Marc Grisaru & The Supergravity Eden

S. James Gates, Jr.

University System of Maryland Regents Professor,
John S. Toll Professor of Physics, and
Center for String and Particle Theory Director

University of Maryland, Physics Department, Rm. 4121, College Park, MD 20742-4111
http://umdphysics.umd.edu/people/faculty/135-gates.html

Part I: Reminiscent of SG Eden

Part II: A Scarecrow's Lament

Part I: Reminiscent of SG Eden


Is STUPOR-SYMMETRY
Achtbein a Damtp Squid?
SAT. 8:30
ZUMINO
I AM A SEA MONSTER
DON'T FORGET TO DOUBLE-CZECH YOUR RESULTS
wot! no graviting?
NO HOPE
THE END
FINIS
LET'S LOOK AT ANOMALIES
IT DOESN'T WORK!!!
MARTIAN ROCHECKERS
Grisaru
Hawking
ACHTBEIN ARACHTNID
THIS MESSAGE.

“Harvard is losing all of its superheroes,”
Howard Schnitzer, 1980

SUPERSPACE

or One thousand and one lessons in supersymmetry

S. James Gates, Jr.

Massachusetts Institute of Technology, Cambridge, Massachusetts
(Present address: *University of Maryland, College Park, Maryland)*
gatess@wam.umd.edu

Marcus T. Grisaru

Brandeis University, Waltham, Massachusetts
(Present address: *McGill University, Montreal, Quebec)*
grisaru@physics.mcgill.ca

Martin Roček

State University of New York, Stony Brook, New York
rocek@insti.physics.sunysb.edu

Warren Siegel

University of California, Berkeley, California
(Present address: *State University of New York)*
warren@wcgall.physics.sunysb.edu

Superspace is the greatest invention since the wheel [1] .

Preface

Said Ψ to Φ, Ξ, and Υ: “Let’s write a review paper.” Said Φ and Ξ: “Great idea!” Said Υ: “Naaa.”

But a few days later Υ had produced a table of contents with 1001 items.

Ξ, Φ, Ψ, and Υ wrote. Then didn’t write. Then wrote again. The review grew; and grew; and grew. It became an outline for a book; it became a first draft; it became a second draft. It became a burden. It became agony. Tempers were lost; and hairs; and a few pounds (alas, quickly regained). They argued about “;” vs. “.”, about “which” vs. “that”, “˜” vs. “ˆ”, “γ” vs. “Γ”, “+” vs. “-”. Made bad puns, drew pictures on the blackboard, were rude to their colleagues, neglected their duties. Bemoaned the paucity of letters in the Greek and Roman alphabets, of hours in the day, days in the week, weeks in the month. Ξ, Φ, Ψ and Υ wrote and wrote.

* * *

Jim Gates

Jim Gates

Warren Siegel

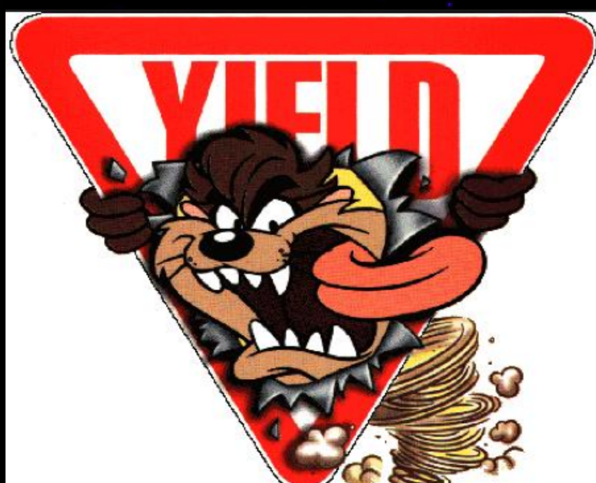
Jim Gates

Warren Siegel

Martin Rocek

Jim Gates

Warren Siegel



Martin Rocek

Robert Brandenberger

Michael Peskin

Michael Peskin

Ulf Lindstrom

Michael Peskin

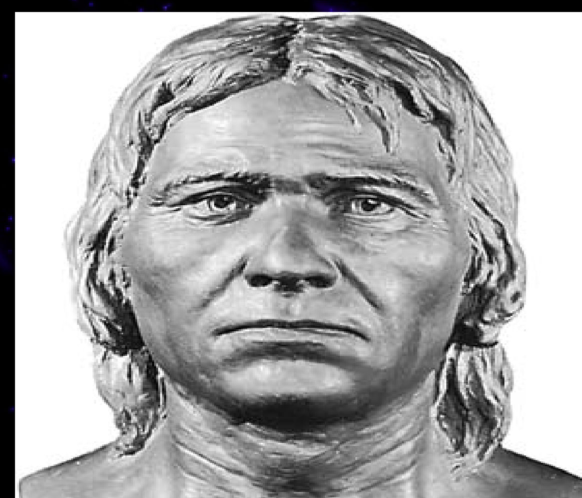
Ulf Lindstrom

Renata Kallosh

Michael Peskin

Ulf Lindstrom

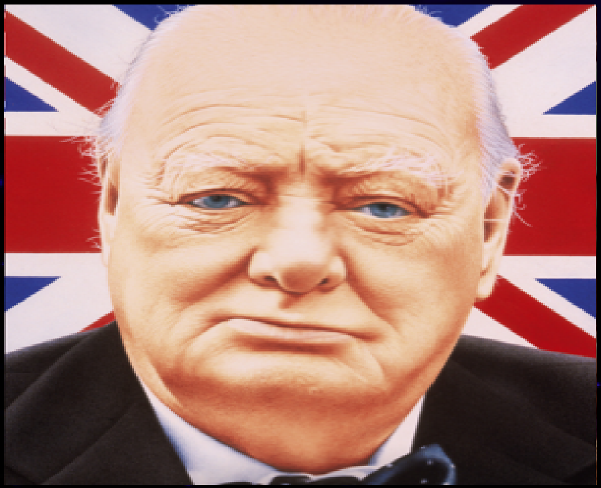
Renata Kallosh



Luca Mezincescu

Paul Townsend

Paul Townsend



Kelly Stelle

Paul Townsend

Kelly Stelle

Bernard de Wit

P. van Nieuwenhuizen

Dan Freedman

Dan Freedman

Michael Duff

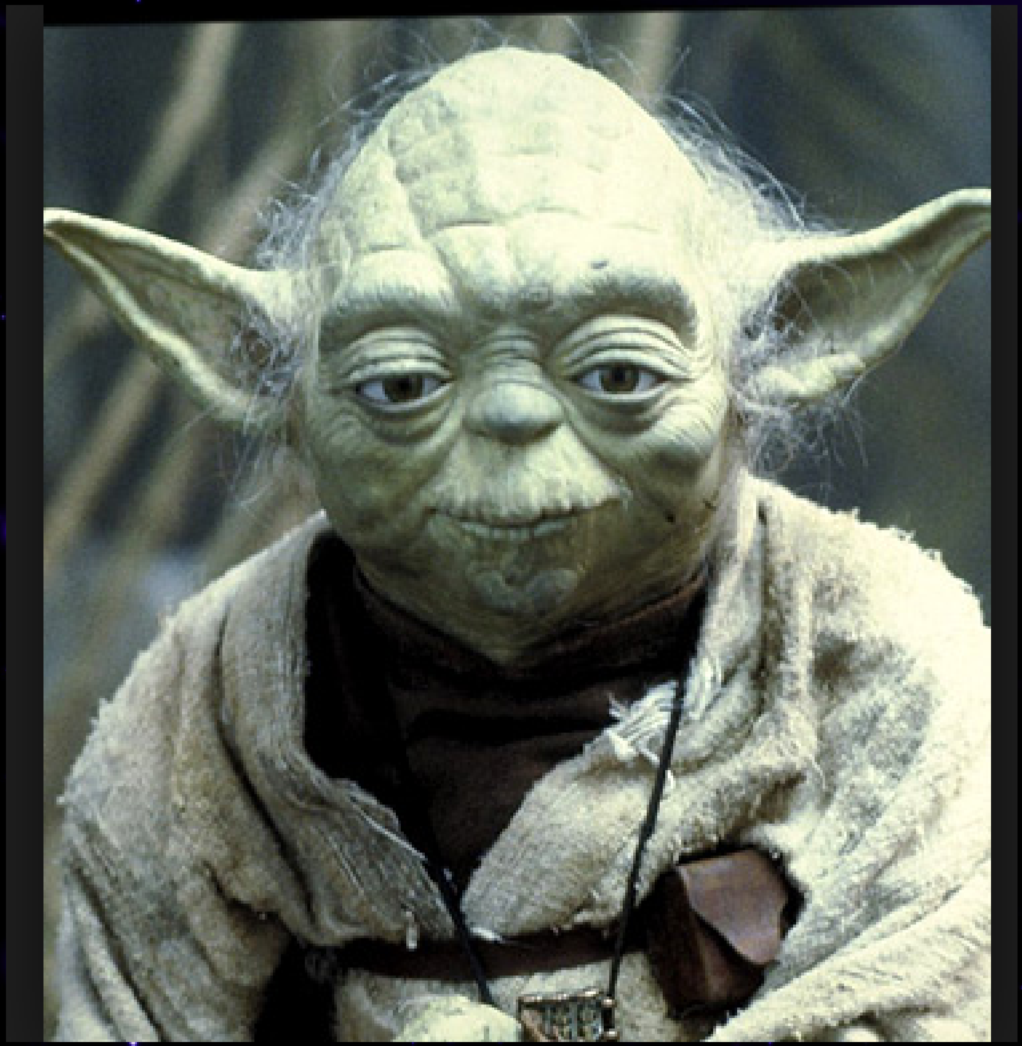
Dan Freedman

Michael Duff

A. van Proeyen

Edward Witten

Edward Witten



Marc Grisaru

You better not shout. You better not cry.
You better not pout. I'm telling you why.
'Marc Grisaru is coming to town.'

Physica 15D (1985) 289-293

STUPERSPACE

V. GATES†, Empty KANGAROO‡, M. ROACHCOCK*, and W.C. GALL∗

California Institute of Technology, Pasadena, CA 91125, USA

†Jolly Good Fellow, supported in part by Ulysses S. Grant #10036.
‡On leave of his senses.
*Work supported by the U.S. Department of Momentum.
∗Permanent address: "Your Royal Highness".

Pentember, 1999

Maybe 1985 UCB-PFFT-85/☺

THE SUPER G-STRING[1]

V. Gates[2], Empty Kangaroo[3], M. Roachcock[4], and W. C. Gall[5]
Departure from Physics, University of Cauliflower, Broccoli, CA 94720

NOT TOO ABSTRACT

We describe a **new** string theory which gives all the phenomenology anybody could or will ever want (and more). It makes use of higher dimensions, higher derivatives, higher spin, higher twist, and hierarchy. It cures the problems of renormalizability of gravity, the cosmological constant, grand unification, supersymmetry breaking, and the common cold.

Part II:
A Scarecrow’s Lament

The MSSM Has Five Higgs-Like Bosons!

Particle type	Particle	Symbol	Spin	Superpartner	Symbol	Spin
Fermions	Quark	**q**	$\frac{1}{2}$	Squark	$\tilde{q}$	0
	Neutrino	**ν**	$\frac{1}{2}$	Sneutrino	$\tilde{\nu}$	0
	Electron	**e**	$\frac{1}{2}$	Selectron	$\tilde{e}$	0
	Muon	**μ**	$\frac{1}{2}$	Smuon	$\tilde{\mu}$	0
	Tau	**τ**	$\frac{1}{2}$	Stau	$\tilde{\tau}$	0
Bosons	W	**W**	1	Wino	$\tilde{W}$	$\frac{1}{2}$
	Z	**Z**	1	Zino	$\tilde{Z}$	$\frac{1}{2}$
	Photon	**γ**	1	Photino	$\tilde{\gamma}$	$\frac{1}{2}$
	Gluon	**g**	1	Gluino	$\tilde{g}$	$\frac{1}{2}$
Higgs bosons	Higgs	$\mathbf{h}, \mathbf{A}, \boldsymbol{H^0}, \boldsymbol{H^{+/-}}$	0	Higgsinos	$\tilde{h}, \tilde{A}, \tilde{H^0}, \tilde{H^{+/-}}$	$\frac{1}{2}$

The NMSSM Has Seven Higgs-Like Bosons!

Particle type	Particle	Symbol	Spin	Superpartner	Symbol	Spin
Fermions	Quark	**q**	$\frac{1}{2}$	Squark	$\tilde{q}$	0
	Neutrino	**ν**	$\frac{1}{2}$	Sneutrino	$\tilde{\nu}$	0
	Electron	**e**	$\frac{1}{2}$	Selectron	$\tilde{e}$	0
	Muon	**μ**	$\frac{1}{2}$	Smuon	$\tilde{\mu}$	0
	Tau	**τ**	$\frac{1}{2}$	Stau	$\tilde{\tau}$	0
Bosons	W	**W**	1	Wino	$\tilde{W}$	$\frac{1}{2}$
	Z	**Z**	1	Zino	$\tilde{Z}$	$\frac{1}{2}$
	Photon	**γ**	1	Photino	$\tilde{\gamma}$	$\frac{1}{2}$
	Gluon	**g**	1	Gluino	$\tilde{g}$	$\frac{1}{2}$
Higgs bosons	Higgs	**h**, **A**, H^0, $H^{+/-}$	0	Higgsinos	$\tilde{h}, \tilde{A}, \tilde{H}^0, \tilde{H}^{+/-}$	$\frac{1}{2}$
Higgs bosons	Higgs	**h**, **A**, H^0, $H^{+/-}$	0	Higgsinos	$\tilde{h}, \tilde{A}, \tilde{H}^0, \tilde{H}^{+/-}$	$\frac{1}{2}$

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	–1	d down	0.006	–1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	–1	s strange	0.1	–1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	–1	b bottom	4.3	–1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0
	125	

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

From: Rolf Heuer <rolf.heuer@cern.ch>
Date: 14 March 2013 11:29:54 AM SAST
To: "cern-personnel (CERN Personnel - Members and Associate Members)" <cern-personnel@cern.ch>
Subject: CERN Press Release: New results indicate that particle discovered at CERN is a Higgs boson

La version française sera disponible ultérieurement ici: http://press.web.cern.ch/fr/press-releases

New results indicate that particle discovered at CERN is a Higgs boson

Geneva, 14 March 2013. At the Moriond Conference today, the ATLAS and CMS collaborations at CERN's Large Hadron Collider (LHC) presented preliminary new results that further elucidate the particle discovered last year. Having analysed two and a half times more data than was available for the discovery announcement in July, they find that the new particle is looking more and more like a Higgs boson, the particle linked to the mechanism that gives mass to elementary particles. It remains an open question, however, whether this is the Higgs boson of the Standard Model of particle physics, **or possibly the lightest of several bosons** predicted in some theories that go beyond the Standard Model. Finding the answer to this question will take time.

Whether or not it is a Higgs boson is demonstrated by how it interacts with other particles, and its quantum properties. For example, a Higgs boson is postulated to have no spin, and in the Standard Model its parity – a measure of how its mirror image behaves – should be positive. CMS and ATLAS have compared a number of options for the spin-parity of this particle, and these all prefer no spin and positive parity. This, coupled with the measured interactions of the new particle with other particles, strongly indicates that it is a Higgs boson.

Review of Minimal Off-shell 4D Supermultiplets

Supercharge & Supersymmetry Algebra

$$\mathrm{D}_a$$

$$\mathrm{D}_a \, \mathrm{D}_b \; + \; \mathrm{D}_b \, \mathrm{D}_b \; = \; i \, 2 (\gamma^\mu)_{a\,b} \, \partial_\mu$$

4D Chiral Supermultiplet

$$(\boldsymbol{A},\ \boldsymbol{B},\ \boldsymbol{\psi_a},\ \boldsymbol{F},\ \boldsymbol{G})$$

$$\mathrm{D}_a A = \psi_a \ ,$$

$$\mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \ ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A - (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B - i\,C_{a\,b}\,F + (\gamma^5)_{a\,b}G \ ,$$

$$\mathrm{D}_a F = (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \ ,$$

$$\mathrm{D}_a G = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \ .$$

4D Vector Supermultiplet

$(\boldsymbol{A_\mu}, \boldsymbol{\lambda_a}, \mathbf{d})$

$$\mathrm{D}_a A_\mu = (\gamma_\mu)_a{}^b \lambda_b \quad ,$$

$$\mathrm{D}_a \lambda_b = -\, i \tfrac{1}{4}([\gamma^\mu, \gamma^\nu])_{ab} (\partial_\mu A_\nu - \partial_\nu A_\mu) + (\gamma^5)_{ab} \mathrm{d} \ ,$$

$$\mathrm{D}_a \mathrm{d} = i\, (\gamma^5 \gamma^\mu)_a{}^b \, \partial_\mu \lambda_b \quad .$$

4D Tensor Supermultiplet

$$(\varphi,\, B_{\mu\nu},\, \chi_a)$$

$$\mathrm{D}_a \varphi = \chi_a \quad ,$$

$$\mathrm{D}_a B_{\mu\nu} = -\tfrac{1}{4}([\,\gamma_\mu\,,\,\gamma_\nu\,])_a{}^b \chi_b \quad ,$$

$$\mathrm{D}_a \chi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu \varphi - (\gamma^5 \gamma^\mu)_{a\,b}\,\epsilon_\mu{}^{\rho\,\sigma\,\tau} \partial_\rho B_{\sigma\,\tau} \quad .$$

Entering Plato's Cave

Zero Brane Reduction

$$A(\tau, x, y, z) \rightarrow A(\tau) \ , \ B(\tau, x, y, z) \rightarrow B(\tau) \ ,$$

$$F(\tau, x, y, z) \rightarrow F(\tau) \ , \ G(\tau, x, y, z) \rightarrow G(\tau) \ ,$$

$$\psi_a(\tau, x, y, z) \rightarrow \psi_a(\tau)$$

$$\begin{aligned}
\mathrm{D}_a A &= \psi_a \quad , \\
\mathrm{D}_a B &= i\,(\gamma^5)_a{}^b\,\psi_b \quad , \\
\mathrm{D}_a \psi_b &= i\,(\gamma^0)_{a\,b}\,\partial_\tau A \ - \ (\gamma^5\gamma^0)_{a\,b}\,\partial_\tau B \ - \ i\,C_{a\,b}\,F \ + \ (\gamma^5)_{a\,b} G \ , \\
\mathrm{D}_a F &= (\gamma^0)_a{}^b\,\partial_\tau\,\psi_b \quad , \\
\mathrm{D}_a G &= i\,(\gamma^5\gamma^0)_a{}^b\,\partial_\tau\,\psi_b \quad .
\end{aligned}$$

‘Node Lowering’

$$F \to \partial_\tau \widehat{F} \quad , \quad G \to \partial_\tau \widehat{G} \quad .$$

$$\begin{aligned}
\mathrm{D}_a A &= \psi_a \quad , \quad \mathrm{D}_a \widehat{F} = (\gamma^0)_a{}^b \psi_b \quad , \\
\mathrm{D}_a B &= i\,(\gamma^5)_a{}^b \psi_b \quad , \quad \mathrm{D}_a \widehat{G} = i\,(\gamma^5\gamma^0)_a{}^b \psi_b \quad , \\
\mathrm{D}_a \psi_b &= i\,(\gamma^0)_{a\,b}\,\partial_\tau A \,-\, (\gamma^5\gamma^0)_{a\,b}\,\partial_\tau B \\
&\quad -\, i\,C_{a\,b}\,\partial_\tau \widehat{F} \,+\, (\gamma^5)_{a\,b}\,\partial_\tau \widehat{G} \quad .
\end{aligned}$$

Valise Formulation

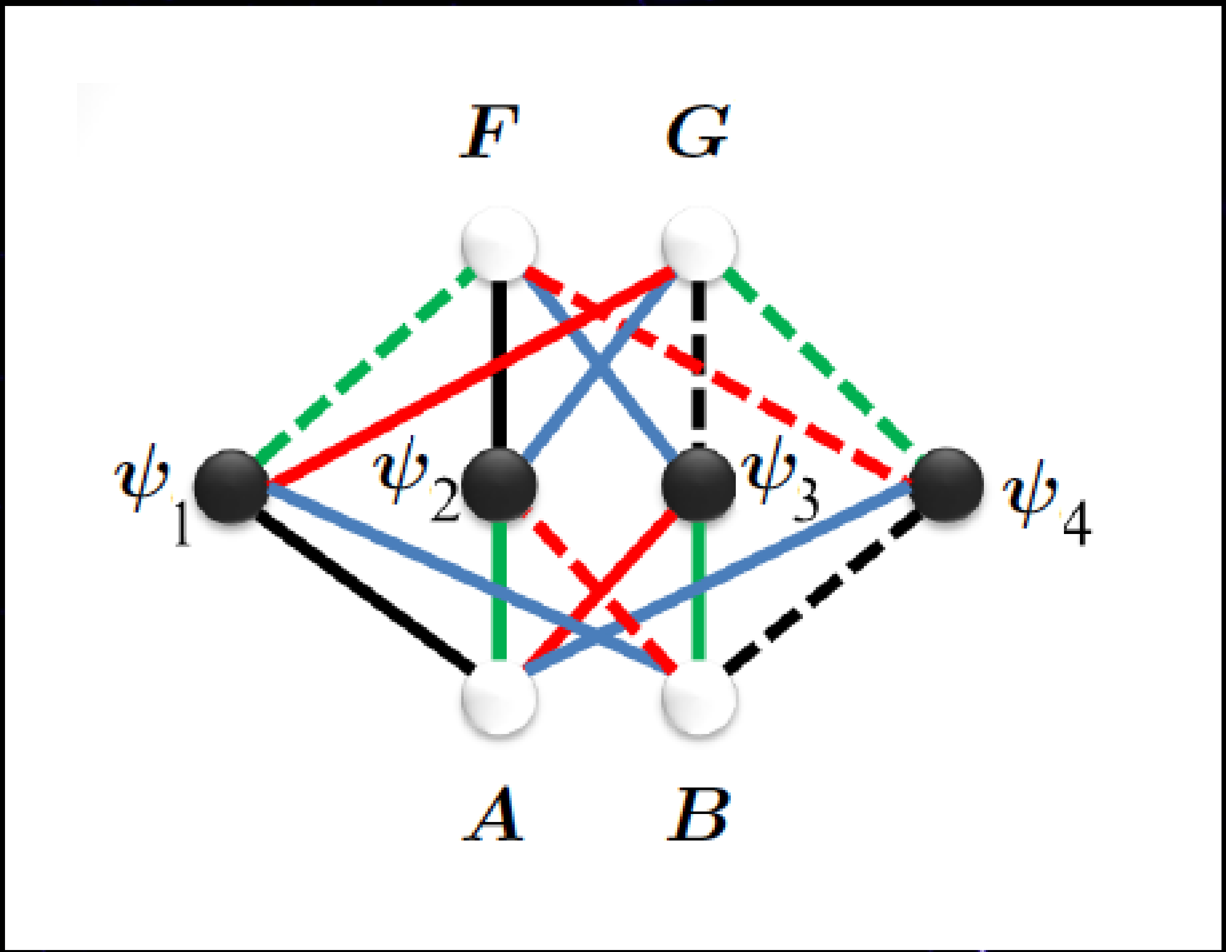
$$\mathrm{D}_a \rightarrow \mathrm{D}_{\mathrm{I}}$$

$$\Phi_1 = A \ , \ \Phi_2 = B \ , \ \Phi_3 = \hat{F} \ , \ \Phi_4 = \hat{G} \ ,$$

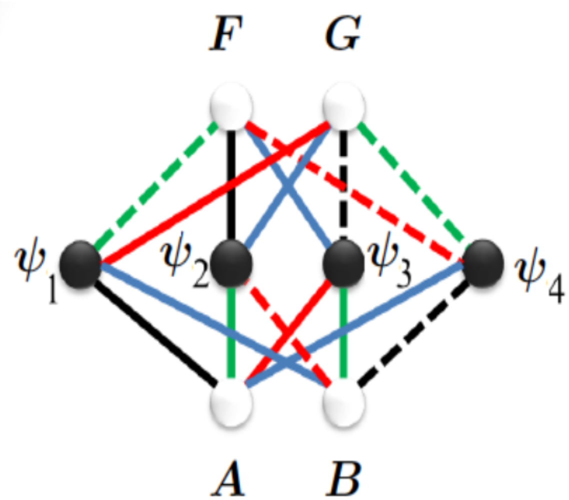
$$\Psi_1 = -i\psi_1 \ , \ \Psi_2 = -i\psi_2 \ , \ \Psi_3 = -i\psi_3 \ , \ \Psi_4 = -i\psi_4 \ ,$$

$$\mathrm{D}_{\mathrm{I}}\Phi_i = i\,(\mathrm{L}_{\mathrm{I}})_{i\hat{k}}\,\Psi_{\hat{k}} \ , \ \mathrm{D}_{\mathrm{I}}\Psi_{\hat{k}} = (\mathrm{R}_{\mathrm{I}})_{\hat{k}i}\,\frac{d}{d\tau}\Phi_i \ .$$

Introducing the CM Adinkra



CM Adinkra -> L-Matrices



$$L_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The 'Garden Algebra'

$\mathcal{GR}(d, N)$ = '*Garden*'

N, $d \times d$ Generalized Real

Pauli/van der Waerden Matrices

$$(\mathrm{L_I})_i{}^{\hat{j}}\,(\mathrm{R_J})_{\hat{j}}{}^{k} + (\mathrm{L_J})_i{}^{\hat{j}}\,(\mathrm{R_I})_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \;\;,$$

$$(\mathrm{R_J})_{\hat{i}}{}^{j}\,(\mathrm{L_I})_j{}^{\hat{k}} + (\mathrm{R_I})_{\hat{i}}{}^{j}\,(\mathrm{L_J})_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \;\;,$$

$$(\mathrm{R_I})_{\hat{j}}{}^{k}\,\delta_{ik} = (\mathrm{L_I})_i{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}} \;\;.$$

The ‘Garden Algebra’

$$\gamma^{I} = \begin{bmatrix} 0 & L^{I} \\ R^{I} & 0 \end{bmatrix}$$

$$\gamma^{I}\gamma^{J} + \gamma^{J}\gamma^{I} = 2\,\delta^{IJ}\,\mathbf{I}$$

For a fixed value of $\mathcal{N}$ there is a minimum value $d_{\mathcal{N}}$ such that $d_{\mathcal{N}} \times d_{\mathcal{N}}$ matrices faithfully represent this algebra. With $\mathcal{N} = 8m + n$, $1 \leq n \leq 8$ and using the definition if $\mathcal{N} = 8k \rightarrow m = k - 1$ for $k = 1, 2, 3, \ldots \infty$, this minimum value is shown in the following table

$$\mathrm{d}_{\mathcal{N}} = 16^m F_{\mathcal{RH}}(n)$$

n	$F_{\mathcal{RH}}(n)$
1	1
2	2
3	4
4	4
5	8
6	8
7	8
8	8

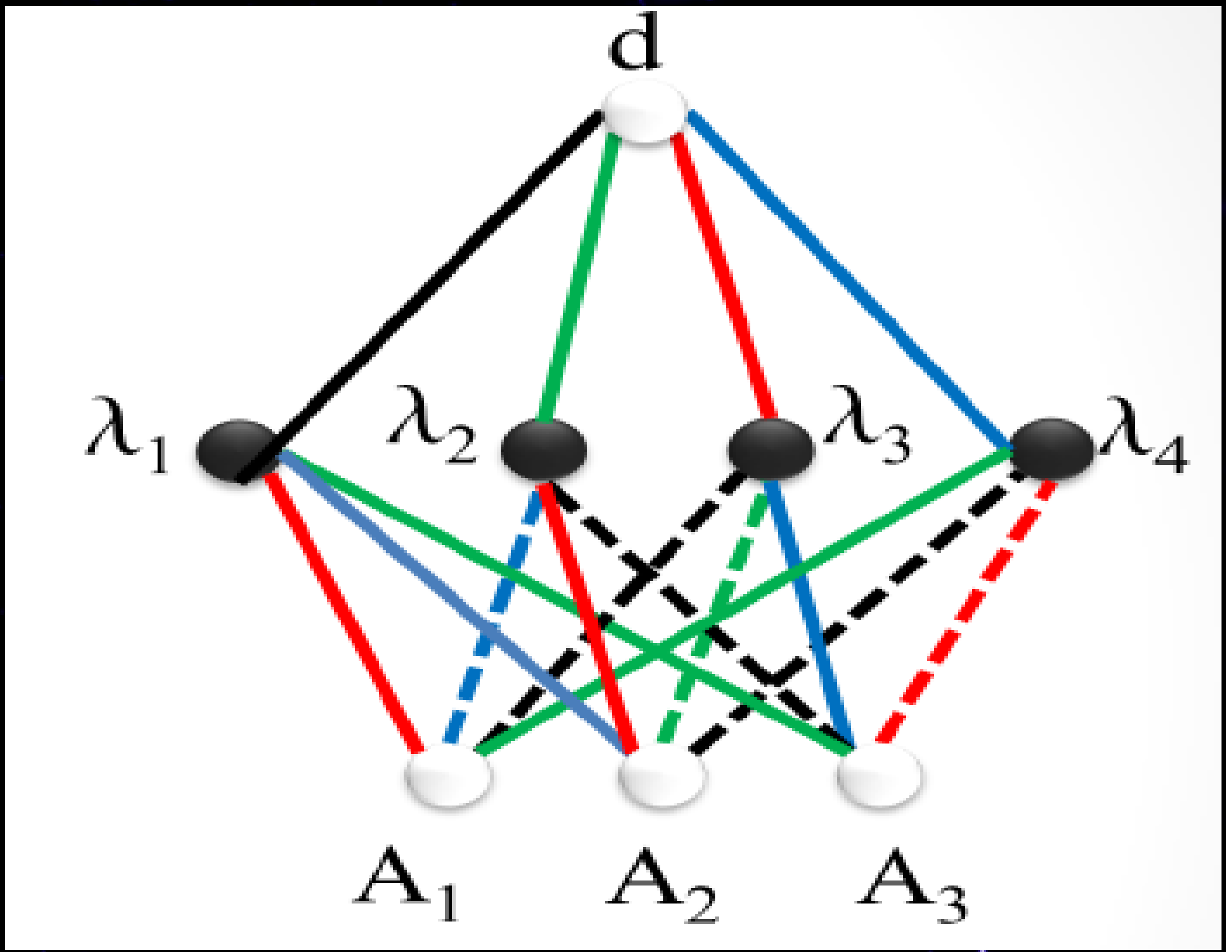
CM'Garden Algebra' Matrices

CM Explicit L-Matrices

$$(\mathrm{L}_1)_{i\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad , \quad (\mathrm{L}_2)_{i\hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$(\mathrm{L}_3)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad , \quad (\mathrm{L}_4)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Introducing the VM Adinkra



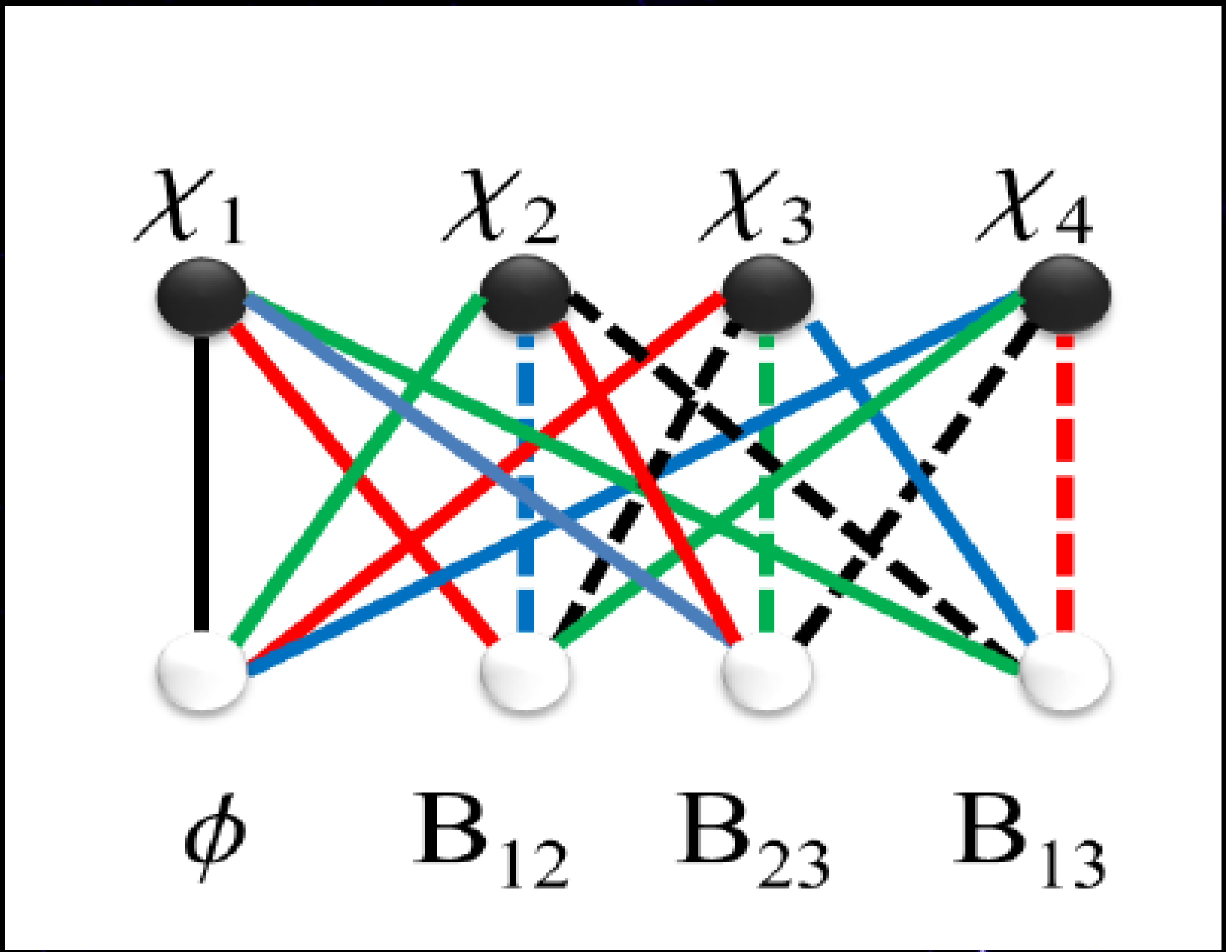
VM'Garden Algebra' Matrices

VM Explicit L-Matrices

$$(\mathrm{L}_1)_{i\hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad , \quad (\mathrm{L}_2)_{i\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$(\mathrm{L}_3)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad , \quad (\mathrm{L}_4)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Introducing the TM Adinkra



TM'Garden Algebra' Matrices

TM Explicit L-Matrices

$$
(\mathrm{L}_1)_{i\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix} , \quad (\mathrm{L}_2)_{i\hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}
$$

$$
(\mathrm{L}_3)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} , \quad (\mathrm{L}_4)_{i\hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
$$

Peering Out of Plato's Cave: Evidence For SUSY Holography

Q: Is This All Just An Accident?

A: Maybe Not.

1D Garden Algebra Valise Supermultiplet

$$\mathrm{D}_{\mathrm{I}}\Phi_i = i\,(\mathrm{L}_{\mathrm{I}})_{i\hat{k}}\,\Psi_{\hat{k}} \quad , \quad \mathrm{D}_{\mathrm{I}}\Psi_{\hat{k}} = (\mathrm{R}_{\mathrm{I}})_{\hat{k}i}\,\frac{d}{d\tau}\Phi_i \quad .$$

Valise Chiral Supermultiplet

$$\begin{aligned}
\mathrm{D}_a A &= \psi_a \quad , \quad \mathrm{D}_a \widehat{F} = (\gamma^0)_a{}^b\,\psi_b \quad , \\
\mathrm{D}_a B &= i\,(\gamma^5)_a{}^b\,\psi_b \quad , \quad \mathrm{D}_a \widehat{G} = i\,(\gamma^5\gamma^0)_a{}^b\,\psi_b \quad , \\
\mathrm{D}_a \psi_b &= i\,(\gamma^0)_{ab}\,\partial_\tau A - (\gamma^5\gamma^0)_{ab}\,\partial_\tau B \\
&\quad - i\,C_{ab}\,\partial_\tau \widehat{F} + (\gamma^5)_{ab}\,\partial_\tau \widehat{G} \quad .
\end{aligned}$$

Valise Vector Supermultiplet

$$\begin{aligned}
\mathrm{D}_a A_i &= (\gamma_i)_a{}^b\,\lambda_b \quad , \quad \mathrm{D}_a \widehat{\mathrm{d}} = i\,(\gamma^5\gamma^0)_a{}^b\,\lambda_b \quad , \\
\mathrm{D}_a \lambda_b &= -\,i\,\tfrac{1}{2}(\gamma^0\gamma^i)_{ab}\,(\,\partial_\tau A_i\,) + (\gamma^5)_{ab}\,\partial_\tau \widehat{\mathrm{d}} \quad .
\end{aligned}$$

4D Tensor Supermultiplet

$$\begin{aligned}
\mathrm{D}_a \varphi &= \chi_a \quad , \quad \mathrm{D}_a B_{ij} = -\,\tfrac{1}{4}([\,\gamma_i\,,\,\gamma_j\,])_a{}^b\,\chi_b \quad , \\
\mathrm{D}_a \chi_b &= i\,(\gamma^0)_{ab}\,\partial_\tau \varphi + (\gamma^5\gamma_i)_{ab}\,\epsilon^{ijk}\partial_\tau B_{jk} \quad .
\end{aligned}$$

The 'Garden Algebra'

$$(\mathrm{L_I})_i{}^{\hat{j}}\,(\mathrm{R_J})_{\hat{j}}{}^{k} + (\mathrm{L_J})_i{}^{\hat{j}}\,(\mathrm{R_I})_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \quad ,$$

$$(\mathrm{R_J})_{\hat{i}}{}^{j}\,(\mathrm{L_I})_j{}^{\hat{k}} + (\mathrm{R_I})_{\hat{i}}{}^{j}\,(\mathrm{L_J})_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \quad ,$$

$$(\mathrm{R_I})_{\hat{j}}{}^{k}\,\delta_{ik} = (\mathrm{L_I})_i{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}} \quad .$$

The 'Garden Algebra'

- Mathematica was used to generate all possible L-matrix solutions of the $\mathcal{GR}$ (4,4) equations.
- The matrices can be decomposed into a binary part and permutation part:

$$(L_I)_i^{\hat{k}} = \left(S^{(I)}\right)_i^{\hat{l}} (P_{(I)})_{\hat{l}}^{\hat{k}}$$

$$L_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The ‘Garden Algebra’

- $$(L_I)_i^{\hat{k}} = \left(S^{(I)}\right)_i^{\hat{l}} (P_{(I)})_{\hat{l}}^{\hat{k}}$$

$$(S^{(I)})_i{}^{\hat{k}} = \begin{bmatrix} (-1)^{p_{I1}} & 0 & 0 & 0 \\ 0 & (-1)^{p_{I2}} & 0 & 0 \\ 0 & 0 & (-1)^{p_{I3}} & 0 \\ 0 & 0 & 0 & (-1)^{p_{I4}} \end{bmatrix}$$

- $(R_I)_b = \ p_{I1}2^0 + p_{I2}2^1 + p_{I3}2^2 + p_{I4}2^3$
- $(P_{(I)})_{\hat{l}}{}^{\hat{k}} \equiv \{\text{matrix representation of } S_4\}$
- $(P_{(I)})_{\hat{l}}{}^{\hat{k}} \longrightarrow (s_1 s_2 s_3 s_4)_p = \langle s_1 s_2 s_3 s_4 \rangle$
- This also leads to a simpler notation for the matrices

$$L_1 = (10)_b (1423)_p = (10)_b \langle 1423 \rangle = \langle 1\bar{4}2\bar{3} \rangle$$

The 'Coxeter Group'

Definition

Formally, a **Coxeter group** can be defined as a group with the presentation

$$\left\langle r_1, r_2, \ldots, r_n \mid (r_i r_j)^{m_{ij}} = 1 \right\rangle$$

where $m_{ii} = 1$ and $m_{ij} \geq 2$ for $i \neq j$. The condition $m_{ij} = \infty$ means no relation of the form $(r_i r_j)^m$ should be imposed.

The pair *(W,S)* where *W* is a Coxeter group with generators $S=\{r_1,\ldots,r_n\}$ is called **Coxeter system**. Note that in general *S* is *not* uniquely determined by *W*. For example, the Coxeter groups of type BC_3 and A_1xA_3 are isomorphic but the Coxeter systems are not equivalent (see below for an explanation of this notation).

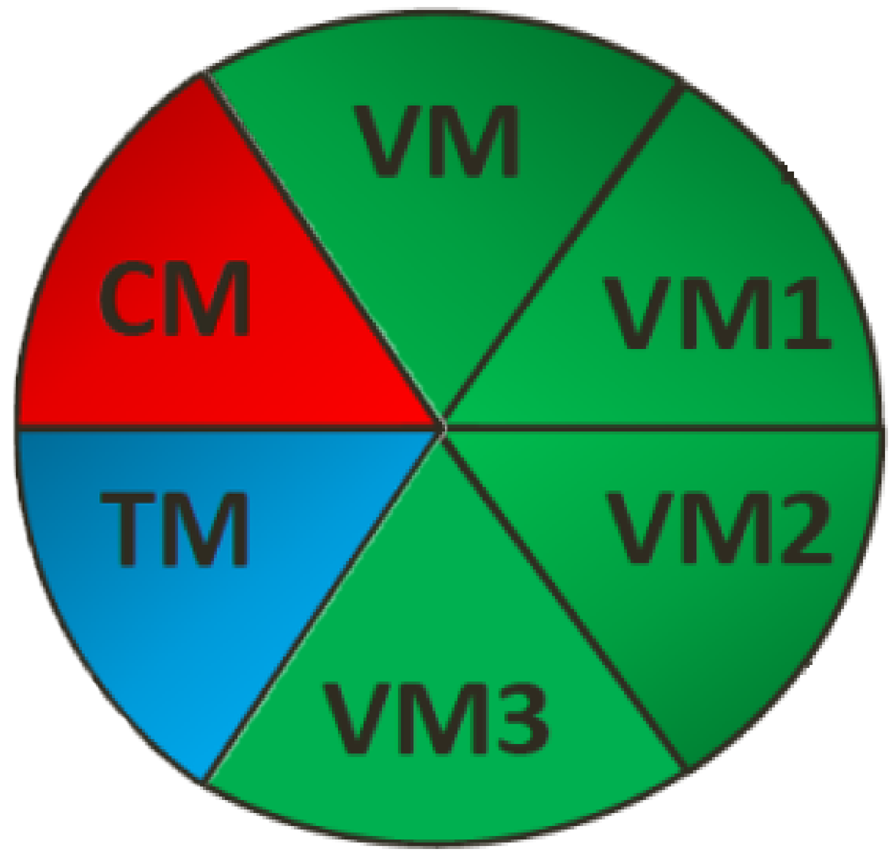
The 'Garden Algebra'

- For d=4,N=4
  - there are $2^4 = 16$ possible combinations of the binary part
  - There are 4! = 24 possible combinations of the permutation part (elements of S_4)
  - There are 16 x 24 = **384** possible matrices in the solution space
  - $2^4 4!$ = order of Coxeter group BC_4 , abstract group of reflections
- Using Mathematica, every combination of the matrices was used in the Garden Algebra equations to find tetrads (sets of 4) of solutions.
  - **1536** tetrad solutions were found
  - By examining the solutions, the number was reduced only considering the even-parity solutions ($(0)_b$, $(2)_b$, $(4)_b$, ...) leaving 96 tetrad solutions.
- The **96** tetrads can be broken into **6** groups...

SUSY Permutation Quartets Within A Coxeter Algebra

	L_1	L_2	L_3	L_4
Chiral	<1423>	<2314>	<3241>	<4132>
Vector	<2413>	<1324>	<4231>	<3142>
Tensor	<1342>	<2431>	<3124>	<4213>
VM1	<4123>	<1432>	<2341>	<3214>
VM2	<3421>	<4312>	<2134>	<1243>
VM3	<3412>	<4321>	<1234>	<2143>

SUSY Permutation Quartets Within A Coxeter Algebra



Electromagnetic/ Hodge Duality

Hodge Duality

$$\vec{\nabla}\cdot\vec{\mathcal{E}} = 0 \quad , \quad \vec{\nabla}\cdot\vec{\mathcal{B}} = 0 \quad ,$$

$$\vec{\nabla}\times\vec{\mathcal{B}} - \frac{1}{c}\frac{\partial\vec{\mathcal{E}}}{\partial t} = 0 \quad , \quad \vec{\nabla}\times\vec{\mathcal{E}} + \frac{1}{c}\frac{\partial\vec{\mathcal{B}}}{\partial t} = 0 \quad .$$

$$\vec{\mathcal{E}}' = \vec{\mathcal{E}}\,cos\alpha + \vec{\mathcal{B}}\,sin\alpha \quad ,$$

$$\vec{\mathcal{B}}' = -\vec{\mathcal{E}}\,sin\alpha + \vec{\mathcal{B}}\,cos\alpha \quad ,$$

$$\vec{\nabla}\cdot\vec{\mathcal{E}}' = 0 \quad , \quad \vec{\nabla}\cdot\vec{\mathcal{B}}' = 0 \quad ,$$

$$\vec{\nabla}\times\vec{\mathcal{B}}' - \frac{1}{c}\frac{\partial\vec{\mathcal{E}}'}{\partial t} = 0 \quad , \quad \vec{\nabla}\times\vec{\mathcal{E}}' + \frac{1}{c}\frac{\partial\vec{\mathcal{B}}'}{\partial t} = 0 \quad .$$

If $\alpha = \pi/2$ then

$$\vec{\mathcal{E}}' = \vec{\mathcal{B}} \quad , \quad \vec{\mathcal{B}}' = -\vec{\mathcal{E}}\,,$$

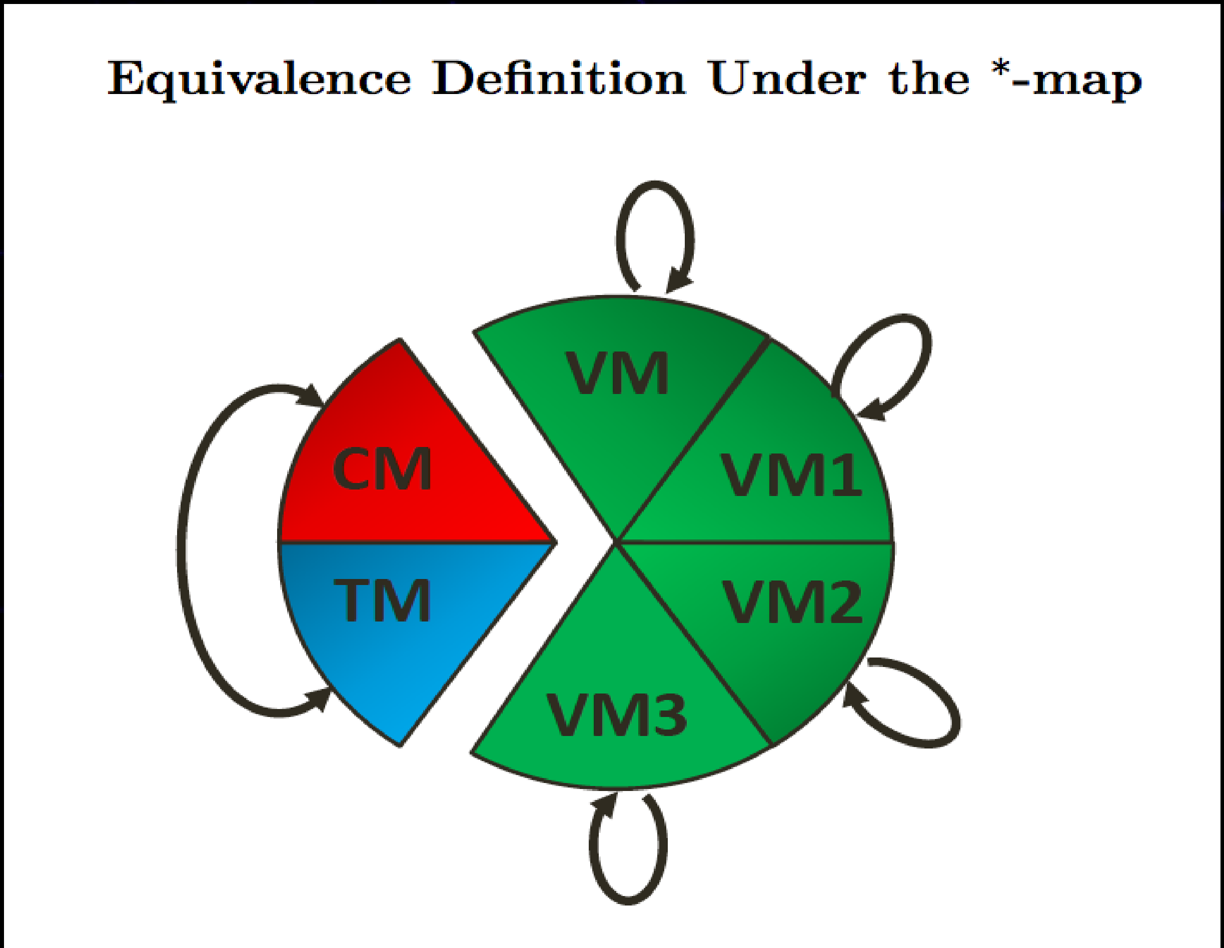
which can be written in the manifestly relativistic notation as

$$F_{\mu\nu}{}' = \equiv \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\eta^{\rho\kappa}\eta^{\sigma\lambda}F_{\kappa\lambda} \quad ,$$

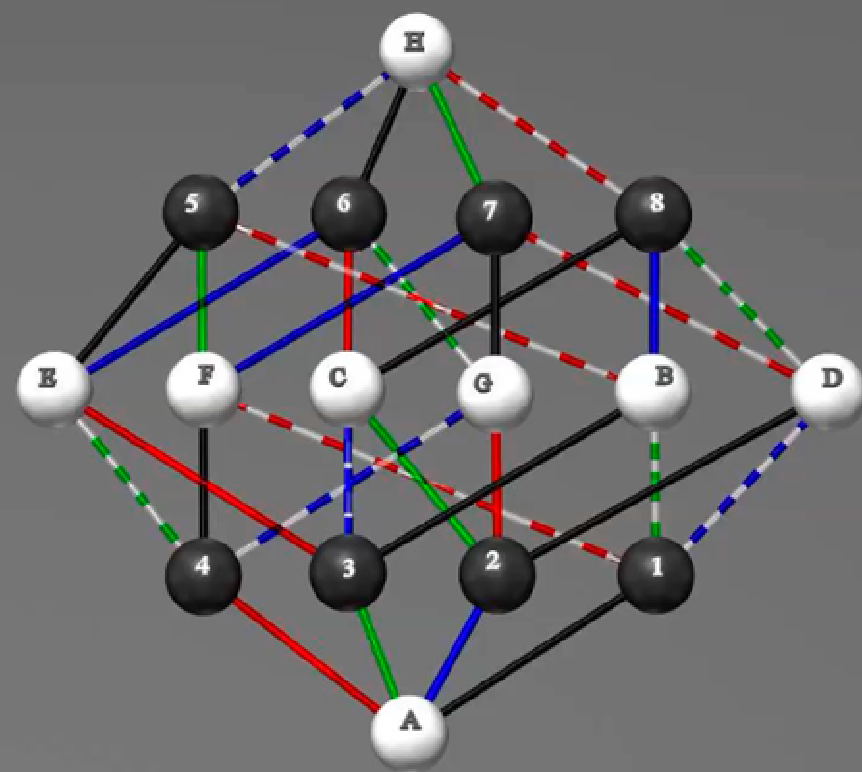
Field	ϕ	A_μ	$B_{\mu\nu}\ \left(B_{\nu\mu} = -B_{\mu\nu}\right)$
Field Strength	$f_\mu = \partial_\mu \phi$	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$	$h_\mu = \frac{1}{3!}\epsilon^{\kappa\lambda\mu\nu}\partial_\lambda B_{\mu\nu}$
Bianchi Identity	$\partial_\mu f_\nu - \partial_\nu f_\mu = 0$	$\epsilon^{\kappa\lambda\mu\nu}\partial_\lambda F_{\mu\nu} = 0$	$\partial^\mu h_\mu = 0$
Action	$S_0 = -\frac{1}{2}\int d^4x\ f^\mu f_\mu$	$S_1 = -\frac{1}{4}\int d^4x\ F^{\mu\nu}F_{\mu\nu}$	$S_2 = \frac{1}{2}\int d^4x\ h^\mu h_\mu$
Equation of Motion	$\frac{\delta S_0}{\delta \phi} = 0\ \rightarrow \partial^\mu f_\mu = 0$	$\frac{\delta S_1}{\delta A_\mu} = 0\ \rightarrow \partial^\mu F_{\mu\nu} = 0$	$\frac{\delta S_2}{\delta \mathrm{B}_{\mu\nu}} = 0\ \rightarrow$ $\partial_\mu h_\nu - \partial_\nu h_\mu = 0$
Duality	$f_\mu \leftrightarrow h_\mu$	$F_{\mu\nu} \leftrightarrow \epsilon_{\mu\nu\kappa\lambda}F^{\kappa\lambda}$	$h_\mu \leftrightarrow f_\mu$

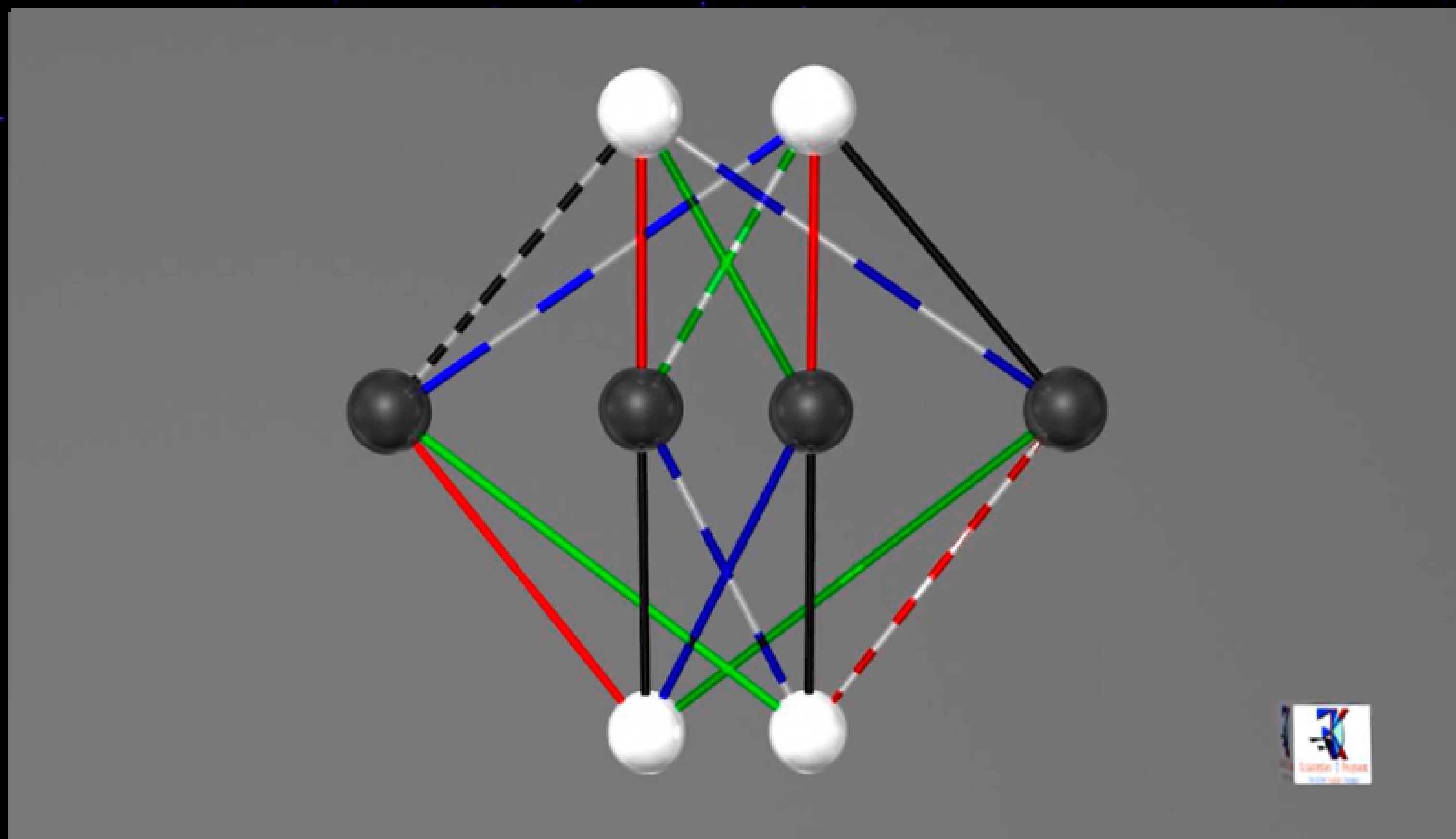
Chiral	Tensor	Vector	*Vector
A	φ	d	d
ψ_a	χ_a	λ_a	λ_a
B,F,G	B_{ij}	A_i	A_i

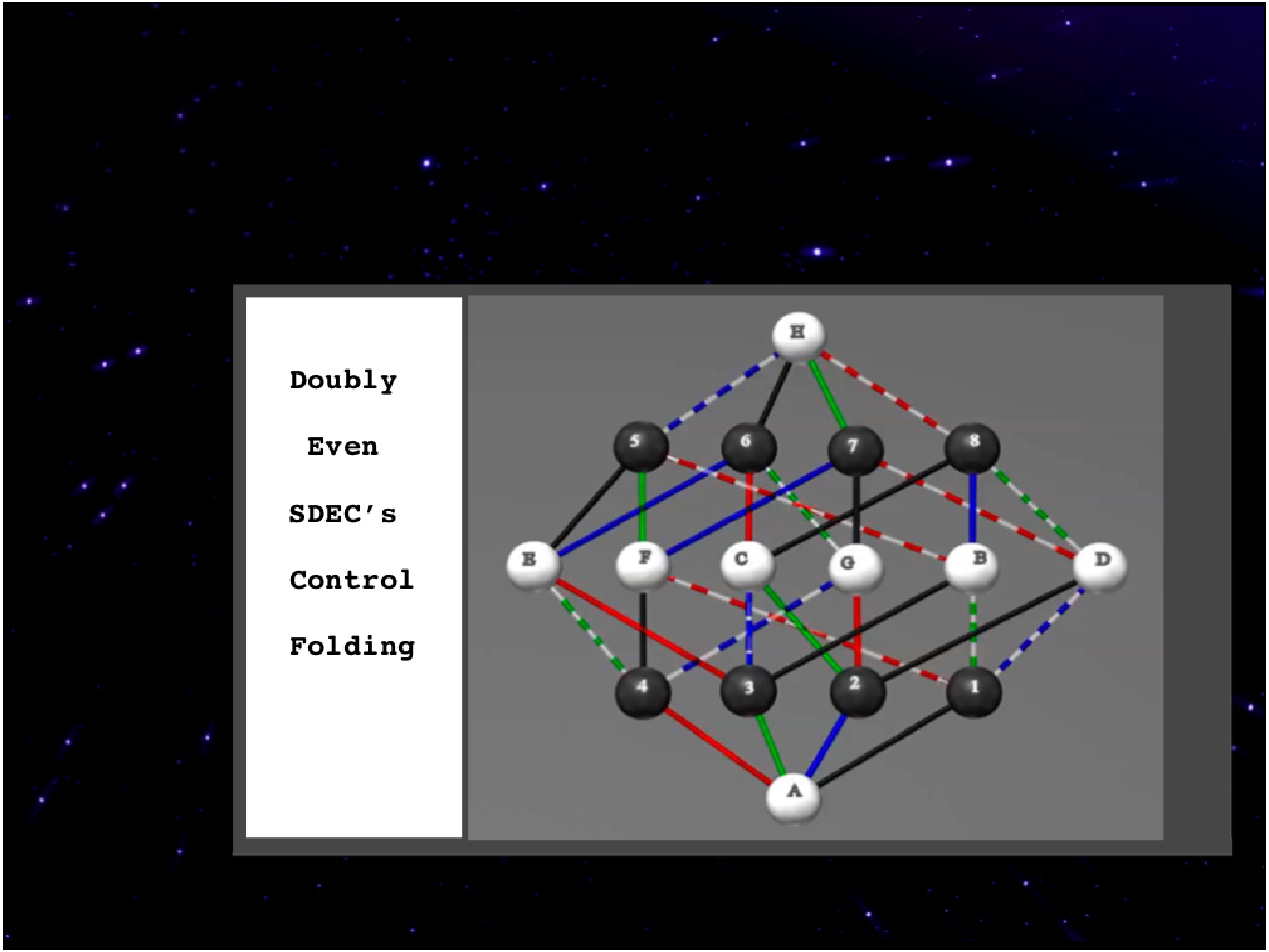
A Proposed Definition of Equivalence Classes



Tesseract Adinkra Folding


H
5
6
7
8
E
F
C
G
B
D
4
3
2
1
A




Doubly
Even
SDEC's
Control
Folding
H
5
6
7
8
E
F
C
G
B
D
4
3
2
1
A

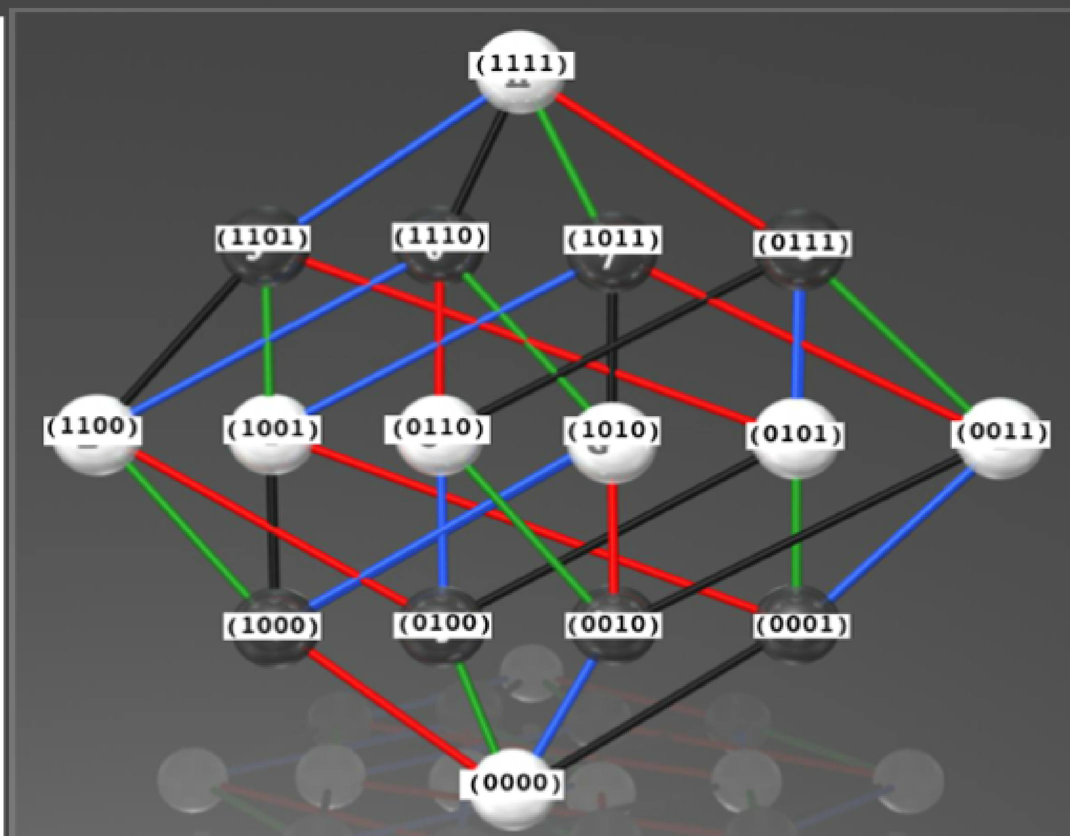
Doubly

Even

SDEC's

Control

Folding



Doubly Even SDEC's Control Folding

Bits Naturally Arise From The Geometry Of Hypercubes

To a small part, the appearance of the SDEC's is not mysterious. Bits naturally appear in any situation where cubical geometry is relevant. The vertices of a cube can always be written in the form

$$(\pm 1, \pm 1, \pm 1, \ldots, \pm 1)$$

or re-written in the form

$$((\pm 1,)^{p_1}, (\pm 1,)^{p_2}, (\pm 1,)^{p_3}, \ldots, (\pm 1,)^{p_d})$$

where the exponents are bits since they take on values 1 or 0. Thus any vertex has an 'address' that is a string of bits

$$(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \ldots, \boldsymbol{p}_d)$$

the information theoretic definition of a 'word.'

Character Functions in Adinkras

Character Functions

$$\tilde{\chi}_\rho = \mathrm{Tr}_\rho \left[(-t)^{2\Delta} \mathrm{e}^{iaM_{12}} \mathrm{e}^{ibM_{34}}\right]$$

$$(M_{\mathrm{IJ}})_i{}^j = \frac{i}{4}\left[(\mathrm{L_I})_i{}^{\hat{k}}(\mathrm{R_J})_{\hat{k}}{}^j - (\mathrm{L_J})_i{}^{\hat{k}}(\mathrm{R_I})_{\hat{k}}{}^j\right]$$

$$\tilde{\chi}_{\mathrm{cis}}(a,b) = 4\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right) + 4\sin\left(\frac{a}{2}\right)\sin\left(\frac{b}{2}\right)$$

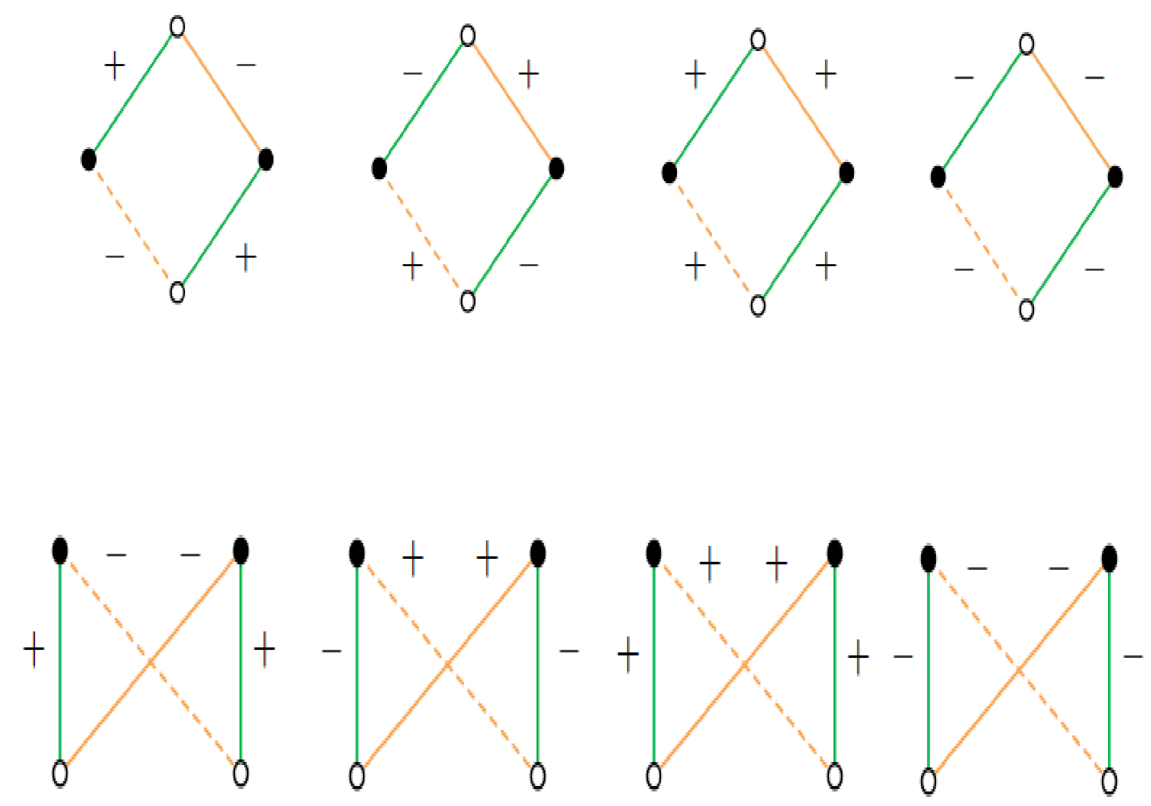
$$\tilde{\chi}_{\mathrm{trans}}(a,b) = 4\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right) - 4\sin\left(\frac{a}{2}\right)\sin\left(\frac{b}{2}\right)$$

$$\tilde{\chi}_{\mathbb{R}}(a,b) = 8\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right)$$

$$\tilde{\chi}(a,b) = 4(n_c + n_t)\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right) + 4(n_c - n_t)\sin\left(\frac{a}{2}\right)\sin\left(\frac{b}{2}\right)$$

Dimensional Enhancement in Adinkras

Calculating Kirchoff Bowties in Adinkras



Kirchoff's Law: $\mathcal{V} = \oint \vec{\mathcal{E}} \cdot d\vec{\ell}$

$$\oint \vec{\mathcal{E}} \cdot d\vec{\ell} \longrightarrow \sum_{links} h(\mathrm{D_I})\,(\,h_f \;-\; h_i\,)\quad,$$

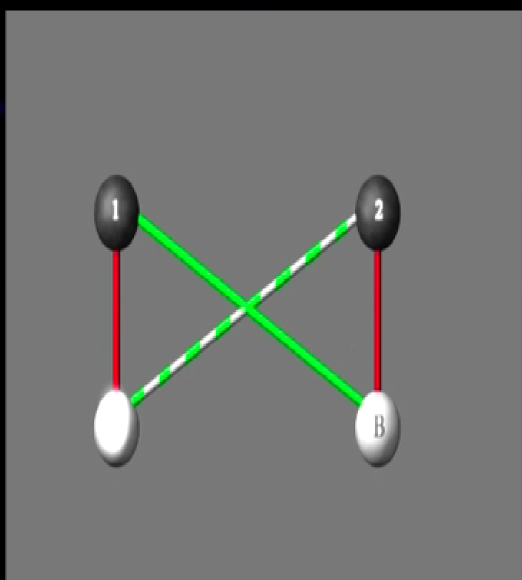
$$\mathcal{V} \longrightarrow \mathcal{B}_N$$

$$\mathcal{B}_N = \sum_{links} h(\mathrm{D_I})\,(\,h_f \;-\; h_i\,)$$

$$\begin{aligned}\mathcal{B}_N = \;& h(\mathrm{D}_1)\,(\,h_f - h_i\,)_1 + h(\mathrm{D}_2)\,(\,h_f - h_i\,)_2 \\ & + h(\mathrm{D}_3)\,(\,h_f - h_i\,)_3 + h(\mathrm{D}_4)\,(\,h_f - h_i\,)_4\end{aligned}$$

$$\begin{aligned}\mathcal{B}_N = \;& h(\mathrm{D})\,(\,h_f - h_i\,)_1 + h(\mathrm{D})\,(\,h_f - h_i\,)_2 \\ & + h(\mathrm{D})\,(\,h_f - h_i\,)_3 + h(\mathrm{D})\,(\,h_f - h_i\,)_4\end{aligned}$$

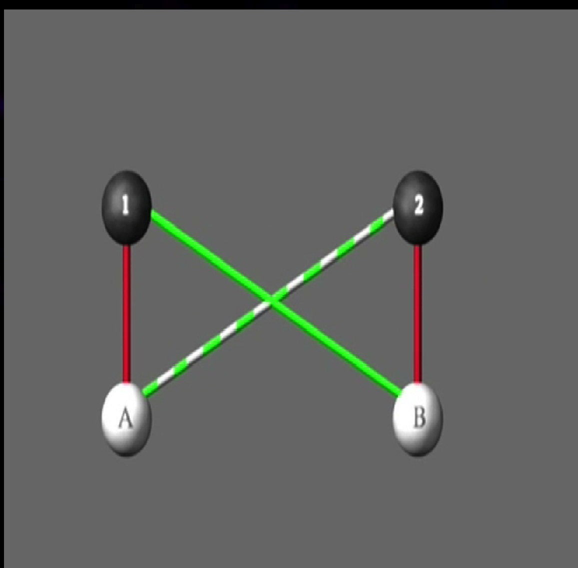
$h(\mathrm{D}) = h(\mathrm{D}) = \pm\frac{1}{2}$



Bow-Tie Number Calculations in the "Ferromagnetic Phase"

$$\mathcal{B}_N = \pm\tfrac{1}{2}\{(h_f - h_i)_1 + (h_f - h_i)_2 + (h_f - h_i)_3 + (h_f - h_i)_4\}$$

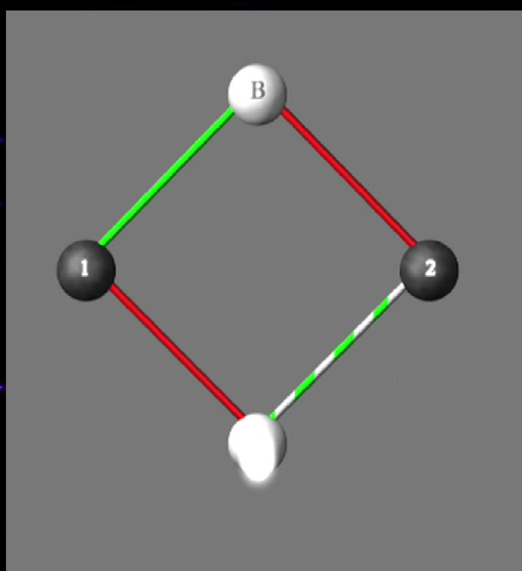
$$h(\mathrm{D}) = h(\mathrm{D}) = \pm\,\frac{1}{2}$$



$$\begin{aligned}
\mathcal{B}_N &= \pm\,\tfrac{1}{2}\{(\,h_f - h_i\,)_1 + (\,h_f - h_i\,)_2 \\
&\qquad + (\,h_f - h_i\,)_3 + (\,h_f - h_i\,)_4\} \\
&= \pm\,\tfrac{1}{2}\{(\,\tfrac{1}{2}\,) + (\,h_f - h_i\,)_2 \\
&\qquad + (\,h_f - h_i\,)_3 + (\,h_f - h_i\,)_4\} \\
&= \pm\,\tfrac{1}{2}\{(\,\tfrac{1}{2}\,) + (\,-\,\tfrac{1}{2}\,) + (\,h_f - h_i\,)_3 \\
&\qquad + (\,h_f - h_i\,)_4\} \\
&= \pm\,\tfrac{1}{2}\{(\,\tfrac{1}{2}\,) + (\,-\,\tfrac{1}{2}\,) + (\,\tfrac{1}{2}\,) \\
&\qquad + (\,h_f - h_i\,)_4\} \\
&= \pm\,\tfrac{1}{2}\{(\,\tfrac{1}{2}\,) + (\,-\,\tfrac{1}{2}\,) + (\,\tfrac{1}{2}\,) + (\,-\,\tfrac{1}{2}\,)\}
\end{aligned}$$

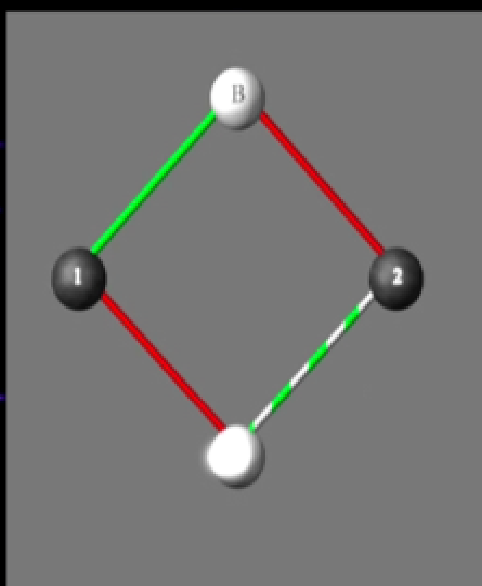
$$\longrightarrow \mathcal{B}_N = 0$$

$$h(\mathrm{D}) \;=\; h(\mathrm{D}) \;=\; \pm\,\frac{1}{2}$$



$$
\begin{aligned}
\mathcal{B}_N \;=\;& \pm\,\tfrac{1}{2}\{\,(\,h_f \,-\, h_i\,)_1 \,+\, (\,h_f \,-\, h_i\,)_2 \\
& \quad +\, (\,h_f \,-\, h_i\,)_3 \,+\, (\,h_f \,-\, h_i\,)_4\,\} \\
=\;& \pm\,\tfrac{1}{2}\{\,(\,\tfrac{1}{2}\,) \,+\, (\,h_f \,-\, h_i\,)_2 \\
& \quad +\, (\,h_f \,-\, h_i\,)_3 \,+\, (\,h_f \,-\, h_i\,)_4\,\} \\
=\;& \pm\,\tfrac{1}{2}\{\,(\,\tfrac{1}{2}\,) \,+\, (\,\tfrac{1}{2}\,) \,+\, (\,h_f \,-\, h_i\,)_3 \\
& \quad +\, (\,h_f \,-\, h_i\,)_4\,\} \\
=\;& \pm\,\tfrac{1}{2}\{\,(\,\tfrac{1}{2}\,) \,+\, (\,\tfrac{1}{2}\,) \,+\, (\,-\,\tfrac{1}{2}\,) \\
& \quad +\, (\,h_f \,-\, h_i\,)_4\,\} \\
=\;& \pm\,\tfrac{1}{2}\{\,(\,\tfrac{1}{2}\,) \,+\, (\,\tfrac{1}{2}\,) \,+\, (\,-\,\tfrac{1}{2}\,) \,+\, (\,-\,\tfrac{1}{2}\,)\,\} \\
& \longrightarrow \mathcal{B}_N \;=\; 0\;\;.
\end{aligned}
$$

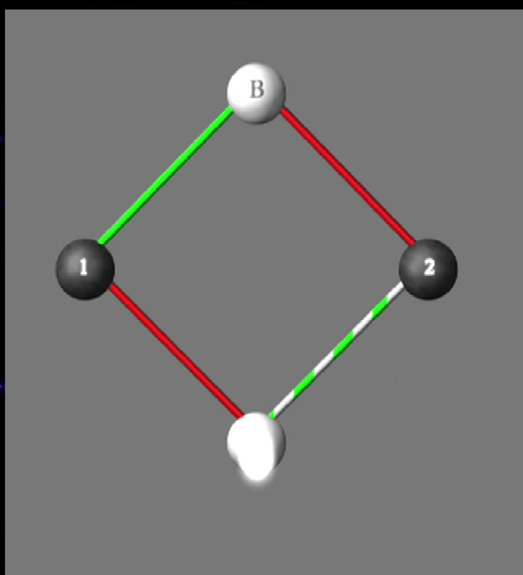
$$h(\mathrm{D}) = -h(\mathrm{D}) = \pm\frac{1}{2}$$



Bow-Tie Number Calculations in the "Anti-Ferromagnetic Phase"

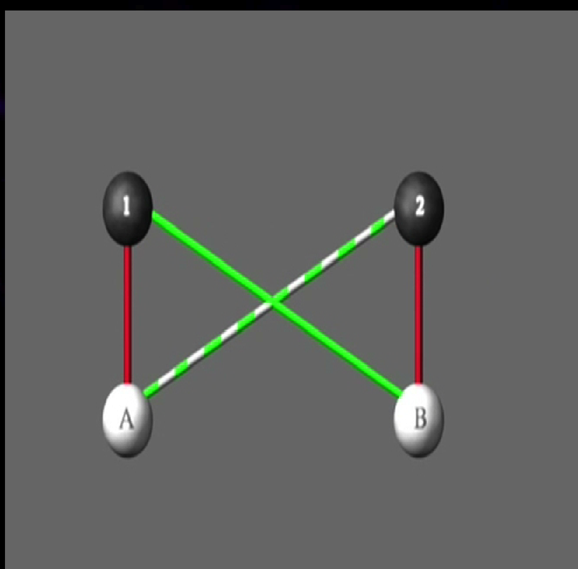
$$\mathcal{B}_N = \pm\frac{1}{2}\{(h_f - h_i)_1 - (h_f - h_i)_2 + (h_f - h_i)_3 - (h_f - h_i)_4\}$$

$$h(\mathrm{D}) = -\,h(\mathrm{D}) = \pm\tfrac{1}{2}$$



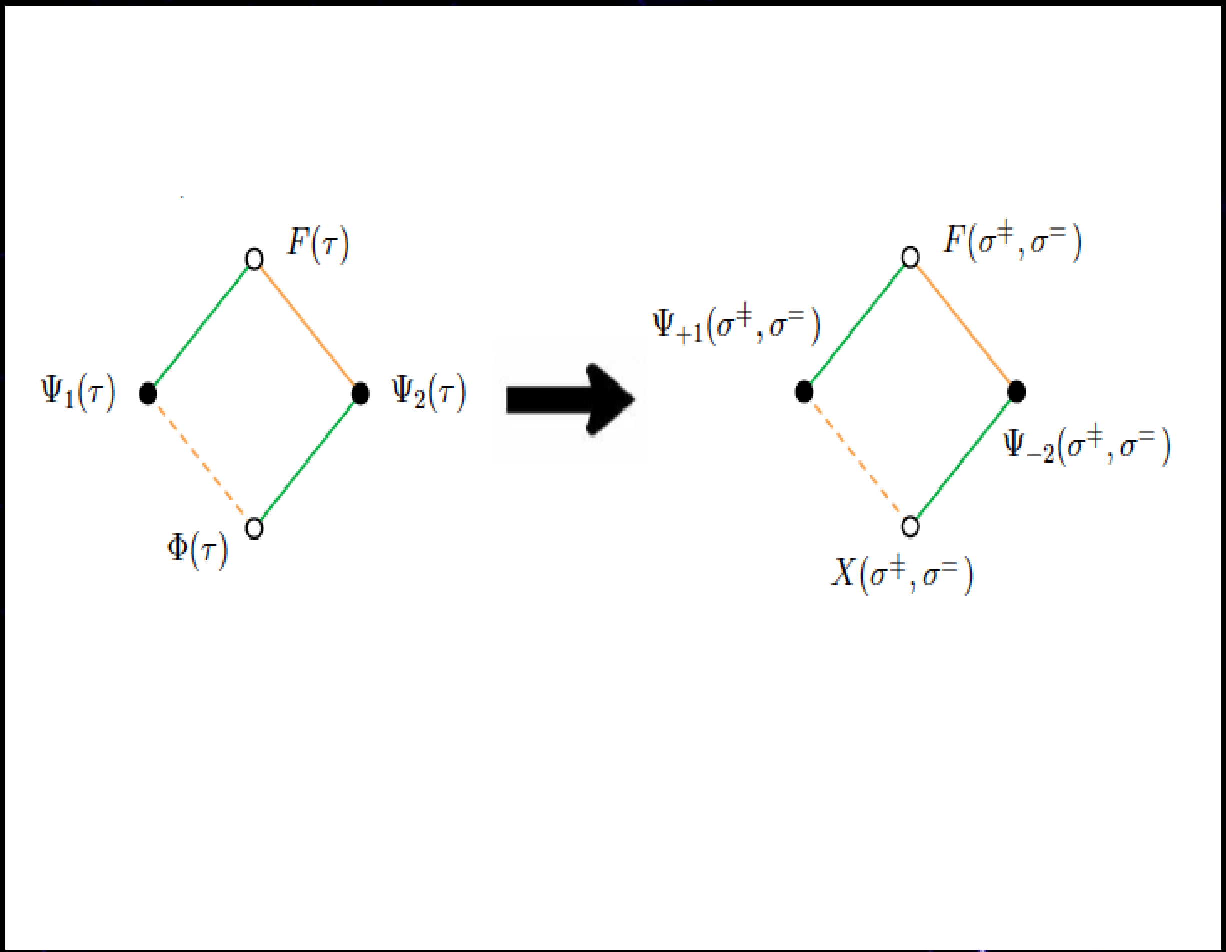
$$
\begin{aligned}
\mathcal{B}_N &= \pm\tfrac{1}{2}\{(h_f - h_i)_1 - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (\tfrac{1}{2}) + (h_f - h_i)_3 \\
&\qquad - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (\tfrac{1}{2}) + (-\tfrac{1}{2}) \\
&\qquad - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (\tfrac{1}{2}) + (-\tfrac{1}{2}) - (-\tfrac{1}{2})\} \\
&\longrightarrow \mathcal{B}_N = 0\ .
\end{aligned}
$$

$$h(\mathrm{D}) = -\,h(\mathrm{D}) = \pm\frac{1}{2}$$



$$
\begin{aligned}
\mathcal{B}_N &= \pm\frac{1}{2}\{(h_f - h_i)_1 - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) - (-\tfrac{1}{2}) \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) - (-\tfrac{1}{2}) \\
&\qquad + (\tfrac{1}{2}) - (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) - (-\tfrac{1}{2}) + (\tfrac{1}{2}) - (-\tfrac{1}{2})\} \\
&\longrightarrow \mathcal{B}_N = \pm 1\ .
\end{aligned}
$$

Dimensional Enhancement



Morse-Like Functions in Adinkras

Morse-like Functions

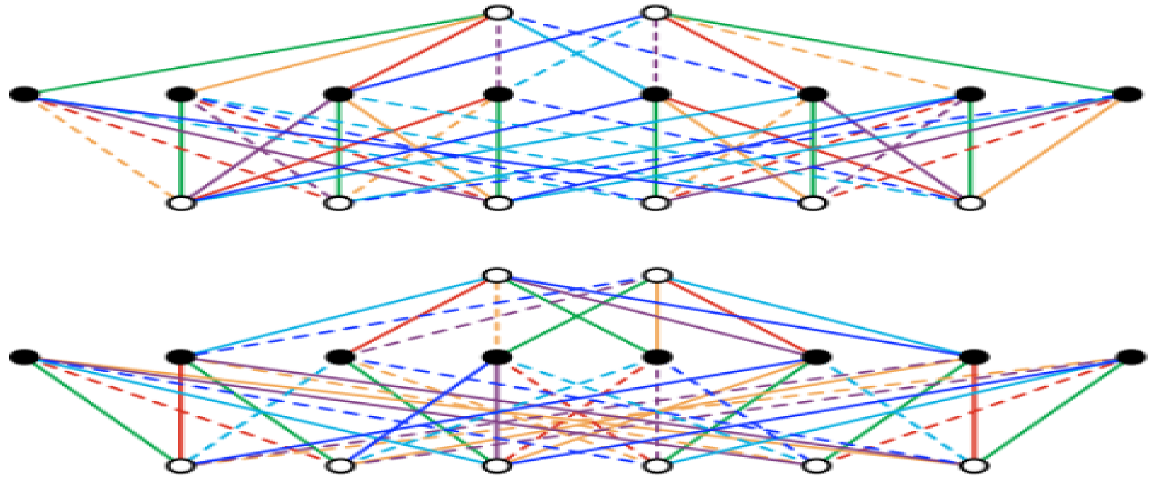
$$\mathbf{C}_{\mathrm{J}} \equiv \begin{pmatrix} 0 & \mathcal{B}_{\mathrm{J}R} \\ \mathcal{B}_{\mathrm{J}L} & 0 \end{pmatrix} \quad ,$$

$$\mathbf{C}_1\mathbf{C}_2 = \begin{pmatrix} 0 & \beta_2\beta_1 & 0 & 0 \\ \beta_2^{-1}\beta_1^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2^{-1}\beta_1 \\ 0 & 0 & \beta_2\beta_1^{-1} & 0 \end{pmatrix} \quad .$$

$$\pm\beta_{\mathrm{I}}\beta_{\mathrm{J}}\beta_{\mathrm{I}}^{-1}\beta_{\mathrm{J}}^{-1} = \pm 1 \quad .$$

$$\pm\beta_{\mathrm{I}}^{-1}\beta_{\mathrm{J}} \ , \ \ \pm\,\beta_{\mathrm{I}}\beta_{\mathrm{J}}^{-1} \quad .$$

Morse-like Functions

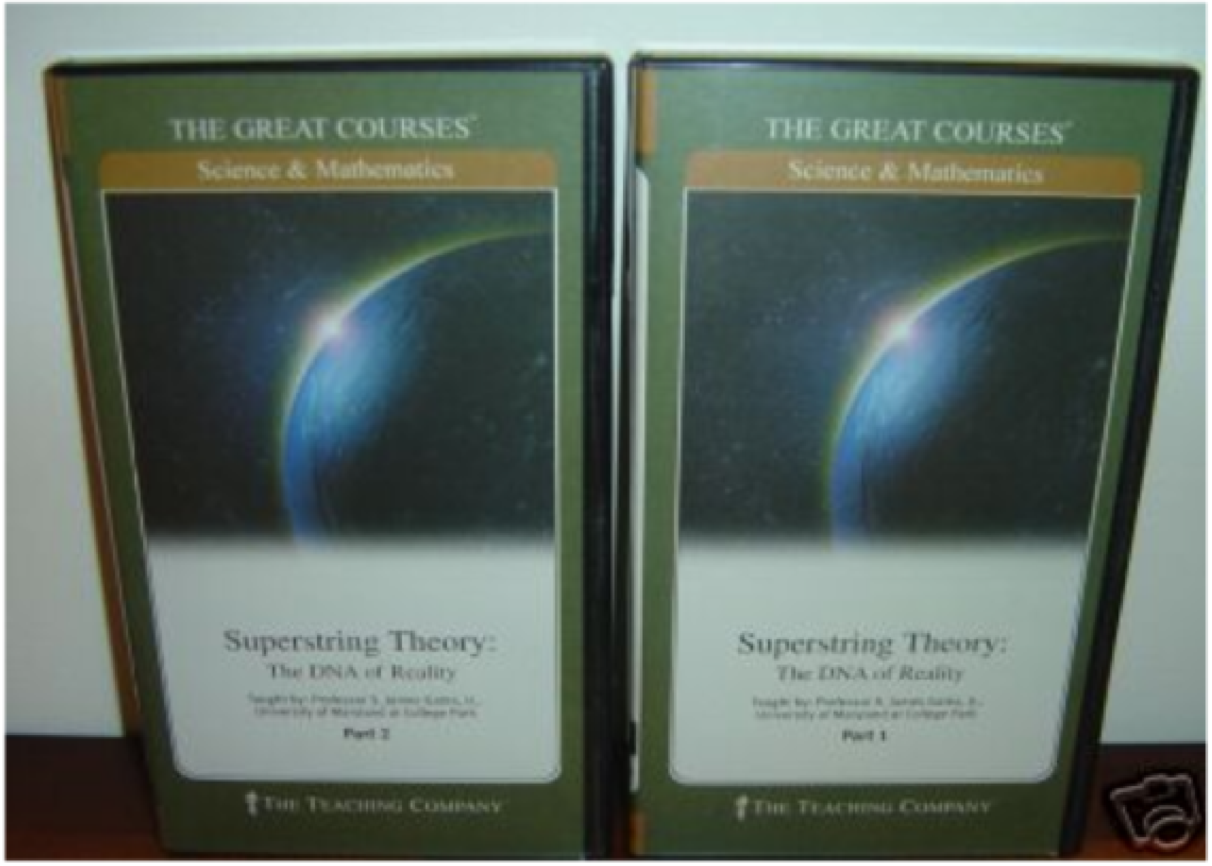


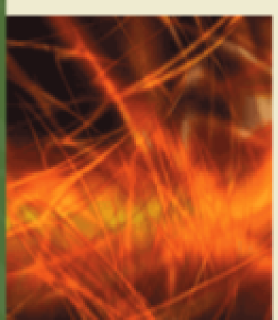
$\|\mathcal{B}_{6L}...\mathcal{B}_{1R}\|$ $\|\mathcal{B}_{6R}...\mathcal{B}_{1L}\|$

$$\pm\beta_2\beta_4\beta_6\beta_1^{-1}\beta_3^{-1}\beta_5^{-1}(\text{3x degenerate}) \ , \ \pm\beta_1\beta_3\beta_5\beta_2^{-1}\beta_4^{-1}\beta_6^{-1} \ ,$$

$$\pm\beta_1\beta_3\beta_5\beta_2^{-1}\beta_4^{-1}\beta_6^{-1} \ , \ \pm\beta_2\beta_3\beta_5\beta_1^{-1}\beta_4^{-1}\beta_6^{-1} \ ,$$

$$\pm\beta_1\beta_4\beta_5\beta_2^{-1}\beta_3^{-1}\beta_6^{-1} \ , \ \pm\beta_1\beta_3\beta_6\beta_2^{-1}\beta_4^{-1}\beta_5^{-1} \ .$$





Superstring Theory: The DNA of Reality

Web:
http://www.teach12.com/ttcx/coursedesclong2.aspx?cid=1284

Acknowledgment

Prof. Gates also wishes to acknowledge
The Teaching Company for the use of
some CGI units that appear in

"Superstring Theory: The DNA of Reality."