PHYS 251   HONOURS CLASSICAL MECHANICS - 2014

Readings and Homework Set 3

Readings: Chapter 4 (Pages 101 - 120)

Problems, due Sept. 19 in class:

1. In class I defined stability of fixed points in phase space under the flow of a dynamical system. Using the exact mathematical language used in class, discuss the stability of the origin in phase space in the case of a
a) spring with friction
b) spring with antifriction.

2. In the two previous examples, find the linearized solutions about the fixed point.

3. In class I introduced the notion of *flow* Φ_{ts}. Show that for an autonomous system (this notion was also defined in class)

$$\Phi_{tr} = \Phi_{ts}\Phi_{sr}$$

for any values of r, s and t obeying $r < s < t$.

4. Consider a ball rolling in the Earth's gravitational field on a surface whose potential energy is

$$V(q) = (q^4 - 1)^2 + \alpha q^2,$$

where $0 < \alpha << 1$:
a) Draw a phase space diagram of the dynamics.
b) Discuss the stability of the fixed points of the dynamics.

5. Consider a circle of circumference L. Consider a series of n mass points of equal mass m, each connected with its nearest neighbor on each side by springs with equal spring constant k.
a) What is the potential energy of the system?
b) What are the resulting equations of motion?
c) Discuss the fixed point(s) of the system.