PHYS 182: Solutions for Homework Set 4

1

Time elapsed according to your watch, $t_0 = \frac{distancetravelled}{speed} = \frac{2\pi R_{earth}}{0.9c}$

$$= \frac{2\pi\ 6371km}{0.9\times3\times10^5 km/s} = 1.48s$$

Time elapsed according to your friend's watch $= t_0\sqrt{1-\frac{v^2}{c^2}} = 1.48\sqrt{1-0.9^2} = 0.645s$

2

The laser is perceived with a lower frequency or shorter wavelength and hence "blue-shifted". We use the equation for the relativistic Doppler shift for a case of blue-shift,

$\nu_{observed} = \nu_b\sqrt{\frac{1+v/c}{1-v/c}} = \nu_b\sqrt{\frac{3+\sqrt{5}}{3-\sqrt{5}}} \approx 2.62\nu_b$
This observed frequency corresponds to ultraviolet radiation.

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As objects approach relativistic speeds, they undergo length contraction. If we consider the garage's frame of reference, the car approaches the garage at relativistic speeds and undergoes length contraction. The length of car appears to be,
$L' = L_{rest}\sqrt{1-\frac{v^2}{c^2}} = 9\sqrt{1-\frac{5c^2}{9c^2}} = 6m$
The observed length of car from garage's frame of reference (at rest) is 6 m which is less than the length of the garage (8 m) and thus it is perceived that the car fits in the garage.
Similarly, in the reference frame of driver (at rest), the garage is moving towards him at relativistic speed and undergoes length contraction. The length of the garage appears to be,
$L' = L_{rest}\sqrt{1-\frac{v^2}{c^2}} = 8\sqrt{1-\frac{5c^2}{9c^2}} = 5.33m$
The observed length of garage from driver's frame of reference (at rest) is 5.33 m which is less than the length of car (9 m) and thus it is perceived that the car will not fit in the garage.

5

No, the stick made of wood does not expand. The electromagnetic and nuclear forces on these small scales dominate over the expansion of the universe, just as you have seen in the raisin-bread analogy.

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From the Hubble velocity–distance relation, v = Hr, we can say that, if galaxies are currently moving away from each other, then it implies they were closer together in the past. This means that, at the beginning, all the matter and radiation in the universe were crammed together into a small volume. Energy density

is energy per unit volume. At time $t_0 \to 0$, implies volume $V_0 \to 0$ which would explain why the energy density would be infinite.
Extra credit:

$H^2 = \frac{8\pi G}{3}\rho$

We have, $H(t) = \dot{a}/a$ and $\rho = \rho_0 a^{-3}$

$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho_0 a^{-3}$

$\frac{\dot{a}}{a} = ka^{-3/2}$

$\int_0^a \sqrt{a}da = \int_0^t kdt$

$kt = (\frac{2}{3})a\sqrt{a}$

as $t \to 0$, we see that $a \to 0$ and by the relation $\rho = \rho_0 a^{-3}$, we conclude that energy density is infinite at the big bang when $t \to 0$.

8

From the uncertainty principle, we have, $\Delta x \Delta p \geq \frac{\hbar}{2}$

$\Delta x \ m_e \Delta v \geq \frac{\hbar}{2}$
This gives, $\Delta v \approx 5.79 \times 10^5 \ m/s$

9

Given, $\Delta x = 10^{-10} m$

From the momentum-position uncertainty principle, we have, $\Delta x \Delta p \geq \frac{\hbar}{2}$, which gives, $\Delta p \approx \frac{\hbar}{2\Delta x}$

Also, from the energy-time uncertainty , we have, $\Delta E \Delta t \approx \frac{\hbar}{2}$

Energy is related to momentum as, $\Delta E \approx \frac{\Delta p^2}{2m}$

Therefore, energy-time uncertainity equation gives us, $\Delta t \approx \frac{4\Delta x^2 m}{\hbar}$

For virtual electron, $\Delta t \approx \frac{4\Delta x^2 m_e}{\hbar} \approx 3.45 \times 10^{-16} \ s$

For virtual quark, $\Delta t \approx \frac{4\Delta x^2 m_p}{\hbar} \approx 6.34 \times 10^{-13} \ s$

10

In the classical model of the Hydrogen atom, an electron revolves around a proton such that the centrifugal force counterbalances the Coulomb force. The uniform circular motion involves a constant acceleration towards the centre. We also know that accelerated charged particles gives off radiation. This means that the electron will lose energy by continuously radiating, decreasing the radius of the orbit, resulting in the electron spiralling into the nucleus. However, we observe the spectrum of Hydrogen atom to be discrete and not continuous. Bohr proposed that the angular momentum, orbit radius and energy are quantized, instead of having continuous values. When an electron transitions from an excited state (higher energy) to a less excited state, or ground state, the difference in energy is emitted as a photon with that corresponding wavelength. Similarly, if a photon is absorbed by an atom, the corresponding photon energy excites an electron from a lower energy orbit to a higher energy orbit. This explains the discrete radiation spectrum of hydrogen.