PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 3

READING: Appendices A & B, Chapter 3 (3.1 - 3.8)

PROBLEMS (due Jan. 30, 2018 in class):

1. Show that the two-dimensional torus T^2 is a manifold, by explicitly constructing an appropriate atlas (set of coordinate patches which cover the manifold).

2. In class I derived the geodesic equation in the case when the path parameter λ is chosen to be an affine parameter. Assume you do not want to make this choice. Find the general form of the geodesic equation valid for arbitrary path parameters.

3. Let $d\Omega = d\theta^2 + sin^2\theta d\phi^2$ denote the line element on a two sphere of unit radius. The metric of flat three-dimensional Euclidean space is $ds^2 = dx^2 + dy^2 + dz^2$. Show that the metric g_{ij} in spherical polar coordinates (r, θ, ϕ) specified by

$$z = rcos\theta, y = rsin\theta sin\phi, x = rsin\theta cos\phi,$$

is given by $ds^2 = dr^2 + r^2 d\Omega$. Calculate the Christoffel symbols Γ^i_{jk} in this coordinate system, write down the components of the geodesic equation, and verify that solutions through the origin correspond to straight lines in Cartesian coordinates.

4. Let X and Y be two vector fields on a manifold M, and $\lambda \in R$. Prove the following properties of the Lie derivative which were stated but not fully derived in class

a) $\mathcal{L}_X(Y) = [X, Y]$

a) $\mathcal{L}_{X + Y} = \mathcal{L}_X + \mathcal{L}_Y$

b) $\mathcal{L}_{\lambda X} = \lambda \mathcal{L}_X$.

5. Show that the metric is covariantly conserved.

6. Carroll, Problem 3.5 (Page 147).