some finite range of control parameter near threshold. Sometimes, particularly in systems with roll type symmetry, the parameters must be chosen quite delicately to yield a stable stationary structure. On the other hand the Taylor-vortex roll structure is found to survive into the strongly chaotic regime, with remarkably clear delineation of the large-scale rolls in a fluid which is strongly chaotic on the small scales. Also we have suggested that, as in equilibrium systems, cellular structures are more robust than striped ones.

Thus, at the laboratory scale we find that the existence of spontaneously broken continuous symmetries, and the relevance of this idea to experimental phenomena (i.e. "Broken Symmetry," not just "broken symmetry" in the language of Anderson, 1981), can be considered to be established.

This conclusion does not address the larger question of whether these structures appearing from nowhere in a dissipative system are an appropriate first step in modeling more exotic (and more interesting!) phenomena such as the emergence of life from the primordial soup. As we have tried to make clear in this review there is no evidence for the existence of any global minimization principles controlling the structure, except as a perturbative statement near threshold. Such a principle would make it easier to generalize from the small scale phenomena of the laboratory (in the sense of number of unit blocks) to the large-scale phenomena of biological complexity. As a modest contribution to the debate we have reviewed tools and ideas which may be relevant to the building blocks of such phenomena. It seems plausible to us (although by no means demonstrated) that reaction-diffusion type mechanisms, perhaps augmented with other phenomena such as forces and flows, *may* provide a mechanism for communicating information encoded at the molecular level up to the cellular level. It is encouraging to note that parameters set by molecular scales can lead naturally to macroscopic length scales, through energy barriers appearing in exponential activation expressions that are large for large molecules. As in many branches of physics, however, it is simply not clear how many conceptual leaps are involved in putting together these building blocks to make the full satisfying edifice.

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APPENDIX A. DERIVATION OF THE AMPLITUDE EQUATION

1. The Swift-Hohenberg equation

We first illustrate the multiple scales approach used to derive amplitude equations on a particularly simple example where the answer can almost be guessed without calculation. We consider the Swift-Hohenberg model (3.27) in two dimensions,

$$\partial_t u = \varepsilon u - (\nabla^2 + q_0^2)^2 u - u^3 , \tag{A1}$$

where for clarity we introduce the scale q_0 in the original equation. Near the bifurcation, i.e. for $|\varepsilon| \ll 1$, we wish to separate fast and slow scales for x and t. We therefore define

$$X = \varepsilon^{1/2} x , \quad Y = \varepsilon^{1/4} y , \quad T = \varepsilon t , \tag{A2}$$

anticipating the anisotropic scaling for roll systems in Eq. (4.3). We will consider $u(\mathbf{x},t)$ to be a product of functions of fast and slow variables. From the chain rule for differentiation we therefore must make the replacements

$$\partial_x \to \partial_x + \varepsilon^{1/2} \partial_X , \quad \partial_y \to \partial_y + \varepsilon^{1/4} \partial_Y , \quad \partial_t \to \partial_t + \varepsilon \partial_T , \tag{A3}$$

etc., where on the right-hand side ∂_x, ∂_y, and ∂_t now only act on the rapid dependences. The differential operator in Eq. (A1) becomes

$$(\nabla^2 + q_0^2) \to \partial_x^2 + 2\varepsilon^{1/2} \partial_x \partial_X + \varepsilon \partial_X^2 + \varepsilon^{1/2} \partial_Y^2 + q_0^2 , \tag{A4}$$

where we assume no rapid y dependence, i.e. we are expanding about a roll state with wave vector along x. Let us now set

$$u = \varepsilon^{1/2} u_0 + \varepsilon u_1 + \varepsilon^{3/2} u_2 , \tag{A5}$$

and insert (A4) and (A5) into Eq. (A1). Collecting orders of $\varepsilon^{1/2}$ we find

$$\varepsilon^{1/2}: \quad (\partial_x^2 + q_0^2)^2 u_0 = 0 , \tag{A6a}$$

$$\varepsilon: \quad (\partial_x^2 + q_0^2)^2 u_1 = -2(2\partial_x \partial_X + \partial_Y^2)(\partial_x^2 + q_0^2) u_0 , \tag{A6b}$$

$$\varepsilon^{3/2}: \quad (\partial_x^2 + q_0^2)^2 u_2 = -2(2\partial_x \partial_X + \partial_Y^2)(\partial_x^2 + q_0^2) u_1 - [\partial_T - 1 + u_0^2 + (2\partial_x \partial_X + \partial_Y^2)^2 + 2\partial_X^2 (\partial_x^2 + q_0^2)] u_0 . \tag{A6c}$$

The first two equations are solved by setting

$$u_0(\mathbf{x},t)=A_0(X,Y,T)\,e^{iq_0x}+\text{c.c.}\ , \tag{A7a}$$

$$u_1(\mathbf{x},t)=A_1(X,Y,T)\,e^{iq_0x}+\text{c.c.}\ , \tag{A7b}$$

since Eq. (A7a) implies

$$(\partial_x^2+q_0^2)\,u_0=0\ , \tag{A8}$$

so that the rhs of Eq. (A6b) vanishes identically. The last equation (A6c), has a nontrivial rhs so the linear operator on the left must be inverted. Since this operator has vanishing eigenvalues we must impose a *solvability condition,* requiring that the vector on the right should not drive any eigenvector with zero eigenvalue (Stakgold, 1979). The simplest example of such a condition occurs for a matrix equation

$$\underset{\approx}{M}V=G\ . \tag{A9}$$

Let C_0 be an eigenvector of the adjoint $\underset{\approx}{M}^\dagger$ with zero eigenvalue. Then clearly

$$(C_0,\underset{\approx}{M}V)=(\underset{\approx}{M}^\dagger C_0,\ V)=(C_0,G)=0\ , \tag{A10}$$

i.e. G is orthogonal to C_0. The Fredholm theorem states that Eq. (A10) is also a sufficient condition, i.e. Eq. (A9) has a solution for V if and only if G is orthogonal to all zero eigenvectors of $\underset{\approx}{M}^\dagger$. This theorem also holds if $\underset{\approx}{M}$ is replaced by a differential operator.

For Eq. (A6c) the operator

$$\mathcal{L}_0=(\partial_x^2+q_0^2)^2 \tag{A11}$$

is self-adjoint and its zero eigenvectors are $\exp(\pm iq_0x)$. The Fredholm theorem thus requires that the coefficients of these terms on the rhs of Eq. (A6c) should vanish identically, i.e. A_0 should satisfy the solvability condition

$$[\partial_T-1+3|A_0|^2+(2iq_0\partial_X+\partial_Y^2)^2]\,A_0=0\ . \tag{A12}$$

Returning to unscaled units

$$A(\mathbf{x},t)=\varepsilon^{1/2}\,A_0(X,Y,T)\ , \tag{A13}$$

we have the general amplitude equation (4.3)

$$\tau_0\partial_t A=\varepsilon A+\xi_0^2[\partial_x-(i/2q_0)\partial_y^2]^2\,A-g_0|A|^2A\ , \tag{A14}$$

with

$$\tau_0=1\ ,\quad \xi_0^2=4q_0^2\ ,\quad g_0=3\ . \tag{A15}$$

[The value of g_0 depends on the arbitrary normalization in Eq. (A7a).]

2. The Kuramoto-Sivashinsky equation

Let us consider the damped KS model (3.31) in one dimension

$$\partial_t u=-\eta u-\partial_x^2u-\partial_x^4u-u\partial_x u\ , \tag{A16}$$

which we rewrite as

$$\partial_t u=\varepsilon u-(\partial_x^2+q_0^2)^2u-u\partial_x u\ , \tag{A17}$$

with $\varepsilon=1/4-\eta$, $q_0^2=1/2$. The equations corresponding to (A6) are

$$\varepsilon^{1/2}:\quad (\partial_x^2+q_0^2)^2\,u_0=0\ , \tag{A18a}$$

$$\varepsilon:\quad (\partial_x^2+q_0^2)^2\,u_1=-4\partial_x\partial_X(\partial_x^2+q_0^2)\,u_0-u_0\partial_x u_0\ , \tag{A18b}$$

$$\begin{aligned}\varepsilon^{3/2}:\quad (\partial_x^2+q_0^2)^2\,u_2=&-4\partial_x\partial_X(\partial_x^2+q_0^2)\,u_1\\ &-[\partial_T-1+4\partial_x^2\partial_X^2+2\partial_X^2(\partial_x^2+q_0^2)]\,u_0-u_0\partial_x u_1-u_1\partial_x u_0-u_0\partial_X u_0\ .\end{aligned} \tag{A18c}$$

These equations are solved by setting

$$u_0(u,t)=A_0(X,T)\,e^{iq_0x}+\text{c.c.}\ , \tag{A19a}$$

$$u_1(x,t)=A_1(X,T)\,e^{iq_0x}+B_1(X,T)\,e^{2iq_0x}+\text{c.c.}\ , \tag{A19b}$$

$$u_2(u,t)=A_2(X,T)\,e^{iq_0x}+B_0+B_2\,e^{2iq_0x}+B_3e^{3iq_0x}+\text{c.c.} \tag{A19c}$$

The function $B_1(X,T)$ can be calculated by setting the coefficient of $\exp(2iq_0x)$ in Eq. (A18b) to zero, yielding

$$B_1=-i\,(9q_0^3)^{-1}\,A_0^2\ . \tag{A20}$$

With these choices Eqs. (A19a) and (A19b) are satisfied identically, and Eq. (A19c) once again requires a solvability condition, which is obtained by setting the coefficient of $\exp(iq_0x)$ on the rhs to zero. The result is

$$[\partial_T-1-4q_0^2\partial_X^2+(9q_0^2)^{-1}|A_0|^2]\,A_0=0\ , \tag{A21}$$

which leads to the general amplitude equation (A14) with

$$\varepsilon=1/4-\eta\ ,\quad \tau_0=1\ ,\quad \xi_0^2=4q_0^2=2\ ,\quad g_0=(9q_0^2)^{-1}=2/9\ . \tag{A22}$$

3. Rayleigh-Bénard convection

A much more involved calculation is necessary to derive the amplitude equation (A14) from the hydrodynamic equations (8.3) for Rayleigh-Bénard convection. We will once again use the method of multiple-scales perturbation theory. An alternative approach, perhaps more familiar to physicists, is the mode expansion or pro-