'Well, you don't think I meant *him*, do you?!' she laughed.

'Er, well I'm not dressed for it. I'll have to go and get my swimming trunks.'

'Of course. I had assumed you would be wearing *something*,' she said with a knowing look.

4 The Notes of the Professor's Lecture on Curved Space

Ladies and gentlemen:

Today's topic is curved space and its relation to the phenomena of gravitation.

There's clearly no problem imagining a curved line or a curved surface. But what could we possibly mean by a curved space – a curved *three*-dimensional space? It is obviously impossible to form a mental picture of what a curved three-dimensional space would look like. To do that one would have somehow to view it from 'outside' so to speak – from some other dimension (in the same way as we view the curvature of a two-dimensional surface by seeing how it extends into the third dimension). However, there is another approach to the investigation of curvature – a mathematical approach, rather than one relying on visualisation.

Take first of all curvature in a two-dimensional surface. We mathematicians call this surface curved if the properties of geometrical figures drawn on it are different from those on a plane. We determine the degree of the curvature by measuring the deviation from the classical rules of Euclid. For example, if you draw a triangle on a flat piece of paper the sum of its angles is equal to two right angles (as you know from elementary geometry). You can bend this paper to give it a cylindrical, a conical, or still more complicated shape, but the sum of the angles in the triangle drawn upon it will always remain equal to two right angles. The geometry of the surface, therefore, does not change with such deformations. From the point of view of the 'internal' or *intrinsic* curvature, the surfaces obtained are just as flat as a plane (even though we would commonly call them 'curved').

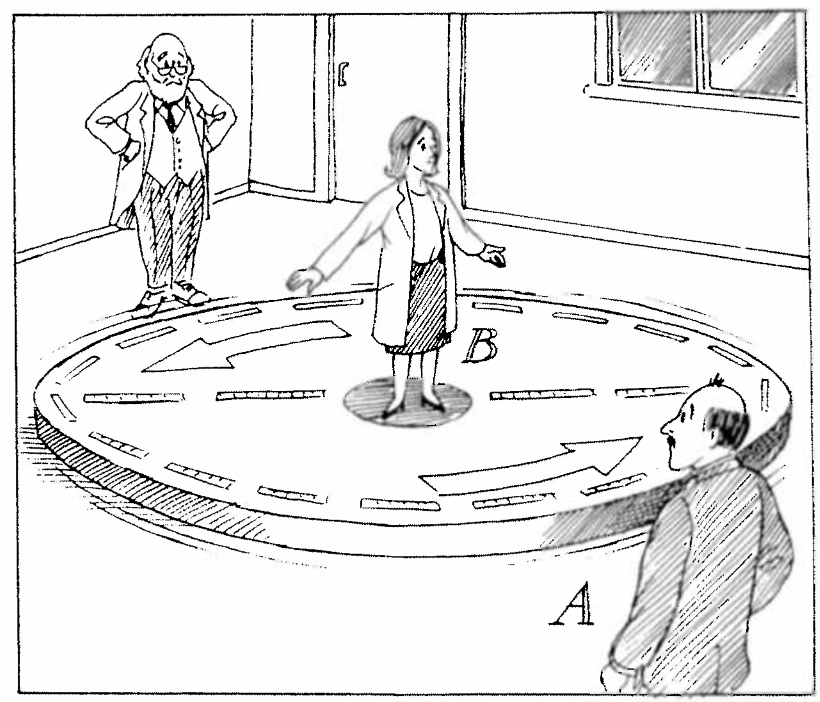
By way of contrast, you cannot fit the sheet of paper on to the surface of a sphere or a saddle – not without squashing or stretching it. This is because the geometry of the surface of a globe, say, is fundamentally different from that of a flat surface. Take for instance a triangle on a globe. To draw a triangle on this surface we would need the equivalent of three 'straight lines'. As on a flat surface, we define a 'straight line' on the curved surface to be the shortest distance between two points. That means we are dealing with arcs of great circles – great circles being the intersection of the spherical surface with planes drawn through the centre of the globe (for example, lines of longitude on the Earth are great circles). If you were to draw a triangle using such arcs, you would find the simple theorems of Euclidean geometry would no longer hold. In fact, a triangle formed, for example, by the northern halves of two meridians and the section of the equator between them, will have two right angles at its base and an arbitrary angle at the top – a sum clearly greater than two right angles.

On the other hand, with a triangle drawn on a saddle surface, you would find that the sum of its angles would always be *smaller* than two right angles.

Thus, to determine the curvature of a surface it is necessary to study the *geometry* on this surface. Merely looking at it from outside can be misleading. By looking you would probably place the surface of a cylinder in the same class as the surface of a globe. But as we have noted, the first is actually the same as a flat surface, and only the second is curved in the sense of having an *intrinsic* curvature. As soon as you get accustomed to this strict mathematical notion of curvature, you should have no difficulty in understanding what a physicist means in discussing whether the three-dimensional space in which we live is curved or not. It is unnecessary to get 'outside' the 3-D space to see whether it 'looks' curved. Rather, we remain within the space, and carry out experiments to see whether the common laws of Euclidean geometry hold or not.

But you might be wondering why we should in any case expect the geometry of space to be anything other than 'commonsense' Euclidean. In order to show you that geometry can indeed depend on physical conditions, let us imagine a large round platform uniformly rotating around its axis like a turntable. Suppose small measuring rulers are placed end-to-end in a straight line along a radius from the centre to a point on the periphery. Additional rulers are placed around the periphery to form a circle.

According to an observer A, stationary relative to the room in which the platform is situated, the rulers placed around the periphery of the turntable are moving in the direction of their lengths as the turntable rotates. They will therefore be length-contracted (as we learned from the first lecture). It therefore takes *more* rulers to complete the circle than would have been necessary if the table had been stationary. The rulers lying along the radius, oriented so as to lie in a



It takes more rulers to complete the circle

direction at right angles to the motion, will not undergo length contraction. It will therefore take the same number to span the distance from the centre of the table to its periphery regardless of the table's motion.

Thus, the distance measured round the circumference, C, (in terms of the number of rulers required) will be greater than the normal $2\pi r$, r being the measured radius.

As we have seen, all this makes perfect sense to observer A in terms of the length contraction produced by the motion of the rulers around the periphery. But what of an observer B, placed at the centre of the turntable and rotating with it? What will she make of it all? She would see the same number of rulers involved as did observer A, and so would likewise conclude that the ratio of circumference to radius did not conform with Euclidean geometry. But suppose the platform were a closed room without windows, she would observe no motion. To what then would she attribute the unusual geometry?

Observer B might not know about the motion, but she would be aware that there was something odd about her surroundings. She would note that objects placed at different locations on the table do not remain stationary. They accelerate away from the centre, the acceleration being dependent on the distance of their location from the centre. In other words, they appear to be subject to a force (a centrifugal force). It is a peculiar force in that it causes all objects to accelerate from any particular location with identically the same acceleration regardless of mass. In other words, the 'force' appears to adjust its strength automatically to match the mass of the object, thus always producing the acceleration characteristic of the location. Observer B concludes that there must be some connection between this 'force' and the non-Euclidean geometry she finds.

Not only that, consider the path taken by a light beam. For the stationary observer A, light always travels in straight lines. But suppose a beam were to skim across the surface of the rotating platform. Though it would continue to move in a straight line according to A, its path as traced out over the surface of the rotating platform would *not* be straight. This is because it takes a finite time for the light to cross

the platform, and in that time, the table rotates through a certain angle. (It is as though you were to pull a sharp knife in a straight line across a rotating disc; the scratch on the surface would be curved rather than straight.) Thus, observer B at the centre of her rotating platform would find that a light beam passing from one side to the other would follow a curved, rather than straight, path. This phenomenon, like the one involving the circumference and the radius, she would have to attribute to the 'force' characterising the special physical conditions at work in her surroundings.

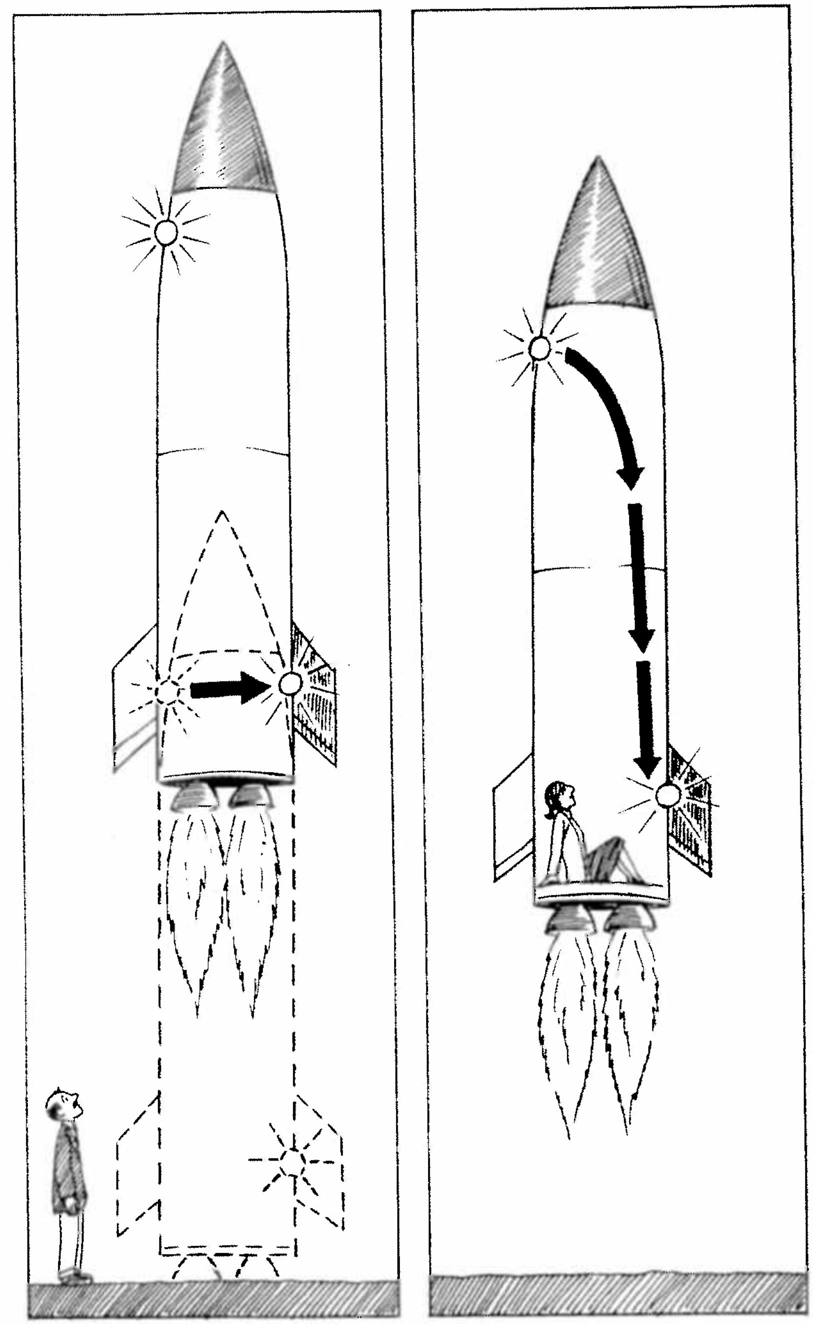
This 'force' not only affects geometry, including the paths of light beams, but also the passage of time. This can be demonstrated by placing a clock on the periphery of the rotating platform. Observer B finds that it runs more slowly than a clock placed at the centre of her platform. This phenomenon is most readily understood from the point of view of the stationary observer A. As far as he is concerned the clock placed at the periphery is moving due to the table's rotation, and is thus time dilated compared with the clock at the centre, which remains at the same position. Observer B, not aware of the motion, must attribute the slowing down of the clock to the presence of the 'force'. Thus, we see that both geometry and the passage of time can be a function of physical circumstances.

We now turn to a different physical situation – that which we find close to the surface of the Earth. All objects are pulled towards the centre of the Earth by the force of gravity. This can be regarded as somewhat similar to the way all objects placed on the rotating platform are pulled towards the periphery. The similarity is strengthened when we further note that the acceleration undergone by the object is independent of its mass; it depends solely on the location. The correspondence between gravity and accelerated motion can be seen even more clearly in the following example:

Suppose a spacecraft floats freely somewhere in space so far from any stars that there is no force of gravity inside it. All objects inside such a craft, including the astronaut travelling in it, will thus have no weight and will float freely. Now the engines are switched on, and the craft gains velocity. What will happen inside? It is easy to see that, as long as the craft is accelerated, all the objects in its interior will show a tendency to move towards the back end of the craft – what we might call the 'floor'. To say the same thing in another way, the floor will be moving towards these objects. If, for example, our astronaut holds an apple in her hand and then lets it go, the apple will continue to move (relative to the surrounding stars) with a constant velocity – the velocity with which the craft was moving at the moment when the apple was released. But the craft itself is accelerated; consequently the floor, moving all the time faster and faster, will overtake the apple and hit it. From this moment on, the apple will remain permanently in contact with the floor, being pressed to it by the steady acceleration.

For the astronaut inside, however, this will look as if the apple 'falls down' with a certain acceleration, and after hitting the floor remains pressed to it by its own 'weight'. Dropping different objects, she will notice furthermore that all of them fall with exactly equal accelerations (neglecting the friction of the air) and will remember that this is exactly the rule of free fall discovered by Galileo Galilei by dropping balls from the leaning tower of Pisa. *In fact the astronaut will not notice any difference between the phenomena in the accelerated cabin and the ordinary phenomena of gravity.* If she chooses, she can use a clock with a pendulum, put books on a shelf without any danger of their floating away, and hang a picture on a nail in the wall. The picture might in fact be a portrait of Albert Einstein – the one who first indicated this equivalence of the acceleration of a system of reference on the one hand, and of a field of gravity on the other. It was on this simple basis that Einstein developed the so-called *general theory of relativity*. His *special* theory of relativity is what we dealt with last time: the effects on space and time of uniform constant motion. The *general* theory adds to this the effects on space and time of gravity. And, as I said, this is done through noting the equivalence of gravity and *accelerated* motion.

For example, take the case of a light beam. We noted that under the conditions of centrifugal acceleration on the rotating platform, a



A light beam crossing an accelerated spacecraft light beam would appear to follow a curved path. The same applies to a light beam crossing an accelerated spacecraft. An outside observer would see such a light beam move in a straight line. The beam starts off lined up with the point exactly opposite on the facing wall. If the craft had been stationary, it would have hit that point. But because of the craft's acceleration during the passage of the beam across the cabin, the far wall moves. As a result, the beam hits a point behind the one at which it had been originally aimed – a point closer to the 'floor' of the craft. The astronaut makes a similar observation: the beam originally starts out aiming for the point directly opposite, but ends up at a point closer to the 'floor' of the craft. As far as she is concerned, the beam followed a *curved* path and 'fell' towards the 'floor'. Not only that but she finds her geometry has gone wrong; the sum of the angles of a triangle formed by three light rays are not equal two right angles, and the ratio of the circumference of a circle to its radius is not equal to 2π.

We come now to the question of greatest importance. We have just seen that in an accelerated system of reference, not only do objects 'fall', but a light beam also 'falls' towards the 'floor', following a curved path. We therefore ask whether, in accordance with the equivalence principle, we are justified in concluding that light beams will be bent by *gravity*.

In order to get a measure for the expected curvature of a light ray in the field of gravity, we consider how much bending we expect in the case of the accelerating spacecraft. If l is the distance across the cabin, then the time t taken by light to cross it is given by

$$t = \frac{l}{c} \qquad (5)$$

During this time, the ship, moving with the acceleration g, will cover the distance L given by the following formula of elementary mechanics:

$$L = \tfrac{1}{2}gt^2 = \tfrac{1}{2}g\frac{l^2}{c^2} \qquad (6)$$

Thus, the angle representing the change of the direction of the light ray is of the order of magnitude

$$\varnothing = \frac{L}{l} = \tfrac{1}{2}gl/c^2 \qquad (7)$$

where angle $\varnothing$ is in radians (1 radian is about 57degrees). We see that $\varnothing$ is greater the larger the distance l which the light has travelled in the gravitational field. Here, the acceleration g of the craft has, of course, to be interpreted as the acceleration due to gravity. If I send a beam of light across this lecture room, I can take l to be roughly 10 metres. The acceleration of gravity g on the surface of the Earth is 9.81 m/s^2, and

$$c = 3 \times 10^8 \text{ m/s, so we get}$$

$$\varnothing = \tfrac{1}{2}(9.81 \times 10)/(3 \times 10^8)^2 = 5 \times 10^{-16} \text{ radians}$$

$$= 10^{-10} \text{ seconds of arc} \qquad (8)$$

Thus you can see that the curvature of light can definitely not be observed under such conditions. However, near the surface of the Sun, g is 270 m/s^2, and the total path travelled in the gravitational field of the Sun is very large. The exact calculations show that the value for the deviation of a light ray passing near the solar surface should be 1.75 seconds of arc. This is indeed the value observed by astronomers for the displacement of the apparent position of stars seen near the solar limb during a total eclipse, compared with their positions at night-time at other times of the year when the Sun is in a different part of the sky. Indeed, since the advent of astronomy using radio emissions from strongly emitting galaxies called *quasars*, one does not even need to wait for an eclipse; radio waves originating from quasars and passing close to the limb of the Sun can be detected without difficulty in broad daylight. It is these observations that give us our most precise measurements of the bending of light.

So we conclude that the bending of light that we found in the accelerated system does indeed apply equivalently to a gravitational field. What about the other strange effect our observer B found on the rotating platform – the one whereby a clock placed at some distance from her on the periphery of the platform was found to be running slow? Would this mean that a clock placed at some distance from us in a gravitational field would behave similarly? In other words, are the effects of acceleration and the effects of gravity not only very similar, but identical?

The answer to this can be given only by direct experiments. And, indeed, these do prove that time can be affected in an ordinary field of gravity. The effects predicted through the equivalence of accelerating motion and gravitational fields are very small: that is why they have been discovered only after scientists started looking specially for them.

Using the example of the rotating platform discussed before, we can easily estimate the order of magnitude of the expected change of the clock rate. It is known from elementary mechanics that the centrifugal force acting on a particle of unit mass, located at a distance r from the centre, is given by the formula

$$F = r\omega^2 \qquad (9)$$

where ω is the constant angular velocity of rotation of our platform. The total work done by this force while moving the particle out from the centre to the periphery is then

$$W = \tfrac{1}{2}R^2\,\omega^2 \qquad (10)$$

where R is the radius of the platform.

According to the above-stated equivalence principle, we have to identify F with the force of gravity on the platform, and W with the difference of gravitational potential between the centre and the periphery.

Now, we must remember that, as we have seen in the previous lecture, the slowing down of the clock moving with the velocity v is given by the factor

$$\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$

This can be approximated by

$$1-\tfrac{1}{2}\frac{v^2}{c^2}+\cdots$$

If v is small as compared with c, we can neglect other terms. According to the definition of the angular velocity we have $v=R\omega$ and the 'slowing-down factor' becomes

$$1-\tfrac{1}{2}\left(\frac{R\omega}{c}\right)^2 = 1-\frac{W}{c^2} \qquad (11)$$

giving the change of rate of the clock in terms of the difference of gravitational potentials at the places of their location.

So, if we imagine placing one clock on the ground and another on the top of the Eiffel tower (about 300 metres high) the difference of potential between them will be so small that the clock on the ground will go slower only by a factor 0.999,999,999,999,97 compared with that at the top.

In fact, an experiment carried out be R. V. Pound and G. A. Rebka has demonstrated this small effect by examining the difference in the rates of atomic vibrations at the top and bottom of a tower 22.5 metres high. The same effect has also been found by comparing the rates of atomic clocks flown in aircraft with those on the ground. Agreement with observation is obtained only if, in addition to the time dilation caused by the aircraft's motion (special relativity), one takes account of the slowing down of the Earth-bound clock compared with the high flying one due to the difference in gravitational potential.

Considerably larger effects than these, however, are to be found once one involves the much stronger gravity of the Sun. The difference of gravitational potential between the surface of the Earth and the surface of the Sun is much larger, giving a slowing down factor of 0.999,999,5. This is much easier to measure, and provided the first confirmation of these ideas. Of course, nobody can place an ordinary clock on the surface of the Sun and watch it go! The physicists have much better means. By using a spectroscope we can observe the periods of vibration of different atoms on the surface of the Sun and compare them with the periods of the atoms of the same elements put into the flame of a Bunsen-burner in the laboratory. The vibrations of atoms on the surface of the Sun should be slowed down by the factor given by equation (11), and as a consequence, the light emitted or absorbed by them should have a somewhat lower frequency than in the case of terrestrial sources, i.e. the frequencies should be shifted towards the red end of the spectrum. This *gravitational red-shift* has been observed in the spectra of the Sun, and of several other stars, and the results agree with the value given by our theoretical formula. This shows that the processes on the Sun really do take place somewhat more slowly than they do on Earth, owing to the difference in gravitational potential.

These observations have therefore demonstrated the equivalence of the effects of acceleration and those of gravitation. So, with this in mind, let me now return once more to the curvature of space:

You remember that we came to the conclusion that the geometry obtained in accelerating systems of reference is different from that of Euclid, and that such spaces should be considered as curved spaces. Since any gravitational field is equivalent to some acceleration of the system of reference, this means also that any space in which the gravitational field is present is a curved space. Or, going a step further, we can say that *a gravitational field is just a physical manifestation of the curvature of space.*

We know that gravity arises in the vicinity of masses. Thus, we would expect that the curvature of space at each point should be determined by the distribution of masses, and would reach maximum values close to heavy objects. I cannot enter into the rather complicated mathematical system describing the properties of curved space and their dependence on the distribution of masses. I will mention only that this curvature is in general determined not by one, but by ten different numbers which are usually known as the components of gravitational potential, $g\mu\nu$, and represent a generalization of the gravitational potential of classical physics which I have previously equated with W in equation (10). Correspondingly, the curvature at each point is described by ten different radii of curvature usually denoted by $R\mu\nu$. Those radii of curvature are connected with the distribution of masses by the fundamental equation of Einstein:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu} \qquad (12)$$

where R is another kind of curvature, and the source term $T_{\mu\nu}$ (representing the *cause* of the curvature) depends on densities, velocities and other properties of the gravitational field produced by masses. G is the familiar gravitational constant.

This equation has been tested out, for example, by studying the motion of the planet Mercury – the planet closest to the Sun, and hence the one with the orbit most sensitively dependent on the details of Einstein's equation. It is found that the perihelion of the orbit (i.e. the point of closest approach to the Sun of the planet as it executes its elongated elliptical path) does not remain fixed in space, but is found with each turn of the orbit to have systematically shifted its orientation relative to the Sun. Part of this precession is attributable to the perturbing gravitational fields of the other planets, and part can be explained in terms of the special relativistic increase in mass due to the planet's motion. But there remains a tiny residual amount of 43 seconds of arc per century which cannot be accounted for by the old Newtonian theory of gravity, but finds a ready explanation in terms of general relativity.

This observation, together with the other experimental results I have mentioned in this lecture, confirm us in our judgement that general relativity is the theory of gravity that best fits what we actually see happening in the Universe.

Before ending this lecture, allow me to indicate two further interesting consequences of equation (12):

If we consider a space uniformly filled with masses, as, for example, our space is filled with stars, galaxies and clusters of galaxies, we must conclude that, apart from localized large curvatures near particular stars or galaxies, space should possess an *overall* curvature due to the combined effect of all the masses – a regular tendency to curve uniformly over large distances. Mathematically there are different solutions. Some of them correspond to space finally closing in upon itself, and thus possessing a finite volume, somewhat similar to a sphere. The others represent a curved space, but not curved sufficiently to cause closure; instead, the space is infinite in extent, having no boundaries – rather analogous to the saddle surface I mentioned at the beginning of this lecture.

A second important consequence of equation (12) is that such curved spaces should be in a state of steady expansion or contraction. This physically means that the particles (the galaxy clusters) filling the space should be flying away from each other, or, on the contrary, approaching each other. Further, it can be shown that for a closed space with a finite volume, the expansion phase will be followed by a contraction phase (with possibly further expansion and contraction phases to follow – thus giving rise to an oscillating Universe). On the other hand, an infinite expanding 'saddle-like' space would continue to expand for ever.

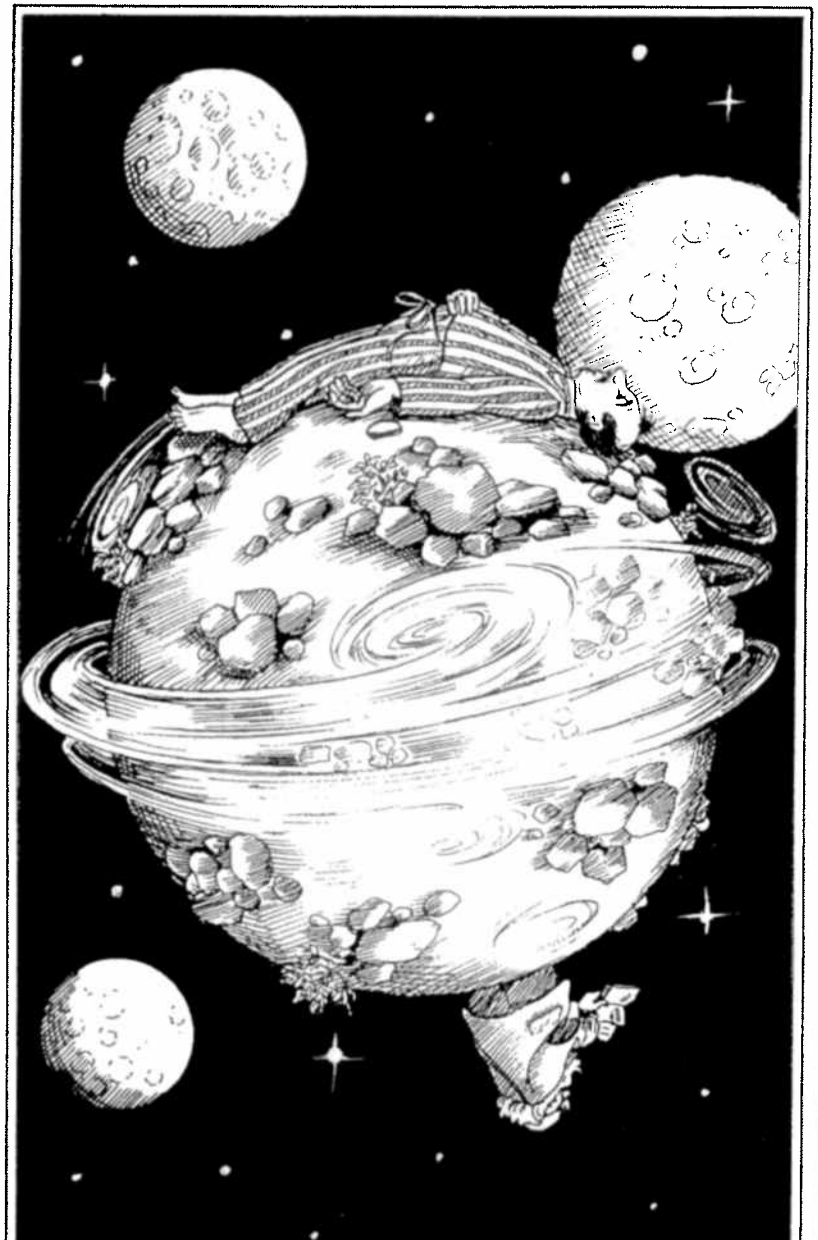
The question of which of these different mathematical possibilities corresponds to the space in which we live is very much a live issue at present. It can only be resolved by experimental observation on the movements of the galaxy clusters (including their rate of slowing down); either that or by accounting for all the mass present in the Universe and calculating how great the slowing down effect will be. At present, the astronomical evidence is unclear. Though it is certain that we are at present in an expanding phase, whether this will ever turn into a contraction (and consequently, whether the space is finite or infinite in size) is not yet definitely settled.

5 Mr Tompkins Visits a Closed Universe

That evening in the Beach Hotel, the professor and his daughter were deep in conversation. They talked freely of both cosmology and art. Mr Tompkins joined in from time to time as best he could, but for the most part was happy just to observe and listen. He was fascinated by Maud; he had never met anyone like her. But in due course he became sleepy and made his excuses. Climbing the stairs and reaching his room, he quickly changed into his pyjamas and collapsed on to the bed, pulling the blanket over his head. His tired brain was all mixed up.

As he lay there, one thought kept recurring. The type of cosmology that really intrigued him was that of a closed Universe – the one where if you go off from the North Pole in a straight line you will end up at the South Pole. At least it would be a Universe with a finite volume (he simply could not get his mind round the infinite volume of an open Universe). Fair enough, the professor seemed to have his reasons for thinking that the density of matter had the critical value, and so you would not be able to make that odd type of journey, and the expansion would not give way to a contraction and a Big Crunch. But what if he were wrong? What if there was a lot more dark matter out there than they had yet accounted for? What if ...?

These thoughts were interrupted as he became aware that he was uncomfortable. He had the strange feeling that instead of lying on a comfy spring mattress he was stretched out on something hard. He pulled back the blankets and peeped out. To his astonishment



HE BECAME AWARE THAT HE WAS UNCOMFORTABLE

he found himself lying on a slab of rock out in the open. The hotel had vanished!

The rock was covered with some green moss, and in a few places little bushes were growing from cracks in the stone. The space above him was illuminated by some glimmering light and was very dusty. In fact, there was more dust in the air than he had ever seen, even in the films representing dust storms in the American mid-west. He tied his handkerchief round his nose to keep from breathing the dust.

But there were more dangerous things than the dust in the surrounding space. Occasionally stones, the size of his head and larger, came whirling through space, hitting the ground around him. He also noticed one or two rocks, about 10 metres across he judged, floating through space at some distance away.

Another strange thing was that there appeared to be no distant horizon – despite his being perched high up. He decided he had better explore his surroundings. So it was he began crawling over the surface. Because the rock curved down quite sharply, he held on grimly to the protruding edges in constant fear of falling off. But then he gradually became aware of something odd; although he had moved down onto a very steep part of the rocky face – so steep he could now no longer see the blanket he had left behind, he did not feel any tendency to fall; he was still being pulled securely onto the surface. Emboldened, he continued crawling. Eventually he reckoned he must have gone through about 180° – in other words he ought to be directly *underneath* his starting point – and still there was no tendency to fall off into the surrounding dusty depths of space. He was presumably now upside-down compared with when he started out. It was then it dawned on him that the rock he was on had no visible means of support. It was a planet! A tiny planet similar to the floating rocks he had seen.

To his great surprise and relief it was at that moment he almost bumped into the legs of a familiar figure. It was the professor. He was standing there busily noting down observations in a note-book.

'Oh, it's you,' observed the professor casually. 'What are you doing down there? Lost something?'

Mr Tompkins sheepishly let go of his hand-hold, and gingerly stood up. To his great relief, not only did he not fall off into space, he did not even feel as though he would drift off into space. He began to understand what was going on. He remembered that he was taught in his schooldays that the Earth is a big round rock moving freely in space around the Sun. Everything is pulled towards its centre, so there is no danger of 'falling off', no matter where you are positioned on its surface. Now he was gently but firmly being pulled towards the centre of this new 'planet' – a planet so tiny its population numbered two.

'Good evening,' said Mr Tompkins, 'What a relief to see you.'

The professor raised his eyes from his note-book. 'There are no evenings here,' he said. 'There is no Sun', and with that he returned again to his note-book.

Mr Tompkins felt uneasy; to meet the only living person in the whole Universe, and to find him so preoccupied! Unexpectedly, one of the little meteorites came to his help. With a crashing sound, the stone hit the book in the hands of the professor and knocked it hard. It flew up into space away from their little planet. 'Oh dear,' said Mr Tompkins, 'I hope that wasn't important. I don't reckon our gravity is strong enough to pull it back.' As they watched, the book continued its journey into the furthest depths of space, getting smaller and smaller.

'Not to worry,' replied the professor. 'You see, the space in which we are now is not infinite in its extension. Oh I know that you were doubtless taught in school that space is infinite, and that two parallel lines never meet. This, however, is not true for the space of this particular Universe – the one we are now in. Our normal Universe is, of course, very large indeed; about 100,000,000,000,000,000,000,000 kilometres across at present, which for most purposes is fairly infinite. If I had lost my book there, it would have taken an incredibly long time to come back – even assuming it were a Universe of the closed type that this one is. Here, however, the situation is rather different. Just before the note-book was torn out of my hands, I had figured out that this space is only about five miles in diameter, though it is expanding. I expect the book back in not more than half an hour.'

'Are you saying that the book is going to do one of those round trip journeys in a straight line,' ventured Mr Tompkins. 'Like the one you said about taking off from the North Pole ...'

'... and landing back at the South Pole? Yes,' replied the professor. 'Precisely. The same thing is going to happen to my book – unless it's hit on its way by some other stone and gets deflected from the straight track.'

'And this has nothing to do with the gravity of our little planet pulling it back?'

'No, nothing at all to do with that. As far as the gravity here is concerned, the book has escaped into space. Here, take these binoculars, and see if you can still see it.'

Mr Tompkins put the binoculars to his eyes, and through the dust which somewhat obscured the whole picture, he managed to see the professor's note-book travelling through space far, far away. He was somewhat surprised by the pink colouring of all the objects, including the book, at that distance. Not only that, 'Your book is returning already,' he cried out excitedly. 'Yes, yes, it's definitely growing larger now.'

'No, no,' said the professor, 'it'll still be going away. Here, give those to me.' He took back the binoculars, and looked intently. 'No, as I said, it's still going away. The fact that it appears to be growing in size – *as if* it were coming back – that's due to a peculiar focussing effect on the rays of light due to the closed, spherical nature of the space.'

He lowered the binoculars and scratched his greying head. 'How can I put it ...? Yes. Suppose we were back on Earth, and let's imagine that horizontal rays of light (aimed at the horizon) could be kept going all the time hugging the curved surface of the Earth (say, by refraction of the atmosphere). Under those circumstances, if an athlete were to run away from us, it wouldn't matter how far she went, we would be able, using powerful binoculars, to see her all the time during her journey. Now, if you think about the globe, you will see that the straightest lines on its surface, the meridians, first diverge from one pole, but, after passing the equator, begin to converge towards the opposite pole. If the rays of light travelled along the meridians, you, located for example at one pole, would see the person going away from you growing smaller and smaller only until she crossed the equator. After this point you would see her growing larger; it would seem to you that she was returning, albeit going backwards. Once she reached the opposite pole, you would see her as large as if she were standing right by your side. You would not be able to touch her, of course, just as you cannot touch the image formed by a spherical mirror.

'Right now,' the professor continued, 'this behaviour of light as it travels over the two-dimensional curved surface of the Earth can be used as an analogy for how light rays behave in this strangely curved three-dimensional space we find ourselves in. In fact I do believe the image of the book is about to arrive.'

As he said that, the image of the book appeared to be only a few yards away, and coming closer. It was big enough now for one no longer to need the binoculars to see it by. However, it looked rather odd; the contours were not sharp, but seemed washed out, and the writing on the cover could hardly be recognized; the whole book looked like a photograph taken out of focus and underdeveloped.

'You can see now it's only an image – not the real thing,' said the professor. 'See how its badly distorted by the light having had to travel halfway across the Universe. And notice how you can see other little planets behind the book – through its pages.'

Mr Tompkins reached out and tried to grab the 'book' as it sped passed, but his hand simply passed through the image without encountering any resistance.

'No, no,' admonished the professor. 'The book itself is now very close to the *opposite* pole of the Universe. As I've said, what you see here is just an image – in fact two images of it. The second image is just behind you and when both images momentarily coincided just then, that was when the real book was exactly at the opposite pole.'

Mr Tompkins didn't hear; he was too deeply absorbed in his thoughts, trying to remember how the images of objects are formed in elementary optics by concave and convex mirrors and lenses. When he finally gave up, the two images were receding in opposite directions.

'And all these strange effects are due to the matter in the Universe?' he eventually asked.

'That's right. The matter we're standing on – our tiny planet – curves the space in our immediate vicinity, and it is this that is responsible for the way we are held onto its surface. But more than that, the gravity of this planet combines with that of all the other masses in the Universe to produce the overall curvature that gives rise to these lensing effects. In fact, in general relativity one dispenses altogether with talk of gravitational 'forces' as such, and simply thinks in terms of curvature.'

'But tell me, if there were no matter, would we have the kind of geometry I was taught at school, and would parallel lines never meet?'

'That's right,' answered the professor, 'but neither would there be any material creature to check it.'

In the meantime, the image of the book went off again far away in the original direction, and started coming back for the second time. Now it was still more damaged than before, and could hardly be recognized at all, which, according to the professor, was due to the fact that this time the light rays had travelled round the whole Universe.

'And if we pop round to the other side of our planet …' he added, grabbing Mr Tompkins by the arm and marching him the few yards it took to get to the other side. 'There,' he declared, pointing in the opposite direction. 'There. Can you see? Here comes my book. It's about to complete its journey round the Universe.' With a triumphant grin, he stretched out his hand, caught the book, and pushed it into his pocket. 'The trouble with this Universe is that there is so much dust and stones around, it makes it almost impossible to see round the world. Notice these shapeless shadows around us? Most probably they're the images of ourselves, and surrounding objects. It's just that they're so distorted by dust and irregularities of the curvature of space that I cannot even tell which is which.'

'Does the same effect occur in our normal Universe – the one we used to live in?' asked Mr Tompkins.

'Probably not – not if we're right about the density being critical. But,' the professor added with a twinkle in his eye.

'you have to admit, it's still fun to think this kind of thing through, don't you agree?'

By now the sky had considerably changed. There seemed to be less dust about, so Mr Tompkins was able to take off the handkerchief from around his face. The small stones were passing much less frequently and hitting the surface of their planet with much less energy. Not only that, but the other planets had drifted much farther away by now and could hardly be seen at this distance.

'Well, I must say life is getting a lot less scary,' he commented. 'Though I must say it's become quite chilly.' He picked up the blanket and wrapped it round him. 'Can you explain the change in our surroundings?' he asked, turning to the professor.

'Very easily; our little Universe is expanding and since we have been here its radius has increased from five to about a hundred miles. As soon as I found myself here, I noticed this expansion from the reddening of the distant objects.'

'Ah. I did notice everything was pink at great distances,' said Mr Tompkins, 'but why does that signify expansion?'

'Oh that's not difficult to see,' said the professor. 'I take it you've noticed that the siren of an approaching ambulance sounds very high, but after the ambulance passes you, the tone is considerably lower? This is the so-called Doppler Effect: the dependence of the pitch (or frequency of the sound) on the velocity of the source. When the whole of space is expanding, every object located in it moves away with a velocity proportional to its distance from the observer. Therefore, the light emitted by such objects is of lower frequency, which in optics corresponds to redder light. The more distant the object is, the faster it moves and the redder it seems to us. In our normal Universe, which is also expanding, this reddening, or the *cosmological red-shift* as we call it, permits astronomers to estimate the distances of the very remote galaxies. For example, one of the nearest galaxies, the Andromeda galaxy, shows a 0.05% reddening; this corresponds to the distance which can be covered by light in eight hundred thousand years. But there are also galaxies just on the limit of present telescopic power which show a reddening of about 500%, corresponding to distances of approximately ten thousand million light years (a 'light year' being – as the name implies – the distance travelled by light in one year). Such light was emitted when the Universe was less than a fifth its present size. The present rate of expansion is about 0.000,000,01% per year. Our little Universe here grows comparatively much faster, gaining in size by about 1% per minute.'

'Will the expansion of this Universe here ever stop?' asked Mr Tompkins.

'Of course it will,' said the professor. 'I told you in one of the lectures that a closed Universe like this one would entail the expansion eventually coming to a halt, this then being followed by the contraction phase. For a Universe this small the expansion phase should, I reckon, last no more than a couple of hours.'

'A couple of hours,' echoed Mr Tompkins. 'But that would mean there can't be long to go before …' His voice trailed off as the implication sank in.

'Yes,' murmured the professor. 'I think we are now observing the state of largest expansion. That's why it's become so cold.'

In fact, the thermal radiation filling up the Universe, and now distributed over a very large volume, was giving only very little heat to their planet; the temperature was at about freezing-point.

'It's lucky for us that there was originally enough radiation to give some heat even at this stage of expansion,' the professor added. 'Otherwise it might become so cold that the air around our rock would condense into liquid and we would freeze to death.'

He peered intently through his binoculars once more. 'Ah, yes,' he said after a while. 'The contraction has already begun. It'll soon be warm again.'

He offered the binoculars to Mr Tompkins, who took them and scanned the heavens. He noticed that all the distant objects had changed their colour from pink to blue. This, according to the professor, was due to the fact that all the stellar bodies had started moving towards them. He also remembered the analogy given by the professor of the high pitch of the whistle of an approaching train.

Rubbing himself to get warm, he commented, 'Well, I'll be glad when it heats up again.' But then a thought struck him. He turned anxiously to the professor. 'If everything is contracting now, shouldn't we expect that soon all the big rocks filling the Universe will come together and that we shall be crushed between them?'

'I wondered how long it would take you to work that out,' answered the professor calmly. 'But not to worry. Just think: well before that happens, the temperature will rise so high that we shall be vapourised! I suggest you just lie down and observe as long as you can.'

'Oh my!' moaned Mr Tompkins. 'I am beginning to feel hot already, even in my pyjamas.'

It was not long before the hot air became unbearable. The dust, which became very dense now, was accumulating around him, and he felt as if he were being choked. He struggled to free himself from the blanket, when suddenly his head emerged into cool air. He swallowed a deep breath.

'What's happening?' he called out to the professor – only to discover that his companion was no longer with him. Instead, in the dim light of morning, he recognised the hotel bedroom. Sighing with relief he disengaged himself from his blanket; it had become entangled after what must have been a very restless night.

'Thank God we're still expanding!' he muttered, as he made his way to the bathroom. 'That's what you might call a close shave,' he thought as he reached for the razor.

6 Cosmic Opera

It was the final evening of their holiday, and Mr Tompkins and Maud were taking one last stroll along the beach by the water's edge. Was it really only a week since they had first met? Though at first he had been quite nervous of speaking with her, he being shy by nature, they knew each other well enough now for the conversation to flow easily. He found it extraordinary that one person should have such wide interests. Not only that, he was delighted to note that she seemed to take as much pleasure being with him as he with her. He could not possibly think why. Except that on one occasion the professor had let slip that his daughter had been badly let down in the past; her engagement to some high-flying executive had been abruptly broken off. Perhaps she just felt safe with him and his rather humdrum, but reassuringly secure life.

He looked up at the Milky Way. 'I must say your father has opened up a whole new world for me. It's sad how most people seem to go through life without ever appreciating just how extraordinary the world is.'

Picking up a handful of pebbles, he lazily aimed them at a rock sticking up out of the water. Then he shot a quick glance at her. 'Why won't you show me your sketches?'

'I've told you. They're not the sort you show anyone. They're *working* sketches – ideas. Just ideas. That's all. They try to capture the *feel* of the place. They wouldn't mean anything to you. It's only when I get back to the studio and work on them something emerges – or not, as the case may be.'

'Then, can I come and visit your studio one day when we get back?' he asked.

'Of course,' she replied. 'I'd be disappointed if you didn't.'

By now they had got back to the hotel. Mr Tompkins ordered drinks, and for the last time they sat on the patio looking out to sea.

'Your father told me there was a time when you were cut out for a career in physics,' he commented.

'Oh, I wouldn't say that,' she laughed. 'Wishful thinking. That was what *he* wanted.'

'Yes, but you were good at physics, weren't you?' he persisted.

She shrugged. 'Yes. You could say that.'

'So why ...?'

'Why?' she repeated, wistfully. 'Oh, I don't know. Rebellious teenager, I suppose. That and the fact that it wasn't easy in those days for a girl to show an interest in science. Biology maybe, but not physics. Peer pressure and all that. It's different now – well, at least it's not quite so bad now.'

'But how come you still know so much physics after all this time?'

'Oh I don't really. Forgot most of it long ago. Except for astronomy and cosmology. Now *that* I have tried to keep up with. Which reminds me ...' she looked at him in amusement.

'Reminds you of what?' he asked.

'Fancy taking me to the opera?'

'OPERA!' he exclaimed. 'What ... what do you mean? What's opera got to do with anything?'

'Oh, it's not a *real* one,' she added with a laugh. 'No, it's an amateur one. It was written ages ago by someone who used to be in Dad's department. It's all about the Big Bang theory versus the Steady State theory ...'

'Steady State? What's that?' he enquired.

'The Steady State theory says that the Universe did not begin with a Big Bang ...'

'But we know it *did*. Your father's told me all about the expansion of the Universe – the way all the galaxies are still flying apart in the aftermath of the big explosion,' Mr Tompkins protested.

'Ah, but that doesn't prove anything. You see there were these physicists, Fred Hoyle, Hermann Bondi and Tommy Gold, who suggested that the Universe could keep on renewing itself. As fast as galaxies moved away, new matter was created in the spaces left behind. This collected together to form new stars and galaxies, which in their turn moved apart, making room for yet more matter, and so on.'

'So, how did all this get started,' asked Mr Tompkins, clearly intrigued.

'Oh, it didn't. There was no start, no beginning. It has always been going on, and always will. It's a world with no beginning and no end. That's why it was called the Steady State theory; the world looks essentially the same at all times.'

'Hey, I like the sound of that,' enthused Mr Tompkins. 'Yes, it's got the right ... the right kind of *feel* about it. You know what I mean? Somehow the Big Bang idea doesn't have that appeal. You find yourself asking why it was supposed to have happened at that particular instant in time; why not some other instant? It seems so ... so *arbitrary* somehow. Now if there's *no* beginning ...'

'Hold on! Hold on!' interrupted Maud. 'Don't get too carried away. The Steady State theory is dead. Dead as a dodo.'

'Oh,' said Mr Tompkins disappointedly. 'Why's that? How can they be so sure?'

But before Maud could reply, her father emerged from the hotel doorway to remind her that they had to make an early start home in the morning. As she took her leave of Mr Tompkins, he hurriedly asked, 'But what about the opera?'

'Oh yes,' she said. 'Saturday evening, 8 o'clock, in the main physics lecture theatre – the one you normally go to for Dad's lectures. The department is reviving the *Cosmic Opera*. It's just a bit of fun. The 50th anniversary of the first proposal of the Steady State theory; I think that's the excuse. See you there.' With that she followed her father into the hotel, briefly turning her head to blow Mr Tompkins a playful goodnight kiss.

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