PHYS 743   Very Early Universe   2019
Problem Set 5 - for Week 7, due Oct. 22, in class

1. Consider the scalar field potential

$$V(\varphi) = \tfrac{1}{4}\varphi^4 - \tfrac{1}{3}(a+b)\varphi^3 + \tfrac{1}{2}ab\varphi^2$$

where a and b are constant coefficients (or mass dimension 1) which obey the condition $a/2 > b > 0$. Consider the corresponding finite temperature effective potential. Find the critical temperature T_c. Sketch the finite temperature effective potential at the critical temperature and at temperatures much higher and much lower than T_c.

2. Consider a theory of a single real scalar field with potential

$$V(\varphi) = \tfrac{1}{4}(\varphi^2 - \eta^2)^2,$$

where η is the symmetry breaking scale, and λ is a positive coupling constant. The theory has domain wall solutions. I argued in class that the width of the domain wall will be proportional to η^{-1}. Find the coefficient.

3.. As I argued in class, theories with domain walls are very tightly constrained since a scaling network of walls can overclose the universe. Demanding that domain walls do not overclose the universe at the time of recombination, fiind the upper bound on the value of η. Your answer may depend on λ. In that case, assume that $\lambda = 1$. Consider the same potential as in the previous problem.

4. Consider a theory which has local monopole solutions. Assume that at the time of the phase transition you form one monopole per Hubble volume, but that these monopoles then do not interact. By demanding that the monopoles do not dominate the energy density of the universe at the time of nucleosynthesis, find the resulting upper bound on η. Note: take the scalar field potential to be the same as in the previous problems, except that φ now has three real components).

5. Consider a scaling solution of cosmic string loops. Find an expression for Ω_{loop}, the fractional energy density in string loops.

6. For what value of $G\mu$ will there be one string loop per galaxy today?