The AdS/CFT dual of F-maximization

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Based on ArXiV: 1302.7310 with Silviu Pufu

L'hymne Nationale de Canada:

Au Canada, on dort en pyjamas

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Ultrabrief Summary:

We constructed AdS/CFT dual of 2+1 dim ABJM theory, deformed by mass-like terms for the chiral matter fields– (Framework in which Jafferis, 1012.3210 developed F-maximization thm.)

Gravity dual gives valid description for large N and strong coupling:

$$N >> k^5 \qquad \lambda = N/k \quad \text{fixed, large}$$

Usually QFT methods are impotent at strong coupling, but in this case, the Free Energy can be calculated using localization.

We calculate the same Free Energy in the gravity dual and thus obtain an unusually detailed match between gravity dual $\leftrightarrow$ QFT (n.b. conformal symmetry is broken).

Usual ideas of AdS/CFT apply in this application:

i) symmetries of bulk gravity and boundary gauge theories match.

ii) map between classical fields of bulk thy. and gauge invariant composite operators of the QFT.

iii) find classical soltn. of gravity thy. with AAdS metric and other fields.

iv) asymptotics at AdS bdy. determine sources and vets. for QFT operators.

But ∃ many complications and subtleties:

a) ABJM thy. has hidden symmetries and subtle dynamics
To be described with poetic license. *PL*

b) the deformed thy. is on Euclidean S^3, so we need Euclidean signature SG dual.

c) For Chern-Simons level $k = 1, 2$, undeformed ABJM has hidden enhanced $\mathcal{N} = 6 \ \rightarrow \ \mathcal{N} = 8$ SUSY. Deformation breaks $\mathcal{N} = 8 \ \rightarrow \ \mathcal{N} = 2$. So we search for gravity dual as a consistent $\mathcal{N} = 2$ truncation of gauged $\mathcal{N} = 8$. [De Wit + Nicolai] The operator map involves issues of SO(8) triality.

d) In the end we find simple truncation and a classical solution involving metric and three complex scalars. But $m^2_{\mathrm{scalar}} = -2/L^2$, in the mass range with quantization ambiguity [Breitenlohner + DZF].

e) In AdS/CFT, $S_{\mathrm{on-shell}}$ of the gravity dual is the bridge to the QFT. But $S_{\mathrm{on-shell}}$ diverges! Method of Holographic Renormalization [Skenderis] determines ∞ counterterms, but we must add a finite CT to maintain SUSY. Because of alternate quantization, it is not $S_{\mathrm{on-shell}}$ which is the QFT generating function, but its Legendre transform [Klebanov + Witten].

ABJM thy. and its deformation:

Undeformed ABJM is an $\mathcal{N} = 6$ superconf. Chern-Simons thy. with product gauge group $U(N)_k \times U(N)_{-k}$ coupled to 4 bi-fundamental chiral multiplets: $Y^A(x),\ \chi^A(x)$ in 4 of SU(4) global symmetry (plus conjugates $Y^\dagger_A(x),\ \chi^\dagger_A(x),$).

Write down $\mathcal{N} = 2$ Euclidean $U(1)_k$ toy model with one $Y(x),\ \chi(x)$:

A. Euclidean C-S action on S^3 of radius a.

$$S_{\rm CS} = \frac{ik}{4\pi}\int d^3x\ \left[\epsilon^{ijk}A_i\partial_j A_k - \sqrt{g}(\lambda^\dagger\lambda + 2i\sigma D)\right]\ ,$$

where A_i is gauge potential, $\sigma,\ D$ are real scalar auxiliaries, and $\chi,\ \chi^\dagger$ are 2-component complex spinors (also auxiliary).
No local degrees of freedom, but a rich dynamics after coupling to matter fields.

B. Matter action

$$S_{1/2} = \int d^3x \sqrt{g} \Big(D^i Y^* D_i Y + \sigma^2 Y^* Y + i\chi^\dagger \gamma^i D_i \chi + i\chi^\dagger \sigma \chi - F^* F$$
$$+ \lambda^T (i\sigma_2) Y^* \chi + \chi^\dagger (i\sigma_2) Y \lambda^* - D Y^* Y + \frac{3}{4a^2} Y^* Y \Big) ,$$

Can recognize $\sigma = A_4$ from dimensional reduction.
The algebra of $\mathcal{N} = 2$ SUSY on S^3 is

$$\{Q, Q^\dagger\} = \gamma^i J_i + (1/a) R$$

The J_i are generators of $SU(2)_L$ or $SU(2)_R$ of S^3.
R is the $U(1)_R$ charge with values

$$R_Y = 1/2,\ R_\chi = -1/2,\ R_F = -3/2 .$$

Jafferis found a deformation that breaks $\mathcal{N}=2$ superconf. to $\mathcal{N}=2$ SUSY on S^3 with new R-charge R' = R + T, and T is a $U(1)$ flavor symmetry under which $Y,\ \chi,\ F$ carry $T=\Delta-1/2$. Deformed action:

$$\begin{aligned} S_\Delta &= S_{1/2}+\int d^3x\,\sqrt{g}[-\frac{1}{a^2}(\Delta-\frac{1}{2})(\Delta-\frac{3}{2})Y^*Y \\ &\qquad +\frac{1}{a}(\Delta-\frac{1}{2})(\chi^\dagger\chi-\sigma Y^*Y)]\,. \end{aligned}$$

Curious point: due to SUSY on S^3, the coupling constants of the permutation are fully determined by the deformed R-charges.

Jafferis considered non-abelian and multi-flavor extensions of this framework:
i. The free energy $F(\Delta_j)$ can be calculated by localization.
ii. $F(\Delta_j)$ is stationary, i.e. $\partial F(\Delta_j)/\partial\Delta_j=0$ at superconformal fixed points of the RG-flow.
iii. The stationary point is a maximum [Closset et al, 1205.4142].
This is the principle of F-maximization.

Extension to ABJM with 4 bi-fundamentals Y^A

Must assign general R-charges $R[Y^A] = 1/2 + T[Y^A]$.
Consistent with constraint $\sum_A R[Y^A] = 2$ due to quartic superpotential $W = Tr(Y^1 Y^2 Y^3 Y^4)$. *PL*

To accomodate this, choose 3 traceless diagonal 4×4 matrices T^α (i.e. in Cartan of SU(4))

$$\begin{aligned} T^1 &= \mathrm{diag}(1,1,-1,-1) \\ T^2 &= \mathrm{diag}(1,-1,1,-1) \\ T^3 &= \mathrm{diag}(1,-1,-1,1). \end{aligned}$$

Then

$$R[Y^A] = \frac{1}{2} + (\delta_1 T^1 + \delta_2 T^2 + \delta_3 T^3)_{AA}$$

The matrices T^α define 3 independent flavor $U(1)'s$ that mix with the canonical $U(1)_R$ in the deformed theory.

Define 3 bilinear Bose and Fermi operators:

$$\begin{aligned} \mathcal{O}_B^\alpha &= Tr(Y^\dagger T^\alpha Y) \qquad \text{scale dim } 1 \\ \mathcal{O}_V^\alpha &= Tr(\chi^\dagger T^\alpha \chi - Y^\dagger T^\alpha Y \sigma) \quad \text{scale dim } 2 \end{aligned}$$

Lagrangian of deformed ABJM:

$$\begin{aligned} \mathcal{L}_\Delta &= \mathcal{L}_{1/2} + \frac{1}{a^2} \sum_\alpha (\delta_\alpha + \delta_1 \delta_2 \delta_3 / \delta_\alpha) \mathcal{O}_B^\alpha \\ &+ \frac{1}{a} \sum_\alpha \delta_\alpha \mathcal{O}_F^\alpha . \qquad *PL* \end{aligned}$$

The Free Energy of the deformed ABJM theory is calculated by matrix model methods at large N. [Jafferis et al 1103.1181]:

$$F = \frac{4\sqrt{2}\pi N^{3/2}}{3} \sqrt{\prod_A R[Y^A]},$$

The gravity dual of ABJM

Dual of unperturbed ABJM, for $k = 1$ is $AdS_4 \times S^7$ soltn. of 11D SG. Its low energy limit is gauged $\mathcal{N} = 8, \; d = 4$ SG.

So we look for the dual of deformed ABJM as a consistent $\mathcal{N} = 2$ truncation of $\mathcal{N} = 8$. The truncation should contain:
i) 3 cx. scalars $z^\alpha(\rho, x)$ whose Re and Im parts are dual to the QFT ops. $\mathcal{O}_B^\alpha(x)$ and $\mathcal{O}_F^\alpha(x)$, plus
ii) 4 bulk gauge fields dual to canonical R-current R_μ and 3 U(1) currents J_μ^α.

Thus we look for an $\mathcal{N} = 2$ truncation of $\mathcal{N} = 8$ with:
i) gravity multiplet $g_{\mu\nu}, \; \psi)\mu^{\alpha=1,2}, \; A_\mu^0$, plus

ii) 3 abelian vector multiplets: $z^\alpha, \; \chi^\alpha, \; A_\mu^\alpha$.

An elaborate SO(8) group theory argument $\implies$ operator map:

$$\begin{aligned}
\mathcal{O}_B^1 + i\mathcal{O}_F^1 &\quad\longleftrightarrow\quad \Sigma_{1234}\,, & \qquad \mathcal{O}_B^1 - i\mathcal{O}_F^1 &\quad\longleftrightarrow\quad \Sigma_{5678}\,,\\
\mathcal{O}_B^2 + i\mathcal{O}_F^2 &\quad\longleftrightarrow\quad \Sigma_{1256}\,, & \qquad \mathcal{O}_B^2 - i\mathcal{O}_F^2 &\quad\longleftrightarrow\quad \Sigma_{3478}\,,\\
\mathcal{O}_B^3 + i\mathcal{O}_F^3 &\quad\longleftrightarrow\quad \Sigma_{1278}\,, & \qquad \mathcal{O}_B^3 - i\mathcal{O}_F^3 &\quad\longleftrightarrow\quad \Sigma_{3456}\,.
\end{aligned}$$

$\Sigma_{abcd}(x)$ are anti-sym 4th rank self-dual tensors of SO(8).

We then set

$$\begin{aligned}
\Sigma_{1234} &= z^1, & \qquad \Sigma_{5678} &= \tilde{z}^1\,,\\
\Sigma_{1256} &= z^2 & \qquad \Sigma_{3478} &= \tilde{z}^2\,,\\
\Sigma_{1278} &= z^3, & \qquad \Sigma_{3456} &= \tilde{z}^3\,,
\end{aligned}$$

Find very simple $\mathcal{N}=2$ truncation deduced from full $\mathcal{N}=8$ results of De Wit + Nicolai:

$$S = \frac{1}{8\pi G_4}\int d^4x\,\sqrt{-g}\left[\frac{1}{2}R - \sum_{\alpha=1}^{3}\frac{|\partial_\mu z^\alpha|^2}{(1-|z^\alpha|^2)^2} + \frac{1}{L^2}\left(-3+\sum_{\alpha=1}^{3}\frac{2}{1-|z^\alpha|^2}\right)\right],$$

1. Scalar kinetic action is Kähler σ-model action on 3-copies of H^2= Poincaré disc. (Known as *stu* model.)

2. Potential $V(|z^\alpha|)$ gives conformal value of scalar mass $m^2=-2/L^2$.

3. The fields z^α can have two alternate quantizations which means that $\mathrm{Re}z^\alpha$ and $\mathrm{Im}z^\alpha$ can be sources for QFT ops. of scale dim. 1 or 2, as we need.

.

$$S = \frac{1}{8\pi G_4}\int d^4x\,\sqrt{-g}\Bigg[\frac{1}{2}R - \sum_{\alpha=1}^{3}\frac{|\partial_\mu z^\alpha|^2}{(1-|z^\alpha|^2)^2} + \frac{1}{L^2}\left(-3+\sum_{\alpha=1}^{3}\frac{2}{1-|z^\alpha|^2}\right)\Bigg],$$

4. Bulk gauge field actions can be found, but not needed because the physics of interest is captured by classical solutions involving only $g_{\mu\nu}(\rho)$ and $z^\alpha(\rho)$.

5. The dynamics of 3 z^α fields is *uncoupled*! Since $\nexists$ vectors, we effectively have an $\mathcal{N}=1$ SG theory. Can express $V(z,\tilde{z})$ in terms of holomorphic $W(z^\alpha)$ using standard relation

$$V = e^K\left(\nabla_\alpha W\;K^{\alpha\bar{\beta}}\nabla_{\bar{\beta}}\bar{W} - 3W\bar{W}\right).$$

We find $W = (1+z^1z^2z^3)/L$. Curious that V is uncoupled, but W is coupled. The BPS eqtns. will be coupled!

Digression on Euclidean SUSY

It is well known that in Euclidean SUSY, "formally conjugate" (Weyl) fermions ψ and ψ^* are not "actually" cx. conjugate, but independent.

The reason is that their trfs under the Euclidean group $SO(4) = SU(2)_L \times SU(2)_R$ are not conjugate. Instead, $\psi \to U\psi$ and $\tilde{\psi} \equiv i\sigma_2\psi^* \to V\tilde{\psi}$, where $U,\ V$ are independent matrices of $SU(2)$.

But SUSY trfs. relate formally conjugate fermions to formally conjugate bosons, e.g.

$$\delta\psi = \gamma^i \partial_i z \tilde{\epsilon} \qquad \delta\tilde{\psi} = \gamma^i \partial_i \tilde{z} \epsilon \,.$$

Therefore, formally conjugate bosons need not be actually conjugate. In Euclidean SG, even the metric tensor $g_{\mu\nu}$ can be complex!

S^3-sliced domain walls from BPS eqtns.

We use two different coord. systems, both with explicit S^3 factors.

$$
\begin{aligned}
1.\ \ ds^2 &= L^2(d\rho^2 + e^{2A(\rho)}d\Omega_3^2)\\
2.\ \ ds^2 &= L^2 e^{2B(r)}(dr^2 + r^2 d\Omega_3^2)\,,
\end{aligned}
$$

1. AdS bdy. at $\rho = \infty$, simplest for AdS/CFT physics.
2. conformally flat. BPS eqtns. can be solved by Mathematica.

Euclidean reference solution– maximally symmetric H_4:

$$
\begin{aligned}
ds^2 &= L^2(d\rho^2 + \sinh^2\rho d\Omega_3^2) \qquad\qquad r = \tanh(\rho/2)\\
&= \frac{4L^2}{(1-r^2)^2}(dr^2 + r^2 d\Omega_3^2)\,, \quad \leftarrow \mathrm{bdy.at} r = 1
\end{aligned}
$$

Please admire our BPS eqtns.!

$$
\begin{aligned}
\delta\psi_\mu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\sigma_{[a}\bar\sigma_{b]} + \frac{1}{4}\sum_{\alpha=1}^{3}\frac{\tilde z^\alpha\partial_\mu z^\alpha - z^\alpha\partial_\mu\tilde z^\alpha}{1-z^\alpha\tilde z^\alpha}\right)\epsilon \\
&+ \frac{1+z^1z^2z^3}{2L\prod_{\beta=1}^{3}\sqrt{1-z^\beta\tilde z^\beta}}\sigma_\mu\tilde\epsilon = 0\,, \\
\delta\tilde\psi_\mu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\bar\sigma_{[a}\sigma_{b]} - \frac{1}{4}\sum_{\alpha=1}^{3}\frac{\tilde z^\alpha\partial_\mu z^\alpha - z^\alpha\partial_\mu\tilde z^\alpha}{1-z^\alpha\tilde z^\alpha}\right)\tilde\epsilon \\
&+ \frac{1+\tilde z^1\tilde z^2\tilde z^3}{2L\prod_{\beta=1}^{3}\sqrt{1-z^\beta\tilde z^\beta}}\bar\sigma_\mu\epsilon = 0\,, \\
\delta\chi^\alpha &= \sigma^\mu\partial_\mu z^\alpha\tilde\epsilon - \frac{(1-z^\alpha\tilde z^\alpha)\left(z^\alpha+\tilde z^1\tilde z^2\tilde z^3/\tilde z^\alpha\right)}{\prod_{\beta=1}^{3}\sqrt{1-z^\beta\tilde z^\beta}}\epsilon = 0\,, \\
\delta\tilde\chi^\alpha &= \bar\sigma^\mu\partial_\mu\tilde z^\alpha\epsilon - \frac{(1-z^\alpha\tilde z^\alpha)\left(\tilde z^\alpha+z^1z^2z^3/z^\alpha\right)}{\prod_{\beta=1}^{3}\sqrt{1-z^\beta\tilde z^\beta}}\tilde\epsilon = 0\,.
\end{aligned}
$$

These highly coupled eqtns. have a remarkably simple solution.

The solution

$$
\begin{aligned}
z^{\alpha}(r) &= c_{\alpha} f(r) \qquad\qquad f(r) = \frac{(1-r)^2}{1+c_1c_2c_3r^2} \\
\tilde{z}^{\alpha}(r) &= -\frac{c_1c_2c_3}{c_{\alpha}} f(r) \\
ds^2 &= \frac{4L^2(1+c_1c_2c_3)(1+c_1c_2c_3r^4)}{(1+c_1c_2c_3r^2)^2}\,\frac{dr^2+r^2d\Omega_3^2}{(1-r^2)^2}\,.
\end{aligned}
$$

Features:

i) common radial function for all z^{α} $\tilde{z}^{\alpha}$

ii) solution has 3 arbitrary complex constants c_{α}.

iii) so $\tilde{z} \neq \zeta^*$, as advertised.

iv) metric is conformal to H_4

v) we check that BPS soltns. also satisfy Lagrangian EOM's and we find Killing spinors.

Results and Interpretation

1. Change radial variable $r = 1 - 2e^{-\rho} + 2e^{-2\rho} + \dots$.
Find expected bdy. behavior for scalar mass $m^2 = -2/L^2$:

$$
\begin{aligned}
z^\alpha(\rho) &= a^\alpha e^{-\rho} + b^\alpha e^{-2\rho} + \dots \\
\tilde{z}^\alpha(\rho) &= \tilde{a}^\alpha e^{-\rho} + \tilde{b}^\alpha e^{-2\rho} + \dots .
\end{aligned}
$$

with

$$
\begin{aligned}
a^\alpha &= \frac{4c_\alpha}{1 + c_1 c_2 c_3} a^\alpha \qquad & b^\alpha = -\frac{8c_\alpha(1 - c_1 c_2 c_3)}{(1 + c_1 c_2 c_3)^2} \\
\tilde{a}^\alpha &= -\frac{c_1 c_2 c_3}{c_\alpha^2} \qquad & \tilde{b}^\alpha = -\frac{c_1 c_2 c_3}{c_\alpha^2}\, b^\alpha .
\end{aligned}
$$

In "normal" AdS/CFT, dominant asymptotic terms, $a^\alpha,\ \tilde{a}^\alpha$ would be sources for the 3+3 =6 QFT ops. $\mathcal{O}_B^\alpha,\ \mathcal{O}_F^\alpha$.

Not correct here b/c the 3 $\mathcal{O}_B^\alpha$ have scale dim. 1,
but the 3 $\mathcal{O}_F^\alpha$ have scale dim. 2.

From scaling under $\rho \to \rho + \delta\rho$, we see that $a^\alpha,\ \tilde{a}^\alpha$ can be sources for dim. 2 ops., and $b^\alpha,\ \tilde{b}^a$ can be sources for dim. 1 ops.

Since the $\mathcal{O}_F^\alpha$ are pseudoscalar, we take $a^\alpha - \tilde{a}^\alpha$ as their sources. This suggests that $b^\alpha + \tilde{b}^a$ are sources for the scalar $\mathcal{O}_B^\alpha$.

A more precise argument

Equivalent to requirement of SUSY for the source term *PL*

$$\int \ldots e^{\int d^3x \left(g_\alpha \mathcal{O}_B^\alpha + f_\alpha \mathcal{O}_F^\alpha\right)}$$

Result: If $f_\alpha \sim a^\alpha,\ \tilde{a}^\alpha$ are the sources for $\mathcal{O}_F^\alpha$, then

$$g_\alpha \sim b^\alpha - \frac{\tilde{a}^1 \tilde{a}^2 \tilde{a}^3}{\tilde{a}^\alpha} + \tilde{b}^\alpha - \frac{a^1 a^2 a^3}{a^\alpha}$$

are the sources for $\mathcal{O}_B^\alpha$. Can rewrite $a, \tilde{a}, b, \tilde{b}$ in terms of the c_α parameters of the soltns. Then compare with deformation of ABJM

$$\int d^3x \left((\delta_\alpha + \delta_1 \delta_2 \delta_3 / \delta_\alpha) \mathcal{O}_B^\alpha + \sum_\alpha \delta_\alpha \mathcal{O}_F^\alpha \right)$$

to identify

$$\delta_\alpha = k \frac{c_\alpha + c_1 c_2 c_3 / c_\alpha}{1 + c_1 c_2 c_3} \,.$$

Finally we have a detailed correspondence between parameters of the QFT and parameters of the SG solution.

Where are we in the AdS/CFT story?

i) discussed the relevant structure of the $\mathcal{N} = 2$ bulk theory .
ii) presented and solved the BPS eqtns. to find $z^\alpha(\rho),\ g_{\mu\nu}(\rho)$.
iii) new SUSY argument to determine sources of $\mathcal{O}^\alpha_B,\ \mathcal{O}^\alpha_F$.
iv) What is left?
$S_{\mathrm{on-shell}}$ Bridge between QFT and its gravity dual. It is divergent and requires Holographic Renormalization, a systematic procedure to cancel divergences by counterterms which are:
i) local invariants formed from bulk fields at the cutoff $\rho = \rho_0$.
ii) UNIVERSAL– they must cancel ∞'s of *a*ll soltns of EOMs, not just the BPS soltns.
iii) Procedure leaves open the possibility of finite CT's
Important because procedure is not always compatible with SUSY!

A SUSY Counterterm

For flat-sliced domain walls in general $\mathcal{N}=1$ Kähler SG model:

$$ds^2 = L^2(d\rho^2 + e^{2A(\rho)}dx^i dx^i) \qquad z^\alpha(\rho, x^i) \to z^\alpha(\rho)\,.$$

Lorentzian sig. action + partial integration (a la Bogomolny)

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[\frac{1}{2}R - K_{\alpha\bar\beta}\partial_\mu z^\alpha \partial_\mu \bar z^{\bar\beta} - V(z,\bar z)\right] \\ V &= |\nabla W|^2 - 3|W|^2\,. \end{aligned}$$

Find sum of quadratic factors + surface term:
Quadratic factors are BPS eqtns:

$$\begin{aligned} \partial_r z^\alpha &= e^{K/2}\sqrt{W/\bar W}\,\nabla^\alpha \bar W \\ \partial_r A &= -e^{K/2}|W|\,. \end{aligned}$$

Surface term:

$$-\int d^4x 2e^{K/2}|W|\,.$$

Surface term can be dropped for some purposes, but because fields in AdS/CFT fall at definite rate at the bdy. One must include CT:

$$\begin{aligned} S_{\mathrm{SUSY}} &= \int d^3x (e^{3A} e^{K/2} |W|)_{\rho=\rho_o} \\ &= = \int d^3x \sqrt{h} \left[1 + \frac{1}{2} \sum_{\alpha} |z^{\alpha}|^2 + \frac{1}{2} (z^1 z^2 z^3 + c.c.) + \ldots \right] \end{aligned}$$

The first two terms are conventional ∞ CT's, but the third is a new finite CT req'd by SUSY,
Remaining step: The free energy is the Legendre transform of $S_{\mathrm{on-shell}}$

Finale

Our goal is to match QFT free energy:

$$F = \frac{4\sqrt{2}\pi N^{3/2}}{3} \sqrt{\prod_A R[Y^A]},$$

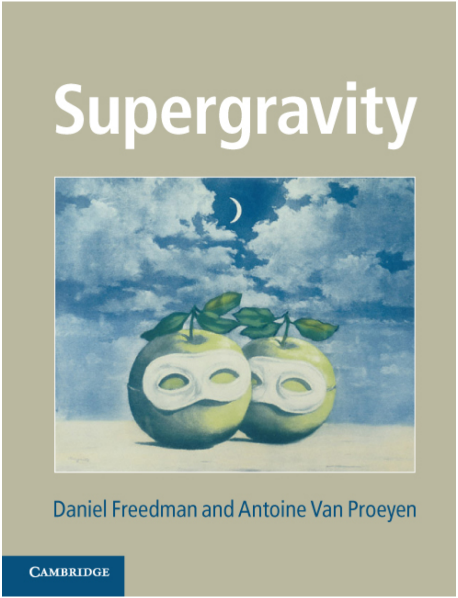
where $R[Y^1] = \frac{1}{2} + \delta_1 + \delta_2 + \delta_3$., etc.

We have discussed: $z^\alpha = c_\alpha f(r), \qquad \tilde{z}^\alpha = -\frac{c_1 c_2 c_3}{c_\alpha} f(r)$

and identified $\delta_\alpha = k \frac{c_\alpha + c_1 c_2 c_3 / c_\alpha}{1 + c_1 c_2 c_3}$. The bulk thy. Legendre trf gives:

$$F = \frac{\sqrt{2}\pi N^{3/2}}{3} \frac{(1-c_1^2)(1-c_2^2)(1-c_3^2)}{(1+c_1 c_2 c_3)^2}$$

Find perfect 3-parameter match if we take $k = 1/2$.



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