PHYS 743   Very Early Universe   2019
Problem Set 4 - for Week 5, due Oct. 8, in class

1. Consider a cosmological model in which there is a period of primordial inflation after which the universe is dominated by radiation. In class I computed the spectrum of cosmological perturbations on super-Hubble scales at the end of inflation. How does this spectrum evolve in the radiation phase? How would the spectrum evolve if the universe were dominated by cold matter after inflation?

2. In class I discussed the quantum theory of cosmological perturbations and applied the analysis to inflationary cosmology. Compute the Bogoliubov coefficients α_k and β_k at time η for modes whose wavelength is larger than the Hubble radius.

3. Verify that the Bogoliubov coefficients obey the relation

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

4. Consider a contracting universe in which matter is dominated by a substance with equation of state parameter $w \gg 1$. Compute the spectra of cosmological perturbations and gravitational waves at late times on super-Hubble scales, assuming vacuum initial conditions at very early times.