402-0389 Physics of the Very Early Universe 2019
Problem Set 1

1. Assume a spatially flat Friedmann-Robertson-Walker-Lemaitre (FRWL) universe. Compute the time dependence of the scale factor if
a) matter is cold (i.e. vanishing pressure)
b) matter is radiation (i.e. $p = 1/3\rho$)
c) matter is a cosmological constant.

2. The equation of motion for a free point particle in Newtonian mechanics can be derived from the action

$$S = \int dt \frac{1}{2}\dot{x}^2 .$$

Use the Einstein Equivalence Principle to write down the corresponding action in an expanding FRWL universe and derive the equation of motion. How does the peculiar velocity (the velocity with respect to the Hubble flow) scale as a function of the scale factor? You can restrict attention to motion along one axis.

3. Another angle on the same physics: in the absence of non-gravitational forces particles move along geodesics. Write down the geodesic equation for motion along one axis in a FRWL metric, and find how the peculiar velocity scales with $a(t)$. You will find that this problem is much more computationally intensive as the previous one.

4. In general, interactions are crucial to establish and maintain thermal equilibrium and hence a black body distribution. The CMB is released at the time t_{rec} of recombination with a black body distribution. Show that the spectrum of CMB photons retains a black body distribution even in the absence of interactions when $t > t_{rec}$.

5. In class I mentioned that for a spatially flat metric one can use the convention that $a(t_0) = 1$, where t_0 is the present time. Can one do this also for a positively or negatively curved cosmological space while maintaining that $k = \pm 1$ for a positively (negatively) curved space?