The Virasoro fusion kernel and its applications

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Quantum Regge Trajectories and the Virasoro Analytic Bootstrap

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Outline

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Conformal block decomposition

All CFTs have OPE (here scalar)

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$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{\sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}} G^{\Delta_\phi}_{\Delta_\mathcal{O},\ell_\mathcal{O}}(z,\bar{z})}{(x_{12})^{2\Delta_\phi}(x_{34})^{2\Delta_\phi}}$$

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Write sum in terms of twist $\tau = \Delta - \ell$

Crossing symmetry

Can do 14 and 23 instead (t-channel) and get same thing

$$\sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}} G^{\Delta_\phi}_{\tau,\ell}(z,\bar{z}) =$$

$$\left(\frac{z\bar{z}}{(1-z)(1-\bar{z})}\right)^{\Delta_\phi} \sum_{\mathcal{O}'} f^2_{\phi\phi\mathcal{O}'} G^{\Delta_\phi}_{\tau',\ell'}(1-z,1-\bar{z})$$

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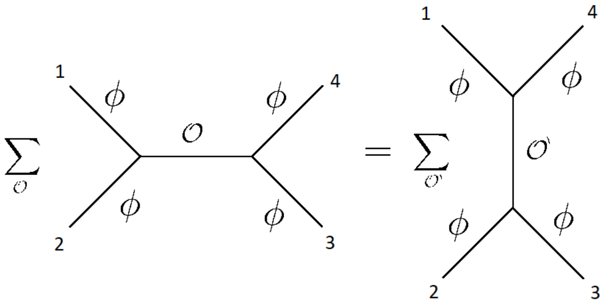

Lightcone limit

Take $\bar{z} \to 1$, t-channel blocks behave as

$$G^{\Delta_\phi}_{\tau',\ell'}(1-z, 1-\bar{z}) \approx (1-\bar{z})^{\frac{\tau'}{2}} K_{\Delta'+\ell'}(1-z)$$

$\Rightarrow$ t-channel dominated by identity!

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Crossing symmetry becomes

$$\sum_{\tau,\ell} f^2_{\phi\phi\mathcal{O}} z^{\frac{\tau}{2}} \log(1-\bar{z}) = \frac{z^{\Delta_\phi}}{(1-\bar{z})^{\Delta_\phi}} + \ldots$$

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Explicitely inverting crossing gives the OPE coefficients

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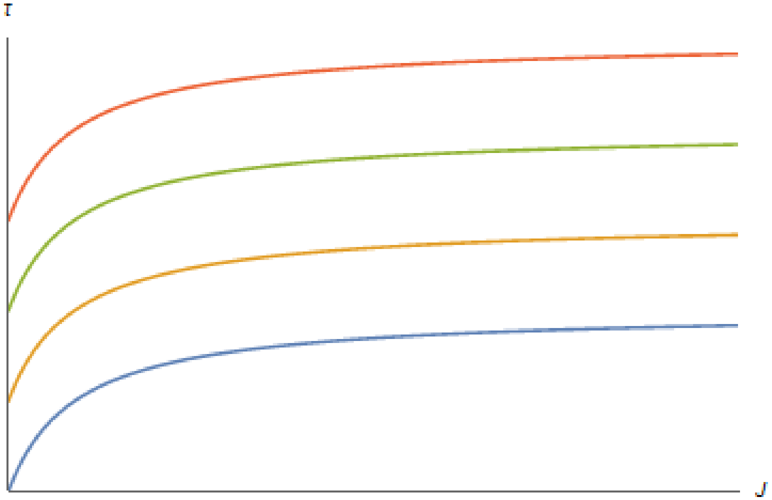
Including subleading operators in t-channel gives corrections to OPE and anomalous dimensions

$$\gamma_{n,\ell} \sim \frac{1}{\ell^{\tau}}$$

Regge trajectories



Inversion formula

Can write 4-point function as

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \sim \sum_{\ell=0}^{\infty}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta C(\Delta,\ell) G_{\Delta,\ell}(z,\bar{z})$$

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Simon's formula inverts this

$$C(\Delta,\ell) \propto \int_0^1\int_0^1 dz d\bar{z}\, M_{\Delta,\ell}(z,\bar{z})\mathrm{dDisc}[\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle]$$

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Inversion of single block = 6j symbol

$\Rightarrow$ 6j symbols rewrite t-channel data into s-channel data

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We will take finite c and reproduce their results.

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Conformal transformations factorize into holomorphic and anti-holomorphic

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Crossing symmetry for $\langle \mathcal{O}_1(0)\mathcal{O}_2(z,\bar{z})\mathcal{O}_2(1)\mathcal{O}_1(\infty)\rangle$ is now

$$\sum_s (f_{12s})^2 \mathcal{F}_S(h_s, z)\bar{\mathcal{F}}_S(\bar{h}_s,\bar{z}) = \sum_t f_{11t} f_{22t} \mathcal{F}_T(h_t, 1-z)\bar{\mathcal{F}}_T(\bar{h}_t, 1-\bar{z})$$

Liouville notation

Need to use new notation:

$$c = 1 + 6Q^2 \quad , \quad Q = b + b^{-1} \quad , \quad h = \alpha(Q - \alpha)$$

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Operators separate in two ranges:

Discrete: $0 < h < \frac{c-1}{24} \longleftrightarrow 0 < \alpha < \frac{Q}{2}$

Continuum: $h \geq \frac{c-1}{24} \longleftrightarrow \alpha = \frac{Q}{2} + iP$

Definition of the kernel

Rewrite t-channel (holomorphic) Virasoro blocks into s-channel blocks

$$\mathcal{F}_T(\alpha_t, 1-z) = \int_C \frac{d\alpha_s}{2i} \mathbb{S}_{\alpha_s \alpha_t} \mathcal{F}_S(\alpha_s, z)$$

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Poles at $\alpha_s = \alpha_1 + \alpha_2 + mb + nb^{-1}$ and reflexions $\alpha \to Q - \alpha$

- For $\alpha_t = 0$, single poles
- For $\alpha_t \neq 0$, double poles

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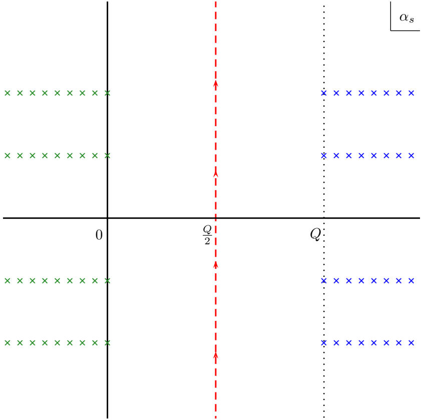

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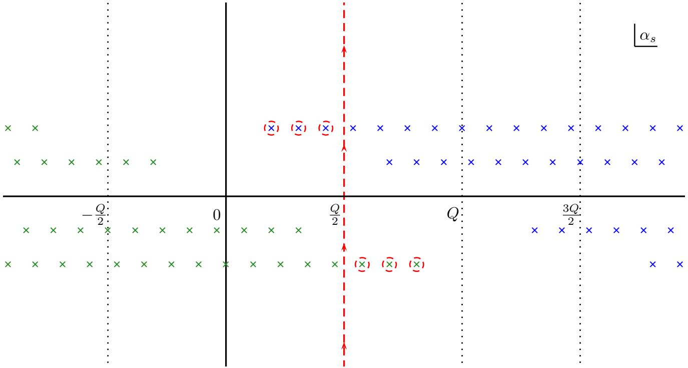

Support of the kernel

- For $\alpha_1 + \alpha_2 > \frac{Q}{2}$,

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- For $\alpha_1 + \alpha_2 < \frac{Q}{2}$,

$$\mathcal{F}_T(\alpha_t) = -2\pi \sum_m \operatorname*{Res}_{\alpha_s = \alpha_m} \{\mathbb{S}_{\alpha_s \alpha_t} \mathcal{F}_S(\alpha_s)\} + \int_0^\infty dP\, \mathbb{S}_{\alpha_s \alpha_t} \mathcal{F}_S\left(\alpha_s = \frac{Q}{2} + iP\right)$$

with sum over $\alpha_m < \frac{Q}{2}$

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Rewrite t-channel into s-channel with kernel tells us what must be there in the s-channel to reproduce what appears in t-channel.

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Consider $\alpha_1+\alpha_2<\frac{Q}{2}$ and $\bar\alpha_1+\bar\alpha_2>\frac{Q}{2}$ and individual t-channel exchange

$$\int_? d\alpha_s d\bar\alpha_s \rho_{12s}\mathcal{F}_S(\alpha_s)\bar{\mathcal{F}}_S(\bar\alpha_s)=\int_0^\infty d\bar P\,\bar{\mathbb{S}}_{\bar\alpha_s\bar\alpha_t}\bar{\mathcal{F}}_S\left(\bar\alpha_s=\frac{Q}{2}+i\bar P\right)$$

$$f_{11t}f_{22t}\left[-2\pi\sum_m \underset{\alpha_s=\alpha_m}{\text{Res}}\{\mathbb{S}_{\alpha_s\alpha_t}\mathcal{F}_S(\alpha_s)\}+\right.$$

$$\left.\int_0^\infty dP\,\mathbb{S}_{\alpha_s\alpha_t}\mathcal{F}_S\left(\alpha_s=\frac{Q}{2}+iP\right)\right]$$

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2. Operators with α and $\bar{\alpha}$ in continuum

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OPE coefficients of Regge operators given by

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This is called Virasoro Mean Field Theory!

Corrections

Assume other operators give small corrections

$$(\rho_{12m}+\delta\rho_{12m})\mathcal{F}_S(\alpha_m+\delta\alpha_m)\bar{\mathcal{F}}_S \approx \bar{\mathcal{F}}_S\,(\rho_{12m}\mathcal{F}_S(\alpha_m)+ \delta\rho_{12m}\mathcal{F}_S(\alpha_m)+ \rho_{12m}\delta\alpha_m\partial\mathcal{F}_S(\alpha_m))$$

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This leads to

$$\delta\alpha_m = f_{11t}f_{22t}\frac{\bar{\mathbb{S}}_{\bar{\alpha}_s\bar{\alpha}_t}}{\bar{\mathbb{S}}_{\bar{\alpha}_s\mathbb{I}}}\frac{\underset{\alpha_s=\alpha_m}{\text{dRes}}\,\mathbb{S}_{\alpha_s\alpha_t}}{\underset{\alpha_s=\alpha_m}{\text{Res}}\,\mathbb{S}_{\alpha_s\mathbb{I}}}$$

$$\delta\rho_{12m} = -2\pi f_{11t}f_{22t}\bar{\mathbb{S}}_{\bar{\alpha}_s\bar{\alpha}_t}\underset{\alpha_s=\alpha_m}{\text{Res}}\,\mathbb{S}_{\alpha_s\alpha_t}$$

where dRes means the coefficient of double pole

Why dRes?

Taylor expanding double pole at $x = x_0$ gives

$$s(x)f(x) = \left(\frac{\mathrm{dRes}(s)}{(x-x_0)^2} + \frac{\mathrm{Res}(s)}{x-x_0} + s(x_0)\right) \times \left(f(x_0) + (x-x_0)f'(x_0)\right)$$

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$$\Rightarrow \mathrm{Res}(s\,f) = f(x_0)\,\mathrm{Res}(s) + f'(x_0)\,\mathrm{dRes}(s)$$

Large spin asymptotics

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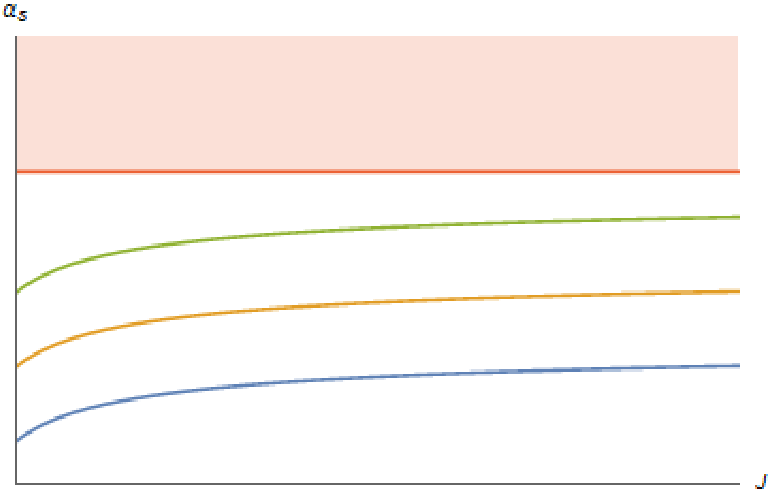
Spectrum of Quantum Regge trajectories at large spin:

$$h_m = h_1 + h_2 + m - 2(\alpha_1 + mb)(\alpha_2 + mb) + m(m+1)b^2 + \delta h_m$$

Quantum Regge trajectories



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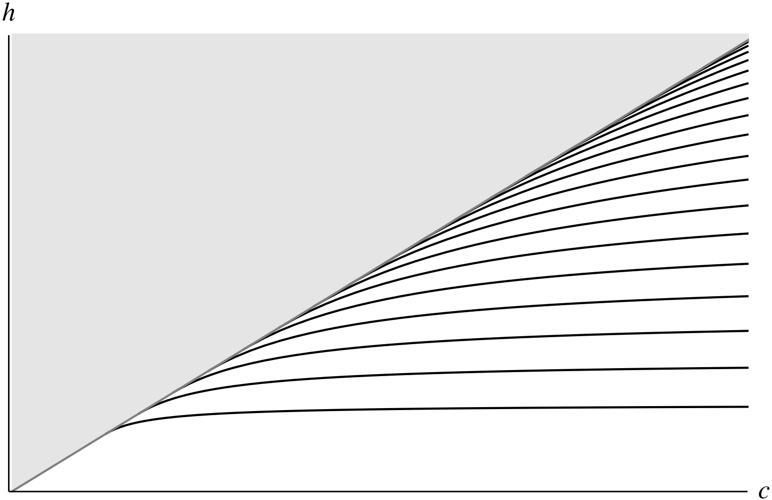
Checks:

1. Reproduce MFT from VMFT (exchange of identity)
2. Other t-channel reproduced
3. Next order in identity exchange gives T

Large c trajectories



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When $m \ll b^{-1} \sim \sqrt{c}$, $h_1 = O(c) < \frac{c}{24}$ and $h_2 = O(1)$, recover

$$h_m \approx h_1 + \sqrt{1 - \frac{24 h_1}{c}}(h_2 + m)$$

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When further take $\frac{h_1}{c} \ll 1$, recover

$$h_m \approx h_1 + h_2 - \frac{12h_1h_2}{c}$$

which can be derived from inversion formula

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5. Large c limits

Other results

Many other applications

1. Virasoro blocks at late time (information paradox)
2. Gravity interpretation
3. $z \to 1$ limit of Virasoro blocks
4. HHLL Virasoro blocks
5. 2d lightcone bootstrap