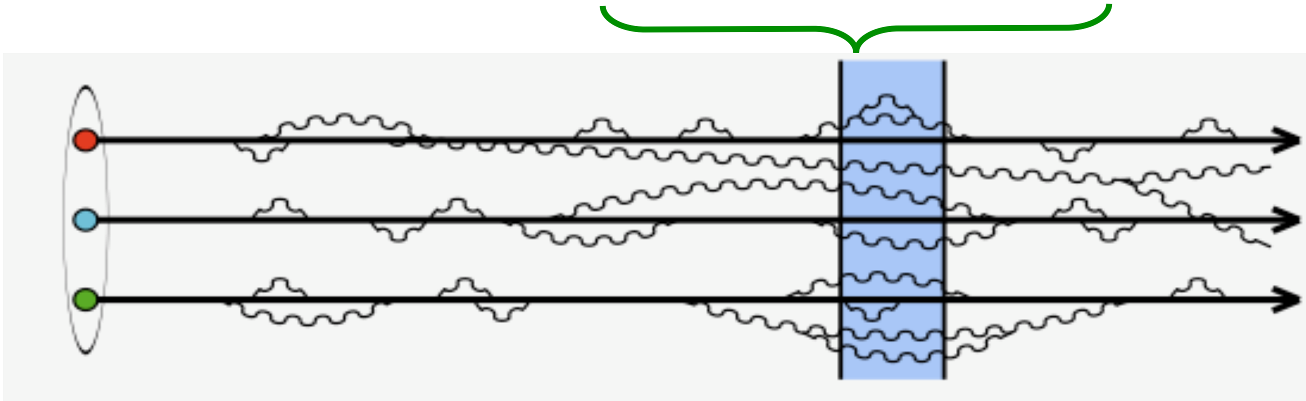
Is there a universal fixed point for the RG evolution of d.o.f

Does the coupling run with Q_s^2 ?

How does saturation transition to chiral symmetry breaking and confinement

The nuclear wavefunction at high energies

|A> = |qqq…q> + … + |qqq…q**gg…g**>



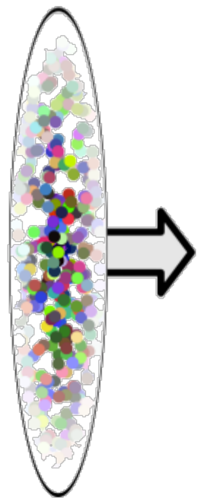
- At high energies, interaction time scales of fluctuations are dilated well beyond typical hadronic time scales

- Lots of short lived (gluon) fluctuations now seen by probe -- proton/nucleus -- dense many body system of (primarily) gluons

- Fluctuations with lifetimes much longer than interaction time for the probe function as static color sources for more short lived fluctuations

Nuclear wave function at high energies is a Color Glass Condensate

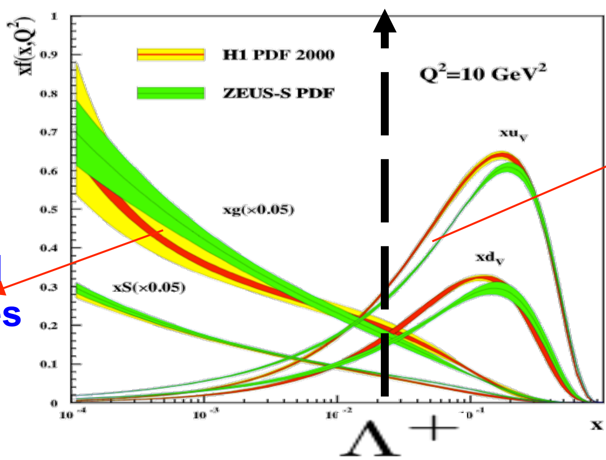
The nuclear wavefunction at high energies



$$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\,\mathbf{gg\ldots gg}\rangle$$

Higher Fock components dominate multiparticle production- construct Effective Field Theory

Dynamical Wee modes

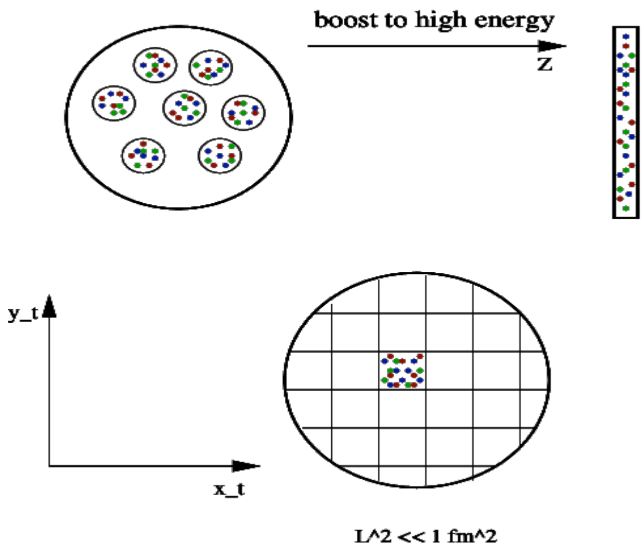


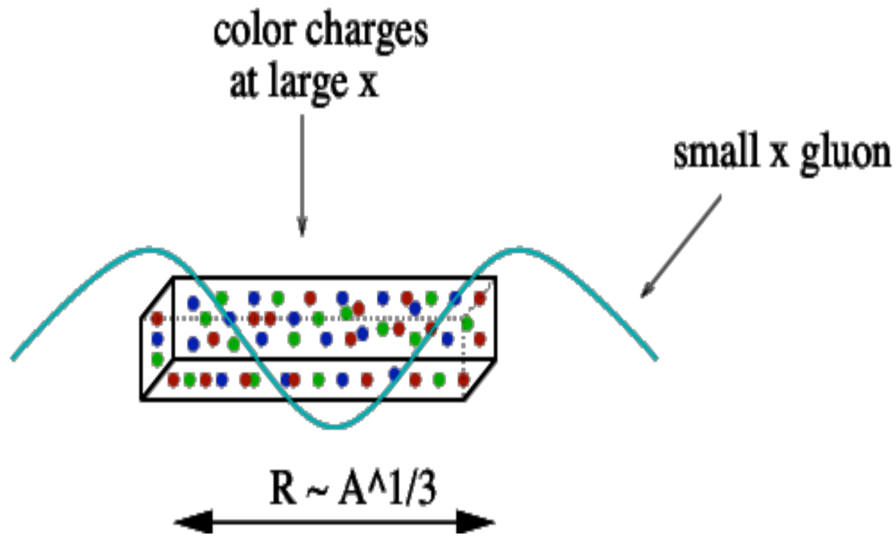
Valence modes- are static sources for wee modes

Born--Oppenheimer LC separation natural for EFT.

RG equations describe evolution of wavefunction with energy

What do sources look like in the IMF ?





$$\lambda_{\text{wee}} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\text{val.}} \equiv \frac{Rm_p}{P^+} => x << A^{-1/3}$$

Wee partons "see" a large density of color sources at small transverse resolutions

Effective Field Theory on Light Front

Susskind
Bardacki-Halpern

Poincare group on LF ⟷ Galilean sub-group of 2D Quantum Mechanics

isomorphism

Eg., LF dispersion relation $P^- = \frac{P_\perp^2}{2P^+}$

Energy (P^-), Momentum ($P_\perp^2$), Mass ($2P^+$)

Large x (P^+) modes: static LF (color) sources ρ^a

Small x ($k^+ \ll P^+$) modes: dynamical fields A_μ^a

McLerran, RV

CGC: Coarse grained many body EFT on LF

$$< P|\mathcal{O}|P > \longrightarrow \int [d\rho^a][dA^{\mu,a}]\, W_{\Lambda^+}[\rho]\, e^{iS_{\Lambda^+}[\rho,A]}\, \mathcal{O}[\rho, A]$$

$W_{\Lambda^+}[\rho]$ non-pert. gauge invariant "density matrix" defined at initial scale Λ_0^+

RG equations describe evolution of W with x

JIMWLK, BK

Classical field of a large nucleus

$$< AA >_\rho = \int [d\rho] A_{\rm cl.}(\rho) A_{\rm cl.}(\rho) W_{\Lambda^+}[\rho]$$

For a large nucleus, A >>1,

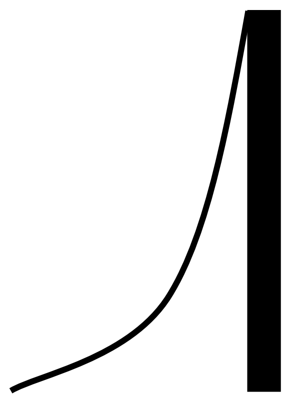
"Pomeron" excitations

"Odderon" excitations

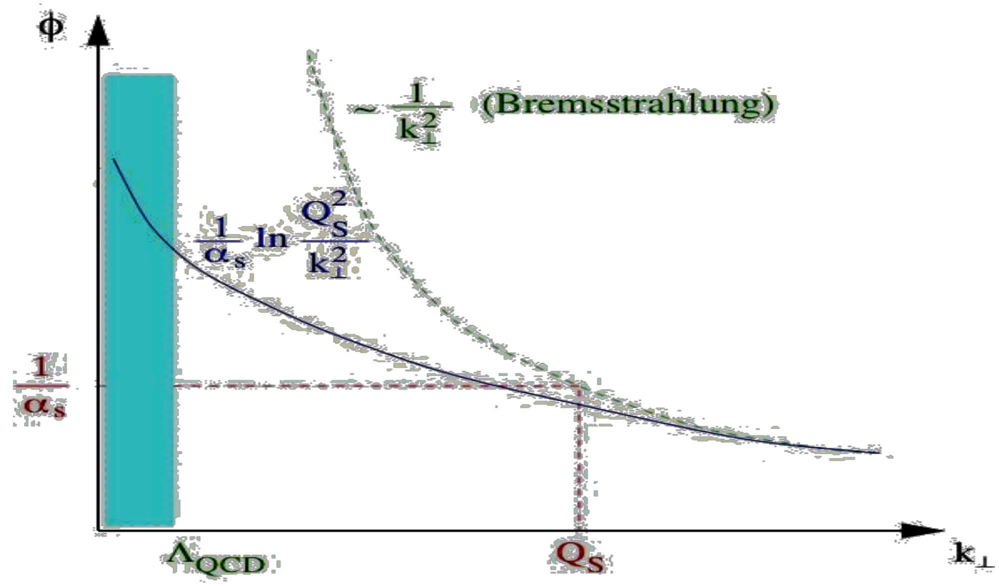
$$W_{\Lambda^+} = \exp\left(-\int d^2x_\perp \left[\frac{\rho^a\rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

McLerran,RV
Kovchegov
Jeon, RV

A_{cl} from $\longrightarrow$ $(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu +}\,\delta(x^-)\,\rho^a(x_\perp)$

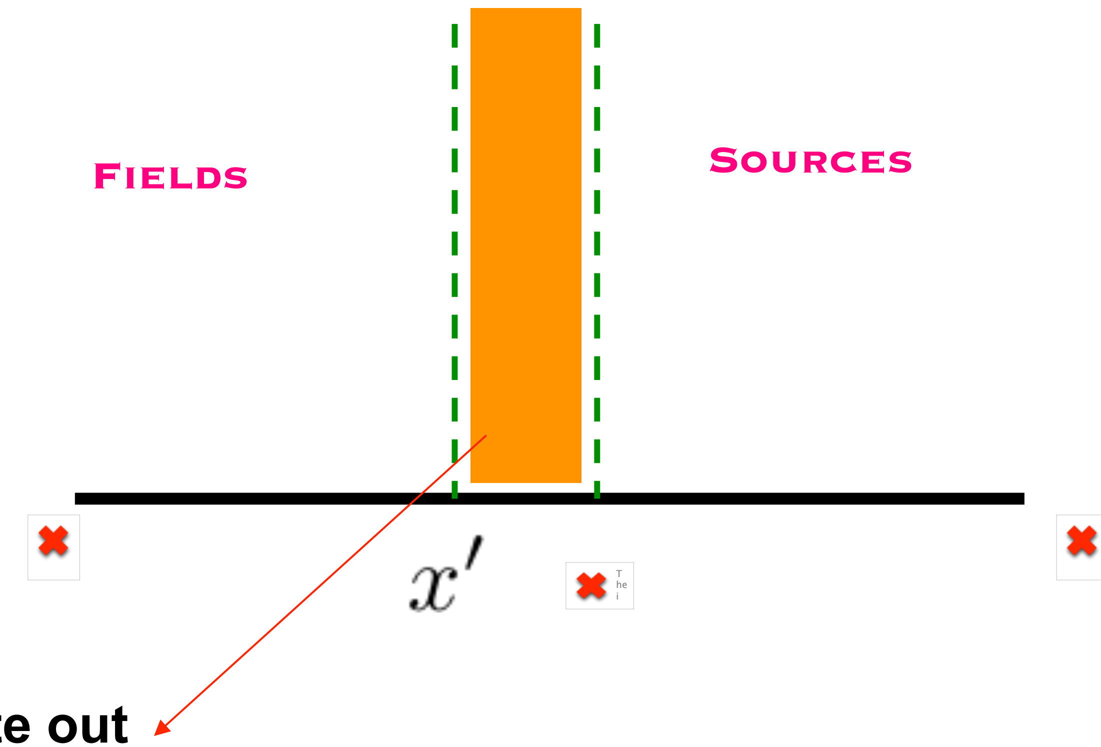


$1/\Lambda_{QCD}$ $\quad$ $2R/\gamma$



Wee parton dist. : $\frac{1}{\Lambda_{\rm QCD}} e^{-\lambda \Delta Y/2}$ determined from RG

Quantum evolution of classical theory: Wilson RG



Integrate out
Small fluctuations => Increase color charge of sources

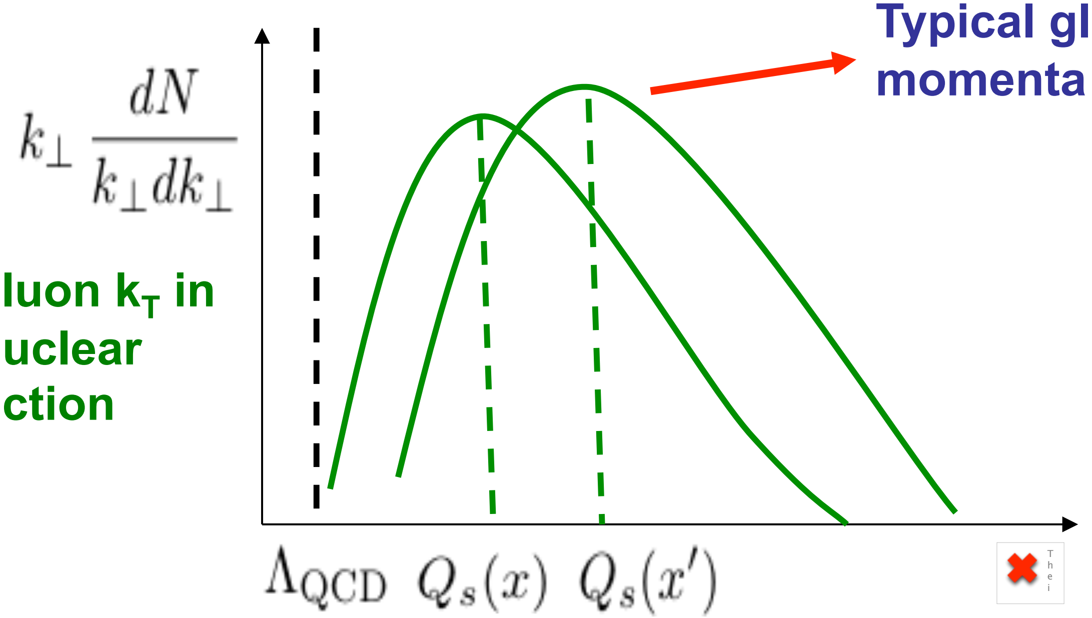
Wilsonian RG equations describe evolution of all N-point correlation functions with energy

JIMWLK Jalilian-marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Saturation scale grows with energy

Typical gluon momenta are large

Typical gluon k_T in hadron/nuclear wave function

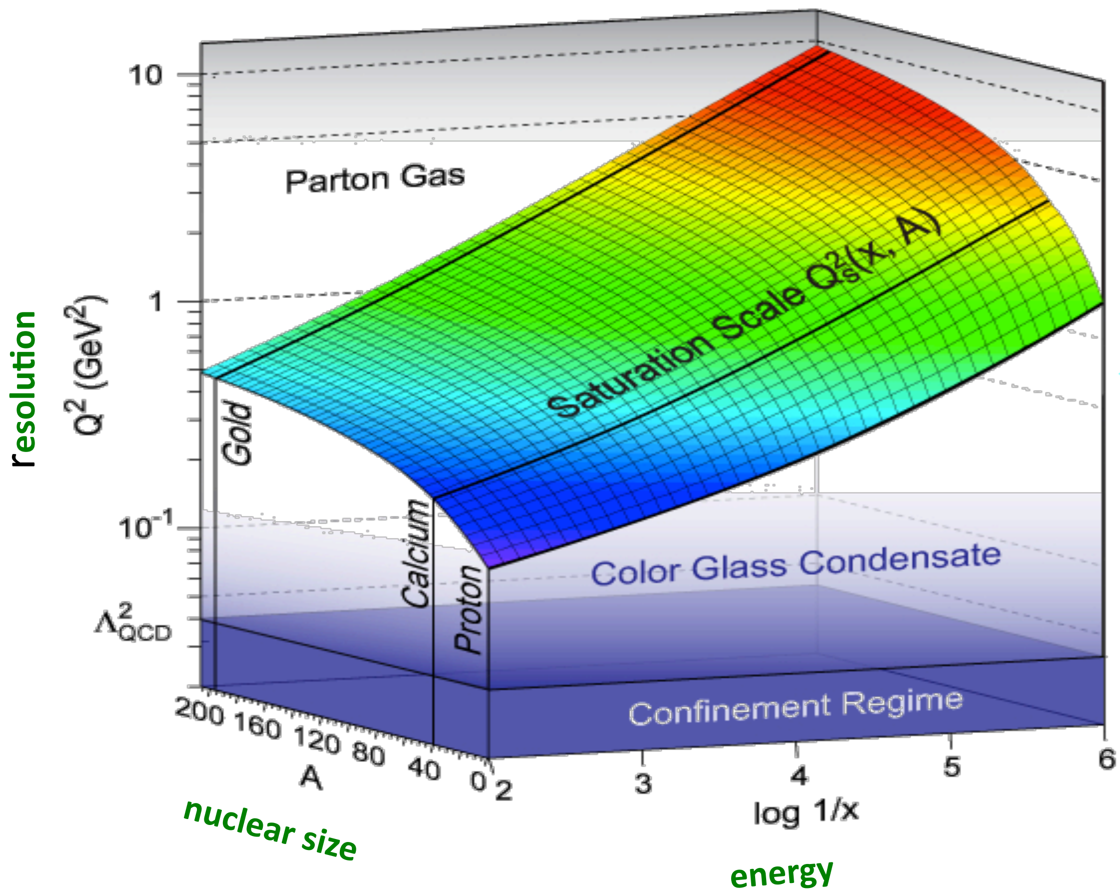


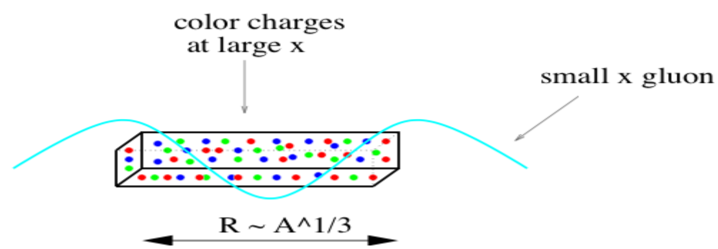
Bulk of high energy cross-sections:

a) **obey dynamics of novel non-linear QCD regime**
b) **Can be computed systematically in weak coupling**

Many-body high energy QCD:
The Color Glass Condensate

Gelis,Iancu,Jalilian-Marian,RV:
Ann. Rev. Nucl. Part. Sci. (2010), arXiv: 1002.0333

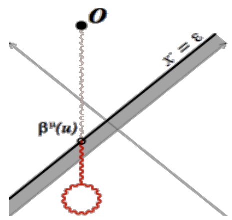




Dynamically generated semi-hard "saturation scale" opens window for systematic weak coupling study of non-perturbative dynamics

JIMWLK RG evolution for a single nucleus:

$$\mathcal{O}_{\rm NLO} = \Bigg($$



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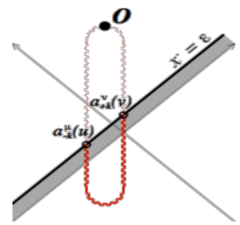


$$\chi(x_\perp, y_\perp)$$

$$\Bigg)\, \mathcal{O}_{\rm LO}$$

$$= \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}\, \mathcal{O}_{\rm LO}$$

(keeping leading log divergences)

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}]\, [\mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO}]$$

$$= \int [d\tilde{\rho}] \left\{ \left[1 + \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H} \right] W_{\Lambda^+} \right\} \mathcal{O}_{\rm LO}$$

LHS independent of Λ^+ **=>**

$$\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H}\, W[\tilde{\rho}]$$

JIMWLK eqn.

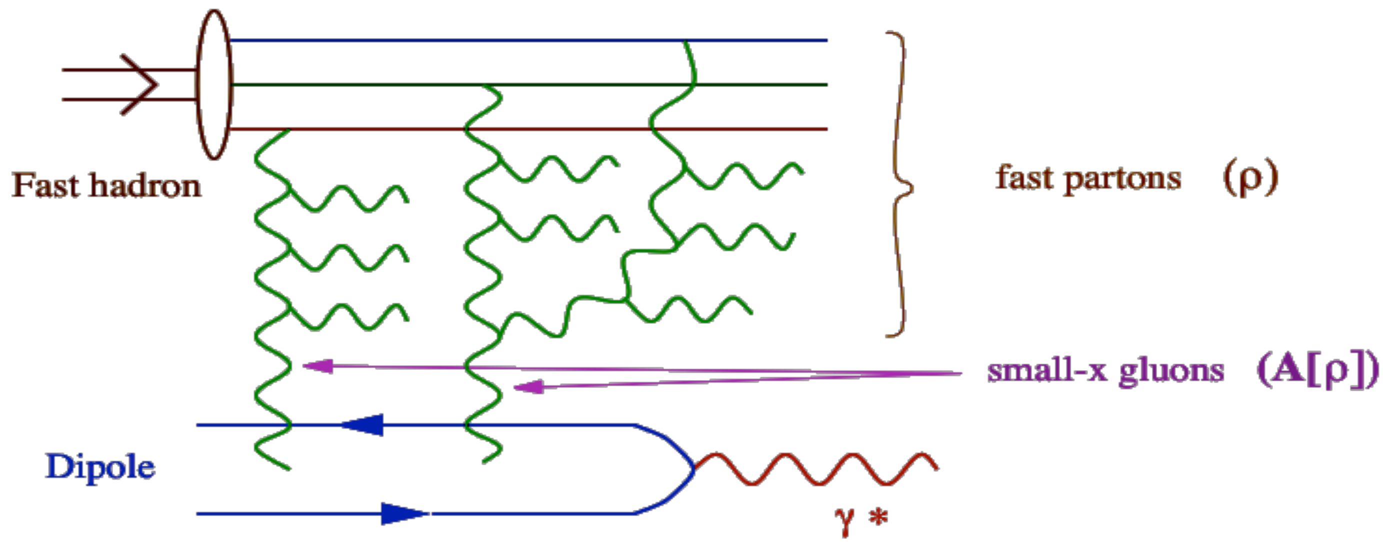
Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner

CGC Effective Theory: B-JIMWLK hierarchy of correlators

$$\frac{\partial}{\partial Y} < O[\alpha] >_Y = < \frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta \alpha_Y^b(y)} O[\alpha] >_Y$$

"time"

"diffusion coefficient"



At high energies, the d.o.f that describe the frozen many-body gluon configurations are novel objects: dipoles, quadrupoles, …

Universal – appear in a number of processes in p+A and e+A; how do these evolve with energy ?

Solving the B-JIMWLK hierarchy

- JIMWLK includes all multiple scattering and leading log evolution in x

- Expectation values of Wilson line correlators at small x satisfy a Fokker-Planck eqn. in functional space

Weigert (2000)

- This translates into a hierarchy of equations for n-point Wilson line correlators

- As is generally the case, Fokker-Planck equations can be re-expressed as Langevin equations – in this case for Wilson lines

Blaizot,Iancu,Weigert
Rummukainen,Weigert

B-JIMWLK hierarchy: Langevin realization

Numerical evaluation of Wilson line correlators on 2+1-D lattices:

$$\left\langle \mathcal{O}[U] \right\rangle_Y = \int D[U]\, W_Y[U]\, \mathcal{O}[U] \longrightarrow \frac{1}{N} \sum_{U \epsilon W} \mathcal{O}[U]$$

Langevin eqn:

Gaussian random variable

$$\partial_Y [V_x]_{ij} = [V_x i t^a]_{ij} \left[\int d^2 y\, [\mathcal{E}^{ab}_{xy}]_k\, [\xi^b_y]_k + \sigma^a_x \right]$$

$$\mathcal{E}^{ab}_{xy} = \left(\frac{\alpha_S}{\pi^2}\right)^{1/2} \frac{(x-y)_k}{(x-y)^2} \left[1 - U^\dagger_x U_y\right]^{ab}$$

"square root" of JIMWLK kernel

$$\sigma^a_x = -i \left(\frac{\alpha_S}{2\pi^2} \int d^2 z \frac{1}{(x-z)^2} \mathrm{Tr}(T^a\, U^\dagger_x U_z) \right)$$

"drag"

- **Initial conditions for V's from the MV model**
- **Daughter dipole prescription for running coupling**

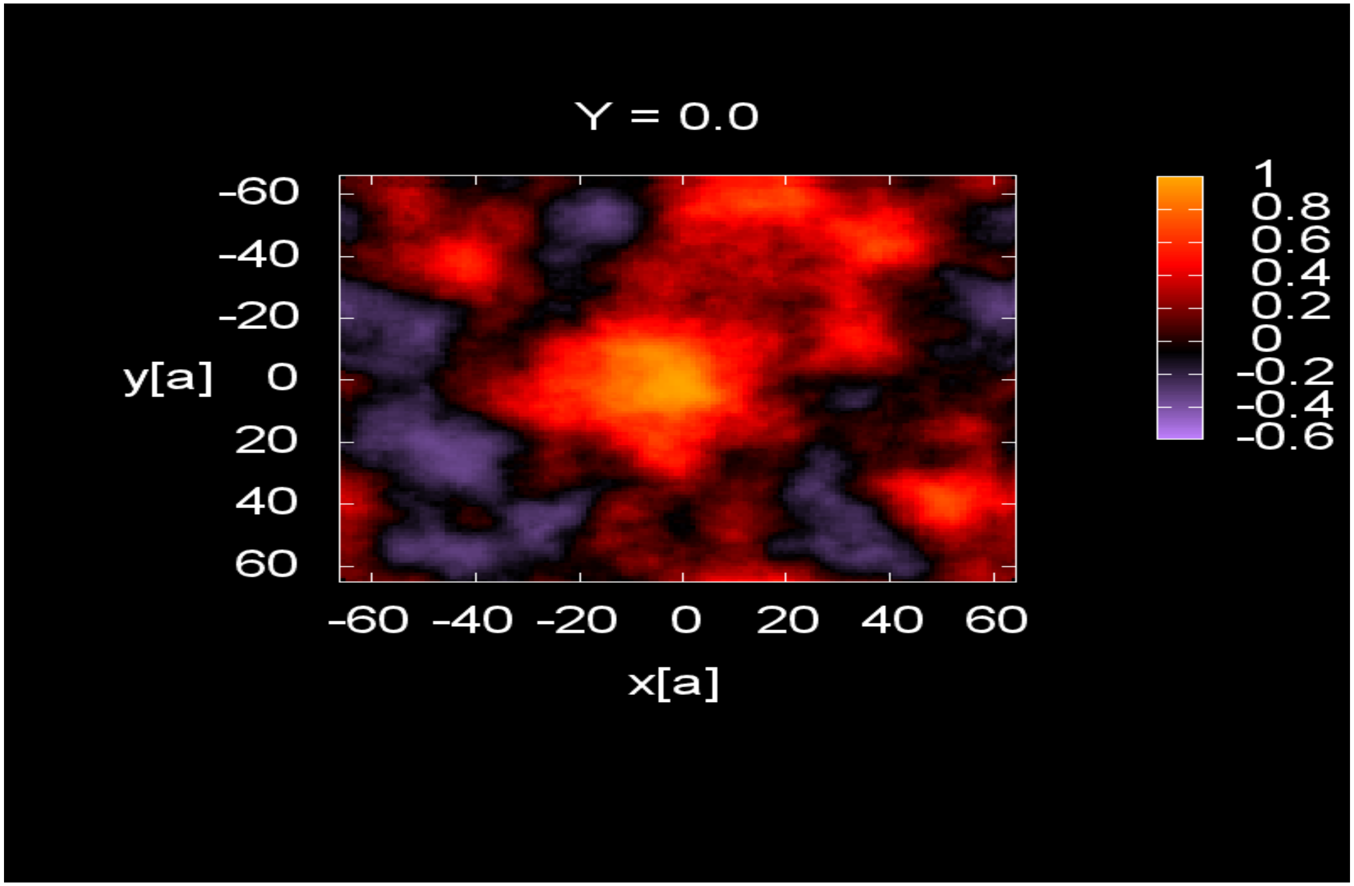
Functional Langevin solutions of JIMWLK hierarchy

Rummukainen,Weigert (2003)

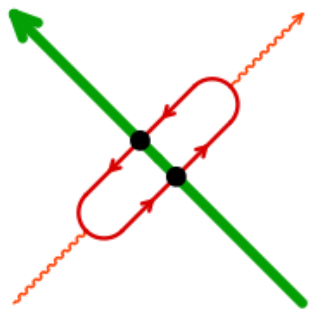
Dumitru,Jalilian-Marian,Lappi,Schenke,RV, PLB706 (2011)219

✓ *We are now able to compute all n-point correlations of a theory of strongly correlated gluons and study their evolution with energy!*

Correlator of Light-like Wilson lines Tr(V(0,0)V^dagger (x,y))



Inclusive DIS: dipole evolution

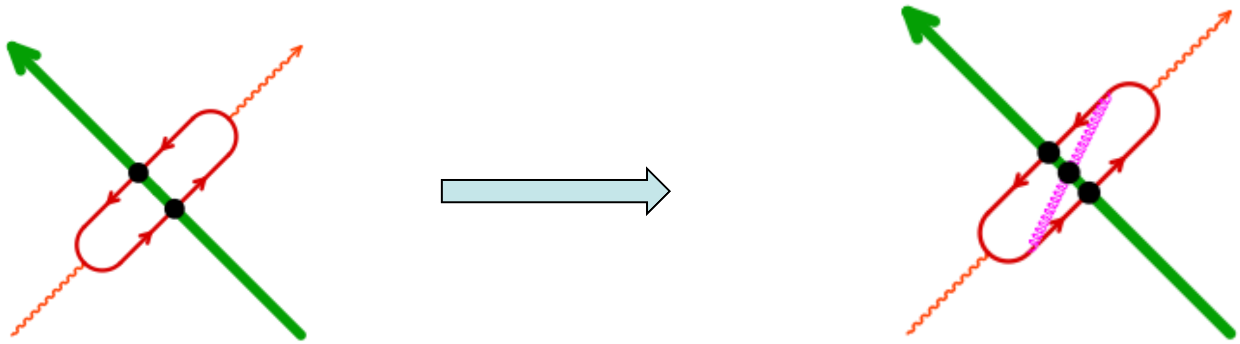


$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_\perp \, |\psi(z, r_\perp)|^2 \sigma_{\text{dipole}}(x, r_\perp)$$

$$\sigma_{\text{dipole}}(x, r_\perp) = 2 \int d^2 b \int [D\rho] \, W_{\Lambda^+}[\rho] \, T(b + \frac{r_\perp}{2}, b - \frac{r_\perp}{2})$$

$$1 - \frac{1}{N_c} \text{Tr} \left(V \left(b + \frac{r_\perp}{2} \right) V^\dagger \left(b - \frac{r_\perp}{2} \right) \right)$$

Inclusive DIS: dipole evolution



B-JIMWLK eqn. for dipole correlator

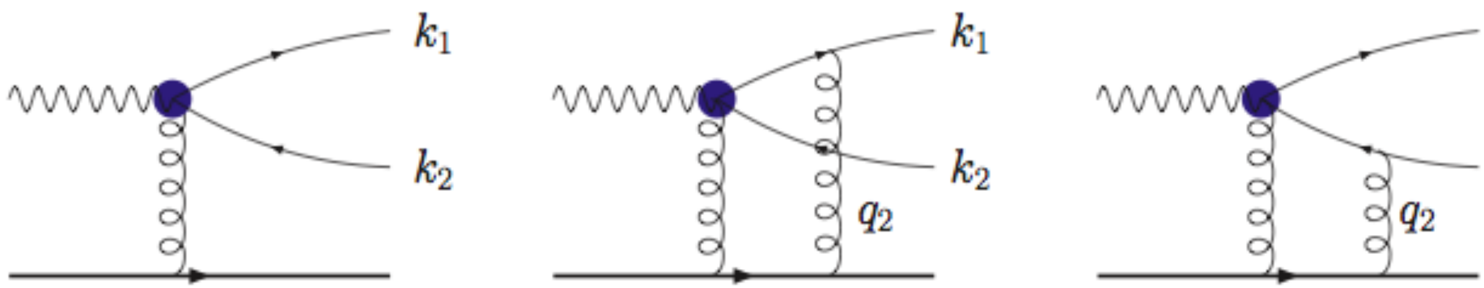
$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\left\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\right\rangle_Y$$

Dipole factorization:

$$\langle \mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger)\rangle_Y \langle \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \qquad N_c \to \infty$$

Resulting closed form eqn. is the Balitsky-Kovchegov eqn.
Widely used in phenomenological applications

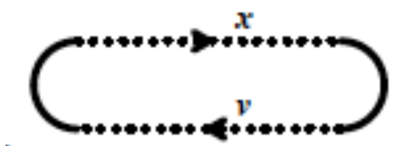
Semi-inclusive DIS: quadrupole evolution

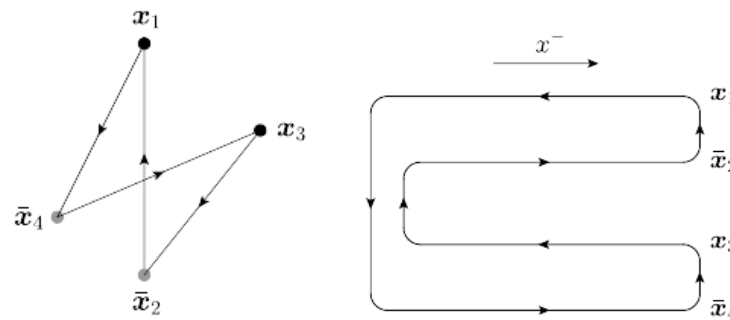


Dominguez,Marquet,Xiao,Yuan (2011)

$$\frac{d\sigma^{\gamma^*_{\mathrm{T,L}}A\to q\bar{q}X}}{d^3k_1d^3k_2} \propto \int_{x,y,\bar{x}\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})}e^{ik_{2\perp}\cdot(y-\bar{y})}\ [1+Q(x,y;\bar{y},\bar{x})-D(x,y)-D(\bar{y},\bar{x})]$$

Semi-inclusive DIS: quadrupole evolution





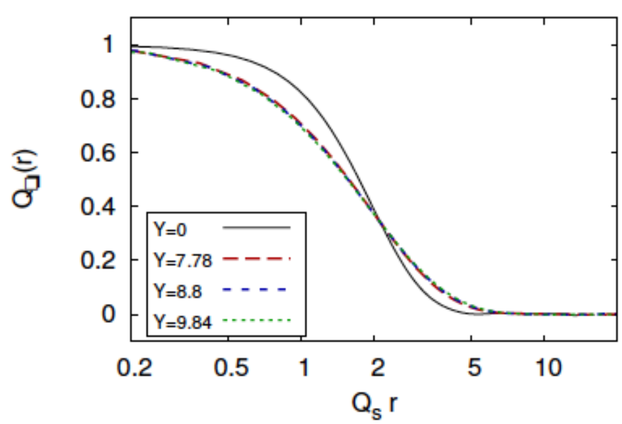
$$D(x,y) = \frac{1}{N_c} \langle \mathrm{Tr}(V_x V_y^\dagger) \rangle_Y$$

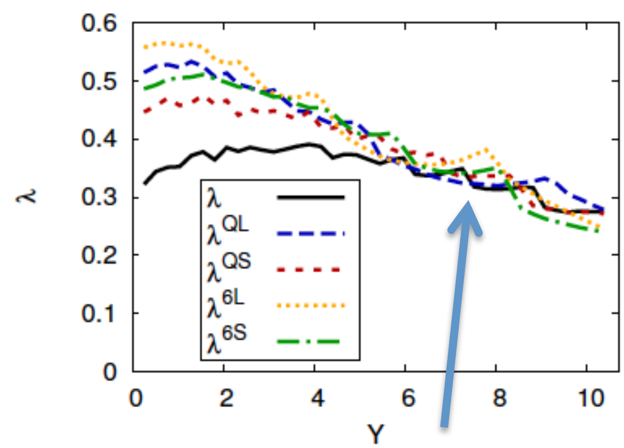
$$Q(x,y;\bar{y},\bar{x}) = \frac{1}{N_c} \langle \mathrm{Tr}(V_x V_{\bar{x}}^\dagger V_{\bar{y}} V_y^\dagger) \rangle_Y$$

RG evolution provides fresh insight into multi-parton correlations

Dumitru,Jalilian-Marian,Lappi,Schenke,RV: arXiv:1108.1764

Quadrupoles, like Dipoles, exhibit Geometrical Scaling

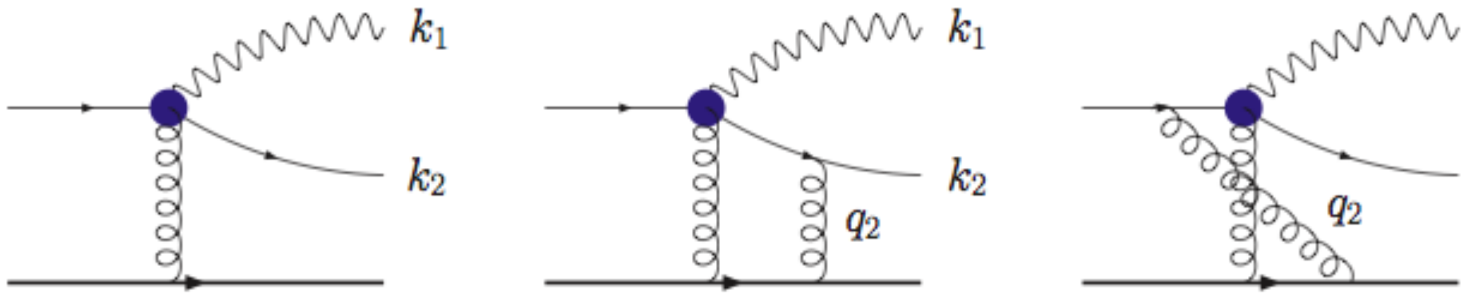




Rate of energy evolution of dipole and quadrupole saturation scales

Iancu,Triantafyllopolous,arXiv:1112.1104

Universality: Di-jets in p/d-A collisions

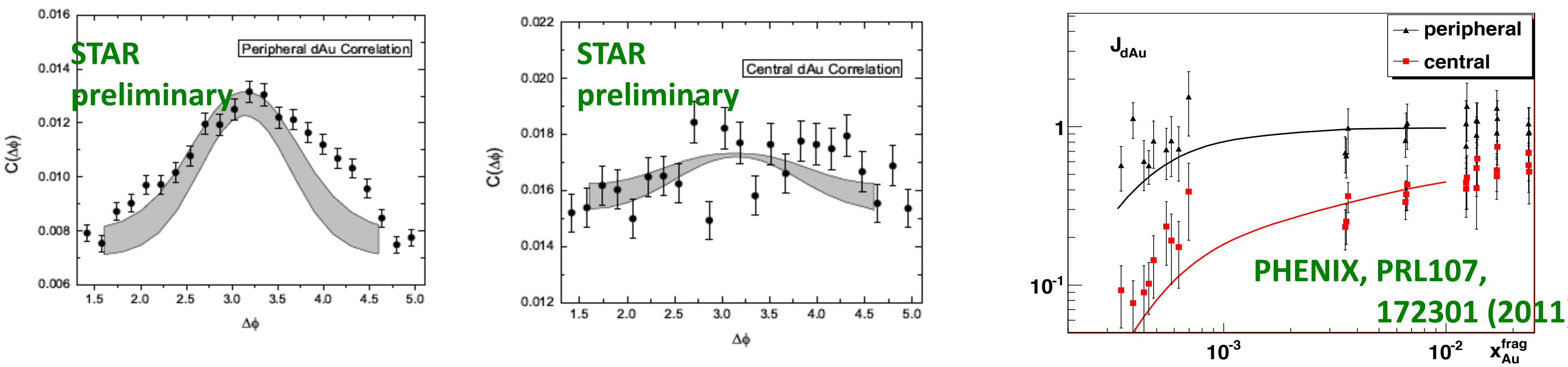


$$\frac{d\sigma^{qA\to qgX}}{d^3k_1d^3k_2} \propto \int_{x,y,\bar{x},\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})}e^{ik_{2\perp}\cdot(y-\bar{y})}\left[S_6(x,y,\bar{x},\bar{y}) - S_4(x,y,v) - ...\right]$$

$$\frac{N_c}{2\,C_F}\left\langle Q(x,y\,\bar{y},\bar{x})D(y,\bar{y}) - \frac{D(x,\bar{x})}{N_c}\right\rangle \quad \frac{N_c}{2\,C_F}\left\langle D(x,y)D(\bar{y},\bar{x}) - \frac{D(x,\bar{x})}{N_c}\right\rangle$$

Jalilian-Marian, Kovchegov (2004)
Marquet (2007), Tuchin (2010)
Dominguez,Marquet,Xiao,Yuan (2011)

Away-side (ΔΦ ~ π) forward-forward di-hadron correlations: measure of strong color fields



Recent computation (Stasto,Xiao,Yuan) includes Pedestal, Shadowing (color screening) and Broadening (multiple scattering) effects in CGC framework

CGC: the state of the art

- Numerical solutions of Leading Log JIMWLK hierarchy – and good analytical approximations Dumitru,Jalilian-Marian,Lappi,Schenke,RV, Iancu,Triantafyllopolous

- Influence of non-Gaussian initial conditions on evolution

  Dumitru,Jalilian-Marian,Petreska,RV,Schenke,Jeon

- Factorization of leading logs in A+A

  Dusling,Gelis,Lappi,RV

- Increasing number of NLO+ computations:
  Structure functions, single inclusive hadron production in p+A

  Balitsky,Chirilli,Kovchegov,Weigert, Gardi, Rummukainen,Kuokkonen,Albacete,Horowitz, Xiao, Yuan, Mueller, Munier, Stasto, Motyka, Triantafyllopolous, Tuchin

- NLO corrections to the BK/JIMWLK kernel beyond running coupling corrections ? Salam,Ciafaloni,Colferai,Stasto,Triantafyllopolous, Sabio-Vera

- Beginnings of global analysis AAMQS collaboration,Rezaiean,Levin,Tribedy,RV